CHAPTER 2

2.1
```
IF x < 100 THEN
  IF x < 50 THEN
    x = 0
  ELSE
    x = 75
  END IF
ELSE
  DO
    IF x ≤ 500 EXIT
    x = x – 50
  END DO
ENDIF
```

2.2
```
DO
  j = j + 1
  x = x + 5
  IF x > 5 THEN
    y = x
  ELSE
    y = 0
  ENDIF
  z = x + y
  IF z > 50 EXIT
ENDDO
```

2.3 Students could implement the subprogram in any number of languages. The following VBA program is one example. It should be noted that the availability of complex variables in languages such as Fortran 90 would allow this subroutine to be made even more concise. However, we did not exploit this feature, in order to make the code more compatible with languages that do not support complex variables. This version is then followed by a MATLAB script and function that does accommodate complex variables.

```
Option Explicit

Sub Rootfind()
Dim ier As Integer
Dim a As Double, b As Double, c As Double
Dim r1 As Double, i1 As Double, r2 As Double, i2 As Double
a = 1: b = 7: c = 2
Call Roots(a, b, c, ier, r1, i1, r2, i2)
If ier = 0 Then
  MsgBox "No roots"
ElseIf ier = 1 Then
  MsgBox "single root=" & r1
ElseIf ier = 2 Then
  MsgBox "real roots = " & r1 & ", " & r2
ElseIf ier = 3 Then
  MsgBox "complex roots =" & r1 & "," & i1 & " i" & "; "_
                         & r2 & "," & i2 & " i"
End If
End Sub

Sub Roots(a, b, c, ier, r1, i1, r2, i2)
Dim d As Double
r1 = 0: r2 = 0: i1 = 0: i2 = 0
If a = 0 Then
```

```
  If b <> 0 Then
    r1 = -c / b
    ier = 1
  Else
    ier = 0
  End If
Else
  d = b ^ 2 - 4 * a * c
  If (d >= 0) Then
    r1 = (-b + Sqr(d)) / (2 * a)
    r2 = (-b - Sqr(d)) / (2 * a)
    ier = 2
  Else
    r1 = -b / (2 * a)
    r2 = r1
    i1 = Sqr(Abs(d)) / (2 * a)
    i2 = -i1
    ier = 3
  End If
End If
End Sub
```

The answers for the 3 test cases are: (*a*) –0.2984, –6.702; (*b*) 0.32; (*c*) –0.4167 + 1.5789*i*; –0.4167 – 1.5789*i*.

Several features of this subroutine bear mention:

- The subroutine does not involve input or output. Rather, information is passed in and out via the arguments. This is often the preferred style, because the I/O is left to the discretion of the programmer within the calling program.
- Note that a variable is passed (`IER`) in order to distinguish among the various cases.

MATLAB:

```
function [r1,r2]=quadroots(a,b,c)
r1 = 0; r2 = 0;
if a == 0
  if b ~= 0
    r1=-c/b;
  else
    r1='Trivial solution';
  end
else
  discr=b^2-4*a*c;
  if discr >= 0
    r1=(-b+sqrt(discr))/(2*a);
    r2=(-b-sqrt(discr))/(2*a);
  else
    r1 =-b/(2*a); i1=sqrt(abs(discr))/(2*a);
    r2=r1-i1*i; r1=r1+i1*i;
  end
end
```

Script:

```
clc
format compact
disp('(a)'),[r1,r2]=quadroots(1,7,2)
disp('(b)'),[r1,r2]=quadroots(0,-5,1.6)
disp('(c)'),[r1,r2]=quadroots(3,2.5,8)
```

Output when script is run

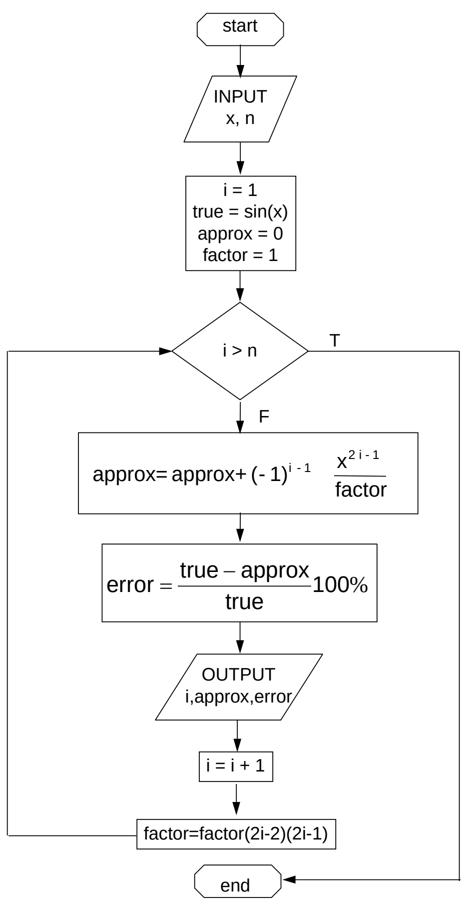
```
(a)
r1 =
   -0.2984
r2 =
   -6.7016
(b)
r1 =
    0.3200
r2 =
     0
(c)
r1 =
  -0.4167 + 1.5789i
r2 =
  -0.4167 - 1.5789i
```

2.4 The development of the algorithm hinges on recognizing that the series approximation of the sine can be represented concisely by the summation,

$$\sum_{i=1}^{n} (-1)^{i-1} \frac{x^{2i-1}}{(2i-1)!}$$

where i = the order of the approximation.

(a) Structured flowchart:



(b) Pseudocode:

```
SUBROUTINE Sincomp(n,x)
i = 1; truth = SIN(x); approx = 0
factor = 1
DO
  IF i > n EXIT
  approx = approx + (-1)^(i-1)·x^(2•i-1) / factor
  error = (truth - approx) / truth) * 100
  PRINT i, truth, approx, error
  i = i + 1
  factor = factor·(2·i-2)·(2·i-1)
END DO
END
```

2.5 Students could implement the subprogram in any number of languages. The following MATLAB M-file is one example. It should be noted that MATLAB allows direct calculation of the factorial through its intrinsic function `factorial`. However, we did not exploit this feature, in order to make the code more compatible with languages such as Visual BASIC and Fortran.

```
function sincomp(x,n)
i = 1; tru = sin(x); approx = 0;
f = 1;
fprintf('\n');
fprintf('order  true value     approximation     error\n');
while (1)
  if i > n, break, end
  approx = approx + (-1)^(i - 1) * x^(2*i-1) / f;
  er = (tru - approx) / tru * 100;
  fprintf('%3d  %14.10f  %14.10f  %12.8f \n',i,tru,approx,er);
  i = i + 1;
  f = f*(2*i-2)*(2*i-1);
end
```

Here is a run of the program showing the output that is generated:

```
>> sincomp(1.5,8)

order  true value     approximation     error
  1    0.9974949866    1.5000000000  -50.37669564
  2    0.9974949866    0.9375000000    6.01456523
  3    0.9974949866    1.0007812500   -0.32945162
  4    0.9974949866    0.9973911830    0.01040643
  5    0.9974949866    0.9974971226   -0.00021414
  6    0.9974949866    0.9974949557    0.00000310
  7    0.9974949866    0.9974949869   -0.00000003
  8    0.9974949866    0.9974949866    0.00000000
```

2.6 (a) The following pseudocode provides an algorithm for this problem. Notice that the input of the quizzes and homeworks is done with logical loops that terminate when the user enters a negative grade:

```
INPUT WQ, WH, WF
nq = 0
sumq = 0
DO
  INPUT quiz (enter negative to signal end of quizzes)
  IF quiz < 0 EXIT
  nq = nq + 1
  sumq = sumq + quiz
```

```
END DO
AQ = sumq / nq
nh = 0
sumh = 0
DO
  INPUT homework (enter negative to signal end of homeworks)
  IF homework < 0 EXIT
  nh = nh + 1
  sumh = sumh + homework
END DO
AH = sumh / nh
DISPLAY "Is there a final exam (y or n)"
INPUT answer
IF answer = "y" THEN
  INPUT FE
  AG = (WQ * AQ + WH * AH + WF * FE) / (WQ + WH + WF)
ELSE
  AG = (WQ * AQ + WH * AH) / (WQ + WH)
END IF
DISPLAY AG
END
```

(b) Students could implement the program in any number of languages. The following VBA code is one example.

```
Option Explicit

Sub Grader()
Dim WQ As Double, WH As Double, WF As Double
Dim nq As Integer, sumq As Double, AQ As Double
Dim nh As Integer, sumh As Double, AH As Double
Dim answer As String, FE As Double
Dim AG As Double, quiz As Double, homework As Double

'enter weights
WQ = InputBox("enter quiz weight")
WH = InputBox("enter homework weight")
WF = InputBox("enter final exam weight")
'enter quiz grades
nq = 0: sumq = 0
Do
  quiz = InputBox("enter negative to signal end of quizzes")
  If quiz < 0 Then Exit Do
  nq = nq + 1
  sumq = sumq + quiz
Loop
AQ = sumq / nq
'enter homework grades
nh = 0: sumh = 0
Do
  homework = InputBox("enter negative to signal end of homeworks")
  If homework < 0 Then Exit Do
  nh = nh + 1
  sumh = sumh + homework
Loop
AH = sumh / nh
'determine and display the average grade
answer = InputBox("Is there a final exam (y or n)")
If answer = "y" Then
  FE = InputBox("final exam:")
  AG = (WQ * AQ + WH * AH + WF * FE) / (WQ + WH + WF)
```

```
Else
  AG = (WQ * AQ + WH * AH) / (WQ + WH)
End If
MsgBox "Average grade = " & AG
End Sub
```

The results should conform to:

AQ = 442/5 = 88.4
AH = 556/6 = 92.667

without final

$$AG = \frac{30(88.4) + 40(92.667)}{30 + 40} = 90.8381$$

with final

$$AG = \frac{30(88.4) + 40(92.667) + 30(91)}{30 + 40 + 30} = 90.8867$$

Here is an example of how a MATLAB script could be developed to solve the same problem:

```
clc
% enter weights
WQ = input('enter quiz weight');
WH = input('enter homework weight');
WF = input('enter final exam weight');
% enter quiz grades
nq = 0; sumq = 0;
while(1)
  quiz = input('enter negative to signal end of quizzes');
  if quiz < 0;break;end
  nq = nq + 1;
  sumq = sumq + quiz;
end
AQ = sumq / nq;
% enter homework grades
nh = 0; sumh = 0;
while(1)
  homework = input('enter negative to signal end of homeworks');
  if homework < 0;break;end
  nh = nh + 1;
  sumh = sumh + homework;
end
AH = sumh / nh;
answer = input('Is there a final exam (y or n)','s');
if answer == 'y'
  FE = input('final exam:');
  AG = (WQ * AQ + WH * AH + WF * FE) / (WQ + WH + WF);
else
  AG = (WQ * AQ + WH * AH) / (WQ + WH);
end
fprintf('Average grade: %8.4f\n',AG)
```

Finally, here is an alternative MATLAB script that solves the same problem, but is much more concise. Note that rather than using interactive input, the script employs vectors to enter the data. In addition, the nonexistence of a final is denoted by entering a negative number for the final exam:

```
clc
WQ=30;WH=40;WF=30;
```

```
QG=[98 95 90 60 99];
HG=[98 95 86 100 100 77];
FE=91;
if FE>0
  AG=(WQ*mean(QG)+WH*mean(HG)+WF*FE)/(WQ+WH+WF);
else
  AG=(WQ*mean(QG)+WH*mean(HG))/(WQ+WH);
end
fprintf('Average grade: %8.4f\n',AG)
```

2.7 (a) Pseudocode:

```
IF a > 0 THEN
  tol = 10^-6
  x = a/2
  DO
    y = (x + a/x)/2
    e = |(y – x)/y|
    x = y
    IF e < tol EXIT
  END DO
  SquareRoot = x
ELSE
  SquareRoot = 0
END IF
```

(b) Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

<table>
<tr><th>VBA Function Procedure</th><th>MATLAB M-File</th></tr>
<tr><td><pre>Option Explicit
Function SquareRoot(a)
Dim x As Double, y As Double
Dim e As Double, tol As Double
If a > 0 Then
  tol = 0.000001
  x = a / 2
  Do
    y = (x + a / x) / 2
    e = Abs((y - x) / y)
    x = y
    If e < tol Then Exit Do
  Loop
  SquareRoot = x
Else
  SquareRoot = 0
End If
End Function</pre></td><td><pre>function s = SquareRoot(a)
if a > 0
  tol = 0.000001;
  x = a / 2;
  while(1)
    y = (x + a / x) / 2;
    e = abs((y - x) / y);
    x = y;
    if e < tol, break, end
  end
  s = x;
else
  s = 0;
end</pre></td></tr>
</table>

2.8 A MATLAB M-file can be written to solve this problem as

```
function futureworth(P, i, n)
nn = 0:n;
F = P*(1+i).^nn;
y = [nn;F];
fprintf('\n  year   future worth\n');
fprintf('%5d %14.2f\n',y);
```

This function can be used to evaluate the test case,

```
>> futureworth(100000,0.04,11)

  year   future worth
    0      100000.00
    1      104000.00
    2      108160.00
    3      112486.40
    4      116985.86
    5      121665.29
    6      126531.90
    7      131593.18
    8      136856.91
    9      142331.18
   10      148024.43
   11      153945.41
```

2.9 A MATLAB M-file can be written to solve this problem as

```
function annualpayment(P, i, n)
nn = 1:n;
A = P*i*(1+i).^nn./((1+i).^nn-1);
y = [nn;A];
fprintf('\n  year   annual payment\n');
fprintf('%5d %14.2f\n',y);
```

This function can be used to evaluate the test case,

```
>> annualpayment(55000,0.066,5)

  year   annual payment
    1       58630.00
    2       30251.49
    3       20804.86
    4       16091.17
    5       13270.64
```

2.10 Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure

```
Option Explicit
Function avgtemp(Tm, Tp, ts, te)
Dim pi As Double, w As Double
Dim Temp As Double, t As Double
Dim sum As Double, i As Integer
Dim n As Integer
pi = 4 * Atn(1)
w = 2 * pi / 365
sum = 0
n = 0
t = ts
For i = ts To te
  Temp = Tm+(Tp-Tm)*Cos(w*(t-205))
  sum = sum + Temp
  n = n + 1
  t = t + 1
Next i
avgtemp = sum / n
End Function
```

MATLAB M-File

```
function Ta = avgtemp(Tm,Tp,ts,te)
w = 2*pi/365;
t = ts:te;
T = Tm + (Tp-Tm)*cos(w*(t-205));
Ta = mean(T);
```

The function can be used to evaluate the test cases. The following show the results for MATLAB,

```
>> avgtemp(22.1,28.3,0,59)

ans =
   16.2148

>> avgtemp(10.7,22.9,180,242)

ans =
   22.2491
```

2.11 The programs are student specific and will be similar to the codes developed for VBA and MATLAB as outlined in sections 2.4 and 2.5. For example, the following MATLAB script was developed to use the function from section 2.5 to compute and tabulate the numerical results for the value at t = 12 s, along with an estimate of the absolute value of the true relative error based on the analytical solution:

```
clc; format compact
m=68.1; cd=12.5;
ti=0; tf=12.;
vi=0;
vtrue=9.81*m/cd*(1-exp(-cd/m*tf))
dt=[2 1 0.5]';
for i = 1:3
  v(i)=euler(dt(i),ti,tf,vi,m,cd);
end
et=abs((vtrue-v)/vtrue*100);
z=[dt v' et']';
fprintf('      dt         v(12)      et(pct)\n')
fprintf('%10.3f %10.3f %10.3f\n',z);
```

Output:

```
vtrue =
   47.5387
      dt         v(12)      et(pct)
     2.000     50.010      5.199
     1.000     48.756      2.561
     0.500     48.142      1.269
```

The general conclusion is that the error is halved when the step size is halved.

2.12 Students could implement the subprogram in any number of languages. The following VBA/Excel and MATLAB programs are two examples based on the algorithm outlined in Fig. P2.12.

<table>
<tr><th>VBA/Excel</th><th>MATLAB</th></tr>
<tr><td><pre>Option Explicit

Sub Bubble(n, b)
Dim m As Integer, i As Integer
Dim switch As Boolean, dum As Double
m = n - 1
Do
  switch = False
  For i = 1 To m
    If b(i) > b(i + 1) Then
      dum = b(i)
      b(i) = b(i + 1)
      b(i + 1) = dum</pre></td><td><pre>

function y = Bubble(x)
n = length(x);
m = n - 1;
b = x;
while(1)
  s = 0;
  for i = 1:m
    if b(i) > b(i + 1)
      dum = b(i);
      b(i) = b(i + 1);
      b(i + 1) = dum;</pre></td></tr>
</table>

`      switch = True`	`      s = 1;`
`    End If`	`    end`
`  Next i`	`  end`
`  If switch = False Then Exit Do`	`  if s == 0, break, end`
`  m = m - 1`	`  m = m - 1;`
`Loop`	`end`
`End Sub`	`y = b;`

Notice how the MATLAB `length` function allows us to omit the length of the vector in the function argument. Here is an example MATLAB script that invokes the function to sort a vector:

```
clc
a=[6 3 4 2 1 5 7];
Bubble(a)

ans =
     1     2     3     4     5     6     7
```

2.13 Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure

```
Option Explicit
Function Vol(R, d)
Dim V1 As Double, V2 As Double
Dim pi As Double
pi = 4 * Atn(1)
If d < R Then
  Vol = pi * d ^ 3 / 3
ElseIf d <= 3 * R Then
  V1 = pi * R ^ 3 / 3
  V2 = pi * R ^ 2 * (d - R)
  Vol = V1 + V2
Else
  Vol = "overtop"
End If
End Function
```

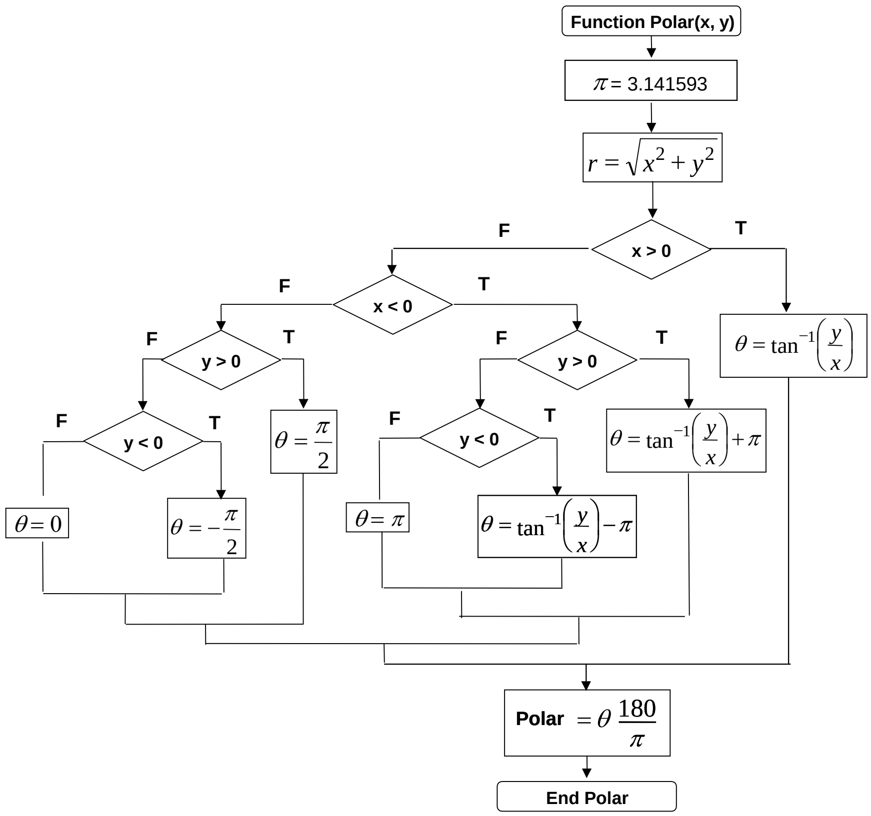
MATLAB M-File

```
function Vol = tankvolume(R, d)
if d < R
  Vol = pi * d ^ 3 / 3;
elseif d <= 3 * R
  V1 = pi * R ^ 3 / 3;
  V2 = pi * R ^ 2 * (d - R);
  Vol = V1 + V2;
else
  Vol = 'overtop';
end
```

The results are:

R	***d***	**Volume**
1	0.5	0.1309
1	1.2	1.675516
1	3	7.330383
1	3.1	overtop

2.14 Here is a flowchart for the algorithm:



Students could implement the function in any number of languages. The following MATLAB M-file is one option. Versions in other languages such as Fortran 90, Visual Basic, or C would have a similar structure.

```
function polar(x, y)
r = sqrt(x .^ 2 + y .^ 2);
n = length(x);
for i = 1:n
  if x(i) > 0
    th(i) = atan(y(i) / x(i));
  elseif x(i) < 0
    if y(i) > 0
      th(i) = atan(y(i) / x(i)) + pi;
    elseif y(i) < 0
      th(i) = atan(y(i) / x(i)) - pi;
    else
      th(i) = pi;
    end
  else
    if y(i) > 0
      th(i) = pi / 2;
    elseif y(i) < 0
      th(i) = -pi / 2;
    else
      th(i) = 0;
    end
  end
  th(i) = th(i) * 180 / pi;
end
ou=[x;y;r;th];
```

```
fprintf('\n     x        y       radius     angle\n');
fprintf('%8.2f %8.2f %10.4f %10.4f \n',ou);
```

This function can be used to evaluate the test cases as in the following script:

```
clc; format compact
x=[1 1 0 -1 -1 -1 0 1 0];
y=[0 1 1 1 0 -1 -1 -1 0];
polar(x,y)
```

When the script is run, the resulting output is

```
     x        y       radius     angle
   1.00     0.00     1.0000     0.0000
   1.00     1.00     1.4142    45.0000
   0.00     1.00     1.0000    90.0000
  -1.00     1.00     1.4142   135.0000
  -1.00     0.00     1.0000   180.0000
  -1.00    -1.00     1.4142  -135.0000
   0.00    -1.00     1.0000   -90.0000
   1.00    -1.00     1.4142   -45.0000
   0.00     0.00     0.0000     0.0000
```

2.15 Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure	**MATLAB M-File**

`Function grade(s)`
`If s >= 90 Then`
`  grade = "A"`
`ElseIf s >= 80 Then`
`  grade = "B"`
`ElseIf s >= 70 Then`
`  grade = "C"`
`ElseIf s >= 60 Then`
`  grade = "D"`
`Else`
`  grade = "F"`
`End If`
`End Function`
	`function grade = lettergrade(score)`
	`if score >= 90`
	`  grade = 'A';`
	`elseif score >= 80`
	`  grade = 'B';`
	`elseif score >= 70`
	`  grade = 'C';`
	`elseif score >= 60`
	`  grade = 'D';`
	`else`
	`  grade = 'F';`
	`end`

2.16 Students could implement the functions in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure	**MATLAB M-File**

(a) Factorial
`Function factor(n)`
`Dim x As Long, i As Integer`
`x = 1`
`For i = 1 To n`
`  x = x * i`
`Next i`
`factor = x`
`End Function`
	
	`function fout = factor(n)`
	`x = 1;`
	`for i = 1:n`
	`  x = x * i;`
	`end`
	`fout = x;`

(b) Minimum
`Function min(x, n)`
`Dim i As Integer`
`min = x(1)`
	
	`function xm = xmin(x)`
	`n = length(x);`
	`xm = x(1);`

`For i = 2 To n`
`  If x(i) < min Then min = x(i)`
`Next i`
`End Function`
	`for i = 2:n`
	`  if x(i) < xm, xm = x(i); end`
	`end`

(c) Average
`Function mean(x, n)`
`Dim sum As Double`
`Dim i As Integer`
`sum = x(1)`
`For i = 2 To n`
`  sum = sum + x(i)`
`Next i`
`mean = sum / n`
`End Function`
	`function xm = xmean(x)`
	`n = length(x);`
	`s = x(1);`
	`for i = 2:n`
	`  s = s + x(i);`
	`end`
	`xm = s / n;`

2.17 Students could implement the functions in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA Function Procedure	MATLAB M-File

(a) Square root sum of squares
`Function SSS(x, n, m)`
`Dim i As Integer, j As Integer`
`SSS = 0`
`For i = 1 To n`
`  For j = 1 To m`
`    SSS = SSS + x(i, j) ^ 2`
`  Next j`
`Next i`
`SSS = Sqr(SSS)`
`End Function`
	`function s = SSS(x)`
	`[n,m] = size(x);`
	`s = 0;`
	`for i = 1:n`
	`  for j = 1:m`
	`    s = s + x(i, j) ^ 2;`
	`  end`
	`end`
	`s = sqrt(s);`

(b) Normalization
`Sub normal(x, n, m, y)`
`Dim i As Integer, j As Integer`
`Dim max As Double`
`For i = 1 To n`
`  max = Abs(x(i, 1))`
`  For j = 2 To m`
`    If Abs(x(i, j)) > max Then`
`      max = x(i, j)`
`    End If`
`  Next j`
`  For j = 1 To m`
`    y(i, j) = x(i, j) / max`
`  Next j`
`Next i`
`End Sub`
	`function y = normal(x)`
	`[n,m] = size(x);`
	`for i = 1:n`
	`  mx = abs(x(i, 1));`
	`  for j = 2:m`
	`    if abs(x(i, j)) > mx`
	`      mx = x(i, j);`
	`    end`
	`  end`
	`  for j = 1:m`
	`    y(i, j) = x(i, j) / mx;`
	`  end`
	`end`
	
	`Alternate version:`
	
	`function y = normal(x)`
	`n = size(x);`
	`for i = 1:n`
	`  y(i,:) = x(i,:)/max(x(i,:));`
	`end`

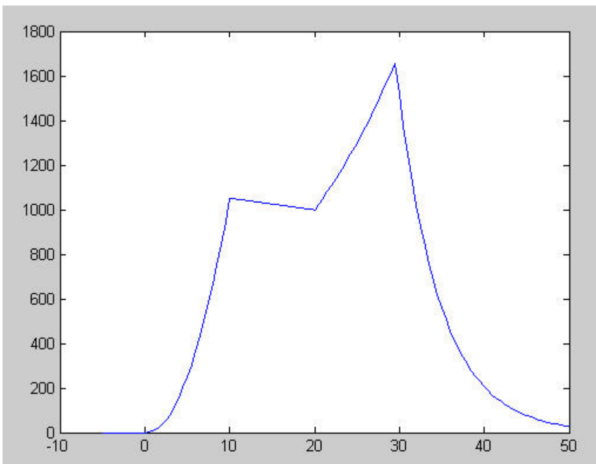
2.18 The following MATLAB function implements the piecewise function:

```
function v = vpiece(t)
if t<0
  v = 0;
```

```
elseif t<10
  v = 11*t^2 - 5*t;
elseif t<20
  v = 1100 - 5*t;
elseif t<30
  v = 50*t + 2*(t - 20)^2;
else
  v = 1520*exp(-0.2*(t-30));
end
```

Here is a script that uses `vpiece` to generate the plot

```
k=0;
for i = -5:.5:50
  k=k+1;
  t(k)=i;
  v(k)=vpiece(t(k));
end
plot(t,v)
```



2.19 The following MATLAB function implements the algorithm:

```
function nd = days(mo, da, leap)
nd = 0;
for m=1:mo-1
  switch m
    case {1, 3, 5, 7, 8, 10, 12}
      nday = 31;
    case {4, 6, 9, 11}
      nday = 30;
    case 2
      nday = 28+leap;
  end
  nd=nd+nday;
end
nd = nd + da;

>> days(1,1,0)
ans =
     1
>> days(2,29,1)
ans =
```

```
    60
>> days(3,1,0)
ans =
    60
>> days(6,21,0)
ans =
   172
>> days(12,31,1)
ans =
   366
```

2.20 The following MATLAB function implements the algorithm:

```
function nd = days(mo, da, year)
leap = 0;
if year / 4 - fix(year / 4) == 0, leap = 1; end
nd = 0;
for m=1:mo-1
  switch m
    case {1, 3, 5, 7, 8, 10, 12}
      nday = 31;
    case {4, 6, 9, 11}
      nday = 30;
    case 2
      nday = 28+leap;
  end
  nd=nd+nday;
end
nd = nd + da;

>> days(1,1,1999)
ans =
     1
>> days(2,29,2000)
ans =
    60
>> days(3,1,2001)
ans =
    60
>> days(6,21,2002)
ans =
   172
>> days(12,31,2004)
ans =
   366
```

2.21 A MATLAB M-file can be written as

```
function Manning(A)
A(:,5)=sqrt(A(:,2))./A(:,1).*(A(:,3).*A(:,4)./(A(:,3)+2*A(:,4))).^(2/3);
fprintf('\n    n         S          B          H          U\n');
fprintf('%8.3f %8.4f %10.2f %10.2f %10.4f\n',A');
```

This function can be run to create the table,

```
>> A=[.035 .0001 10 2
.020 .0002 8 1
.015 .001 20 1.5
.03 .0007 24 3
.022 .0003 15 2.5];
```

```
>> Manning(A)

    n          S           B          H          U
  0.035   0.0001      10.00       2.00     0.3624
  0.020   0.0002       8.00       1.00     0.6094
  0.015   0.0010      20.00       1.50     2.5167
  0.030   0.0007      24.00       3.00     1.5809
  0.022   0.0003      15.00       2.50     1.1971
```

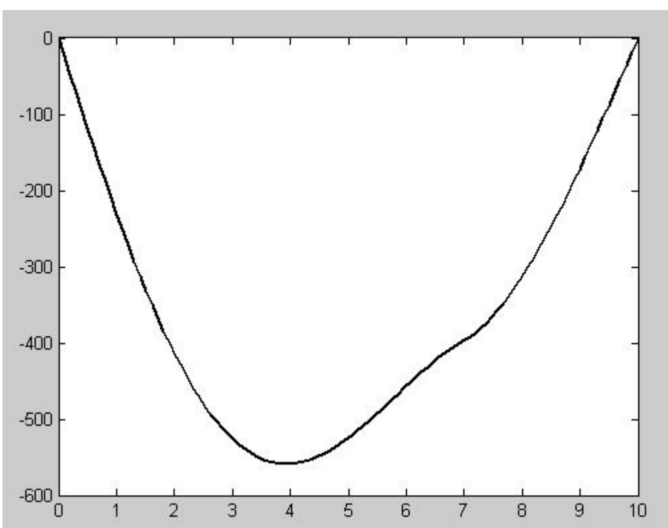
2.22 A MATLAB M-file can be written as

```
function beam(x)
xx = linspace(0,x);
n=length(xx);
for i=1:n
  uy(i) = -5/6.*(sing(xx(i),0,4)-sing(xx(i),5,4));
  uy(i) = uy(i) + 15/6.*sing(xx(i),8,3) + 75*sing(xx(i),7,2);
  uy(i) = uy(i) + 57/6.*xx(i)^3 - 238.25.*xx(i);
end
plot(xx,uy)

function s = sing(xxx,a,n)
if xxx > a
  s = (xxx - a).^n;
else
  s=0;
end
```

This function can be run to create the plot,

```
>> beam(10)
```

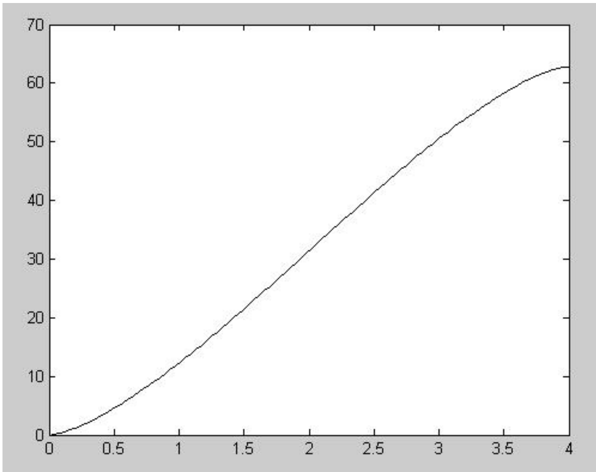


2.23 A MATLAB M-file can be written as

```
function cylinder(r, L)
h = linspace(0,2*r);
V = (r^2*acos((r-h)./r)-(r-h).*sqrt(2*r*h-h.^2))*L;
plot(h, V)
```

This function can be run to create the plot,

```
>> cylinder(2,5)
```



2.24 Before the chute opens ($t < 10$), Euler's method can be implemented as

$$v(t+\Delta t) = v(t) + \left[9.8 - \frac{10}{80}v(t)\right]\Delta t$$

After the chute opens ($t \geq 10$), the drag coefficient is changed and the implementation becomes

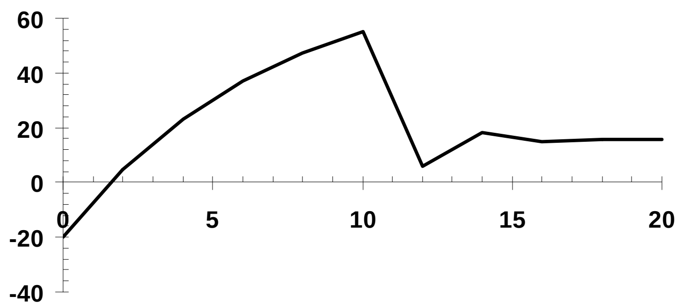
$$v(t+\Delta t) = v(t) + \left[9.8 - \frac{50}{80}v(t)\right]\Delta t$$

You can implement the subprogram in any number of languages. The following MATLAB M-file is one example. Notice that the results are inaccurate because the stepsize is too big. A smaller stepsize should be used to attain adequate accuracy.

```
function parachute
g = 9.81;
m = 80; c = 10;
ti = 0; tf = 20; dt = 2;
vi = -20;
tc = 10; cc = 50;
np = (tf - ti) / dt;
t = ti; v = vi;
tout(1) = t; vout(1) = v;
for i = 1:np
  if t < tc
    dvdt = g - c / m * v;
  else
    dvdt = g - cc / m * v;
  end
  v = v + dvdt * dt;
  t = t + dt;
  tout(i+1) = t; vout(i+1) = v;
end
plot(tout,vout)
z=[tout;vout]
fprintf('    t        v\n');
```

```
fprintf('%5d %10.3f\n',z);

    t        v
    0    -20.000
    2      4.620
    4     23.085
    6     36.934
    8     47.320
   10     55.110
   12      5.842
   14     18.159
   16     15.080
   18     15.850
   20     15.658
```



2.25 Students could implement the function in any number of languages. The following VBA and MATLAB codes are two possible options.

VBA/Excel	**MATLAB**
`Option Explicit`	
`Function fac(n)`	`function f = fac(n)`
`Dim x As Long, i As Integer`	
`If n >= 0 Then`	`if n >= 0`
`  x = 1`	`  x = 1;`
`  For i = 1 To n`	`  for i = 1: n`
`    x = x * i`	`    x = x * i;`
`  Next i`	`  end`
`  fac = x`	`  f = x;`
`Else`	`else`
`  MsgBox "value must be positive"`	`  error 'value must be positive'`
`  End`	`end`
`End If`	
`End Function`	

2.26 (a) Pseudocode:

```
FUNCTION height(t)
IF t < 0 THEN
  y = 0
ELSE IF t < 15 THEN
  y = 38.1454t + 0.13743t^3
ELSE IF t < 33 THEN
  y = 1036   + 130.909(t – 15) + 6.18425(t – 15)^2 - 0.428 (t – 15)^3
ELSE
  y = 2900   - 62.468(t – 33) - 16.9274(t – 33)^2 + 0.41796 (t – 33)^3
END IF
IF y < 0 THEN y = 0
height = y
```

```
END
```

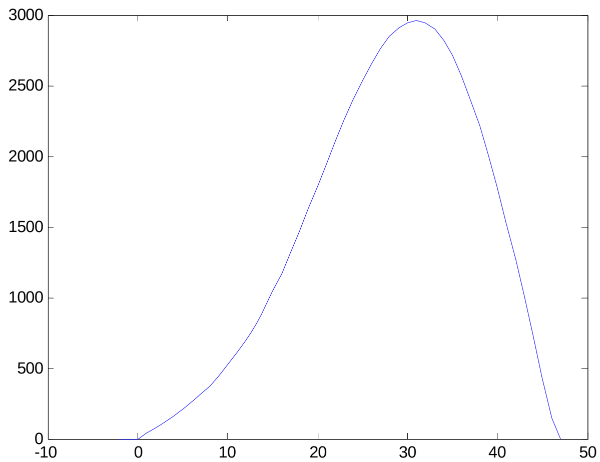
(b) MATLAB:

```
function y = height(t)
%Function to compute height of rocket from piecewise function
%   y = height(t)
% input:
%   t = time
% output:
%   y = height

if  t < 0
  y = 0;
elseif  t < 15
  y = 38.14544*t + 0.137428*t^3;
elseif  t < 33
  y = 1036 + 130.909*(t - 15) + 6.18425*(t - 15)^2 - 0.428*(t - 15)^3;
else
  y = 2900 - 62.468*(t - 33) - 16.9274*(t - 33)^2 + 0.41796*(t - 33)^3;
end
if  y < 0, y = 0; end
end
```

Here is a script that uses the function to generate a plot:

```
clc,clf
t=[-2:47];
for i=1:length(t)
  y(i)=height(t(i));
end
plot(t,y)
```



VBA:

```
Option Explicit

Function height(t)
If t < 0 Then
  y = 0
ElseIf t < 15 Then
  y = 38.14544 * t + 0.137428 * t ^ 3
ElseIf t < 33 Then
```

```
  y = 1036 + 130.909 * (t - 15) + 6.18425 * (t - 15) ^ 2 _
      - 0.428 * (t - 15) ^ 3
Else
  y = 2900 - 62.468 * (t - 33) - 16.9274 * (t - 33) ^ 2 _
       + 0.41796 * (t - 33) ^ 3
End If
If y < 0 Then y = 0
height = y
End Function
```

2.27 We must first identify the general formulas for the volumes. For example, for the full cylinder

$$V = \pi r_1^2 H_1 \qquad (1)$$

and for the volume of the full circular cone frustum

$$V = \frac{\pi H_2}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right) \qquad (2)$$

With this knowledge we can come up with the other cases that can occur:

Case 1: Full tank or overflowing tank.

$$V = \pi r_1^2 H_1 + \frac{\pi H_2}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

Case 2: The depth, $h \le 0$. $V = 0$

Case 3: Partially-full cylinder ($0 < h < H_1$)

$$V = \pi r_1^2 h$$

Case 4: Full cylinder with partially-full frustum ($H_1 \le h < H_1 + H_2$)

$$V = \pi r_1^2 H_1 + \frac{\pi (h - H_1)}{3}\left(r_1^2 + r_2(h)^2 + r_1 r_2(h)\right)$$

where $r_2(h)$ = the radius of the top of the partially-filled frustum. This quantity can be computed using the problem parameters via linear interpolation as

$$r_2(h) = r_1 + \frac{r_2 - r_1}{H_2}\left(h - H_1\right)$$

We can then use an if/then/elseif control structure to logically combine these cases as in

$$V = \pi r_1^2 H_1 + \frac{\pi H_2}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

IF $h \le 0$ *THEN*
  $V = 0$
ELSEIF $h < H_1$ *THEN*
  $V = \pi r_1^2 h$

ELSEIF h < H_1 + H_2 THEN

$$r_2(h) = r_1 + \frac{r_2 - r_1}{H_2}\left(h - H_1\right)$$

$$V = \pi r_1^2 H_1 + \frac{\pi(h - H_1)}{3}\left(r_1^2 + r_2(h)^2 + r_1 r_2(h)\right)$$

ENDIF

Notice how Eqs. (1) and (2) are used several times, but with different arguments. This suggests that we should represent them as independent functions that would be called by the main function. We do this in the following codes.

VBA/Excel.

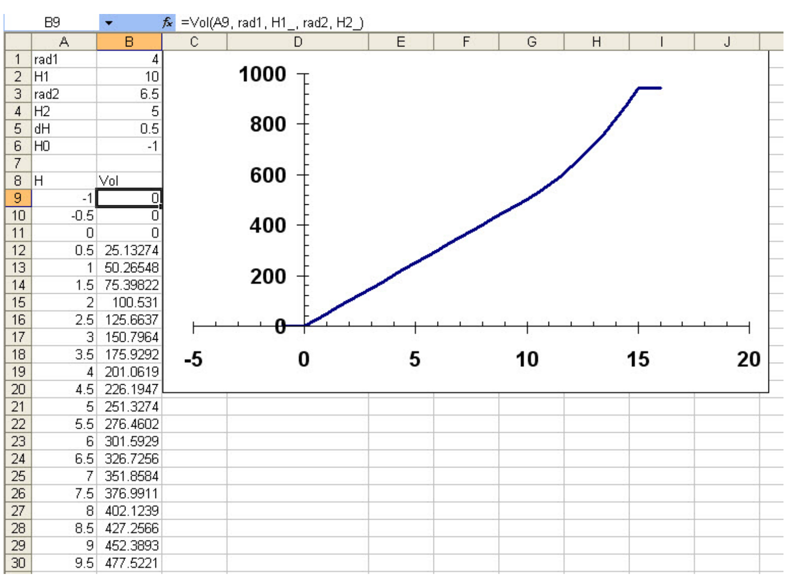
```
Option Explicit

Const pi As Double = 3.14159265358979

Function Vol(h, r1, h1, r2, h2)
Dim r2h As Double
Vol = VCyl(r1, h1) + VFus(r1, r2, h2)
If h <= 0 Then
  Vol = 0
ElseIf h < h1 Then
  Vol = VCyl(r1, h)
ElseIf h < h1 + h2 Then
  r2h = r1 + (r2 - r1) / h2 * (h - h1)
  Vol = VCyl(r1, h1) + VFus(r1, r2h, h - h1)
End If
End Function

Function VCyl(r, y)
VCyl = pi * r ^ 2 * y
End Function

Function VFus(r1, r2, h2)
VFus = pi * h2 / 3 * (r1 ^ 2 + r2 ^ 2 + r1 * r2)
End Function
```



	A	B
1	rad1	4
2	H1	10
3	rad2	6.5
4	H2	5
5	dH	0.5
6	H0	-1
7		
8	H	Vol
9	-1	0
10	-0.5	0
11	0	0
12	0.5	25.13274
13	1	50.26548
14	1.5	75.39822
15	2	100.531
16	2.5	125.6637
17	3	150.7964
18	3.5	175.9292
19	4	201.0619
20	4.5	226.1947
21	5	251.3274
22	5.5	276.4602
23	6	301.5929
24	6.5	326.7256
25	7	351.8584
26	7.5	376.9911
27	8	402.1239
28	8.5	427.2566
29	9	452.3893
30	9.5	477.5221

MATLAB. Here are the functions:

```
function V=Vol(h, r1, h1, r2, h2)
V = VCyl(r1, h1) + VFus(r1, r2, h2);
if h <= 0
  V = 0;
elseif h < h1
  V = VCyl(r1, h);
elseif h < h1 + h2
  r2h = r1 + (r2 - r1) / h2 * (h - h1);
  V = VCyl(r1, h1) + VFus(r1, r2h, h - h1);
end
end

function V=VCyl(r, y)
V = pi * r ^ 2 * y;
end

function V=VFus(r1, r2, h2)
V = pi * h2 / 3 * (r1 ^ 2 + r2 ^ 2 + r1 * r2);
end
```

Here is a script that uses the functions to develop a plot of volume versus height:

```
clc,clf
h=[-1:0.5:16];
r1=4; H1=10; r2=6.5; H2=5;
n=length(h);
vol=zeros(n);
for i=1:n
  vol(i)=Vol(h(i),r1, H1, r2, H2);
end
plot(h,vol)
```