# **CHAPTER 2**

**2.1** Two possible versions can be developed:

```
IF x \ge 10 THEN
                                          IF x \ge 10 THEN
         DO
                                            DO
           x = x - 5
                                              x = x - 5
           IF x < 50 EXIT
                                              IF x < 50 EXIT
          END DO
                                            END DO
        ELSE
                                          ELSEIF x < 5
          IF x < 5 THEN
                                            x = 5
           x = 5
                                          ELSE
          ELSE
                                            x = 7.5
                                          ENDIF
           x = 7.5
         END IF
        ENDIF
2.2
   DO
     i = i + 1
     IF z > 50 EXIT
     x = x + 5
     IF x > 5 THEN
       y = x
     ELSE
       y = 0
     ENDIF
     z = x + y
   ENDDO
```

2.3 Students could implement the subprogram in any number of languages. The following VBA program is one example. It should be noted that the availability of complex variables in languages such as Fortran 90 would allow this subroutine to be made even more concise. However, we did not exploit this feature, in order to make the code more compatible with languages that do not support complex variables.

1

```
Option Explicit
Sub Rootfind()
Dim ier As Integer
Dim a As Double, b As Double, c As Double
Dim r1 As Double, i1 As Double, r2 As Double, i2 As Double
a = 7: b = 6: c = 2
Call Roots(a, b, c, ier, r1, i1, r2, i2)
If ier = 0 Then
 MsqBox "No roots"
ElseIf ier = 1 Then
 MsgBox "single root=" & r1
ElseIf ier = 2 Then
 MsgBox "real roots = " & r1 & ", " & r2
ElseIf ier = 3 Then
 MsgBox "complex roots =" & r1 & "," & i1 & " i" & "; "_
                         & r2 & "," & i2 & " i"
End If
End Sub
Sub Roots(a, b, c, ier, r1, i1, r2, i2)
Dim d As Double
r1 = 0: r2 = 0: i1 = 0: i2 = 0
```

```
If a = 0 Then
  If b <> 0 Then
    r1 = -c / b
    ier = 1
  Else
    ier = 0
  End If
Else
  d = b ^2 - 4 * a * c
  If (d >= 0) Then
    r1 = (-b + Sqr(d)) / (2 * a)
    r2 = (-b - Sqr(d)) / (2 * a)
    ier = 2
  Else
    r1 = -b / (2 * a)
    r2 = r1
    i1 = Sqr(Abs(d)) / (2 * a)
    i2 = -i1
    ier = 3
  End If
End If
End Sub
```

The answers for the 3 test cases are: (a) -0.3542, -5.646; (b) 0.4; (c) -0.4167 + 1.4696i; -0.4167 - 1.4696i.

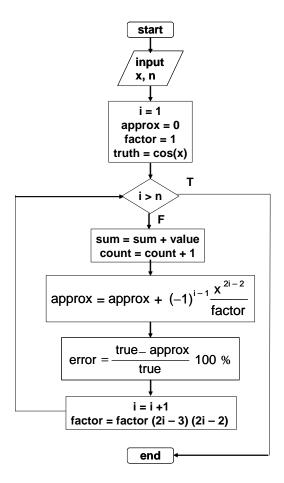
Several features of this subroutine bear mention:

- The subroutine does not involve input or output. Rather, information is passed in and out via the arguments. This is often the preferred style, because the I/O is left to the discretion of the programmer within the calling program.
- Note that a variable is passed (IER) in order to distinguish among the various cases.
- **2.4** The development of the algorithm hinges on recognizing that the series approximation of the cosine can be represented concisely by the summation,

$$\sum_{i=1}^{n} (-1)^{i-1} \frac{x^{2i-2}}{(2i-2)!}$$

where i = the order of the approximation.

(a) Structured flowchart



(b) Pseudocode:

```
SUBROUTINE Coscomp(n,x) i = 1 approx = 0 factor = 1 truth = cos(x) DO

IF i > n EXIT factor = 1 f
```

2.5 Students could implement the subprogram in any number of languages. The following MATLAB M-file is one example. It should be noted that MATLAB allows direct calculation of the factorial through its intrinsic function factorial. However, we did not exploit this feature, in order to make the code more compatible with languages such as Visual BASIC and Fortran.

```
function coscomp(x,n)
i = 1;
tru = cos(x);
approx = 0;
f = 1;
```

```
fprintf('\n');
fprintf('order true value approximation error\n');
while (1)
  if i > n, break, end
  approx = approx + (-1)^(i - 1) * x^(2*i-2) / f;
  er = (tru - approx) / tru * 100;
  fprintf('%3d %14.10f %14.10f %12.8f\n',i,tru,approx,er);
  i = i + 1;
  f = f*(2*i-3)*(2*i-2);
end
```

Here is a run of the program showing the output that is generated:

**2.6** (a) The following pseudocode provides an algorithm for this problem. Notice that the input of the quizzes and homeworks is done with logical loops that terminate when the user enters a negative grade:

```
INPUT WQ, WH, WF
nq = 0
sumq = 0
DO
 INPUT quiz (enter negative to signal end of quizzes)
 IF quiz < 0 EXIT
 nq = nq + 1
 sumq = sumq + quiz
END DO
AQ = sumq / nq
nh = 0
sumh = 0
  INPUT homework (enter negative to signal end of homeworks)
 TF homework < 0 EXTT
 nh = nh + 1
 sumh = sumh + homework
END DO
AH = sumh / nh
DISPLAY "Is there a final grade (y or n)"
INPUT answer
IF answer = "y" THEN
  TNPUT FE
  AG = (WQ * AQ + WH * AH + WF * FE) / (WQ + WH + WF)
  AG = (WQ * AQ + WH * AH) / (WQ + WH)
END IF
DISPLAY AG
END
```

(b) Students could implement the program in any number of languages. The following VBA code is one example.

```
Sub Grader()
Dim WQ As Double, WH As Double, WF As Double
Dim nq As Integer, sumq As Double, AQ As Double
Dim nh As Integer, sumh As Double, AH As Double
Dim answer As String, FE As Double
Dim AG As Double
'enter weights
WQ = InputBox("enter quiz weight")
WH = InputBox("enter homework weight")
WF = InputBox("enter final exam weight")
'enter quiz grades
nq = 0
sumq = 0
Dο
  quiz = InputBox("enter negative to signal end of quizzes")
 If quiz < 0 Then Exit Do
 nq = nq + 1
  sumq = sumq + quiz
AQ = sumq / nq
'enter homework grades
nh = 0
sumh = 0
  homework = InputBox("enter negative to signal end of homeworks")
  If homework < 0 Then Exit Do
 nh = nh + 1
  sumh = sumh + homework
AH = sumh / nh
'determine and display the average grade
answer = InputBox("Is there a final grade (y or n)")
If answer = "y" Then
  FE = InputBox("final grade:")
  AG = (WQ * AQ + WH * AH + WF * FE) / (WQ + WH + WF)
  AG = (WQ * AQ + WH * AH) / (WQ + WH)
MsgBox "Average grade = " & AG
End Sub
The results should conform to:
AO = 437/5 = 87.4
AH = 541/6 = 90.1667
without final
                  AG = \frac{35(87.4) + 30(90.1667)}{35 + 30} = 88.677
with final
                  AG = \frac{35(87.4) + 30(90.1667) + 35(92)}{35 + 30 + 35} = 89.84
```

#### 2.7 (a) Pseudocode:

$$IF a > 0 THEN$$

$$tol = 10^{-5}$$

```
x = a/2
DO
y = (x + a/x)/2
e = |(y - x)/y|
x = y
IF e < tol EXIT
END DO
SquareRoot = x
ELSE
SquareRoot = 0
END IF
```

(b) Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

```
VBA Function Procedure
                                           MATLAB M-File
Option Explicit
                                           function s = SquareRoot(a)
Function SquareRoot(a)
                                          if a > 0
                                            tol = 0.00001;
Dim x As Double, y As Double
Dim e As Double, tol As Double
                                            x = a / 2;
If a > 0 Then
                                            while(1)
 tol = 0.00001
                                              y = (x + a / x) / 2;
 x = a / 2
                                              e = abs((y - x) / y);
                                              x = y;
   y = (x + a / x) / 2
                                              if e < tol, break, end
    e = Abs((y - x) / y)
                                            end
   x = y
                                            s = x_i
    If e < tol Then Exit Do
                                          else
                                            s = 0;
 qool
  SquareRoot = x
                                          end
Else
 SquareRoot = 0
End If
End Function
```

## **2.8** A MATLAB M-file can be written to solve this problem as

```
function futureworth(P, i, n)
nn = 0:n;
F = P*(1+i).^nn;
y = [nn;F];
fprintf('\n year future worth\n');
fprintf('%5d %14.2f\n',y);
```

This function can be used to evaluate the test case,

>> futureworth(100000,0.06,5)

```
year future worth
0 100000.00
1 106000.00
2 112360.00
3 119101.60
4 126247.70
5 133822.56
```

**2.9** A MATLAB M-file can be written to solve this problem as

```
function annualpayment(P, i, n)
nn = 1:n;
```

```
A = P*i*(1+i).^nn./((1+i).^nn-1);
y = [nn;A];
fprintf('\n year annual payment\n');
fprintf('%5d %14.2f\n',y);
```

This function can be used to evaluate the test case,

```
>> annualpayment(55000,0.066,5)
```

```
year annual payment
1 58630.00
2 30251.49
3 20804.86
4 16091.17
5 13270.64
```

**2.10** Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure	MATLAB M-File
Option Explicit	<pre>function Ta = avgtemp(Tm,Tp,ts,te)</pre>
Function avgtemp(Tm, Tp, ts, te)	w = 2*pi/365;
Dim pi As Double, w As Double	t = ts:te;
Dim Temp As Double, t As Double	T = Tm + (Tp-Tm)*cos(w*(t-205));
Dim sum As Double, i As Integer	Ta = mean(T);
Dim n As Integer	
pi = 4 * Atn(1)	
w = 2 * pi / 365	
sum = 0	
n = 0	
t = ts	
For i = ts To te	
Temp = Tm + (Tp-Tm) * Cos(w*(t-205))	
sum = sum + Temp	
n = n + 1	
t = t + 1	
Next i	
avgtemp = sum / n	
End Function	

The function can be used to evaluate the test cases. The following show the results for MATLAB,

```
>> avgtemp(22.1,28.3,0,59)
ans =
    16.2148
>> avgtemp(10.7,22.9,180,242)
ans =
    22.2491
```

**2.11** The programs are student specific and will be similar to the codes developed for VBA and MATLAB as outlined in sections 2.4 and 2.5. The numerical results for the different time steps are tabulated below along with an estimate of the absolute value of the true relative error at t = 12 s:

Step	<i>v</i> (12)	$ \varepsilon_t $ (%)
2	49.96	5.2
1	48.70	2.6

0.5 48.09 1.3

The general conclusion is that the error is halved when the step size is halved.

**2.12** Students could implement the subprogram in any number of languages. The following VBA/Excel and MATLAB programs are two examples based on the algorithm outlined in Fig. P2.15.

```
VBA/Excel
                                             MATLAB
Option Explicit
Sub Bubble(n, b)
                                             function y = Bubble(x)
Dim m As Integer, i As Integer
                                             n = length(x);
Dim switch As Boolean, dum As Double
                                             m = n - 1;
m = n - 1
                                             b = x_i
                                             while(1)
                                               s = 0;
  switch = False
  For i = 1 To m
                                               for i = 1:m
    If b(i) > b(i + 1) Then
                                                 if b(i) > b(i + 1)
      dum = b(i)
                                                   dum = b(i);
      b(i) = b(i + 1)
                                                   b(i) = b(i + 1);
      b(i + 1) = dum
                                                   b(i + 1) = dum;
      switch = True
                                                   s = 1;
    End If
                                                 end
  Next i
                                               end
  If switch = False Then Exit Do
                                               if s == 0, break, end
                                               m = m - 1;
  m = m - 1
good
                                             end
End Sub
                                             y = b;
```

Notice how the MATLAB length function allows us to omit the length of the vector in the function argument. Here is an example MATLAB session that invokes the function to sort a vector:

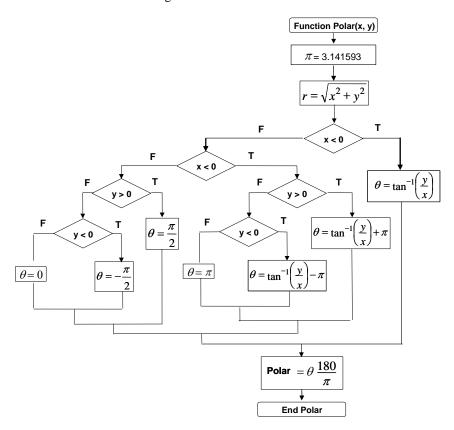
**2.13** Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

```
VBA Function Procedure
                                      MATLAB M-File
Option Explicit
                                      function Vol = tankvolume(R, d)
Function Vol(R, d)
                                      if d < R
Dim V1 As Double, V2 As Double
                                        Vol = pi * d ^ 3 / 3;
Dim pi As Double
                                      elseif d <= 3 * R
                                        V1 = pi * R ^ 3 / 3;
pi = 4 * Atn(1)
                                        V2 = pi * R ^ 2 * (d - R);
If d < R Then
                                        Vol = V1 + V2;
 Vol = pi * d ^ 3 / 3
ElseIf d <= 3 * R Then</pre>
                                      else
  V1 = pi * R ^ 3 / 3
                                        Vol = 'overtop';
  V2 = pi * R ^ 2 * (d - R)
                                      end
  Vol = V1 + V2
Else
  Vol = "overtop"
End If
End Function
```

The results are:

R	d	Volume
1	0.5	0.1309
1	1.2	1.675516
1	3	7.330383
1	3.1	overtop

#### **2.14** Here is a flowchart for the algorithm:



Students could implement the function in any number of languages. The following MATLAB M-file is one option. Versions in other languages such as Fortran 90, Visual Basic, or C would have a similar structure.

```
function polar(x, y)
r = sqrt(x .^2 + y .^2);
n = length(x);
for i = 1:n
  if x(i) > 0
    th(i) = atan(y(i) / x(i));
  elseif x(i) < 0
    if y(i) > 0
      th(i) = atan(y(i) / x(i)) + pi;
    elseif y(i) < 0
      th(i) = atan(y(i) / x(i)) - pi;
    else
      th(i) = pi;
    end
  else
    if y(i) > 0
```

```
th(i) = pi / 2;
elseif y(i) < 0
    th(i) = -pi / 2;
else
    th(i) = 0;
end
end
th(i) = th(i) * 180 / pi;
end
ou=[x;y;r;th];
fprintf('\n x y radius angle\n');
fprintf('\%8.2f %8.2f %10.4f %10.4f\n',ou);</pre>
```

This function can be used to evaluate the test cases.

```
>> x=[1 1 1 -1 -1 -1 0 0 0];
>> y=[1 -1 0 1 -1 0 1 -1 0];
>> polar(x,y)
                               angle
    х
                     radius
             У
   1.00
            1.00
                     1.4142
                               45.0000
   1.00
           -1.00
                     1.4142
                              -45.0000
            0.00
   1.00
                     1.0000
                               0.0000
   -1.00
           1.00
                     1.4142
                             135.0000
  -1.00
           -1.00
                     1.4142
                            -135.0000
   -1.00
           0.00
                     1.0000
                             180.0000
   0.00
           1.00
                     1.0000
                               90.0000
   0.00
           -1.00
                     1.0000
                              -90.0000
   0.00
            0.00
                     0.0000
                                0.0000
```

**2.15** Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure	MATLAB M-File
Function grade(s)	<pre>function grade = lettergrade(score)</pre>
If s >= 90 Then	if score >= 90
grade = "A"	grade = 'A';
ElseIf s >= 80 Then	elseif score >= 80
grade = "B"	grade = 'B';
ElseIf s >= 70 Then	elseif score >= 70
grade = "C"	grade = 'C';
ElseIf s >= 60 Then	elseif score >= 60
grade = "D"	grade = 'D';
Else	else
grade = "F"	grade = 'F';
End If	end
End Function	

**2.16** Students could implement the functions in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure	MATLAB M-File
(a) Factorial	
Function factor(n)	<pre>function fout = factor(n)</pre>
Dim x As Long, i As Integer	x = 1;
x = 1	for i = 1:n
For i = 1 To n	x = x * i;
x = x * i	end
Next i	fout = x;
factor = x	

```
End Function
(b) Minimum
Function min(x, n)
                                     function xm = xmin(x)
Dim i As Integer
                                     n = length(x);
min = x(1)
                                     xm = x(1);
For i = 2 To n
                                     for i = 2:n
 If x(i) < min Then min = x(i)
                                       if x(i) < xm, xm = x(i); end
Next i
End Function
(c) Average
                                     function xm = xmean(x)
Function mean(x, n)
Dim sum As Double
                                     n = length(x);
Dim i As Integer
                                     s = x(1);
sum = x(1)
                                     for i = 2:n
For i = 2 To n
                                       s = s + x(i);
 sum = sum + x(i)
                                     end
Next i
                                     xm = s / n;
mean = sum / n
End Function
```

**2.17** Students could implement the functions in any number of languages. The following VBA and MATLAB codes are two possible options.

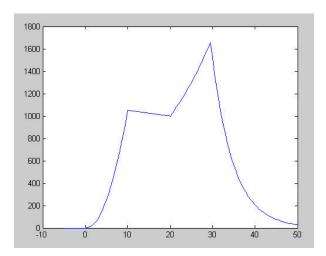
```
MATLAB M-File
VBA Function Procedure
(a) Square root sum of squares
                                      function s = SSS(x)
Function SSS(x, n, m)
Dim i As Integer, j As Integer
                                      [n,m] = size(x);
                                      s = 0;
SSS = 0
For i = 1 To n
                                      for i = 1:n
 For j = 1 To m
                                        for j = 1:m
   SSS = SSS + x(i, j) ^ 2
                                         s = s + x(i, j) ^ 2;
 Next j
                                        end
Next i
                                      end
SSS = Sqr(SSS)
                                      s = sqrt(s);
End Function
(b) Normalization
                                      function y = normal(x)
Sub normal(x, n, m, y)
Dim i As Integer, j As Integer
                                      [n,m] = size(x);
                                      for i = 1:n
Dim max As Double
For i = 1 To n
                                        mx = abs(x(i, 1));
  max = Abs(x(i, 1))
                                        for j = 2:m
                                          if abs(x(i, j)) > mx
  For j = 2 To m
    If Abs(x(i, j)) > max Then
                                            mx = x(i, j);
      max = x(i, j)
                                          end
    End If
                                        end
 Next j
                                        for j = 1:m
                                         y(i, j) = x(i, j) / mx;
  For j = 1 To m
    y(i, j) = x(i, j) / max
                                        end
  Next j
Next i
End Sub
                                      Alternate version:
                                      function y = normal(x)
                                      n = size(x);
                                      for i = 1:n
                                        y(i,:) = x(i,:)/max(x(i,:));
```

## 2.18 The following MATLAB function implements the piecewise function:

```
function v = vpiece(t)
if t<0
    v = 0;
elseif t<10
    v = 11*t^2 - 5*t;
elseif t<20
    v = 1100 - 5*t;
elseif t<30
    v = 50*t + 2*(t - 20)^2;
else
    v = 1520*exp(-0.2*(t-30));
end</pre>
```

Here is a script that uses vpiece to generate the plot

```
k=0;
for i = -5:.5:50
  k=k+1;
  t(k)=i;
  v(k)=vpiece(t(k));
end
plot(t,v)
```



# **2.19** The following MATLAB function implements the algorithm:

```
function nd = days(mo, da, leap)
nd = 0;
for m=1:mo-1
  switch m
    case {1, 3, 5, 7, 8, 10, 12}
      nday = 31;
    case {4, 6, 9, 11}
      nday = 30;
    case 2
      nday = 28+leap;
   end
   nd=nd+nday;
end
nd = nd + da;
```

# **2.20** The following MATLAB function implements the algorithm:

```
function nd = days(mo, da, year)
leap = 0;
if year /4 - fix(year /4) == 0, leap = 1; end
nd = 0;
for m=1:mo-1
 switch m
   case {1, 3, 5, 7, 8, 10, 12}
     nday = 31;
    case {4, 6, 9, 11}
     nday = 30;
    case 2
     nday = 28 + leap;
  end
 nd=nd+nday;
end
nd = nd + da;
>> days(1,1,1999)
ans =
    1
>> days(2,29,2000)
ans =
>> days(3,1,2001)
ans =
   60
>> days(6,21,2002)
ans =
  172
>> days(12,31,2004)
ans =
```

# 2.21 A MATLAB M-file can be written as

This function can be run to create the table,

```
>> A=[.035 .0001 10 2
.020 .0002 8 1
.015 .001 20 1.5
.03 .0007 24 3
.022 .0003 15 2.5];
>> Manning(A)
              S
                         В
                                     Η
                                                 U
   n
   0.035
           0.0001
                        10.00
                                    2.00
                                              0.3624
   0.020
           0.0002
                         8.00
                                    1.00
                                              0.6094
   0.015
           0.0010
                        20.00
                                    1.50
                                              2.5167
                        24.00
   0.030
           0.0007
                                    3.00
                                              1.5809
                        15.00
                                    2.50
   0.022
           0.0003
                                              1.1971
```

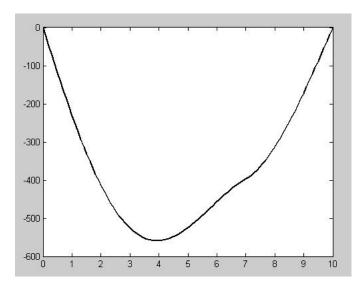
#### 2.22 A MATLAB M-file can be written as

```
function beam(x)
xx = linspace(0,x);
n=length(xx);
for i=1:n
    uy(i) = -5/6.*(sing(xx(i),0,4)-sing(xx(i),5,4));
    uy(i) = uy(i) + 15/6.*sing(xx(i),8,3) + 75*sing(xx(i),7,2);
    uy(i) = uy(i) + 57/6.*xx(i)^3 - 238.25.*xx(i);
end
plot(xx,uy)

function s = sing(xxx,a,n)
if xxx > a
    s = (xxx - a).^n;
else
    s=0;
end
```

This function can be run to create the plot,

## >> beam(10)



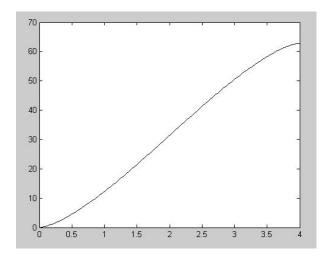
#### 2.23 A MATLAB M-file can be written as

function cylinder(r, L)

```
 \begin{array}{lll} h &=& 1 & \text{inspace(0,2*r);} \\ V &=& (r^2 + a & \cos((r-h)./r) - (r-h). + sqrt(2 + r + h - h.^2)) + L; \\ plot(h, V) & & & \end{array}
```

This function can be run to create the plot,

>> cylinder(2,5)



**2.24** Before the chute opens (t < 10), Euler's method can be implemented as

$$v(t + \Delta t) = v(t) + \left[9.8 - \frac{10}{80}v(t)\right]\Delta t$$

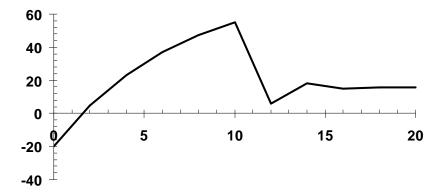
After the chute opens ( $t \ge 10$ ), the drag coefficient is changed and the implementation becomes

$$v(t + \Delta t) = v(t) + \left[9.8 - \frac{50}{80}v(t)\right]\Delta t$$

You can implement the subprogram in any number of languages. The following MATLAB M-file is one example. Notice that the results are inaccurate because the stepsize is too big. A smaller stepsize should be used to attain adequate accuracy.

```
function parachute
g = 9.81;
m = 80; c = 10;
ti = 0; tf = 20; dt = 2;
vi = -20;
tc = 10; cc = 50;
np = (tf - ti) / dt;
t = ti; v = vi;
tout(1) = t; vout(1) = v;
for i = 1:np
  if t < tc
    dvdt = g - c / m * v;
  else
    dvdt = g - cc / m * v;
  v = v + dvdt * dt;
  t = t + dt;
```

```
tout(i+1) = t; vout(i+1) = v;
plot(tout,vout)
z=[tout;vout]
fprintf('
                       v\n');
           t
fprintf('%5d %10.3f\n',z);
    t
    0
          -20.000
    2
            4.620
    4
           23.085
    6
           36.934
           47.320
    8
   10
           55.110
            5.842
   12
   14
           18.159
   16
           15.080
   18
           15.850
   20
           15.658
```



**2.25** Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA/Excel	MATLAB
Option Explicit	
Function fac(n)	function $f = fac(n)$
Dim x As Long, i As Integer	
If $n \ge 0$ Then	if n >= 0
x = 1	x = 1;
For $i = 1$ To n	for i = 1: n
x = x * i	x = x * i;
Next i	end
fac = x	f = x;
Else	else
MsgBox "value must be positive"	error 'value must be positive'
End	end
End If	
End Function	