Contents

Chapter 1
Basic Concepts of Medical Instrumentation

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1.1 The following table shows % reading and % full scale for each data point. There is no need to do a least squares fit.

Inputs	0.50	1.50	2.00	5.00	10.00	
Outputs	0.90	3.05	4.00	9.90	20.50	
Ideal Output	1.00	3.00	4.00	10.00	20.00	
Difference	– 0.10	+0.05	0.0	– 0.10	+0.50	
% Reading	11.1	1.6 %	0%	– 1.0%	+2.4 %	= Difference/Output × 100
Full Scale	– 0.5%	0.25%	0%	– 0.50%	+ 2.5%	= Difference/20 × 100

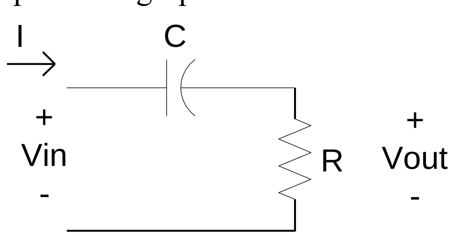
Inspection of these data reveals that all data points are within the "funnel" (Fig. 1.4b) given by the following independent nonlinearity = ±2.4% reading or ±0.5% of full scale, whichever is greater. Signs are not important because a symmetrical result is required. Note that simple % reading = ±11.1% and simple % full scale = 2.5%.

1.2 The following table shows calculations using equation (1.8).

Inputs X_i	0.50	1.50	2.00	5.00	10.00	$\bar{X} = 3.8$	
Outputs Y_i	0.90	3.05	4.00	9.90	20.50	$\bar{Y} = 7.6$	
$X_i - \bar{X}$	– 3.3	– 2.3	– 1.8	1.2	6.2		
$Y_i - \bar{Y}$	– 6.7	– 4.55	– 3.6	2.3	12.9		
$(X_i - \bar{X})(Y_i - \bar{Y})$	22.11	10.465	6.48	2.76	79.98	$\Sigma = 121.795$	
$(X_i - \bar{X})^2$	10.89	5.29	3.24	1.44	38.44	$\Sigma = 59.3$	$(59.3)^{1/2} = 7.701$
$(Y_i - \bar{Y})^2$	44.89	20.7	12.96	5.29	166.41	$\Sigma = 250.25$	$(250.25)^{1/2} = 15.8$
						2	

$$r = \frac{121.795}{(7.701)(15.82)} = 0.9997$$

1.3 The simple RC high-pass filter:



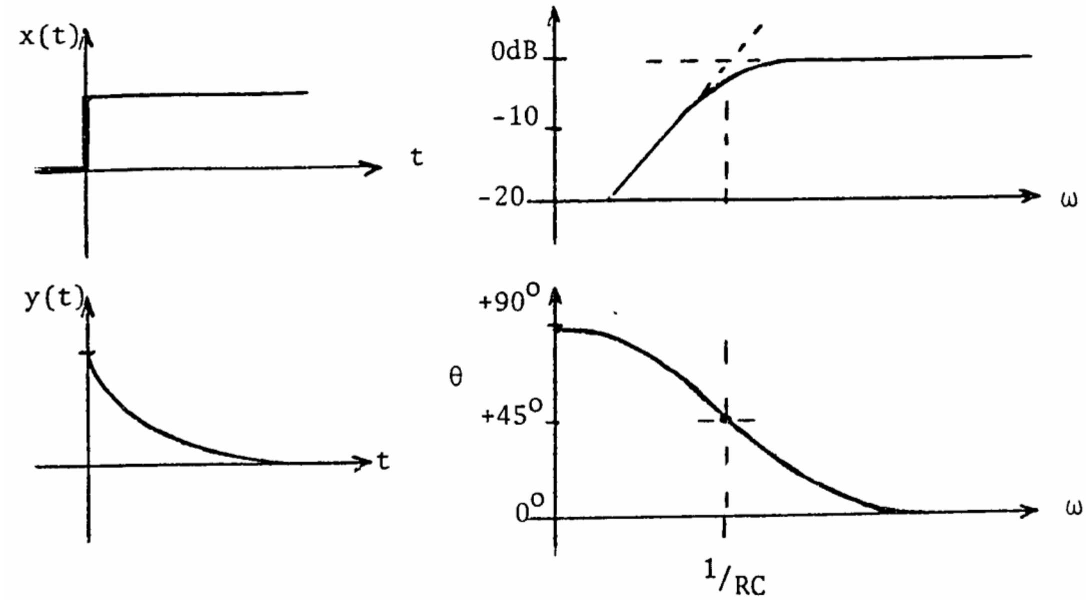
The first order differential equation is:

$$C\frac{d[x(t) - y(t)]}{dt} = \frac{y(t)}{R}$$

$$\left(CD + \frac{1}{R}\right) y(t) = (CD)\, x(t$$

$$\frac{y(D)}{x(D)} = \frac{D}{D + \frac{1}{RC}}$$ Operational transfer functi

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{j\omega RC}{j\omega RC + 1} = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} \quad \angle\phi = \text{Arctan}\frac{1}{\omega RC}$$



;where 1/*RC* is the corner frequency in rad/s.

1.4 For sinusoidal wing motion the low-pass sinusoidal transfer function is

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{(j\omega\tau + 1)}$$

For 5% error the magnitude must not drop below 0.95 K or

$$\left|\frac{K}{j\omega\tau + 1}\right| = \frac{K}{\sqrt{\omega^2\tau^2 + 1}} = 0.95\text{ K}$$

Solve for τ with $\omega = 2\pi f = 2\pi(100)$

$$(\omega^2\tau^2 + 1)\,(.95)^2 = 1$$

$$\tau = \left[\frac{1 - (0.95)^2}{(0.95)^2(2\pi 100)^2}\right]^{1/2} = 0.52\text{ ms}$$

Phase angle $\phi = \tan^{-1}(-\omega\tau)$ at 50 Hz

$$\phi 50 = \tan^{-1}(-2\pi \times 50 \times 0.0005) = -9.3\,°$$

at 100 Hz

$$\phi 100 = \tan^{-1}(-2\pi \times 100 \times 0.0005) = -18.2\,°$$

1.5 The static sensitivity will be the increase in volume of the mercury per °C divided by the cross-sectional area of the thin stem

$$K = \frac{\gamma_{Hg} V_b}{A_c} = 2 \text{mm/°C}$$

where

$$\gamma_{Hg} V_b = 1.82 \times 10^{-4} \frac{\text{cm}^3}{\text{cm}^3 \, °\text{C}}$$

V_b = unknown volume of the bulb

A_c = cross-sectional area of the column

$A_c = \pi(0.1 \text{ mm})^2 = \pi \times 10^{-4} \text{ cm}^2$

Thus

$$V_b = \frac{A_c K}{\gamma_{Hg}} = \frac{\pi\left(10^{-4}\text{cm}^2\right) 0.2 \text{ cm/°C}}{1.82 \times 10^{-4} \frac{\text{cm}^3}{\text{cm}^3 °\text{C}}} = 0.345 \text{ cm}^3$$

1.6 Find the spring scale (Fig. 1.11a) transfer function when the mass is negligible. Equation 1.24 becomes

$$B \frac{dy(t)}{dt} + K_s y(t) = x(t)$$

When M = 0. This is a first order system with

K = static sensitivity $= \frac{1}{K_s}$

τ = time constant $= \frac{B}{K_s}$

Thus the operational transfer function is

$$\frac{y(D)}{x(D)} = \frac{1/K_s}{1 + \frac{B}{K_s} D} = \frac{1}{K_s + BD}$$

and the sinusoidal transfer function becomes

$$\frac{y(j\omega)}{x(j\omega)} = \frac{1/K_s}{1 + j\omega \frac{B}{K_s} D} = \frac{1/K_s}{\sqrt{1 + \frac{\omega^2 B^2}{K_s^2}}} \quad \angle \phi = \tan^{-1}\left(-\omega \frac{B}{K_s}\right)$$

1.7

$$a \frac{dy}{dt} + bx + c + dy = e\frac{dy}{dt} + fx + g$$

$$(a - e)\frac{dy}{dt} + dy = (b + f)x + (g - c)$$

This has the same form as equation 1.15 if g = c.

$(\tau D + 1)y = Kx$

$$\left[\frac{a-e}{d}D + 1\right] y = \frac{b+f}{d} x$$

Thus $\tau = \dfrac{a-e}{d}$

1.8 For a first order instrument

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{(j\omega\tau + 1)}$$

$$\left|\frac{K}{(j\omega\tau + 1)}\right| = \frac{K}{\sqrt{\omega^2\tau^2 + 1}} = 0.93\ K$$

$$\left(\omega^2\tau^2 + 1\right)(0.93)^2 = 1$$

$$f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\sqrt{\frac{1-(0.93)^2}{(0.93)^2\,(.02)^2}} = 3.15\ \text{Hz}$$

$$\phi = \tan^{-1}(-\omega\tau) = -21.6^\circ$$

1.9

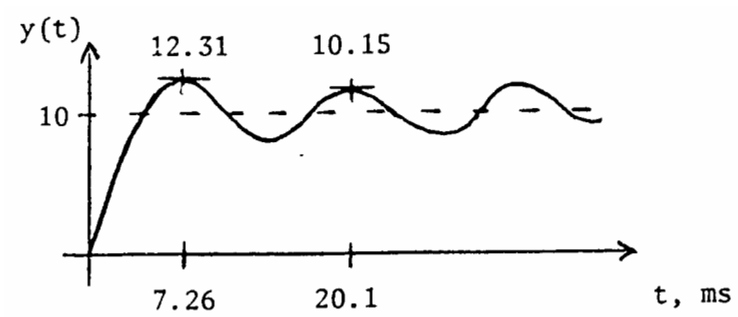
$$y(t) = K\,\frac{Ke^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\ \sin\left(\sqrt{1-\zeta^2}\ \omega_n t + \phi\right) \quad \text{where } \phi = \sin^{-1}\sqrt{1-\zeta^2}$$

$\zeta = 0.4$; $f_n = 85$ Hz

$$t_n = \frac{\frac{3\pi}{2} - \phi}{\omega_n\sqrt{1-\zeta^2}} = 7.26\ \text{ms} \qquad t_{n+1} = \frac{\frac{7\pi}{2} - \phi}{\omega_n\sqrt{1-\zeta^2}} = 20.1\ \text{ms}$$

$$y(t_n) = 10 + \frac{10}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t_n} \qquad y(t_{n+1}) = 10 + \frac{10}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t_{n+1}}$$

$= 12.31$ $\qquad$ $= 10.15$



1.10. At the maxima y_n, y_{n+2}, y_{n+4}: $\sin(\ ) = -1$ at $\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}$

and $\sqrt{1-\zeta^2}\ \omega_n t_n + \phi = \dfrac{3\pi}{2}$ $\qquad$ $t_n = \dfrac{\frac{3\pi}{2} - \phi}{\omega_n\sqrt{1-\zeta^2}}$

at the minima $y_{n+1}, y_{n+3} \ldots$: $\sin(\ ) = +1$ at $\frac{5\pi}{2}, \frac{9\pi}{2} \ldots$

and $\sqrt{1-\zeta^2}\ \omega_n t_{n+1} + \phi = \frac{5\pi}{2}$ $\qquad t_{n+1} = \dfrac{\frac{5\pi}{2} - \phi}{\omega_n \sqrt{1-\zeta^2}}$

then form the ratio

$$\frac{y_n}{y_{n+1}} = \frac{\dfrac{K}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t_n}}{\dfrac{K}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t_{n+1}}} = \exp \frac{-\frac{3\pi}{2} - \frac{5\pi}{2}}{\sqrt{1-\zeta^2}} = \exp \frac{+\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\Gamma = \ln \frac{y_n}{y_{n+1}} = \frac{\pi\zeta}{\sqrt{1-\zeta^2}}$$

Solve for ζ

$$\zeta = \frac{\Gamma}{\sqrt{\pi^2 + \Gamma^2}}$$

Chapter 2
Basic Sensors and Principles

Robert A. Peura and John G. Webster

2.1 Let the wiper fraction $F = x_i/x_t$

$$v_o/v_i = \frac{R_m||F\,R_p}{R_m||F\,R_P + (1-F)R_P}$$

$$= \frac{\dfrac{R_m\,F\,R_p}{R_m + F\,R_p}}{\dfrac{R_m\,F\,R_P}{R_m + F\,R_p} + (1-F)R_P}$$

$$= \frac{1}{1 + \dfrac{R_m\,F\,R_p}{R_m + F\,R_p}\ (1-F)R_P}$$

$$= \frac{1}{\dfrac{R_mF + R_m + F\,R_p - F\,R_m - FFR_p}{R_mF}}$$

$$= \frac{1}{\dfrac{1 + F\,R_p/R_m - FFR_P/R_m}{F}}$$

$$= \frac{1}{\frac{1}{F} + \frac{R_p}{R_m}(1-F)}$$

Let $\varepsilon = R_p/R_m$

error $= F - v_o/v_i$

$$= F - \frac{1}{1/F + \varepsilon(1-F)}$$

$$= F - \frac{F}{1 + \varepsilon F - \varepsilon F^2}$$

$$= F - (F)(1 + \varepsilon F - \varepsilon F^2)^{-1}$$

$$d/dF(\text{error}) = 0 = 1-(1)(1+\varepsilon F-\varepsilon F^2)^{-1} - (F)(-1)(1+\varepsilon F-\varepsilon F^2)^{-2}(\varepsilon-2\varepsilon F)$$

multiply by $(1+\varepsilon F-\varepsilon F^2)^2$

$$0 = (1+\varepsilon F-\varepsilon F^2)^2 - (1+\varepsilon F-\varepsilon F^2) + (F)(\varepsilon - 2\varepsilon F)$$

expand, ignoring terms of ε^2, ε^3, ...

$$0 = 1 + 2\varepsilon F - 2\varepsilon F^2 - 1 - \varepsilon F + eF^2 + \varepsilon F - 2\varepsilon F^2$$

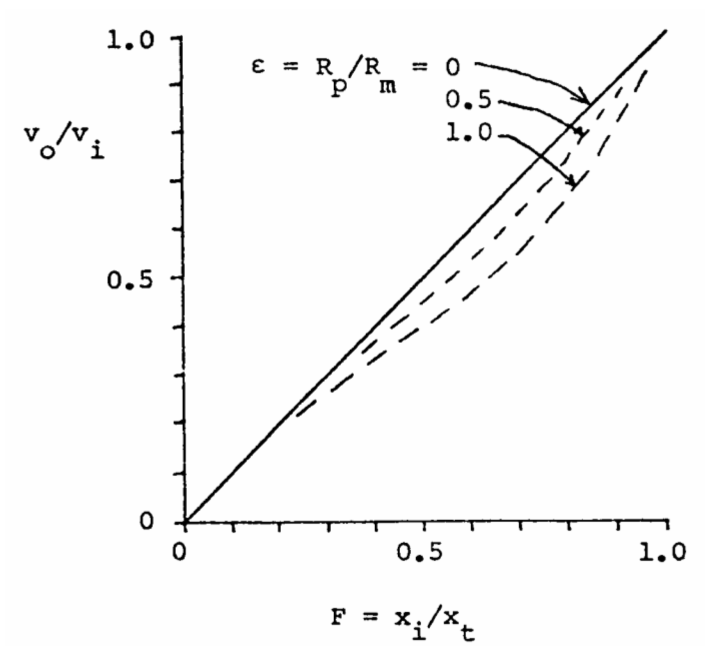
$$0 = -3\varepsilon F^2 + 2\varepsilon F = (\varepsilon)(F)(2-3F)$$

$$F = 0, 2/3$$

$$\text{error} = 0.67 - \frac{1}{1/0.67 + \varepsilon(1 - 0.67)}$$

$$= 0.67 - \frac{1}{1.5 + 0.33\varepsilon}$$

$$= \frac{0.22\varepsilon}{1.5 + 0.33\varepsilon} \approx 0.15\varepsilon = 0.15\ R_p/R_m$$



2.2 The resolution of the translational potentiometer is 0.05 to 0.025 mm. The angular resolution is a function of the diameter, D, of the wiper arm and would = (translational resolution/πD) × 360°. In this case the resolution is 2.87/D to 5.73/D degrees where D is in mm.

A multiturn potentiometer may be used to increase the resolution of a rotational potentiometer. The increased resolution is achieved by the gearing between the shaft whose motion is measured and the potentiometer shaft.

2.3. The elastic-resistance strain gage is nonlinear for large extensions (30%), has a dead band linearity due to slackness and is subject to long-term creep. Continuity in the mercury column and between the column and electrodes may be a problem. The gage has a high temperature drift coefficient. The dynamic response and finite mechanical resistance may cause distortion. These problems may be minimized by carefully selecting the proper size gage for the extremity. The gage should be slightly extended at minimum displacement when applied to eliminate the slackness problem. Mercury continuity checks may be made using an ohmmeter. The temperature drift problems may be minimized with continual calibration or by making measurements in a controlled temperature environment.

2.4 From (2.21)

$$E = 38.7T + (0.082/2)T^2 = 38.7T + 0.041T^2$$

T °C	38.7T μV	$0.41T^2$ μV	E μV
0	0	0	0
10	387	4	391
20	774	16	790
30	1161	37	1196
40	1548	66	1614
50	1935	102	2037

The second term is small. The curve is almost linear but slightly concave upward.

2.5 From (2.22)

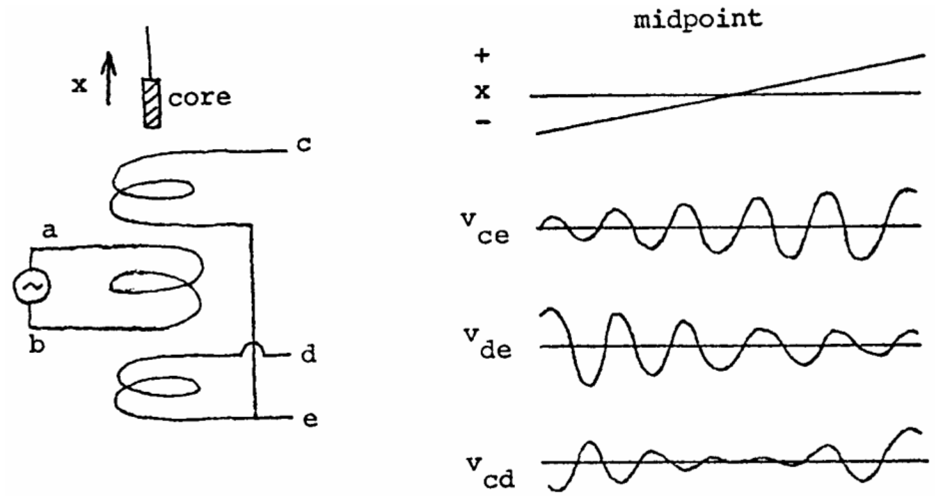
$\alpha = dE/dT = a + bt = 38.7 + 0.082T$ μV/°C

$= 38.7 + 0.082(37) = 41.7$ μV/°C

2.6 From (2.24)

$$\alpha = -\beta/T^2 = \frac{-4000}{(300)^2} = -4.4\%/\mathrm{K}$$

2.7 There is always a voltage induced in each secondary, because it acts as the secondary of an air-core transformer. This voltage increases when the core is inside it.



2.8 In Example 2.3 C = 500 pF for the piezoelectric transducer. The amplifier input impedance = 5 MΩ.

$$F = 0.05\ \text{Hz} = \frac{1}{2\pi R C_{equivalent}}$$

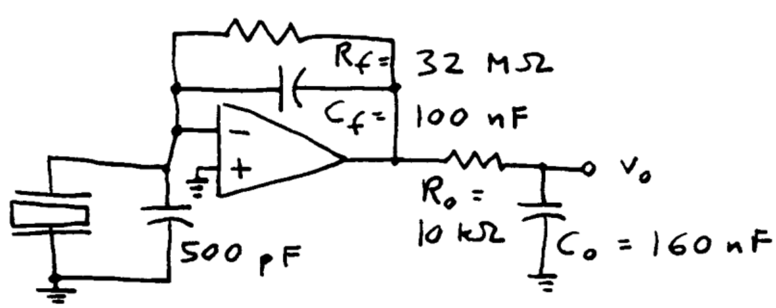
Thus $C_{equivalent} = 0.637 \times 10^{-6} = C_{piezoelectric} + C_{shunt}$

$C_{shunt} = 0.636\ \mu F = 636\ nF$

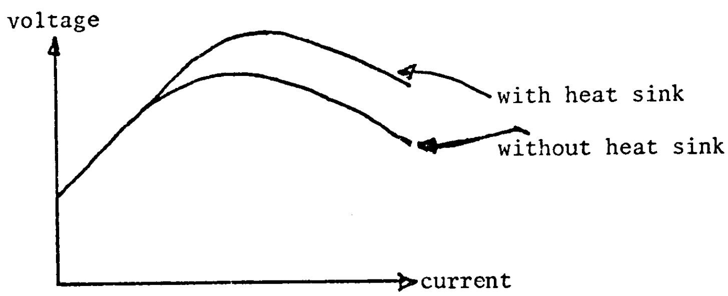
Q = CV, where charge Q is fixed, capacitance C increases by 636 nF/0.5 nF = 1272 times. Voltage V (sensitivity) decreases by 1/1272.

The sensitivity will be decreased by a factor of 1272 due to increase in the equivalent capacitance.

2.9 Select a feedback C_f = 100 nF (much larger than 500 pF). To achieve low corner frequency, add $R_f = 1/(2\pi f_c C_f) = 1/(2\pi \cdot 0.05 \cdot 100\ nF) = 32\ M\Omega$. To achieve high corner frequency add separate passive filter or active filter with $R_o = 10\ k\Omega$ and $C_o = 1/(2\pi f_c R_o) = 1/(2\pi \cdot 100 \cdot 10\ k\Omega) = 160\ nF$.



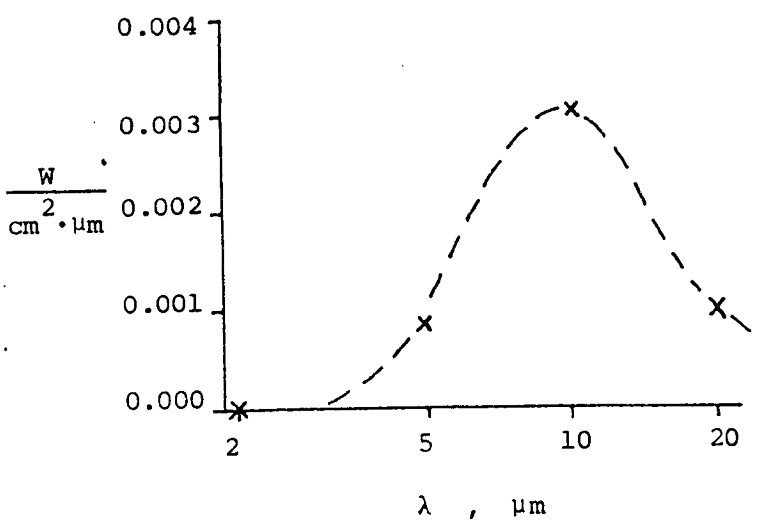
2.10 Typical thermistor V–i characteristics with and without a heat sink are shown below.



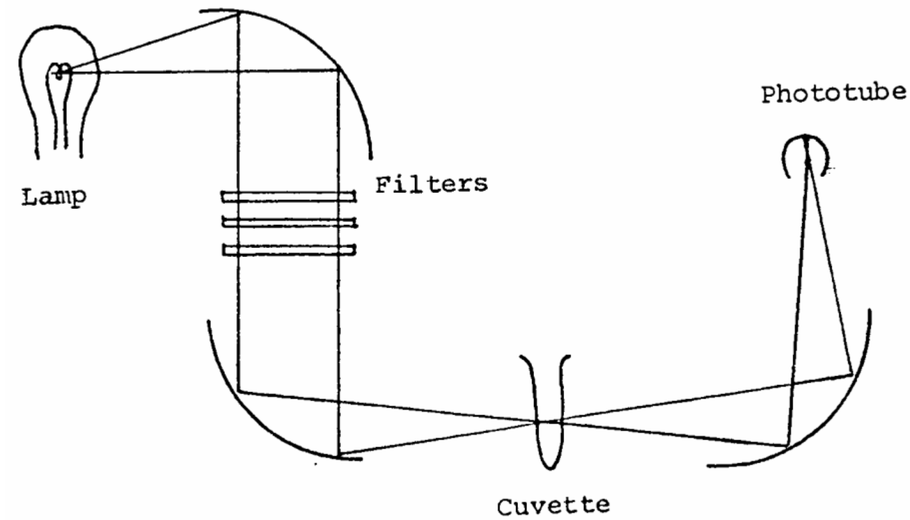
For low currents Ohm's law applies and the current is directly proportional to the applied voltage in both cases. The thermistor temperature is that of its surroundings. The system with a heat sink can reach higher current levels and still remain in a linear portion of the v–i curve since the heat sink keeps the thermistor at approximately the ambient temperature. Eventually the thermistor–heat sink combination will self heat and a negative-resistance relationship will result.

2.11 Assume $\varepsilon = 1.0$ and use (2.25).

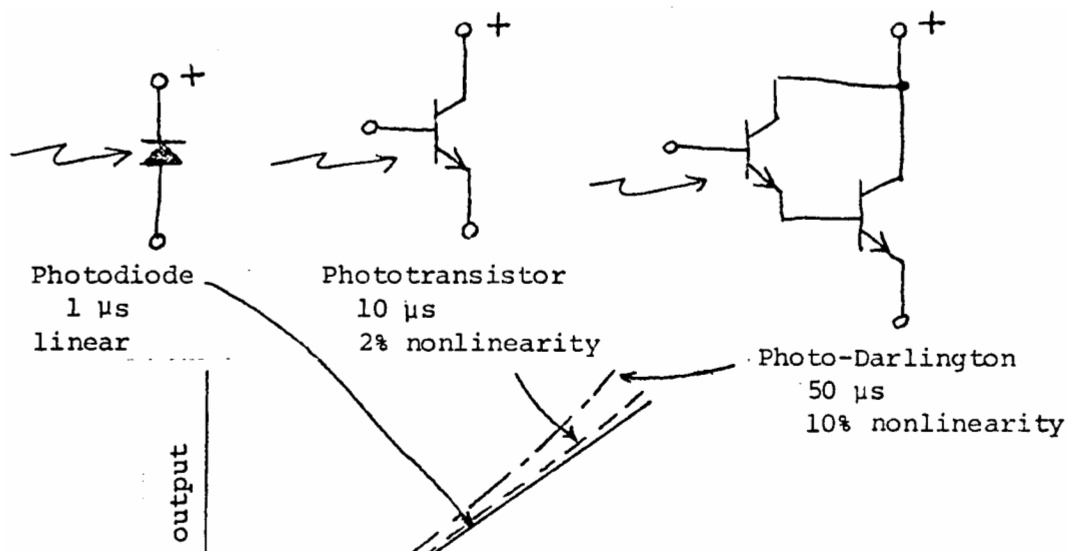
$$W_\lambda = 37400/[\lambda^5(\exp(14400/\lambda 300) - 1)]$$
$$W_\lambda = 37400/[\lambda^5(\exp(48/\lambda) - 1)]$$
$$W_2 = 37400/[(32)(225 \times 10^6)] = 0.000005$$
$$W_5 = 37400/[3125)(15000] = 0.0008$$
$$W_{10} = 37400/[100000)(120)] = 0.003$$
$$W_{20} = 37400/[3.2 \times 10^6)(11 - 1)] = 0.001$$



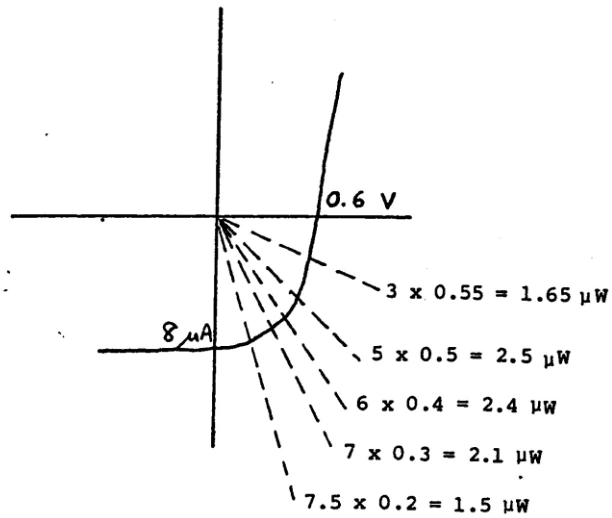
2.12 Infrared and ultraviolet are passed better by mirrors because the absorption in the glass lenses is eliminated.



2.13 See section 2.16, photojunction devices. For small currents, beta, the current gain, increases with collector current. This produces the concave nonlinearity shown. Both nonlinearity and response time increase in the photo-Darlington because two transistors are involved.



2.14 Try several load resistors as shown by the dashed load lines following. The maximum power is 2.5 µW. The load resistor, $R = V/I = (0.5\ \text{V})/(5\ \mu\text{A}) = 100\ \text{k}\Omega$.



2.15 (a) shows the problem—the RC product is too high. (b) shows the simplest solution—the transistor input resistance is much lower than R. (c) shows that an op amp provides a virtual ground that provides a low input resistance. (d) shows that if R is divided by 10, the gain may be achieved by a noninverting amplifier. Active components must have adequate speed.