OM6 Supplementary Chapter B: Queuing Analysis

Problems, Activities, and Discussions

(1) Trucks using a single-server loading dock have a mean arrival rate of 14 per day. The loading/unloading rate is 19 per day.
a. What is the probability that the truck dock will be idle?
b. What is the average number of trucks waiting for service?
c. What is the average time a truck waits for the loading or unloading service?
d. What is the probability that a new arrival will have to wait?
e. What is the probability that more than three trucks are waiting for service?

Single Server Queueing Model	
Lambda	14
Mu	19.00
Probability system is empty	0.26
Average number in queue	2.06
Average number in system	2.80
Average time in queue	0.15
Average waiting time in system	0.20
Probability arrival has to wait	0.74

a. 0.26
b. 2.06
c. 0.15 days
d. 0.74
e. P(0) = .26
P(1) = $(14/19)^1(.26)$ = 0.192
P(2) = $(12/19)^2(.26)$ = 0.141
P(3) = $(14/19)^3(.26)$ = 0.104
P(trucks <= 3) = 0.697; therefore, the probability of more than three waiting = 1 – 0.697 = 0.303

(2) Trosper Tire Company has decided to hire a new mechanic to handle all tire changes for customers ordering new tires. Two mechanics are available for the job. One mechanic has limited experience and can be hired for $7 per hour. It is expected that this mechanic can service an average of three customers per hour. A mechanic with several years of experience is also being considered for the job. This mechanic can service an average of four customers per hour, but must be paid $10 per hour. Assume that customers arrive at the Trosper garage at the rate of two per hour.

a. Compute waiting-line operating characteristics for each mechanic.
b. If the company assigns a customer-waiting cost of $15 per hour, which mechanic provides the lower operating cost?

a.

Single Server Queueing Model		
Lambda	2	2
Mu	3.00	4.00
Probability system is empty	0.33	0.50
Average number in queue	1.33	0.50
Average number in system	2.00	1.00
Average time in queue	0.67	0.25
Average waiting time in system	1.00	0.50
Probability arrival has to wait	0.67	0.50

b. New mechanic = $15(L) + $7 = 15(2) + 7 = $37 per hour
Experienced mechanic = $125(L) + $10 = 15(1) + 10 = $25 per hour

(3) Agan Interior Design provides home and office decorating assistance. In normal operation an average of 3 customers arrive per hour. One design consultant is available to answer customer questions and make product recommendations. The consultant averages 12 minutes with each customer.

a. Compute operating characteristics for the customer waiting line.
b. Service goals dictate that an arriving customer should not wait for service more than an average of 5 minutes. Is this goal being met? What action do you recommend?
c. If the consultant can reduce the average time spent with customers to 8 minutes, will the service goal be met?

a. 10 minutes = 60/12 = 5 customers per hour service rate

Single Server Queueing Model	
Lambda	3
Mu	5.00
Probability system is empty	0.40
Average number in queue	0.90
Average number in system	1.50
Average time in queue	0.30
Average waiting time in system	0.50
Probability arrival has to wait	0.60

b. No, average waiting time is .3 hours or 18 minutes. Should try to increase the mean service rate for the consultant or hire a second person.

c. 8 minutes = 60/8 = 7.5 customers per hour service rate

Single Server Queueing Model	
Lambda	3
Mu	7.50
Probability system is empty	0.60
Average number in queue	0.27
Average number in system	0.67
Average time in queue	0.09
Average waiting time in system	0.22
Probability arrival has to wait	0.40

The average time in queue is 0.09 hours or about five and a half minutes, so the service goal is being met.

(4) Keuka Park Savings and Loan currently has one drive-in teller window. Cars arrive at a mean rate of 10 per hour. The mean service rate is 12 cars per hour.
a. What is the probability that the service facility will be idle?
b. If you were to drive up to the facility, how many cars would you expect to see waiting and being serviced?
c. What is the average time waiting for service?
d. What is the probability an arriving car will have to wait?

e. What is the probability that more than four vehicles are waiting for service?
f. As a potential customer of the system, would you be satisfied with these waiting-line characteristics? How do you think managers could go about assessing its customers' feelings about the current system?

Single Server Queueing Model	
Lambda	10
Mu	12.00
Probability system is empty	0.17
Average number in queue	4.17
Average number in system	5.00
Average time in queue	0.42
Average waiting time in system	0.50
Probability arrival has to wait	0.83

a. 0.17

c. .42 hours
d. 0.83

e. P(0) = .17
$P(1) = (10/12)^1(.17) = 0.142$
$P(2) = (10/12)^2(.17) = 0.118$
$P(3) = (10/12)^3(.17) = 0.098$
$P(4) = (10/12)^4(.17) = 0.082$
P(cars <= 4) = 0.44; therefore, the probability of more than four waiting = 1 – 0.44 = 0..56

f. Probably not. The number waiting and waiting times are quite high. Managers can easily instruct tellers to ask customers, but we suspect they will receive quite a large number of complaints! Tellers should convey this issue to management.

(5) To improve its customer service, Keuka Park Savings and Loan (Problem 4) wants to investigate the effect of a second drive-in teller window. Assume a mean arrival rate of 10 cars per hour. In addition, assume a mean service rate of 12 cars per hour for each window. What effect would adding a new teller window have on the system? Does this system appear acceptable?

Multiple Server Queueing Model	
Lambda	10
Mu	12
Number of servers	2
Probability system is empty	0.412
Average number in queue	0.175
Average number in system	1.008
Average time in queue	0.018
Average waiting time in system	0.101
Probability arrival must wait	0.245

The system has improved considerably. The average number of cars in the system is reduced from 5 to 1 and the average waiting time from 0.5 hours to 0.101 hours.

(6) Consider a two-server waiting line with a mean arrival rate of 40 per hour and a mean service rate of 60 per hour for each server.
a. What is the probability that both servers are idle?
b. What is the average number of cars waiting for service?
c. What is the average time waiting for service?
d. What is the average time in the system?
e. What is the probability of having to wait for service?

Multiple Server Queueing Model	
Lambda	40.000
Mu	60.000
Number of servers	2
Probability system is empty	0.500
Average number in queue	0.083
Average number in system	0.750
Average time in queue	0.002
Average time in system	0.019
Probability arrival must wait	0.167

a. 0.5

b. 0.083
c. 0.002 hours
d. 0.019 hours
e. 0.167

(7) Big Al's Quickie Carwash has two wash bays. Each bay can wash 15 cars per hour. Cars arrive at the carwash at the rate of 15 cars per hour on the average, join the waiting line, and move to the next open bay when it becomes available.
a. What is the average time waiting for a bay?
b. What is the probability that a customer will have to wait?
c. As a customer of Big Al's, do you think the system favors the customer? If you were Al, what would be your attitude toward this service level?

Multiple Server Queueing Model	
Lambda	15
Mu	15
Number of servers	2
Probability system is empty	0.333
Average number in queue	0.333
Average number in system	1.333
Average time in queue	0.022
Average waiting time in system	0.089
Probability arrival must wait	0.333

a. 0.022 hours
b. 0.333
c. Only 1 of 3 customers have to wait, so this is probably acceptable.

(8) Refer to the Agan Interior Design situation in Problem 3. Agan is evaluating two alternatives:
1. use one consultant with an average service time of 8 minutes per customer;
2. expand to two consultants, each of whom has an average service time of 10 minutes per customer.
If the consultants are paid $16 per hour and the customer waiting time is valued at $25 per hour, should Agan expand to the two-consultant system? Explain.

1 consultant:

Single Server Queueing Model	
Lambda	3
Mu	7.50
Probability system is empty	0.60
Average number in queue	0.27
Average number in system	0.67
Average time in queue	0.09
Average waiting time in system	0.22
Probability arrival has to wait	0.40

Total cost = \$25(.09) + 16 = \$18.25 per hour

2 consultants:

Multiple Server Queueing Model	
Lambda	3.000
Mu	6.000
Number of servers	2
Probability system is empty	0.600
Average number in queue	0.033
Average number in system	0.533
Average time in queue	0.011
Average time in system	0.178
Probability arrival must wait	0.100

Total cost = \$25(.011) + 2(16) = \$32.275 per hour

Because the waiting times are not that significantly different, the one consultant system is clearly superior.

(9) Design a spreadsheet similar to Exhibit B.3 to study changes in the mean service rate from 10 to 15 for λ = 9 passengers per minute.

Single Server Queueing Model						
Lambda	9	9	9	9	9	9
Mu	10.00	11.00	12.00	13.00	14.00	15.00
Probability system is empty	0.10	0.18	0.25	0.31	0.36	0.40
Average number in queue	8.10	3.68	2.25	1.56	1.16	0.90
Average number in system	9.00	4.50	3.00	2.25	1.80	1.50
Average time in queue	0.90	0.41	0.25	0.17	0.13	0.10
Average waiting time in system	1.00	0.50	0.33	0.25	0.20	0.17
Probability arrival has to wait	0.90	0.82	0.75	0.69	0.64	0.60

(10) Using the spreadsheet in Exhibit B.6 (Multiple-Server Queue.xlsx), determine the effect of increasing passenger arrival rates of 10, 12, 14, 16, and 18 on the operating characteristics of the airport security screening example.

Multiple Server Queueing Model	
Lambda	9
Mu	10
Number of servers	2
Probability system is empty	0.379
Average number in queue	0.229
Average number in system	1.129
Average time in queue	0.025
Average waiting time in system	0.125
Probability arrival must wait	0.279

Multiple Server Queueing Model	
Lambda	9
Mu	12
Number of servers	2
Probability system is empty	0.455
Average number in queue	0.123
Average number in system	0.873
Average time in queue	0.014
Average waiting time in system	0.097
Probability arrival must wait	0.205

Multiple Server Queueing Model	
Lambda	9
Mu	14
Number of servers	2
Probability system is empty	0.514
Average number in queue	0.074
Average number in system	0.717
Average time in queue	0.008
Average waiting time in system	0.080
Probability arrival must wait	0.156

Multiple Server Queueing Model	
Lambda	9
Mu	16
Number of servers	2
Probability system is empty	0.561
Average number in queue	0.048
Average number in system	0.611
Average time in queue	0.005
Average waiting time in system	0.068
Probability arrival must wait	0.123

Multiple Server Queueing Model	
Lambda	9
Mu	18
Number of servers	2
Probability system is empty	0.600
Average number in queue	0.033
Average number in system	0.533
Average time in queue	0.004
Average waiting time in system	0.059
Probability arrival must wait	0.100

Bourbon County Court Case Study Teaching Note

Overview

A government service, a county court house with a budget deficit, has only one photocopying machine for court use. Students must first do a basic single and multiple server queuing model analysis and interpret the results assuming a Poisson arrival distribution and an exponential service time distribution. In addition, the cost of different people waiting and the cost of the machine are also given and the student must evaluate the economics of the situation.

An advanced and optional assignment (you decide) is to graph the actual arrival and service time data or use a software package such as Stat Fit to evaluate the fit of the real data to the assumptions of queuing models. The arrival data does fit the assumptions of the Poisson distribution but the service time distribution does not fit the assumptions of an exponential distribution very well. Hence, the queuing analysis may not be accurate and this leads the modeler to consider using simulation instead of queuing models. The instructor might also want to demonstrate the use of a software package to analyze this issue as a demo during class. In Supplemental Chapter D simulation you have the opportunity to model this simple service delivery system using a simulation model of your choice. Therefore, the case can require students to apply their queuing model knowledge and begin to understand why the modeler sometimes needs simulation instead of queuing models.

Case Questions and Brief Answers

(1) Assuming a Poisson arrival distribution and an exponential service time distribution, apply queuing models to the case situation and evaluate the results.

Single Server Queue Model
Bourbon County Court

Lambda	8.92
Mu	10.91
Probability system is empty	0.18
Average number in queue	3.66
Average number in system	4.48
Average time in queue	0.41
Average waiting time in system	0.50
Probability arrival has to wait	0.82

The average wait time in the queue is .41 hours or 24.6 minutes which is an unacceptable service level. The other queuing performance statistics are equally bad. Other useful information for the single server model to use in class includes: P_0 = .1824, P_1 = .1491, P_2 = .1219, P_3 = .0997, P_4 = .0815, P_5 = .0666, P_6 = .00545, P_7 = .0445.

Multiple Server Queuing Model
Bourbon County Court

Lambda	8.920		
Mu	10.910		
Number of servers	2	3	4

Probability system is empty	0.420	0.439	0.441
Average number in queue	0.164	0.021	0.003
Average number in system	0.982	0.838	0.820
Average time in queue	0.018	0.002	0.000
Average waiting time in system	0.110	0.094	0.092
Probability arrival must wait	0.237	0.055	0.010

For a two server system, the average wait time in the queue is .018 hours or 1.08 minutes which is an acceptable service level. The other queuing performance statistics show a dramatic improvement in system performance. You might point out to students at some point these results indicate a nonlinear performance relationships inherent in the queuing models so do not expect result to be linear in nature. Other useful information for the two server model to use in class includes: P_0 = .4196, P_1 = .3431, P_2 = .1403, P_3 = .0573, P_4 = .0234, P_5 = .0096, P_6 = .0039, P_7 = .00016.

(2) What are the economics of the situation using queuing model analysis?

Case Exhibit B.10 requires some knowledge of cost accounting and confronts the student with "What cost data do we use?"

<u>Cost of Copying Machine</u>
Some may assume the $18,600 is a sunk cost and use only the variable cost of $5/hour. But we will use a full cost model so
($18,600 per year/250 days/year) = $74.4/day and assuming 9 hours/day = $8.27/hour.
Total copier cost/hour = $8.27 + $5.00 = $13.27

<u>Cost of Customers Waiting</u>
Customer opportunity cost of waiting/hour = (.50)($18.75 + (.2)($22.50) + (.1)($28.40) + (.1)($30.80) + (.1)($100.00) = $29.80

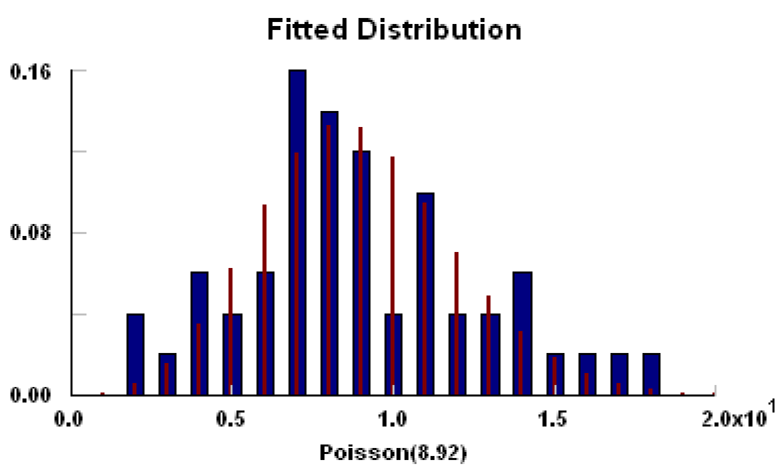
For single server system, cost of waiting/customer = ($29.80/hour)(.41 hours waiting) = $12.22. For single server system, cost of server = $13.27. Total cost is $25.49. With a two server system, cost of waiting = ($29.80/hour)(.018 hours waiting) = $0.54. For the two server system, cost of servers = $26.54. Total cost is $26.08.

(3) What are your final recommendations using queuing model analysis.

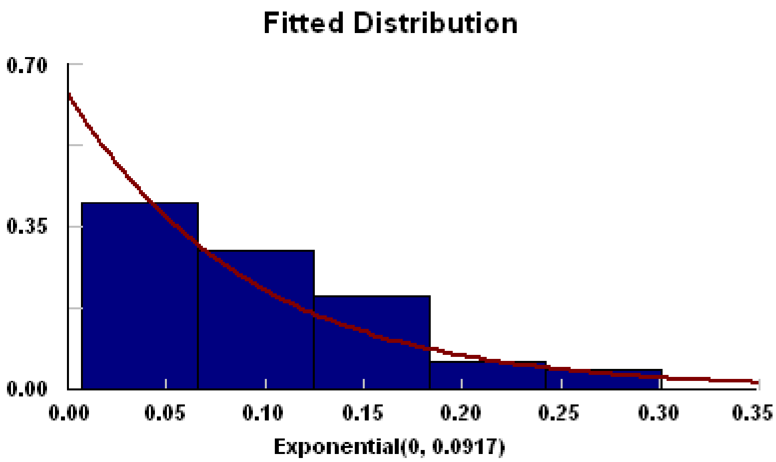
From a purely economic standpoint, a single server is the lowest cost solution (about $1 cheaper) but since the total costs are very close, it really comes down to a policy decision. Does Bourbon County Court want a peripheral service--copying--to disrupt the primary service--court cases and system? These disruption and anxiety costs are not in the current economic analysis. Court is tough enough as it is without this annoying peripheral service. Most students will recommend buying a second photocopying machine based on similar economic and qualitative criteria. Of course, their assumptions will direct their final recommendations.

(4) *Advanced Assignment (requires the use of a statistical package).* Do the customer arrival and service empirical (actual) distributions in the case match the theoretical distributions assumed in queuing models?

Students can set up frequency categories and develop graphs of the data in case Exhibits B.8 and B.9. Then they can look at the shapes of the Poisson and Exponential theoretical distributions and make some inferences by observation and overlaying graphs. More advanced statistical tests of how well the empirical and theoretical distributions match are possible using methods such as Chi-square tests. Most software statistical packages such as Stat Fit provide many advanced statistical tests and graphical ways to analyze these data.



The thin vertical bars indicate the shape of a Poisson distribution so the case arrival data in broader vertical bars is closely aligned with the theoretical distribution.



The thin curved line in the previous graph indicates the shape of a theoretical exponential distribution and the case service time data in histogram form is not so well aligned with the theoretical distribution. Therefore, simulation is a way to model this service system using the actual arrival and service time case data. SC D on simulation models this system using ProcessModel.

Teaching Plan

(1) Assuming a Poisson arrival distribution and an exponential service time distribution, apply queuing models to the case situation and evaluate the results.

(2) What are the economics of the situation using queuing model analysis?

(3) What are your final recommendations using queuing model analysis.

(4) *Advanced Assignment (requires the use of a statistical package).* Do the customer arrival and service empirical (actual) distributions in the case match the theoretical distributions assumed in queuing models?

OM6 Supplementary Chapter B: Queuing Analysis

Problems, Activities, and Discussions

(1) Trucks using a single-server loading dock have a mean arrival rate of 14 per day. The loading/unloading rate is 19 per day.
a. What is the probability that the truck dock will be idle?
b. What is the average number of trucks waiting for service?
c. What is the average time a truck waits for the loading or unloading service?
d. What is the probability that a new arrival will have to wait?
e. What is the probability that more than three trucks are waiting for service?

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Probability arrival has to wait	0.74

a. 0.26
b. 2.06
c. 0.15 days
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e. P(0) = .26
P(1) = $(14/19)^1(.26)$ = 0.192
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a. Compute waiting-line operating characteristics for each mechanic.
b. If the company assigns a customer-waiting cost of $15 per hour, which mechanic provides the lower operating cost?

a.

Single Server Queueing Model		
Lambda	2	2
Mu	3.00	4.00
Probability system is empty	0.33	0.50
Average number in queue	1.33	0.50
Average number in system	2.00	1.00
Average time in queue	0.67	0.25
Average waiting time in system	1.00	0.50
Probability arrival has to wait	0.67	0.50

b. New mechanic = $15(L) + $7 = 15(2) + 7 = $37 per hour
Experienced mechanic = $125(L) + $10 = 15(1) + 10 = $25 per hour

(3) Agan Interior Design provides home and office decorating assistance. In normal operation an average of 3 customers arrive per hour. One design consultant is available to answer customer questions and make product recommendations. The consultant averages 12 minutes with each customer.

a. Compute operating characteristics for the customer waiting line.
b. Service goals dictate that an arriving customer should not wait for service more than an average of 5 minutes. Is this goal being met? What action do you recommend?
c. If the consultant can reduce the average time spent with customers to 8 minutes, will the service goal be met?

a. 10 minutes = 60/12 = 5 customers per hour service rate

Single Server Queueing Model	
Lambda	3
Mu	5.00
Probability system is empty	0.40
Average number in queue	0.90
Average number in system	1.50
Average time in queue	0.30
Average waiting time in system	0.50
Probability arrival has to wait	0.60

b. No, average waiting time is .3 hours or 18 minutes. Should try to increase the mean service rate for the consultant or hire a second person.

c. 8 minutes = 60/8 = 7.5 customers per hour service rate

Single Server Queueing Model	
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Probability arrival has to wait	0.40

The average time in queue is 0.09 hours or about five and a half minutes, so the service goal is being met.

(4) Keuka Park Savings and Loan currently has one drive-in teller window. Cars arrive at a mean rate of 10 per hour. The mean service rate is 12 cars per hour.
a. What is the probability that the service facility will be idle?
b. If you were to drive up to the facility, how many cars would you expect to see waiting and being serviced?
c. What is the average time waiting for service?
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e. What is the probability that more than four vehicles are waiting for service?
f. As a potential customer of the system, would you be satisfied with these waiting-line characteristics? How do you think managers could go about assessing its customers' feelings about the current system?

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Average waiting time in system	0.50
Probability arrival has to wait	0.83

a. 0.17

c. .42 hours
d. 0.83

e. P(0) = .17
$P(1) = (10/12)^1(.17) = 0.142$
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$P(3) = (10/12)^3(.17) = 0.098$
$P(4) = (10/12)^4(.17) = 0.082$
P(cars <= 4) = 0.44; therefore, the probability of more than four waiting = 1 – 0.44 = 0..56

f. Probably not. The number waiting and waiting times are quite high. Managers can easily instruct tellers to ask customers, but we suspect they will receive quite a large number of complaints! Tellers should convey this issue to management.

(5) To improve its customer service, Keuka Park Savings and Loan (Problem 4) wants to investigate the effect of a second drive-in teller window. Assume a mean arrival rate of 10 cars per hour. In addition, assume a mean service rate of 12 cars per hour for each window. What effect would adding a new teller window have on the system? Does this system appear acceptable?

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The system has improved considerably. The average number of cars in the system is reduced from 5 to 1 and the average waiting time from 0.5 hours to 0.101 hours.

(6) Consider a two-server waiting line with a mean arrival rate of 40 per hour and a mean service rate of 60 per hour for each server.
a. What is the probability that both servers are idle?
b. What is the average number of cars waiting for service?
c. What is the average time waiting for service?
d. What is the average time in the system?
e. What is the probability of having to wait for service?

Multiple Server Queueing Model	
Lambda	40.000
Mu	60.000
Number of servers	2
Probability system is empty	0.500
Average number in queue	0.083
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Average time in queue	0.002
Average time in system	0.019
Probability arrival must wait	0.167

a. 0.5

b. 0.083
c. 0.002 hours
d. 0.019 hours
e. 0.167

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c. As a customer of Big Al's, do you think the system favors the customer? If you were Al, what would be your attitude toward this service level?

Multiple Server Queueing Model	
Lambda	15
Mu	15
Number of servers	2
Probability system is empty	0.333
Average number in queue	0.333
Average number in system	1.333
Average time in queue	0.022
Average waiting time in system	0.089
Probability arrival must wait	0.333

a. 0.022 hours
b. 0.333
c. Only 1 of 3 customers have to wait, so this is probably acceptable.

(8) Refer to the Agan Interior Design situation in Problem 3. Agan is evaluating two alternatives:
1. use one consultant with an average service time of 8 minutes per customer;
2. expand to two consultants, each of whom has an average service time of 10 minutes per customer.
If the consultants are paid $16 per hour and the customer waiting time is valued at $25 per hour, should Agan expand to the two-consultant system? Explain.

1 consultant:

Single Server Queueing Model	
Lambda	3
Mu	7.50
Probability system is empty	0.60
Average number in queue	0.27
Average number in system	0.67
Average time in queue	0.09
Average waiting time in system	0.22
Probability arrival has to wait	0.40

Total cost = $25(.09) + 16 = $18.25 per hour

2 consultants:

Multiple Server Queueing Model	
Lambda	3.000
Mu	6.000
Number of servers	2
Probability system is empty	0.600
Average number in queue	0.033
Average number in system	0.533
Average time in queue	0.011
Average time in system	0.178
Probability arrival must wait	0.100

Total cost = $25(.011) + 2(16) = $32.275 per hour

Because the waiting times are not that significantly different, the one consultant system is clearly superior.

(9) Design a spreadsheet similar to Exhibit B.3 to study changes in the mean service rate from 10 to 15 for λ = 9 passengers per minute.

Single Server Queueing Model						
Lambda	9	9	9	9	9	9
Mu	10.00	11.00	12.00	13.00	14.00	15.00
Probability system is empty	0.10	0.18	0.25	0.31	0.36	0.40
Average number in queue	8.10	3.68	2.25	1.56	1.16	0.90
Average number in system	9.00	4.50	3.00	2.25	1.80	1.50
Average time in queue	0.90	0.41	0.25	0.17	0.13	0.10
Average waiting time in system	1.00	0.50	0.33	0.25	0.20	0.17
Probability arrival has to wait	0.90	0.82	0.75	0.69	0.64	0.60

(10) Using the spreadsheet in Exhibit B.6 (Multiple-Server Queue.xlsx), determine the effect of increasing passenger arrival rates of 10, 12, 14, 16, and 18 on the operating characteristics of the airport security screening example.

Multiple Server Queueing Model	
Lambda	9
Mu	10
Number of servers	2
Probability system is empty	0.379
Average number in queue	0.229
Average number in system	1.129
Average time in queue	0.025
Average waiting time in system	0.125
Probability arrival must wait	0.279

Multiple Server Queueing Model	
Lambda	9
Mu	12
Number of servers	2
Probability system is empty	0.455
Average number in queue	0.123
Average number in system	0.873
Average time in queue	0.014
Average waiting time in system	0.097
Probability arrival must wait	0.205

Multiple Server Queueing Model	
Lambda	9
Mu	14
Number of servers	2
Probability system is empty	0.514
Average number in queue	0.074
Average number in system	0.717
Average time in queue	0.008
Average waiting time in system	0.080
Probability arrival must wait	0.156

Multiple Server Queueing Model	
Lambda	9
Mu	16
Number of servers	2
Probability system is empty	0.561
Average number in queue	0.048
Average number in system	0.611
Average time in queue	0.005
Average waiting time in system	0.068
Probability arrival must wait	0.123

Multiple Server Queueing Model	
Lambda	9
Mu	18
Number of servers	2
Probability system is empty	0.600
Average number in queue	0.033
Average number in system	0.533
Average time in queue	0.004
Average waiting time in system	0.059
Probability arrival must wait	0.100

Bourbon County Court Case Study Teaching Note

Overview

A government service, a county court house with a budget deficit, has only one photocopying machine for court use. Students must first do a basic single and multiple server queuing model analysis and interpret the results assuming a Poisson arrival distribution and an exponential service time distribution. In addition, the cost of different people waiting and the cost of the machine are also given and the student must evaluate the economics of the situation.

An advanced and optional assignment (you decide) is to graph the actual arrival and service time data or use a software package such as Stat Fit to evaluate the fit of the real data to the assumptions of queuing models. The arrival data does fit the assumptions of the Poisson distribution but the service time distribution does not fit the assumptions of an exponential distribution very well. Hence, the queuing analysis may not be accurate and this leads the modeler to consider using simulation instead of queuing models. The instructor might also want to demonstrate the use of a software package to analyze this issue as a demo during class. In Supplemental Chapter D simulation you have the opportunity to model this simple service delivery system using a simulation model of your choice. Therefore, the case can require students to apply their queuing model knowledge and begin to understand why the modeler sometimes needs simulation instead of queuing models.

Case Questions and Brief Answers

(1) Assuming a Poisson arrival distribution and an exponential service time distribution, apply queuing models to the case situation and evaluate the results.

Single Server Queue Model
Bourbon County Court

Lambda	8.92
Mu	10.91
Probability system is empty	0.18
Average number in queue	3.66
Average number in system	4.48
Average time in queue	0.41
Average waiting time in system	0.50
Probability arrival has to wait	0.82

The average wait time in the queue is .41 hours or 24.6 minutes which is an unacceptable service level. The other queuing performance statistics are equally bad. Other useful information for the single server model to use in class includes: P_0 = .1824, P_1 = .1491, P_2 = .1219, P_3 = .0997, P_4 = .0815, P_5 = .0666, P_6 = .00545, P_7 = .0445.

Multiple Server Queuing Model
Bourbon County Court

Lambda	8.920		
Mu	10.910		
Number of servers	2	3	4

Probability system is empty	0.420	0.439	0.441
Average number in queue	0.164	0.021	0.003
Average number in system	0.982	0.838	0.820
Average time in queue	0.018	0.002	0.000
Average waiting time in system	0.110	0.094	0.092
Probability arrival must wait	0.237	0.055	0.010

For a two server system, the average wait time in the queue is .018 hours or 1.08 minutes which is an acceptable service level. The other queuing performance statistics show a dramatic improvement in system performance. You might point out to students at some point these results indicate a nonlinear performance relationships inherent in the queuing models so do not expect result to be linear in nature. Other useful information for the two server model to use in class includes: $P_0 = .4196$, $P_1 = .3431$, $P_2 = .1403$, $P_3 = .0573$, $P_4 = .0234$, $P_5 = .0096$, $P_6 = .0039$, $P_7 = .00016$.

(2) What are the economics of the situation using queuing model analysis?

Case Exhibit B.10 requires some knowledge of cost accounting and confronts the student with "What cost data do we use?"

Cost of Copying Machine

Some may assume the $18,600 is a sunk cost and use only the variable cost of $5/hour. But we will use a full cost model so

($18,600 per year/250 days/year) = $74.4/day and assuming 9 hours/day = $8.27/hour.

Total copier cost/hour = $8.27 + $5.00 = $13.27

Cost of Customers Waiting

Customer opportunity cost of waiting/hour = (.50)($18.75 + (.2)($22.50) + (.1)($28.40) + (.1)($30.80) + (.1)($100.00) = $29.80

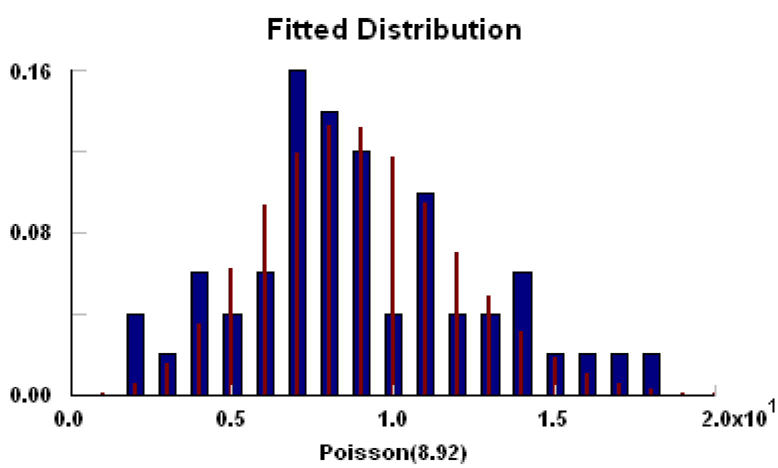
For single server system, cost of waiting/customer = ($29.80/hour)(.41 hours waiting) = $12.22. For single server system, cost of server = $13.27. Total cost is $25.49. With a two server system, cost of waiting = ($29.80/hour)(.018 hours waiting) = $0.54. For the two server system, cost of servers = $26.54. Total cost is $26.08.

(3) What are your final recommendations using queuing model analysis.

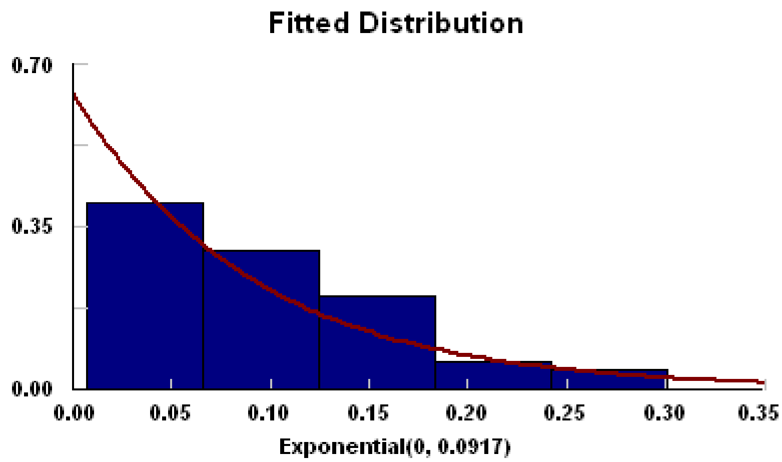
From a purely economic standpoint, a single server is the lowest cost solution (about $1 cheaper) but since the total costs are very close, it really comes down to a policy decision. Does Bourbon County Court want a peripheral service--copying--to disrupt the primary service--court cases and system? These disruption and anxiety costs are not in the current economic analysis. Court is tough enough as it is without this annoying peripheral service. Most students will recommend buying a second photocopying machine based on similar economic and qualitative criteria. Of course, their assumptions will direct their final recommendations.

(4) *Advanced Assignment (requires the use of a statistical package).* Do the customer arrival and service empirical (actual) distributions in the case match the theoretical distributions assumed in queuing models?

Students can set up frequency categories and develop graphs of the data in case Exhibits B.8 and B.9. Then they can look at the shapes of the Poisson and Exponential theoretical distributions and make some inferences by observation and overlaying graphs. More advanced statistical tests of how well the empirical and theoretical distributions match are possible using methods such as Chi-square tests. Most software statistical packages such as Stat Fit provide many advanced statistical tests and graphical ways to analyze these data.



The thin vertical bars indicate the shape of a Poisson distribution so the case arrival data in broader vertical bars is closely aligned with the theoretical distribution.



The thin curved line in the previous graph indicates the shape of a theoretical exponential distribution and the case service time data in histogram form is not so well aligned with the theoretical distribution. Therefore, simulation is a way to model this service system using the actual arrival and service time case data. SC D on simulation models this system using ProcessModel.

Teaching Plan

(1) Assuming a Poisson arrival distribution and an exponential service time distribution, apply queuing models to the case situation and evaluate the results.

(2) What are the economics of the situation using queuing model analysis?

(3) What are your final recommendations using queuing model analysis.

(4) *Advanced Assignment (requires the use of a statistical package).* Do the customer arrival and service empirical (actual) distributions in the case match the theoretical distributions assumed in queuing models?

B Queuing Analysis

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LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- **B-1** Describe the key elements and underlying mathematical concepts of analytical queuing models.
- **B-2** Explain and compute the operating characteristic formulas associated with the single-server queuing model.
- **B-3** Apply the operating characteristic formulas for a multiple-server queuing model.
- **B-4** Explain the economic trade-offs associated with designing and managing queuing systems.
- **B-5** Explain the psychology of waiting for designing and managing queuing systems.

An electrical utility company uses six customer service representatives (CSRs) at its call center to handle telephone calls and inquiries from its top 350 business customers. The next tier of 700 business customers is also handled by six CSRs. Based on the customer's code, the call center routes business customers to different queues and CSRs. A manager at the utility explains: "We don't ignore anyone, but our biggest customers certainly get more attention than the rest."[1]

WHAT DO YOU THINK?

Do you think that this decision is good or bad? Should all customers be treated the same and be considered as important as any other?

This example highlights a growing practice of segmenting customers so that premium service is provided to a few high-value customers while many low-value customers get less attention and organizational resources. The electric utility's call center assigns the same number of CSRs—six—to the top 350 customers and the next 700 customers based on value. Many organizations would gladly see customers that generate marginal profits leave. **Value-based queuing** *is a method that allows organizations to prioritize customer calls based on customers' long-term value to the organization.* Low-profitability customers are often encouraged to serve themselves on the company's website rather than tie up expensive telephone representatives. Such decisions are similar to the notion of segmenting high-value inventory using ABC analysis that we discussed in Chapter 11.

Value-based queuing is a method that allows organizations to prioritize customer calls based on customers' long-term value to the organization.

This supplementary chapter introduces basic concepts and methods of queuing analysis that have wide applicability in manufacturing and service organizations.


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We focus only on simple models; other textbooks devoted exclusively to management science develop more complex models.

B-1 ANALYZING QUEUES USING ANALYTICAL MODELS

Many analytical queuing models exist, each based on unique assumptions about the nature of arrivals, service times, and other aspects of the system. Some of the common models are

1. Single- or multiple-channel with Poisson arrivals and exponential service times. (This is the most elementary situation.)
2. Single-channel with Poisson arrivals and arbitrary service times. (Service times may follow any probability distribution, and only the average and the standard deviation need to be known.)
3. Single-channel with Poisson arrivals and deterministic service times. (Service times are assumed to be constant.)
4. Single- or multiple-channel with Poisson arrivals, arbitrary service times, and no waiting line. (Waiting is not permitted. If the server is busy when a unit arrives, the unit must leave the system but may try to reenter at a later time.)
5. Single- or multiple-channel with Poisson arrivals, exponential service times, and a finite calling population. (A finite population of units is permitted to arrive for service.)

We illustrate the development of the basic queuing model for the problem of designing an automated check-in kiosk for passengers at an airport. Suppose that process design and facility-layout activities are currently being conducted for a new terminal at a major airport. One particular concern is the design and layout of the passenger check-in system. Most major airlines now use automated kiosks to speed up the process of obtaining a boarding pass with an electronic ticket. Passengers either enter a confirmation number or scan their electronic ticket to print a boarding pass. A queuing analysis of the system will help determine if the system will provide adequate service to the airport passengers. To develop a queuing model, we must identify some important characteristics of the system: (1) the arrival distribution of the passengers, (2) the service-time distribution for the check-in operation, and (3) the waiting-line, or queue, discipline for the passengers.

B-1a Arrival Distribution

Defining the arrival distribution for a waiting line consists of determining how many customers arrive for service in given periods of time, for example, the number of passengers arriving at the check-in kiosk during each 1-, 10-, or 60-minute period. Because the number of passengers arriving each minute is not a constant, we need to define a probability distribution that will describe the passenger arrivals. The choice of time period is arbitrary—as long as the same time period is used consistently—and is often determined based on the rate of arrivals and the ease by which the data can be collected. Generally, the slower the rate of arrivals, the longer the time period chosen.

For many waiting lines, the arrivals occurring in a given period of time appear to have a random pattern—that is, although we may have a good estimate of the total number of expected arrivals, each arrival is independent of other arrivals, and we cannot predict when it will occur. In such cases, a good description of the arrival pattern is obtained from the Poisson probability distribution:

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad \text{for } x = 0, 1, 2, \ldots \qquad [\text{B.1}]$$

where

x = number of arrivals in a specific period of time
λ = average, or expected, number of arrivals for the specific period of time
$e \approx 2.71828$

For the passenger check-in process, the wide variety of flight schedules and the variation in passenger arrivals for the various flights cause the number of passengers arriving to vary substantially. For example, data collected from the actual operation of similar facilities show that in some instances, 20 to 25 passengers arrive during a 10-minute period. At other times, however, passenger arrivals drop to three or fewer passengers during a 10-minute period. Because passenger arrivals cannot be controlled and appear to occur in an unpredictable fashion, a random arrival pattern appears to exist. Thus the Poisson probability distribution should provide a good description of the passenger-arrival pattern.

Airport planners have projected passenger volume through the year and estimate that passengers will arrive at an average rate of nine passengers per 10-minute period during the peak activity periods. Note that the choice of time period is arbitrary. We could have used an equivalent rate of 54 passengers per hour or 0.9 passengers per minute—as long as we are consistent in using the same time period in our analysis. Using the average, or mean, arrival rate ($\lambda = 9$), we can use the Poisson distribution defined in Equation B.1 to compute the probability of x passenger arrivals in a 10-minute period.

$$P(x) = \frac{9^x e^{-9}}{x!} \qquad \text{for } x = 0, 1, 2, \ldots$$

Sample calculations for $x = 0$, 5, and 10 passenger arrivals during a one-minute period follow:

$$P(0) = \frac{9^0 e^{-9}}{0!} = .0001$$

$$P(5) = \frac{9^5 e^{-9}}{5!} = .0607$$

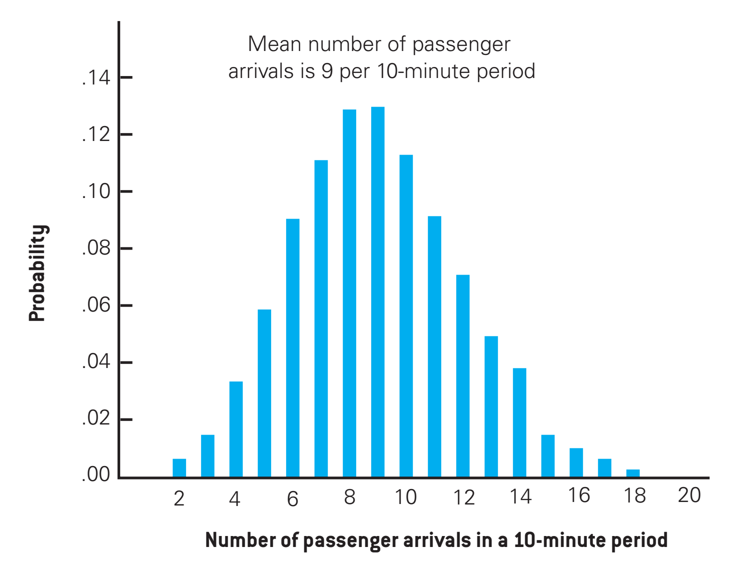
$$P(10) = \frac{9^{10} e^{-9}}{10!} = .1186$$

Using the Poisson probability distribution, we expect it to be very rare to have a 10-minute period in which no passengers ($x = 0$) arrive for screening, as $P(0) = .0001$. Five passenger arrivals occur with a probability $P(5) = .0607$, and 10 with a probability of $P(10) = .1186$. The probabilities for other numbers of passenger arrivals can also be computed. Exhibit B.1 shows the arrival distribution for passengers based on the Poisson distribution. In practice, you would want to record the actual number of arrivals per time period for several days or weeks, and then compare the frequency distribution of the observed number of arrivals to the Poisson distribution to see if the Poisson distribution is a good approximation of the arrival distribution.

B-1b Service-Time Distribution

A service-time probability distribution is needed to describe how long it takes to check in a passenger at the kiosk. This length of time is referred to as the *service time* for the passenger. Although many passengers will complete the check-in process in a relatively short time, others might take a longer time because of unfamiliarity

EXHIBIT B.1 Poisson Distribution of Passenger Arrivals



with the kiosk operation, ticketing problems, flight changes, and so on. Thus we expect service times to vary from passenger to passenger. In the development of waiting-line models, operations researchers have found that the exponential probability distribution can often be used to describe the service-time distribution. Equation B.2 defines the exponential probability distribution

$$f(t) = \mu e^{-\mu t} \quad \text{for } t \geq 0 \qquad \text{[B.2]}$$

where

t = service time (expressed in number of time periods)
μ = average or expected number of units that the service facility can handle in a specific period of time
$e \approx 2.71828$

It is important to use the same time period used for defining arrivals in defining the average service rate. If we use an exponential service-time distribution, the probability of a service being completed within t time periods is given by

$$P(\text{Service time} \leq t \text{ Time periods}) = 1 - e^{-\mu t} \qquad \text{[B.3]}$$

By collecting data on service times for similar check-in systems in operation at other airports, we find that the system can handle an average of 10 passengers per 10-minute period. Using a mean service rate of $\mu = 10$ customers per 10-minute period in Equation B.3, we find that the probability of a check-in service being completed within t 10-minute periods is

$$P(\text{Service time} \leq t \text{ 10-minute time periods}) = 1 - e^{-10t}$$

Now we can compute the probability that a passenger completes the service within any specified time, t. For example, for 1 minute, we set $t = 0.1$ (as a fraction of a 10-minute period). Some example calculations are

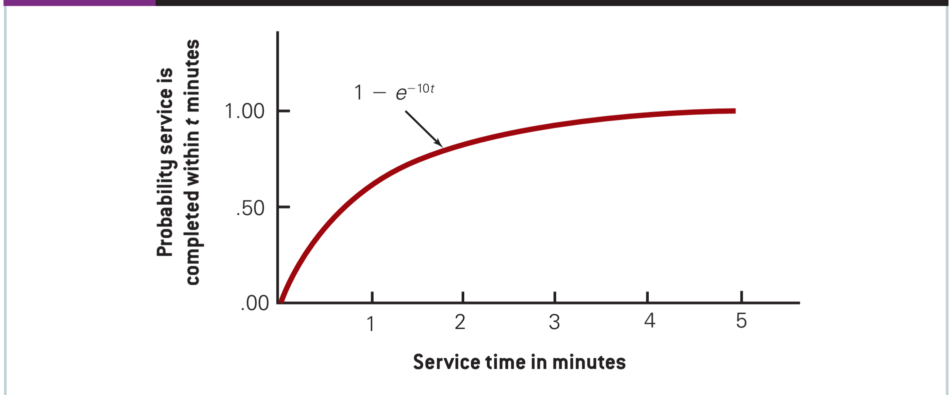
$$P\,(\text{Service time} \leq 1 \text{ minute}) = 1 - e^{-10(0.1)} = 1 - e^{-1} = .6321$$

$$P\,(\text{Service time} \leq 2.5 \text{ minutes}) = 1 - e^{-10(0.25)} = 1 - e^{-2.5} = .9179$$

Thus, using the exponential distribution, we would expect 63.21 percent of the passengers to be serviced in 1 minute or less, and 91.79 percent in 2½ minutes or less. Exhibit B.2 shows graphically the probability that t minutes or less will be required to service a passenger.

In the analysis of a specific waiting line, we want to collect data on actual service times to see if the exponential distribution assumption is appropriate. If you find other service-time patterns (such as a normal service-time probability distribution or a constant service-time distribution), the exponential distribution should not be used.

EXHIBIT B.2 Probability That a Passenger Will Be Serviced in *t* Minutes



B-1c Queue Discipline

A ***queue discipline*** *is the manner in which new arrivals are ordered or prioritized for service.* For the airport problem, and in general for most customer-oriented waiting lines, the waiting units are ordered on a first-come,

A **queue discipline** is the manner in which new arrivals are ordered or prioritized for service.

first-served (FCFS) basis—referred to as an FCFS queue discipline. Other types of queue disciplines are also prevalent. These include the following:

- *Shortest processing time (SPT)*, which we discussed in Chapter 14. SPT tries to maximize the number of units processed, but units with long processing times must wait long periods of time to be processed, if they are processed at all.
- *A random queue discipline* provides service to units at random regardless of when they arrived for service. In some cultures, a random queue discipline is used for serving people instead of the FCFS rule.
- *Triage* is used by hospital emergency departments based on the criticality of patients' injuries as they arrive. That is, a patient with a broken neck receives top priority over another patient with a cut finger.
- *Preemption* is the use of a criterion that allows new arrivals to displace members of the current queue and become the first to receive the service. This criterion could be wealth, society status, age, government position, and so on. Triage is a form of preemption based on the patient's degree and severity of medical need.
- *Reservations and appointments* allocate a specific amount of capacity at a specific time for a specific customer or processing unit. Legal and medical services, for example, book their day using appointment queuing disciplines.

A few of these queue disciplines are modeled analytically, but most require simulation models to capture system queuing behavior. We will restrict our attention in this chapter to waiting lines with an FCFS queue discipline.

B-1d Queuing Behavior

People's behavior in queues and service encounters is often unpredictable. **Reneging** *is the process of a customer entering the waiting line but later deciding to leave the line and server system*. **Balking** *is the process of a customer evaluating the waiting line and server system and deciding not to enter the queue.* In both situations, the customer leaves the system, may not return, and a current sale or all future sales may be lost. Most analytical models assume the customer's behavior is patient and steady and that customers will not renege or balk, as such situations are difficult to model without simulation.

Reneging is the process of a customer entering the waiting line but later deciding to leave the line and server system.

Balking is the process of a customer evaluating the waiting line and server system and deciding not to enter the queue.

B-2 SINGLE-SERVER QUEUING MODEL

The queuing model presented in this section can be applied to waiting-line situations that meet these assumptions or conditions:

1. The waiting line has a single server.
2. The pattern of arrivals follows a Poisson probability distribution.
3. The service times follow an exponential probability distribution.
4. The queue discipline is first-come, first-served (FCFS).
5. No balking or reneging.

Because we have assumed that these conditions are applicable to the airport check-in problem, we can use this queuing model to analyze the operation. We have already concluded that the mean arrival rate is $\lambda = 9$ passengers per 10-minute period, and that the mean service rate is $\mu = 10$ passengers per 10-minute period. Using the assumptions of Poisson arrivals and exponential service times, quantitative analysts have developed the following expressions to define the operating characteristics of a single-channel waiting line:

1. The probability that the service facility is idle (that is, the probability of 0 units in the system):
$$P_0 = (1 - \lambda/\mu) \quad \text{[B.4]}$$
2. The probability of n units in the system:
$$P_n = (\lambda/\mu)^n P_0 \quad \text{[B.5]}$$
3. The average number of units waiting for service:
$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \quad \text{[B.6]}$$
4. The average number of units in the system:
$$L = L_q + \lambda/\mu \quad \text{[B.7]}$$
5. The average time a unit spends waiting for service:
$$W_q = L_q/\lambda \quad \text{[B.8]}$$
6. The average time a unit spends in the system (waiting time plus service time):
$$W = W_q + 1/\mu \quad \text{[B.9]}$$
7. The probability that an arriving unit has to wait for service:
$$P_w = \lambda/\mu \quad \text{[B.10]}$$

The values of the mean arrival rate, λ, and the mean service rate, μ, are clearly important components in these formulas. From Equation B.10, we see that the ratio of these two values, λ/μ, is simply the probability that an arriving unit must wait because the server is busy. Thus, λ/μ is often referred to as the *utilization factor* for the waiting line. The formulas for determining the operating characteristics of a single-server waiting line presented in Equations B.4 through B.10 are applicable only when the utilization factor, λ/μ, is less than 1. This condition occurs when the mean service rate, μ, is greater than the mean arrival rate, λ, and hence when the service rate is sufficient to process or service all arrivals.

Returning to the airport check-in problem, we see that with λ = 9 and μ = 10, we can use Equations B.4 through B.10 to determine the operating characteristics of the screening operation. This is done as follows:

$$P_0 = (1 - \lambda/\mu) = (1 - 9/10) = .10$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{9^2}{10(10-9)} = 81/10 = 8.1 \text{ passengers}$$

$$L = L_q + \lambda/\mu = 8.1 + 9/10 = 9.0 \text{ passengers}$$

$W_q = L_q/\lambda = 8.1/9 = .9$ (Note that this refers to the number of 10-minute periods, or, equivalently, 9 minutes per passenger)

$W = W_q + 1/\mu = 0.9$ hour + 1/10 hour = one 10-minute period, or, equivalently, 10 minutes per passenger

$$P_w = \lambda/\mu = 9/10 = .90$$

Using this information, we can learn several important things about the check-in operation. In particular, we see that passengers wait an average of 9 minutes at the kiosk. With this as the average, many passengers wait even longer. In airport operations with passengers rushing to meet plane connections, this waiting time might be judged to be undesirably high. In addition, the fact that the average number of passengers waiting in line is 8.1 and that 90 percent of the arriving passengers must wait to check in might suggest to the operations manager that something should be done to improve the efficiency of the process.

These operating characteristics are based on the assumption of an arrival rate of 9 and a service rate of 10 per 10-minute period. As the figures are based on airport planners' estimates, they are subject to forecasting errors. It is easy to examine the effects of a variety of assumptions about arrival and service rates on the operating characteristics by using a spreadsheet such as the Excel Single-Server Queuing Model template in Exhibit B.3. You may use this spreadsheet to examine the effect of changes in the mean arrival rate.

EXHIBIT B.3 Spreadsheet from Excel Single-Server Queuing Model Template

	A	B	C	D	E	F	G
1	Single Server Queueing Model						
2	Enter the data only in the yellow cells.						
3							
4	Lambda	9					
5	Mu	10					
6							
7	Probability system is empty	0.10					
8	Average number in queue	8.10					
9	Average number in system	9.00					
10	Average time in queue	0.90					
11	Average waiting time in system	1.00					
12	Probability arrival has to wait	0.90					

For example, varying λ from 7 to 10 while keeping μ = 10 yields the following:

Lambda (λ):	7	8	9	10
Probability system is empty	0.30	0.20	0.10	0.00
Average number in queue	1.63	3.20	8.10	∞
Average number in system	2.33	4.00	9.00	∞
Average time in queue	0.23	0.40	0.90	∞
Average waiting time in system	0.33	0.50	1.00	∞
Probability arrival has to wait	0.70	0.80	0.90	1.00

The data in this figure tell us that if the mean arrival rate is 7 passengers per period, the system functions acceptably. On average, only 1.63 passengers are waiting and the average waiting time of 0.23(10 minutes) = 2.3 minutes appears acceptable. However, we see that the mean arrival rate of 9 passengers per period provides undesirable waiting characteristics, and if the rate increases to 10 passengers per period, the system as proposed is completely inadequate. When λ = μ, the operating characteristics are not defined (i.e., dividing any number by zero equals infinity), meaning that these times and numbers of passengers grow infinitely large (that is, when λ = μ, L and $W \to \infty$). These results show that airport planners need to consider design modifications that will improve the efficiency of the check-in process.

If a new process can be designed that will improve the passenger-service rate, Equations B.4 through B.10 can be used to predict operating characteristics under any revised mean service rate, μ. Developing a spreadsheet with alternative mean service rates provides the information to determine which, if any, of the screening facility designs can handle the passenger volume acceptably.

Computing the probability of more or less than x units arriving requires us to use Equation B.5 and the following two equations:

$$P(\text{Number of arrivals} > x) = 1 - P(\text{Number of arrivals} \leq x) \quad \text{[B.11]}$$

$$P(\text{Number of arrivals} < x) = 1 - P(\text{Number of arrivals} \geq x) \quad \text{[B.12]}$$

These equations are used to simplify the calculations. For example, to find the probability that more than 4 customers are waiting for service, we would need to sum the probabilities associated with 5, 6, 7, . . . up to a (theoretically) infinite number. From Equation B.11, we simply need to find the probabilities associated with 4 or less, sum them up, and subtract them from 1.0. With $\lambda = 9$ and $\mu = 10$ and using Equation B.5, we have

$$P_0 = (1 - \lambda/\mu) = (1 - 9/10) = .1000$$
$$P_1 = (\lambda/\mu)^1 P_0 = (9/10)^1(.1) = .0900$$
$$P_2 = (\lambda/\mu)^2 P_0 = (9/10)^2(.1) = .0810$$
$$P_3 = (\lambda/\mu)^3 P_0 = (9/10)^3(.1) = .0729$$
$$P_4 = (\lambda/\mu)^4 P_0 = (9/10)^4(.1) = .0656$$
$$P(\text{Number of arrivals} \leq 4) = .4095$$

Therefore, the probability that more than 4 customers would be waiting for service is $1 - .4095 = .5905$.

SOLVED PROBLEM B.1

The reference desk of a large library receives requests for assistance at a mean rate of 10 requests per hour, and it is assumed that the desk has a mean service rate of 12 requests per hour.

a. What is the probability that the reference desk is idle?
b. What is the average number of requests that will be waiting for service?
c. What is the average number of requests in the system?
d. What is the average waiting time plus service time for a request for assistance?
e. What is the utilization factor?
f. What is the probability of more than three requests?

Solution

a. $P_0 = (1 - \lambda/\mu) = (1 - 10/12) = .1667$ (Equation B.4)

b. $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = \dfrac{10^2}{12(12 - 10)} = 4.1667$ requests (Equation B.6)

c. $L = L_q + \lambda/\mu = 4.1667 + 10/12 = 5.000$ requests (Equation B.7)

d. $W_q = L_q/\lambda = 4.1667/10 = 0.4167$ hour Equation B.8)

$W = W_q + 1/\mu$
$= 0.4167$ hour $+ 1/12$ hour
$= 0.5$ hour, or 30 minutes (Equation B.9)

e. $P_w = \lambda/\mu = 10/12 = .8333$ (Equation B.10)

f. Use Equation B.5 to compute the following:

$P_0 = (1 - \lambda/\mu) = (1 - 10/12) = .1667$
$P_1 = (\lambda/\mu)^1 P_0 = (10/12)^1(.1667) = .1389$
$P_2 = (\lambda/\mu)^2 P_0 = (10/12)^2(.1667) = .1157$
$P_3 = (\lambda/\mu)^3 P_0 = (10/12)^3(.1667) = .0965$
$P(\text{number of requests} \leq 3) = .5178$

Using Equation B.13, we sum the probabilities from P_0 to P_3 and then subtract from 1 to arrive at the probability of more than three requests waiting for service of $1 - .5178 = .4822$. Exhibit B.4 shows these calculations using the Excel Single-Server Queuing Model template.

EXHIBIT B.4 Excel Single-Server Queuing Model Template for Solved Problem Calculations

	A	B
1	**Single Server Queueing Model**	
2	**Enter the data only in the yellow cells.**	
3		
4	**Lambda**	10
5	**Mu**	12
6		
7	**Probability system is empty**	0.17
8	**Average number in queue**	4.17
9	**Average number in system**	5.00
10	**Average time in queue**	0.42
11	**Average waiting time in system**	0.50
12	**Probability arrival has to wait**	0.83

L.L. Bean: Using Queuing Theory for Customer Service

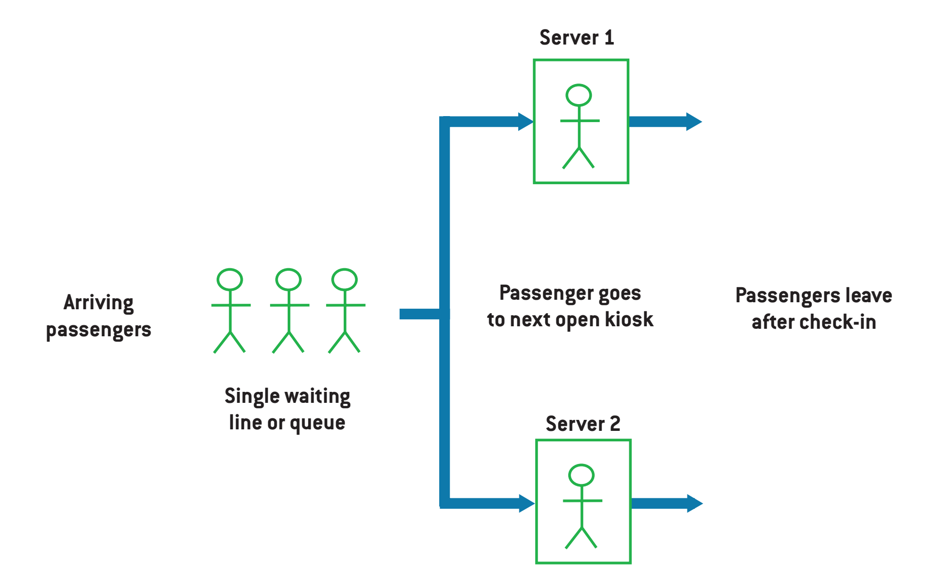
The retailer L.L. Bean is widely known for retailing high-quality outdoor goods and apparel, with more than 85 percent of sales generated through mail orders and telephone orders via 800-number service, which was introduced in 1986. About 65 percent of the total annual sales volume is generated through orders taken at two telemarketing centers located in Maine. L.L. Bean estimated that in 1988 it lost at least $10 million of profit by allocating telemarketing resources—the number of trunk lines, agents, and maximum number of wait positions for telephone calls—suboptimally. Customer service had become unacceptable; in some half-hours, 80 percent of the calls dialed received a busy signal because the trunks were full. Those customers who got through might have waited 10 minutes for an available agent. As a consequence, the company launched a project to better allocate resources and manage its queues. Using a mathematical queuing model to help evaluate the impact of changing the number of trunk lines, agents, and wait positions, L.L. Bean gained improved performance. Calls answered increased 24 percent, revenues increased 16.3 percent, the percentage of calls spending less than 20 seconds in the queue increased by 208 percent, the percentage of abandoned callers fell by 81.3 percent, and the average answer rate fell from 93 seconds to 15 seconds.[2]

B-3 MULTIPLE-SERVER QUEUING MODEL

A logical extension of a single-server waiting line is to have multiple servers, similar to those you are familiar with at many banks. By having more than one server, the check-in process can be dramatically improved. In this situation, customers wait in a single line and move to the next available server. Note that this is a different situation from one in which each server has a distinct queue, such as with highway tollbooths, bank teller windows, or supermarket checkout lines. In such situations, customers might "jockey" for position between servers (channels). **Jockeying** *is the process of customers leaving one waiting line to join another in a multiple-server (channel) configuration.* The model we present assumes that all servers are fed from a single waiting line. Exhibit B.5 is a diagram of this system.

In this section, we present formulas that can be used to compute various operating characteristics for a multiple-server waiting line. The model we will use can be applied to situations that meet these assumptions:

EXHIBIT B.5 A Two-Server Queuing System



1. The waiting line has two or more identical servers.
2. The arrivals follow a Poisson probability distribution with a mean arrival rate of λ.
3. The service times have an exponential distribution.
4. The mean service rate, μ, is the same for each server.

Jockeying is the process of customers leaving one waiting line to join another in a multiple-server (channel) configuration.

5. The arrivals wait in a single line and then move to the first open server for service.
6. The queue discipline is on a first-come, first-served (FCFS) basis.
7. No balking or reneging is allowed.

Using these assumptions, operations researchers have developed formulas for determining the operating characteristics of the multiple-server waiting line. Let

k = number of channels
λ = mean arrival rate for the system
μ = mean service rate for each channel

The following equations apply to multiple-server waiting lines for which the overall mean service rate, $k\mu$, is greater than the mean arrival rate, λ. In such cases, the service rate is sufficient to process all arrivals.

1. the probability that all k service channels are idle (that is, the probability of zero units in the system):

$$P_0 = \frac{1}{\left(\sum_{n=0}^{k-1} \frac{(\lambda/\mu)^n}{n!}\right) + \frac{(\lambda/\mu)^k}{k!}\,\frac{k\mu}{k\mu - \lambda}} \qquad [B.13]$$

2. the probability of n units in the system:

$$P_n = \frac{(\lambda/\mu)^n}{k!k^{n-k}} P_0 \quad \text{for } n > k$$

$$P_n = \frac{(\lambda/\mu)^n}{n!} P_0 \quad \text{for } 1 \le n \le k \qquad [B.14]$$

3. the average number of units waiting for service:

$$L_q = \frac{(\lambda/\mu)^k \lambda\mu}{(k-1)!(k\mu - \lambda)^2} \qquad [B.15]$$

4. the average number of units in the system:

$$L = L_q + \lambda/\mu \qquad [B.16]$$

5. the average time a unit spends waiting for service:

$$W_q = L_q/\lambda \qquad [B.17]$$

6. the average time a unit spends in the system (waiting time plus service time):

$$W = W_q + 1/\mu \qquad [B.18]$$

7. the probability that an arriving unit must wait for service:

$$P_w = \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu - \lambda} P_0 \qquad [B.19]$$

Although the equations describing the operating characteristics of a multiple-server queuing model with Poisson arrivals and exponential service times are somewhat more complex than the single-server equations, they provide the same information and are used exactly as we used the results from the single-channel model. To simplify the use of Equations B.13 through B.19, Exhibit B.6 shows values of P_0 for selected values of λ/μ. Note that the values provided correspond to cases for which $k\mu > \lambda$; hence the service rate is sufficient to service all arrivals.

For an application of the multiple-server waiting-line model, we return to the airport check-in problem and consider the desirability of expanding the screening facility to provide two kiosks. How does this design compare to the single-server alternative?

We answer this question by applying Equations B.13 through B.19 for $k = 2$ servers. Using an arrival rate of $\lambda = 9$ passengers per period and $\mu = 10$ passengers per period for each of the kiosks, we have these operating characteristics:

$$P_0 = .3793 \text{ (from Exhibit B.6 for } \lambda/\mu = .9 \text{ and } k = 2)$$

$$L_q = \frac{(9/10)^2(9)(10)}{(2-1)!(20-9)^2}(.3793) = 0.23 \text{ passengers}$$

$$L = 0.23 + \frac{9}{10} = 1.13 \text{ passengers}$$

$$W_q = \frac{0.23}{9} = 0.026 \text{ multiples of 10-minute periods, or } 0.26 \text{ minutes/passenger}$$

$$W = 0.026 + \frac{1}{10} = 0.126 \text{ multiples of 10-minute periods, or } 1.26 \text{ minutes/passenger}$$

$$P_w = \frac{1}{2!}\left(\frac{9}{10}\right)^2\left(\frac{20}{20-9}\right)(.3793) = .279$$

These operating characteristics suggest that the two-server operation would handle the volume of passengers extremely well. Specifically, note that the total time in the system is an average of only 1.26 minutes per passenger, which is excellent. The percentage waiting is 27.9 percent, which is acceptable, especially in light of the short average waiting time.

Exhibit B.7 is the spreadsheet from the Excel Multiple-Server Queuing Model template designed to compute operating characteristics for up to eight servers in the multiple-server queuing model using the arrival and service rates for the security-screening example. With three servers, we see a significant improvement over two servers in the operating characteristics; beyond this, the improvement is negligible. In addition, we can use the spreadsheet to show that even if the mean arrival rate for passengers exceeds the estimated 9 passengers per hour, the two-channel system should operate nicely.

EXHIBIT B.6 Values of P_0 for Multiple-Server Queuing Model				
	Number of Servers (k)			
Ratio λ/μ	2	3	4	5
0.15	0.8605	0.8607	0.8607	0.8607
0.20	0.8182	0.8187	0.8187	0.8187
0.25	0.7778	0.7788	0.7788	0.7788
0.30	0.7391	0.7407	0.7408	0.7408
0.35	0.7021	0.7046	0.7047	0.7047
0.40	0.6667	0.6701	0.6703	0.6703
0.45	0.6327	0.6373	0.6376	0.6376
0.50	0.6000	0.6061	0.6065	0.6065
0.55	0.5686	0.5763	0.5769	0.5769
0.60	0.5385	0.5479	0.5487	0.5488
0.65	0.5094	0.5209	0.5219	0.5220
0.70	0.4815	0.4952	0.4965	0.4966
0.75	0.4545	0.4706	0.4722	0.4724
0.80	0.4286	0.4472	0.4491	0.4493
0.85	0.4035	0.4248	0.4271	0.4274
0.90	0.3793	0.4035	0.4062	0.4065
0.95	0.3559	0.3831	0.3863	0.3867
1.00	0.3333	0.3636	0.3673	0.3678
1.20	0.2500	0.2941	0.3002	0.3011
1.40	0.1765	0.2360	0.2449	0.2463
1.60	0.1111	0.1872	0.1993	0.2014
1.80	0.0526	0.1460	0.1616	0.1646
2.00		0.1111	0.1304	0.1343
2.20		0.0815	0.1046	0.1094
2.40		0.0562	0.0831	0.0889
2.60		0.0345	0.0651	0.0721
2.80		0.0160	0.0521	0.0581
3.00			0.0377	0.0466
3.20			0.0273	0.0372
3.40			0.0186	0.0293
3.60			0.0113	0.0228
3.80			0.0051	0.0174
4.00				0.0130
4.20				0.0093
4.40				0.0063
4.60				0.0038
4.80				0.0017

What is the probability that less than 4 customers are waiting for service when $k = 2$, $\lambda = 9$, and $\mu = 10$ passengers per time period? Using the spreadsheet, we can calculate the probabilities when $x = 0$ as .3793, $x = 1$ as .3414, $x = 2$ as .1536, $x = 3$ as .0691, and $x = 4$ as .0311. Using Equation B.13, we sum the probabilities from P_0 to P_4 and then subtract from 1 to arrive at the probability of 4 or more customers

EXHIBIT B.7 Spreadsheet from Excel Multiple-Server Queuing Model Template

	A	B	C	D	E	F	G	H	I	J
1	Multiple Server Queueing Model									
2										
3	Lambda	9								
4	Mu	10								
5										
6	Number of servers	2	3	4	5	6	7	8		
7	Probability system is empty	0.379	0.403	0.406	0.407	0.407	0.407	0.407		
8	Average number in queue	0.229	0.030	0.004	0.001	0.000	0.000	0.000		
9	Average number in system	1.129	0.930	0.904	0.901	0.900	0.900	0.900		
10	Average time in queue	0.025	0.003	0.000	0.000	0.000	0.000	0.000		
11	Average time in system	0.125	0.103	0.100	0.100	0.100	0.100	0.100		
12	Probability arrival must wait	0.279	0.070	0.014	0.002	0.000	0.000	0.000		
13										
14										
15	Lookup table (do not modify)	0	1	2	3	4	5	6	7	8
16		1.000	0.900	0.405	0.122	0.027	0.005	0.001	0.000	0.000
17				1.900	2.305	2.427	2.454	2.459	2.459	2.460
18				0.405	0.122	0.027	0.005	0.001	0.000	0.000

waiting for service at 1 − .9745 = .0255. Adding the second server greatly improves system performance, as the results show.

B-4 THE ECONOMICS OF WAITING-LINE ANALYSIS

As we have shown, queuing models can be used to determine operating performance of a waiting-line system. In the economic analysis of waiting lines, we will use the information provided by the queuing model to develop a cost model for the waiting line under study. Then we can use the model to help the manager balance the cost of customers having to wait for service against the cost of providing the service. This is a vital issue for all operations managers (see the feature box on airport security screening).

In developing a cost model for the check-in problem, we will consider the cost of passenger time, both waiting time and servicing time, and the cost of operating the system. Let C_W = the waiting cost per hour per passenger and C_S = the hourly cost associated with each server. Clearly, the passenger waiting-time cost cannot be accurately determined; managers must estimate a reasonable value that might reflect the potential loss of future revenue should a passenger switch to another airport or airline because of perceived unreasonable delays. This is called the *imputed cost of waiting*. Suppose C_W is estimated to be \$50 per hour, or \$0.83 per minute. The cost of operating each service facility is more easily determined because it consists of the wages of any personnel and the cost of equipment, including maintenance. For automated systems, this is usually quite small. Let us assume that C_S = \$10 per hour, or \$0.167 per minute. Therefore, the total cost per minute is $C_W L + C_S k = 0.83\,L + 0.167\,k$, where L = average number of passengers in the system and k number of servers. Exhibit B.8 summarizes the cost for the one- and two-server scenarios. We clearly see the economic advantages of a two-server system.

EXHIBIT B.8 Economic Analysis of Check-In System Design

System	*k*	System Cost	*L*	Passenger Cost	Total Cost
Single-server	1	0.167(1) = 0.167	9	0.83(9) = 7.47	\$7.64
Two-server	2	0.167(2) = 0.334	1.13	0.83(1.13) = 0.94	\$1.27

Airport Security Wait Times

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The U.S. Transportation Security Administration (TSA) set up a website that provides airport-by-airport information on the average wait times. The site provides hourly and daily average wait times based on last month's data. A longer-term goal is to provide real-time, hourly updates. In 2004, the longest (maximum) time waiting in line to get to the metal detector was 36 minutes at a major U.S. airport. Average waiting times range from a few minutes to 30 minutes. For example, the TSA recorded that the wait at the main security checkpoint at Hartsfield International Airport in Atlanta at 7 a.m. on Monday, August 9, 2004, averaged 26 minutes. Monday morning is a peak time for most airports. The data are collected by security screeners who give passengers a card with their arrival time on it, which is collected when they get to the metal detector.[3]

B-5 THE PSYCHOLOGY OF WAITING

Customers become frustrated when a person enters a line next to them and receives service first. Of course, *that* customer feels a certain sense of satisfaction. People expect to be treated fairly; in queuing situations that usually means on a "first-come, first-served" basis. In the mid-1960s, Chemical Bank was one of the first firms to switch to a serpentine line (one line feeding into several servers) from multiple, parallel lines. American Airlines copied this at its airport counters, and most others followed suit. Studies have shown that customers are happier when they wait in a serpentine line, rather than in parallel lines, even if that type of line increases their wait.

Understanding the psychological perception of waiting is as important in addressing queuing problems as are analytical approaches. Creative solutions that do not rely on technical approaches can be quite effective. One example involved complaints of tenants waiting for elevators in a high-rise building. Rather than pursuing the expensive technical solution of installing a faster elevator, the building manager installed mirrors in the elevator lobbies to help the tenants pass the time. This is commonly found in many hotels today. In other elevator lobbies, art or restaurant menus are often used to distract patrons. Another example occurred at the Houston airport. Passengers complained about long waits when picking up their baggage. The airline solved the problem by moving the baggage to the farthest carousel from the planes. While the total time to deliver the baggage did not change, the fact that passengers had to walk farther and wait less eliminated the complaints.

Nothing is worse than not knowing when the next bus will arrive. Not knowing how long a wait will be creates anxiety. To alleviate this kind of uncertainty, the Disney theme parks inform people how long a wait to expect by placing signs at various points along the queue. Chemical Bank pays $5 to customers who wait in line more than 7 minutes. This interval was chosen because research indicated that waits up to 10 minutes were tolerable. Customers have provided good feedback; they do not seem to mind waiting longer if they receive something for it.

Florida Power and Light developed a system that informed customers of the estimated waiting time for telephone calls, allowing customers to call back later if the wait would be too long.[4] Consumer research revealed that customers would wait 94 seconds without knowing the length of wait. It also showed that customers began to be dissatisfied after waiting about 2 minutes. But when customers knew the length of wait, they were willing to wait 105 seconds longer—a total of 199 seconds! Thus, Florida Power and Light knew that it could buy more time, without sacrificing customer satisfaction, by giving customers a choice of holding for a predicted period of time or deferring the call to a later time. The system, called Smartqueue, was implemented, and virtually all customers considered it helpful in subsequent satisfaction surveys. From the company's perspective, Smartqueue increased the time customers were willing to wait without being dissatisfied by an appreciable amount.

Other methods of changing customers' perceptions involve distractions. Time spent without anything to do seems longer than occupied time. Airlines and rental car firms divide their processes into stages to make the process seem shorter, with breaks in service for both the service provider's and customer's benefit. Hospitals try to reduce the perception of waiting all day in the hospital by separating patient parking, admission, blood test, X-rays, examination, and other areas from one another. Guests waiting for a ride at Disney World seldom see the entire queue, which can have hundreds of people. Amusement parks might also have roving entertainers to distract the waiting crowds. As early as 1959, the Manhattan Savings Bank offered live entertainment and even dog and boat shows during the busy lunchtime hours.

Supermarkets place "impulse" items such as candy, batteries, and other small items, as well as magazines, near checkouts to grab customers' attention. The Postal Service has been experimenting with video displays that not only distract customers but also inform them of postal procedures that can speed up their transactions.

Technology is alleviating queuing in many service industries today. For example, rental car firms use automatic tellers for fast check-in and check-out and are working on radio frequency technology to entirely skip waiting in lines to get or return a vehicle. Airlines allow their customers to print out boarding passes at airport kiosks or on their own printer to speed the check-in process. Thus, queuing in operations management entails much more than some analytical calculations and requires good management skills.

PROBLEMS, ACTIVITIES, AND DISCUSSIONS

Note: An asterisk denotes problems for which a template in the *OM6 Spreadsheet Templates* at *OM6* Online may be used.

The following waiting-line problems are all based on the assumptions of Poisson arrivals and exponential service times.

1.* Trucks using a single-server loading dock have a mean arrival rate of 14 per day. The loading/unloading rate is 19 per day.
   a. What is the probability that the truck dock will be idle?
   b. What is the average number of trucks waiting for service?
   c. What is the average time a truck waits for the loading or unloading service?
   d. What is the probability that a new arrival will have to wait?
   e. What is the probability that more than three trucks are waiting for service?

2.* Trosper Tire Company has decided to hire a new mechanic to handle all tire changes for customers ordering new tires. Two mechanics are available for the job. One mechanic has limited experience and can be hired for $7 per hour. It is expected that this mechanic can service an average of three customers per hour. A mechanic with several years of experience is also being considered for the job. This mechanic can service an average of four customers per hour, but must be paid $10 per hour. Assume that customers arrive at the Trosper garage at the rate of two per hour.
   a. Compute waiting-line operating characteristics for each mechanic.
   b. If the company assigns a customer-waiting cost of $15 per hour, which mechanic provides the lower operating cost?

3.* Agan Interior Design provides home and office decorating assistance. In normal operation, an average of three customers will arrive per hour. One design consultant is available to answer customer questions and make product recommendations. The consultant averages 12 minutes with each customer.
   a. Compute operating characteristics for the customer waiting line.
   b. Service goals dictate that an arriving customer should not wait for service for more than an average of five minutes. Is this goal being met? What action do you recommend?
   c. If the consultant can reduce the average time spent with customers to eight minutes, will the service goal be met?

4.* Keuka Park Savings and Loan currently has one drive-in teller window. Cars arrive at a mean rate of 10 per hour. The mean service rate is 12 cars per hour.

  a. What is the probability that the service facility will be idle?

  b. If you were to drive up to the facility, how many cars would you expect to see waiting and being serviced?

  c. What is the average time waiting for service?

  d. What is the probability that an arriving car will have to wait?

  e. What is the probability that more than four vehicles are waiting for service?

  f. As a potential customer of the system, would you be satisfied with these waiting-line characteristics? How do you think managers could go about assessing customers' feelings about the current system?

5.* To improve its customer service, Keuka Park Savings and Loan (problem 4) wants to investigate the effect of a second drive-in teller window. Assume a mean arrival rate of 10 cars per hour. In addition, assume a mean service rate of 12 cars per hour for each window. What effect would adding a new teller window have on the system? Does this system appear acceptable?

6.* Consider a two-server waiting line with a mean arrival rate of 40 per hour and a mean service rate of 60 per hour for each server.

  a. What is the probability that both servers are idle?

  b. What is the average number of cars waiting for service?

  c. What is the average time waiting for service?

  d. What is the average time in the system?

  e. What is the probability of having to wait for service?

7.* Big Al's Quickie Carwash has two wash bays. Each bay can wash 15 cars per hour. Cars arrive at the carwash at the rate of 15 cars per hour on average, join the waiting line, and move to the next open bay when it becomes available.

  a. What is the average time waiting for a bay?

  b. What is the probability that a customer will have to wait?

  c. As a customer of Big Al's, do you think the system favors the customer? If you were Al, what would be your attitude toward this service level?

8.* Refer to the Agan Interior Design situation in problem 3. Agan is evaluating two alternatives:

  1. use one consultant with an average service time of 8 minutes per customer; or

  2. expand to two consultants, each of whom has an average service time of 10 minutes per customer.

  If the consultants are paid $16 per hour and the customer waiting time is valued at $25 per hour, should Agan expand to the two-consultant system? Explain.

9. Design a spreadsheet similar to Exhibit B.3 to study changes in the mean service rate from 10 to 15 for $\lambda = 9$ passengers per minute.

10.* Using the spreadsheet in the Multiple-Server Queuing Model template (Exhibit B.7), determine the effect of increasing passenger arrival rates of 10, 12, 14, 16, and 18 on the operating characteristics of the airport security screening example.

Bourbon County Court Case Study

"Why don't they buy another copying machine for this office? I waste a lot of valuable time fooling with this machine when I could be preparing my legal cases," noted H. C. Morris, as he waited in line. The self-service copying machine was located in a small room immediately outside the entrance of the courtroom. Morris, the county attorney, often copied his own papers, as did other lawyers, to keep his legal cases and work confidential. This protected the privacy of his clients as well as his professional and personal ideas about the cases.

He also felt awkward at times standing in line with secretaries, clerks of the court, other attorneys, police officers and sheriffs, building permit inspectors, and the dog warden—all trying, he thought, to see what he was copying. The line for the copying machine often extended out into the hallways of the courthouse.

Morris mentioned his frustration with the copying machine problem to Judge Hamlet and his summer intern, Dot Gifford. Gifford was home for the summer and working toward a joint MBA/JD degree from a leading university.

"Mr. Morris, there are ways to find out if that one copying machine is adequate to handle the demand. If you can get the judge to let me analyze the situation, I think I can help out. We had a similar problem at the law school with word processors and at the business school with student lab microcomputers."

The next week, Judge Hamlet gave Gifford the go-ahead to work on the copying machine problem. He asked her to write a management report on the problem with recommendations so he could take it to the Bourbon County Board of Supervisors for their approval. The board faced deficit spending last fiscal year, so the trade-offs between service and cost must be clearly presented to the board.

Gifford's experience with analyzing similar problems at school helped her know what type of information and data were needed. After several weeks of working on this project, she developed the information contained in Exhibits B.9, B.10, and B.11.

Gifford was not quite as confident in evaluating this situation as others because the customer mix and associated labor costs seemed more uncertain in the county courthouse. In the law school situation, only secretaries used the word-processing terminals; in the business school situation, students were the ones complaining about long waiting times to get on a microcomputer terminal. Moreover, the professor guiding these two past school projects had suggested using queuing models for one project and simulation for the other project. Gifford was never clear on how the method of analysis was chosen. Now she wondered which methodology she should use for the Bourbon County Court situation.

To organize her thinking, Gifford listed a few of the questions she needed to address as follows:

Case Questions for Discussion

1. Assuming a Poisson arrival distribution and an exponential service-time distribution, apply queuing models to the case situation and evaluate the results.
2. What are the economics of the situation using queuing model analysis?
3. What are your final recommendations using queuing model analysis?
4. *Advanced Assignment* (requires a statistical package that performs chi-square and curve fitting tests): Do the customer arrival and service empirical (actual) distributions in the case match the theoretical distributions assumed in queuing models?

EXHIBIT B.9 Bourbon County Court—Customer Arrivals per Hour*

	Customer Arrivals in One Hour		Customer Arrivals in One Hour		Customer Arrivals in One Hour		Customer Arrivals in One Hour		Customer Arrivals in One Hour
1	5	11	10	21	3	31	11	41	14
2	9	12	17	22	9	32	8	42	7
3	7	13	18	23	11	33	9	43	4
4	13	14	14	24	10	34	8	44	7
5	7	15	11	25	12	35	6	45	7
6	7	16	16	26	4	36	8	46	2
7	7	17	5	27	8	37	14	47	4
8	11	18	6	28	9	38	12	48	7
9	8	19	8	29	9	39	11	49	2
10	6	20	13	30	9	40	15	50	8

*A sample of customer arrivals at the copying machine was taken for five consecutive 9-hour workdays plus 5 hours on Saturday for a total of 50 observations. The mean arrival rate is 8.92 arrivals per hour.

EXHIBIT B.10 Bourbon County Court—Copying Service Times*

Obs. No.	Hours per Job	Obs. No.	Hours per Job	Obs. No.	Hours per Job	Obs. No.	Hours per Job	Obs. No.	Hours per Job
1	0.0700	11	0.1253	21	0.1754	31	0.0752	41	0.2005
2	0.1253	12	0.1754	22	0.0700	32	0.1002	42	0.0501
3	0.0752	13	0.0301	23	0.1253	33	0.0250	43	0.0150
4	0.2508	14	0.1002	24	0.0752	34	0.0752	44	0.0501
5	0.0226	15	0.0752	25	0.2508	35	0.0501	45	0.0527
6	0.1504	16	0.3009	26	0.0752	36	0.0301	46	0.1203
7	0.0501	17	0.0752	27	0.0752	37	0.0752	47	0.1253
8	0.0250	18	0.0376	28	0.1002	38	0.0501	48	0.1053
9	0.0150	19	0.0501	29	0.0388	39	0.0075	49	0.1253
10	0.2005	20	0.0226	30	0.0978	40	0.0602	50	0.0301

*A sample of customers served at the copying machine was taken for five consecutive 9-hour workdays plus 5 hours on Saturday for a total of 50 observations. The average service time is 0.0917 hour per copying job, or 5.499 minutes per job. The equivalent service rate is 10.91 jobs per hour (that is, 10.91 jobs/hour = [60 minutes/hour]/5.5 minutes/job).

EXHIBIT B.11 Bourbon County Court—Cost and Customer Mix*

Resource Category	Mix of Customers in Line (%)	Cost of Average Direct Wages per Hour
Lease and maintenance cost of copying machine per year @ 250 days/year	na	$18,600
Average hourly copier variable cost (electric, ink, paper, etc.)	na	$5/hour
Secretaries	50%	$18.75
Clerks of the court	20%	$22.50
Building inspectors and dog warden	10%	$28.40
Police officers and sheriffs	10%	$30.80
Attorneys	10%	$100.00

*The mix of customers standing in line was collected at the same time as the data in Exhibits B.9 and B.10. Direct wages do include employee benefits, but not work opportunity costs, ill-will costs, etc.

ENDNOTES

1. D. Brady, "Why Service Stinks," *BusinessWeek*, October 23, 2000, pp. 118–128. This episode is partially based on this article.
2. Adapted from Quinn et al., "Allocating Telecommunications Resources at L.L. Bean, Inc," *Interfaces*, 21, 1, January/February, 1991, pp. 75–91.
3. D. Machalaba, "Taking the Slow Train: Amtrak Delays Rise Sharply," *The Wall Street Journal*, August 10, 2004, pp. D1–D2.
4. A. Schatz, "Airport Security-Checkpoint Wait Times Go Online," *The Wall Street Journal*, August 10, 2004, p. D2.

OM6 Chapter 2: Measuring Performance in Operations and Value Chains

Discussion Questions

1. What types of performance measurements might be used to evaluate a fraternity or student organization?

   Metrics might include attendance at key events, total membership each academic term, gains and losses in membership, fundraising amounts, operations costs, number of professional or social events held each term, grade point average of members, number of intramural sporting events participated in, number of guest speakers, student (member) satisfaction, projects completed on time and on budget, and so on.

2. Select an organization you are familiar with or have an interest in and write a short two-page paper describing key performance metrics in that industry and firm using the format of Exhibit 2.1.

   Students will develop some interesting tables for different industries and firms of interest to them. A few questions you might pose during discussion of this question are as follows:

   - What criteria are missing? Explain
   - Does the measurement support our mission?
   - Will the measurement be used to manage change?
   - Is it important to our customers?
   - Is it effective in measuring performance? (Is it actionable?) **Actionable measures** *provide the basis for decisions at the level at which they are applied*—the value chain, organization, process, department, workstation, job, and service encounter. They should be meaningful to the user, timely, and reflect how the organization generates value to customers.)
   - Is it effective in forecasting results?
   - Is it easy to understand/simple?
   - Is the data easy/cost-efficient to collect? (How would the data be collected? Who would do it? How long would it take? What would the cost be?)
   - Does the measurement have validity, integrity, and timeliness?
   - Does the measurement have an owner? (Who will ensure that the data do get collected, analyzed, and disseminated as needed?)

   Good performance measures are actionable. **Actionable measures** *provide the basis for decisions at the level at which they are applied*—the value chain, organization, process, department, workstation, job, and service encounter. They should be meaningful to the user, timely, and reflect how the organization generates value to customers.

3. Discuss some analytical or graphical approaches that organizations can use for analyzing performance data based on your experience and previous coursework.

   These methods might include simple charts that you would find in Microsoft Excel, such as bar charts, scatter plots, pie charts, and line charts for time series data. Other approaches would be basic statistical techniques such as frequency distributions and histograms, basic statistical measures such as means and standard deviations, statistical process control charts, Pareto (ABC) analysis, regression and correlation analysis, and so on.

4. Under which perspective of the balanced scorecard would you classify each of the following measurements?

   a. On-time delivery to customers (customer perspective)
   b. Time to develop the next generation of products (innovation and learning perspective)
   c. Manufacturing yield (internal perspective)
   d. Engineering efficiency (internal perspective)
   e. Quarterly sales growth (customer perspective if units; financial perspective if dollars)
   f. Percent of products that equal 70 percent of sales (innovation and learning perspective)
   g. Cash flow (financial perspective)
   h. Number of customer partnerships (customer, perspective)
   i. Increase in market share (customer perspective)
   j. Unit cost of products (financial perspective)

   Arguments can be made for other perspectives. Some measures may not clearly fall into a particular category; however, what is more important is that the organization takes a broad view of the most important measures across the enterprise, rather than just focusing on financial results.

5. When the value of a loyal customer (VLC) market segment is high, should these customers be given premium goods and services for premium prices? If the VLC is low, should they be given less service? Explain.

   This question can trigger significant differences in student opinions. For example, should banking customers with average bank deposits of over $100,000 have to stand in the same teller line as a bank customer with average bank deposits of $1,000? That is, should the bank set up a premium service channel for premium customers? In the early 1990s when a New York bank set up a separate bank teller window (and line) for customers with bank deposits over $100,000, the outcry from other bank customers resulted in the bank closing the premium teller window for premium customers three days after it opened. Yet, hotels have VIP and loyal customer suites and floors, airlines give premium customers first choice at airline seats and flights plus VIP lounges and first class services, some automobile dealerships give free loaner cars to their top customers while not offering these extra services to less valuable customers, and so on. *The reality is that when a small percentage of customers (say 20%) account for a large percentage of total revenue (say 65%) it is profitable to segment markets based on the value of a loyal customer or customers, and provide premium service for A customers.*

Problems and Activities

(Note: an asterisk denotes problems for which an Excel spreadsheet template on the CourseMate Web site may be used.)

1. Interview managers at a local company to identify the key business measures (financial, market, supplier, employee, process, information, innovation, etc.) for that company. What quality indicators does that company measure? What cause and effect (interlinking) performance relationships would be of interest to the organization?

   It is always interesting to see what organizations really measure. In many cases, don't be surprised to see simply a heavy emphasis on financial results without a "balanced scorecard" as such. Quality indicators are often the traditional ones (defects, yield). Many smaller companies don't measure the cost of quality or customer satisfaction. Does the firm measure time, product and service quality, or what? Highlight OM metrics and issues. This question can be used to generate discussion on what *should* be measured and why (a good lead in to ideas of strategy in the next chapter). For small firms all performance measurement is sometimes done by observation of the owner(s). So make sure the size of the firm is identified upfront.

2. Each day, a FedEx competitor processes approximately 70,000 shipments. Suppose that they use the same Service Quality Index as FedEx and identified the following numbers of errors during a 5-day week (see the "FedEx: Measuring Service Performance" box). These values are hypothetical and do not reflect any real company's actual performance.

   Complaints reopened: 125
   Damaged packages: 18
   International: 102
   Invoice adjustments: 282
   Late pickup stops: 209
   Lost packages: 2
   Missed proof of delivery: 26
   Right date late: 751
   Traces: 115
   Wrong day late: 15

Compute the Service Quality Indicator by finding the weighted sum of errors as a percentage of total shipments. How might such an index be used in other organizations such as a hotel or automobile service facility?

OM6 Chapter 2 Problem #2 Fed Ex Problem

Number of Shipments/Day	70,000			
Total Number of Shipments Over 5 Days	350,000			
	Weight	Percent of Total Weight	Number of Errors	Weighted Average Errors
Complaints Reopen	3	0.079	125	9.87
Damaged Pkgs	10	0.263	18	4.74
International	1	0.026	102	2.68
Invoice Adjustments	1	0.026	282	7.42
Late Pickup Stops	3	0.079	209	16.50
Lost Packages	10	0.263	2	0.53
Missed Proof of Delivery	1	0.026	26	0.68
Right Date Late	1	0.026	751	19.76
Traces	3	0.079	115	9.08
Wrong Day Late	5	0.132	15	1.97
Total	38	1	1645	73.24

Wt Average Percent of Total Shipments 0.000209248* 0.020924812+

Service Quality Indicator (SQI) 99.979^

*73.24/350,000 = 0.000209248
+0.000209248*100 = 0.020924812
^100-0.020924812 = 99.979

Over this 5-day period FE delivery performance was almost perfect on a percent basis, yet 1,645 customers experienced some type of service upset. You might point out that the U.S. Postal Service has good performance too (not as good as above) and that the huge volumes hide the number of impacts on customers.

3. Research and write a short paper on how some organization applies the five dimensions of service quality.

SERVQUAL was originally measured on 10 aspects of service quality: reliability, responsiveness, competence, access, courtesy, communication, credibility, security, understanding the customer and tangibles (background -- using factor analysis). It measures the gap between customer expectations and experience. By the early nineties the authors had refined (combined) the SERVQUAL model to the useful acronym RATER (these five dimensions are in the chapter):

- Reliability

- Assurance
- Tangibles
- Empathy, and
- Responsiveness

If students search SEVQUAL and/or the GAP model (in OM4 C15) they will find many applications. The SERVQUAL has been tested in banking, credit cards, repair and maintenance, and long distance telephone service. Hospitals, for example, (see web reference below) have also used these five measures of service quality to measure their performance.

http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1069855/pdf/hsresearch00075-0070.pdf

4. A major airline is attempting to evaluate the effect of recent changes it has made in scheduling flights between New York City and Los Angeles. Data available are shown below.

	Number of Flights	Number of Passengers
Month prior to schedule change	16	8,795
Month after schedule change	27	15,653

Using passengers per flight as a productivity indicator, comment on the apparent effect of the schedule change.

Computing passengers per flight, we obtain (after rounding)

Month prior to schedule change: 8795/16 = 550
Month after schedule change: 15,563/27 = 576

Productivity increased by 4.7 percent (26/550) after the schedule change. This could be due to more convenient flight times, better schedules or some other intervening variable. Here the productivity metric is output per flight. Other possible productivity indicators for airlines might include flights/labor dollar, passengers/labor dollar, total passenger revenue/total cost of all flights, total number of passengers/total cost of all flights.

5. Revenue or costs per passenger mile are two key performance measures in the airline industry. Research their use in this industry and prepare a one-page paper summarizing how they are used and why they are so important.

These two metrics drive profitability in the airline industry. Few industries have so few and simple summary metrics yet they are very powerful. Southwest Airlines, for example, normally has the widest gap between these two metrics, and therefore, generates profits, while older airlines such as United often have costs per passenger mile equal to or higher than revenue per passenger mile. Your students will find many interesting ways to use these productivity metrics for this industry. Your students will also discover energy and labor costs are huge components of total airline costs.

6. A hamburger factory produces 60,000 hamburgers each week. The equipment used costs $10,000 and will remain productive for 4 years. The labor cost per year is $13,500.

   a. What is the productivity measure of "units of output per dollar of input" averaged over the four-year period?

      Productivity = total units produced divided by the total labor cost plus total equipment cost = 60,000(52)(4)/[13,500(4)+10,000] = 195 hamburgers/dollar

   b. We have the option of $13,000 equipment, with an operating life of 5 years. It would reduce labor costs to $11,000 per year. Should we consider purchasing this equipment (using productivity arguments alone)?

      For the expensive machine, productivity = 60,000(52)(5)/[11,000(5) + 13,000] = 229.4 hamburgers/dollar input. Because the productivity of the expensive machine is higher, it would be a good investment based on this single criterion.

7. A fast-food restaurant has a drive-through window and during peak lunch times can handle a maximum of 50 cars per hour with one person taking orders, assembling them, and acting as cashier. The average sale per order is $9.00. A proposal has been made to add two workers and divide the tasks among the three. One will take orders, the second will assemble them, and the third will act as cashier. With this system it is estimated that 70 cars per hour can be serviced. Use productivity arguments to recommend whether or not to change the current system.

   Productivity = revenue/labor dollar

   For system 1, productivity = 50($9.00)/x = 450/x
   For system 2, productivity = 70($9.00)/3x = 210/x

   where x is the prevailing minimum wage. With the additional two workers, productivity drops by more than on-half (i.e., too much labor for system 2). Thus, it is not advisable to change the current system (i.e., keep system 1). System #2 simply uses too much labor.

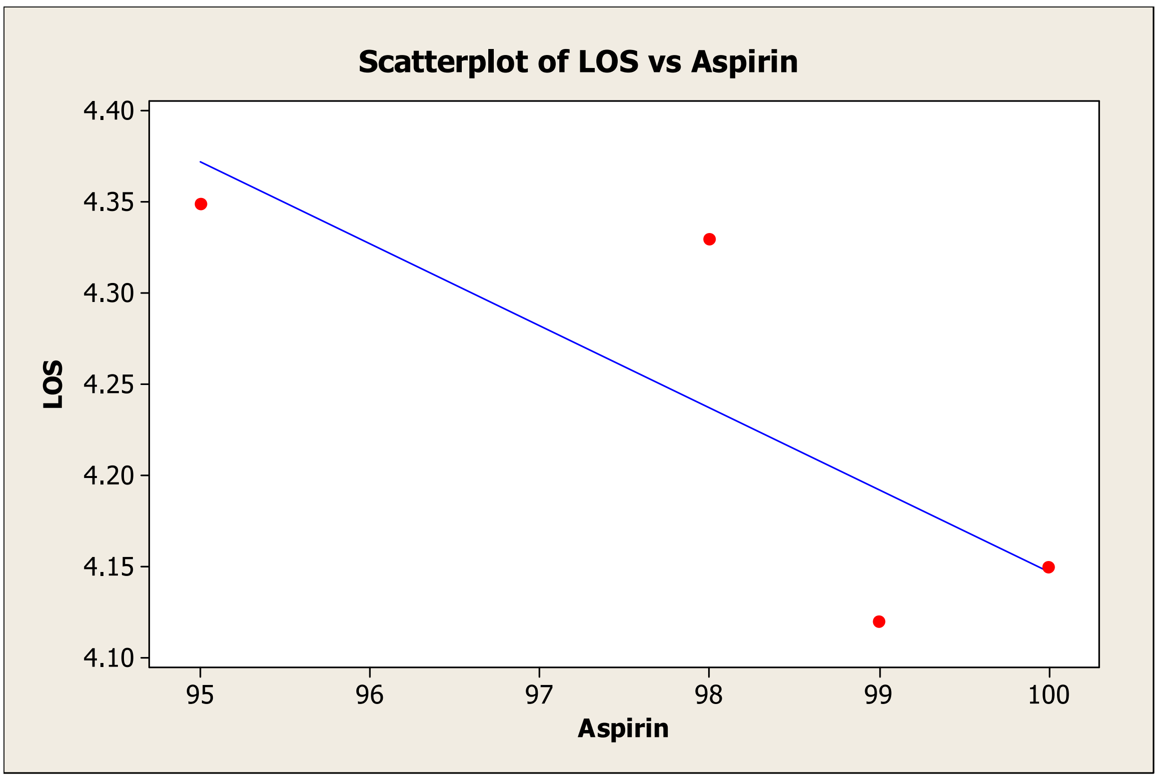
8. A key hospital outcome measure of clinical performance is length of stay (LOS); that is, the number of days a patient is hospitalized. For patients at one hospital with acute myocardial infarction (heart attack), the length of stay over the past four years has consistently decreased. The hospital also has data for various treatment options such as the percentage of patients who received aspirin upon arrival and cardiac

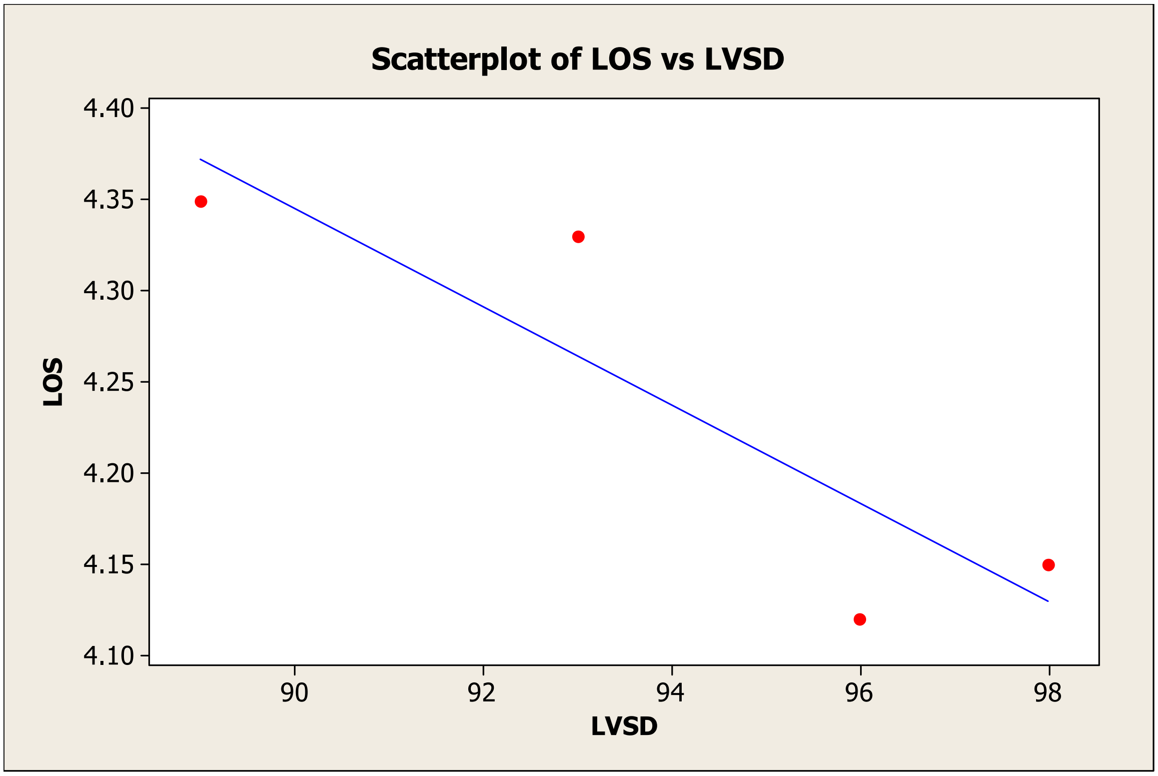
medication for Left Ventricular Systolic Dysfunction (*LVSD*). The data are shown below:

Year	Average LOS	Aspirin on arrival	LVSD medication
2007	4.35 days	95%	89%
2008	4.33 days	98%	93%
2009	4.12 days	99%	96%
2010	4.15 days	100%	98%

Illustrate the interlinking relationships by constructing scatter using Excel showing the LOS as a function of the other variables. What do these models tell you?

The charts below show that as the percentage of aspirin on arrival and LVSD medications increase, the average LOS decreases, suggesting that these interventions reduce hospitalization which is good. Instructors might wish to illustrate how to add a trendline to a scatter chart (right click the data series and choose Add Trendline).





Descriptive Statistics: LOS, Aspirin, LVSD

```
Variable    Mean  SE Mean   StDev  Minimum  Median  Maximum
LOS       4.2375   0.0596  0.1193   4.1200  4.2400   4.3500
Aspirin    98.00     1.08    2.16    95.00   98.50   100.00
LVSD       94.00     1.96    3.92    89.00   94.50    98.00
```

Correlations: LOS, Aspirin, LVSD

```
             LOS    Aspirin
Aspirin   -0.815
           0.185
LVSD      -0.885    0.985
           0.115    0.015
Cell Contents: Pearson correlation
               P-Value
```

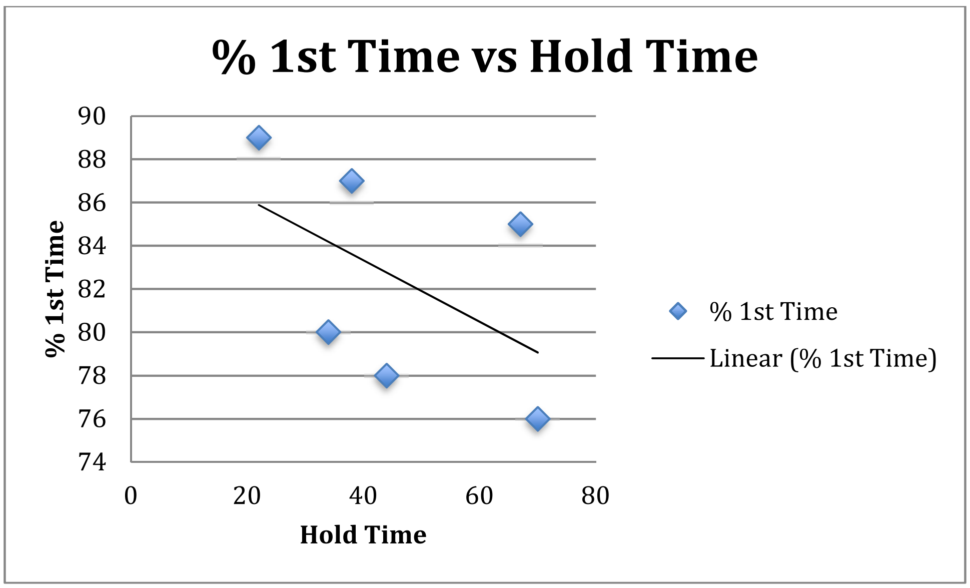
9. Customers call a call center to make room reservations for a small chain of 42 motels located throughout the southwestern part of the United States. Business analytics is used to determine how and if the following performance metrics are related: time by quarter, average time on hold (seconds) before a customer reaches a company customer service representative, percent of time the customer inquiry is solved the first time (called first pass quality) and customer satisfaction with the overall call center experience.

Quarter	Average Hold Time	Percent Solved First Time	Overall Customer Satisfaction Percent
Q1	22 seconds	89%	96%

Q2	34 seconds	80%	92%
Q3	44 seconds	78%	82%
Q5	67 seconds	85%	84%
Q6	38 seconds	87%	90%
Q7	70 seconds	76%	80%
Q8	86 seconds	67%	74%

Develop a graphical interlinking model by constructing scatter charts showing the relationships between each pair of variables. What do results tell you?

The charts below suggest that as the average hold time increases, both the percent solved the first time and customer satisfaction decreases (suggesting that service reps are probably rushing due to high call volumes). Instructors might wish to illustrate how to add a trendline to a scatter chart (right click the data series and choose Add Trendline).



Below are basic statistics and variable correlations in case you need them during a class discussion.

Descriptive Statistics: Hold Time, % 1st Time, Cust Sat %

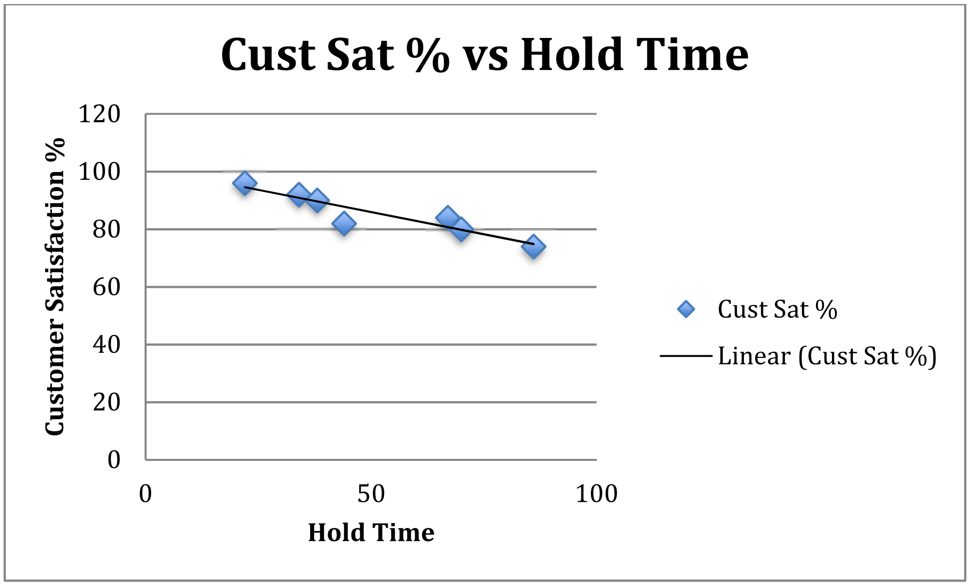
```
Variable      Mean  SE Mean  StDev  Minimum  Median  Maximum
Hold Time    51.57     8.71  23.05    22.00   44.00    86.00
% 1st Time   80.29     2.86   7.57    67.00   80.00    89.00
Cust Sat %   85.43     2.89   7.63    74.00   84.00    96.00
```

Correlations: Hold Time, % 1st Time, Cust Sat %

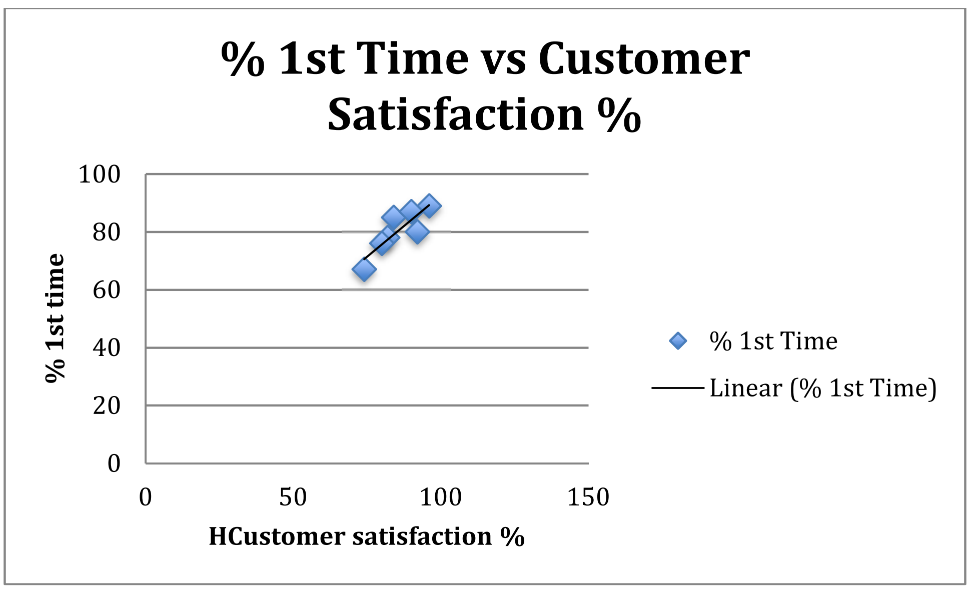
```
             Hold Time  % 1st Time
% 1st Time      -0.755
                 0.050

Cust Sat %      -0.928       0.857
                 0.003       0.014

Cell Contents: Pearson correlation
               P-Value
```



There also appears to be a positive relationship between Percent Solved the First Time and Customer Satisfaction as shown below.



10.* What is the average value of a loyal customer (VLC) in a target market segment if the average purchase price is $75 per visit, the frequency of repurchase is six times per year, the contribution margin is 10 percent, and the average customer defection rate is 25 percent?

VLC = P*CM*RF*BLC, where P = the revenue per unit, CM = contribution margin to profit and overhead expressed as a fraction (i.e., 0.45, 0.5, and so on), RF = repurchase frequency = 6 times/year, BLC = buyer's life cycle, computed as 1/defection rate, expressed as a fraction (1/0.25 = 4 years)

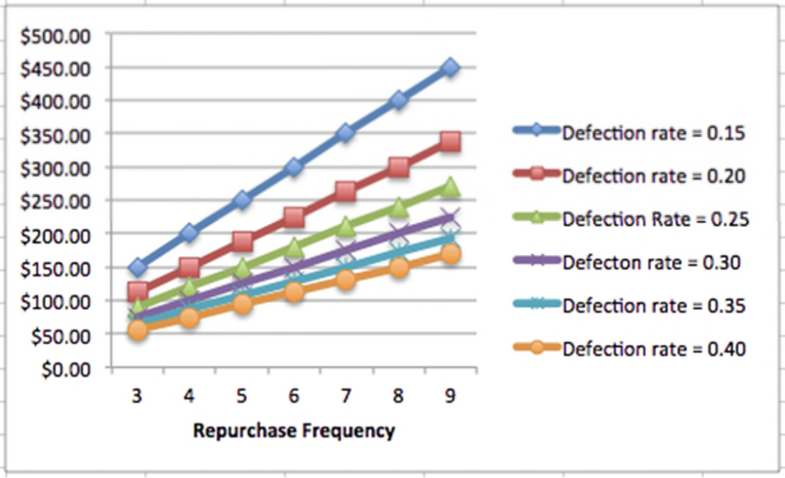
VLC = P*CM*RF*BLC = ($75)(.10)(6)(4) = $180

We may also use the spreadsheet template VLC:

Value of a Loyal Customer	
Enter data only in yellow cells.	
Revenue per unit	$75.00
Percent contribution margin to profit and overhead	10%
Repurchase frequency (purchases/year)	6
Defection rate	0.25
Buyer's life cycle	4.00
VLC	$180.00

11.* Using the base case data in question 10, analyze how the value of a loyal customer (VLC) will change if the average customer defection rate varies between 15 and 40 percent (in increments of 5 percent) and the frequency of repurchase varies between 3 and 9 times per year (in increments of 1 year). Sketch graphs (or use Excel charts) to illustrate the impact of these assumptions on the VLC.

Defection Rate	Repurchase Frequency						
	3	4	5	6	7	8	9
0.15	$150.00	$200.00	$250.00	$300.00	$350.00	$400.00	$450.00
0.20	$112.50	$150.00	$187.50	$225.00	$262.50	$300.00	$337.50
0.25	$90.00	$120.00	$150.00	$180.00	$210.00	$240.00	$270.00
0.30	$75.00	$100.00	$125.00	$150.00	$175.00	$200.00	$225.00
0.35	$64.29	$85.71	$107.14	$128.57	$150.00	$171.43	$192.86
0.40	$56.25	$75.00	$93.75	$112.50	$131.25	$150.00	$168.75



12.* What is the average defection rate for grocery store shoppers in a local area of a large city if they spend $45 per visit, shop 52 weeks per year, the grocery store has a

4 percent gross margin, and the value of a loyal customer is estimated at $3,500 per year?

VLC = P*CM*RF*BLC = ($45)(.04)(52)(1/DR)
$3,500 = $93.6/DR
$3,500 DR = $93.6

DR = 0.0267 (The average customer defection rate is 2.7%.)

The VLC spreadsheet template may also be used either by experimentation or using Excel's Goal Seek tool:

Value of a Loyal Customer

Enter data only in yellow cells.

Revenue per unit	$45.00
Percent contribution margin to profit and overhead	4%
Repurchase frequency (purchases/year)	52
Defection rate	0.02674
Buyer's life cycle	37.40
VLC	$3,500.37

13. Research and write a short paper on how sports analytics is used by some professional team.

A recent Google search on "sports analytics" results in 57,700,000 hits including conferences, data hubs, methods, blogs, jobs, video, and consulting firms. Business analytics at work!

Today, coaches, players, investors, and owners need to take full advantage of modern analytical methods and digital video software capabilities to make the most efficient use of a team's resources. For example, the economic impact of Division I NCAA basketball exceeds $14 billion in the United States. During the 2009-2010 season the NCAA signed a 14 year $10.8 billion dollar contract with CBS television to cover the NCAA tournament through 2024. In addition, more than $3 billion changed hands with gamblers during the 2010 NCAA tournament alone.

Similar economic statistics document the importance of the National Football League (NFL), National Basketball Association (NBA), Major League Baseball (MLB), NASCAR, and the National Hockey League (NHL). The USA is a "sports nation" and global events like the Olympics and World Cup Soccer demand that we analyze the performance of these sports organizations as rigorously as world-class corporations analyze their goods, services, processes, people, and supply chains.

14. Go to the Baldrige Web site and find the links to the most recent award recipients. Review one of the application summaries and describe the types of performance measures that these companies use.

    The Baldrige application summaries are excellent sources of information to learn about best practices. Categories 4 and 7 provide good examples of the types of measures that leading companies use. Instructors might also wish to ask students to compare measures used by small versus large companies, manufacturing versus service, and differences with not-for-profit education and health care sectors

15. The balanced scorecard was originally developed by Arthur M. Schneiderman at Analog Devices. Visit his Web site, www.schneiderman.com, and read the articles to answer the following questions:

    a. How was the first balanced scorecard developed? (Click The Scorecard link under the Contents link. Find "ADI: The First Balanced Scorecard)

    b. What steps should an organization follow to build a good balanced scorecard? (Find "How to Build a Balanced Scorecard")

    c. Why do balanced scorecards fail? (Find "Why Balanced Scorecards Fail")

    This Web site provides interesting history about the balanced scorecard and a host of other information developed by Mr. Schneiderman, including numerous articles on the subject.

Case Teaching Notes: Rapido Burrito

Overview

Rapido Burrito is a small regional chain of quick service restaurants. Rather than wait in a cafeteria style line, customers check boxes for their choice of ingredients, sauce, and so on paper menus at their table. The food is prepared quickly and then delivered to the tables. Lately, one of the store managers has been hearing customer complaints, such as: "The tortillas are too thin"; "The food is not hot"; "Everytime I get a burrito it seems to be a different size"; and "I got the wrong ingredients on my burrito." Many complaints were submitted through the corporate website. The district manager was most concerned with the comments about the consistency of size. One of the staff designed a customer survey using the questions in Exhibit 2.9, based on a 5-point Likert scale [5 = excellent, or strongly agree; 1 = poor or strongly disagree] for the first 10 questions. The last two questions were coded as a 1, 2, 3, or 4. They administered the questionnaire to 25 random customers. The restaurant also gathered data on the weights of 50 samples of 3 burritos (a total of 150). (Both the survey data and weight data are available on spreadsheet *Rapido Burrito Case Data*.)

Exhibit 2.9 Customer Survey Questions

1. Was the menu easy to read?
2. Was order prepared correctly?
3. Was the food tasty?
4. Was the food served hot?
5. Were employees courteous and polite?
6. Was the restaurant clean?
7. In your opinion, did you receive a good value for the price you paid?
8. What was your level of satisfaction?
9. How likely are you to dine with us again?
10. How likely are you to recommend us to your friends/family?
11. How often do you eat at Sizzleking?
12. First time, less than once/month, 1-3 times a month, weekly?
13. What was the main ingredient in your burrito: chicken, beef, pork, beans?

Case Questions and Analysis

1. What conclusions do you reach when you calculate descriptive statistics for the answers to each of the survey questions in the database?

Portions of the spreadsheet Rapido Burrito Case Soln.xlsx are shown below. A frequency count of the 25 customers who were surveyed is evenly divided, from first timers to those who eat there weekly.

- ✓ The survey averages show that customers were most satisfied with the menu and order preparation.
- ✓ Courtesy of employees, restaurant cleanliness, and value for price hovered around a 4.
- ✓ Tastiness of the food and overall satisfaction averaged around 3.8 for all respondents.
- ✓ Respondents were less enthusiastic about the food being served hot at 3.60.
- ✓ The likelihood of the customer dining again is only 3.56.
- ✓ The standard deviations for all of the questions appear to be close to equal for the menu, order preparation, employee courtesy, restaurant cleanliness, and overall satisfaction.
- ✓ There was much more variation in the answers to the questions about food served hot, value vs. price, and likelihood to dine again and to recommend the restaurant to friends.

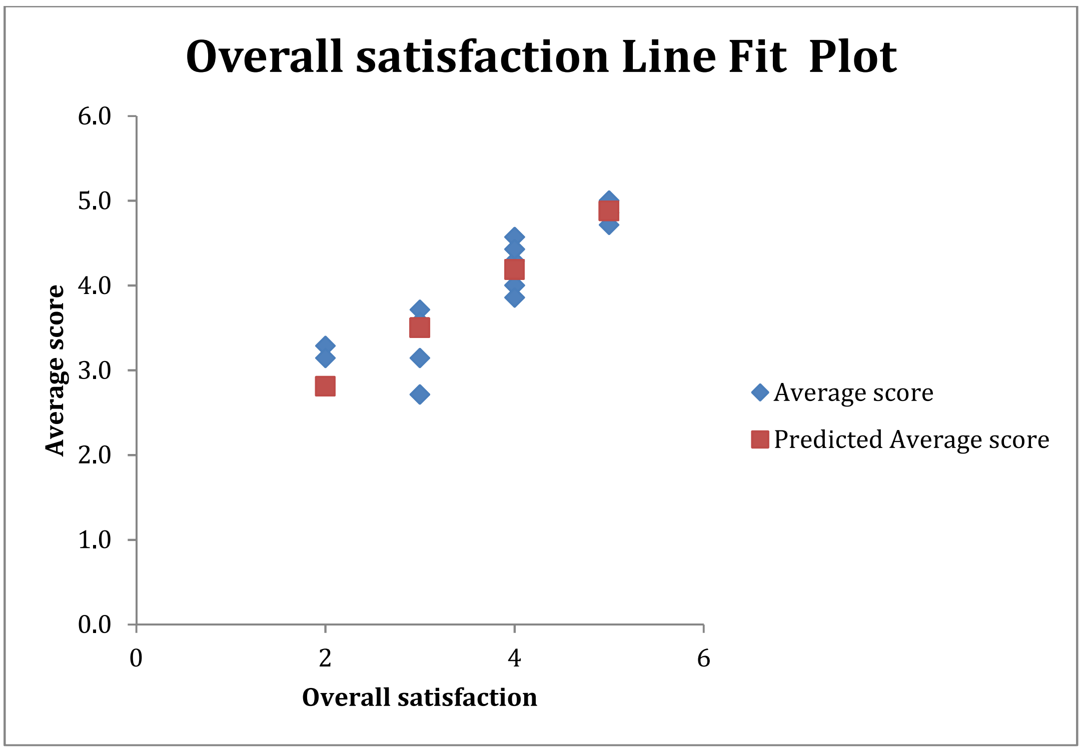
Customer survey responses	Avg	Std. dev.
Menu was easy to read	4.64	0.70
Order was prepared correctly	4.28	0.74
Food was tasty	3.84	0.94
Food was served hot	3.60	1.38
Employees were courteous and polite	4.04	0.61
Restaurant was clean	4.04	0.79

Value for price paid	3.92	1.19
Overall satisfaction	3.80	0.87
Likely to dine with us again?	3.56	1.08
Likely to recommend us to friends?	3.44	1.23

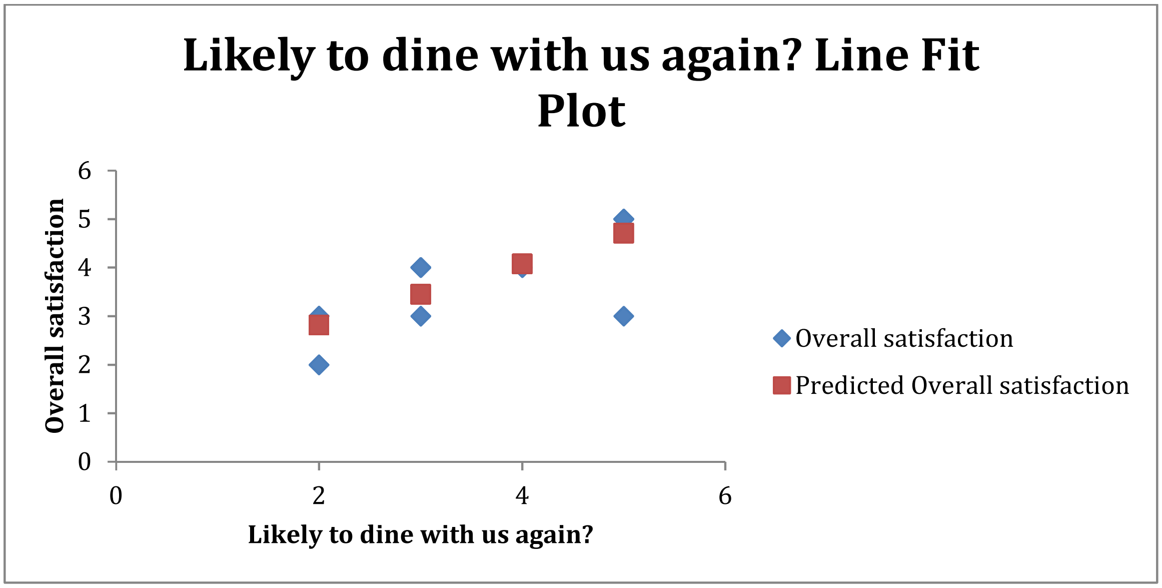
2. If you average the responses to the first seven questions by customer, how closely are those averages correlated to the satisfaction score? Include a scatter chart in your analysis.

The first graph is overall satisfaction versus the average score on the first seven survey questions. The second graph is the survey question scores (a) likely to dine with us again versus (b) the overall satisfaction score. The second graph is for your information only and was not asked in the case assignment questions.

The average responses to the first seven questions by customers, are well correlated with their satisfaction scores. The $R^2 = 0.869$, which indicates a fairly close correlation [correlation coefficient = $\sqrt{0.869} = 0.932$] between the average score and the overall satisfaction score, can be visualized on the scatter chart, below.



The likelihood of "the customer dining again" at Rapido Burrito can be predicted by using the "satisfaction score" and regression analysis by customer. The likelihood of customer's dining again is moderately correlated to the satisfaction score. The $R^2 =$ 0.625, which does not indicate an extremely close correlation between the average score and the overall satisfaction score, as seen on the scatter chart, below.



3. Analyze the data on burrito weights using descriptive statistical measures such as the mean and standard deviation, and tools such as a frequency distribution and histogram. What do your results tell you about the consistency of the food servings?

   The descriptive statistics for burrito weights show that the mean $\bar{x} = 1.100$ and standard deviation, s = 0.048. The frequency distribution and histogram show that the sample is somewhat normal in shape. The range and standard deviation show that the food servings are somewhat variable. The range is 0.24, or ¼ pound difference between the lowest and highest values. This could be due to the nature of the burrito product, where the customer specifies ingredients, which add more or less weight to the burrito.

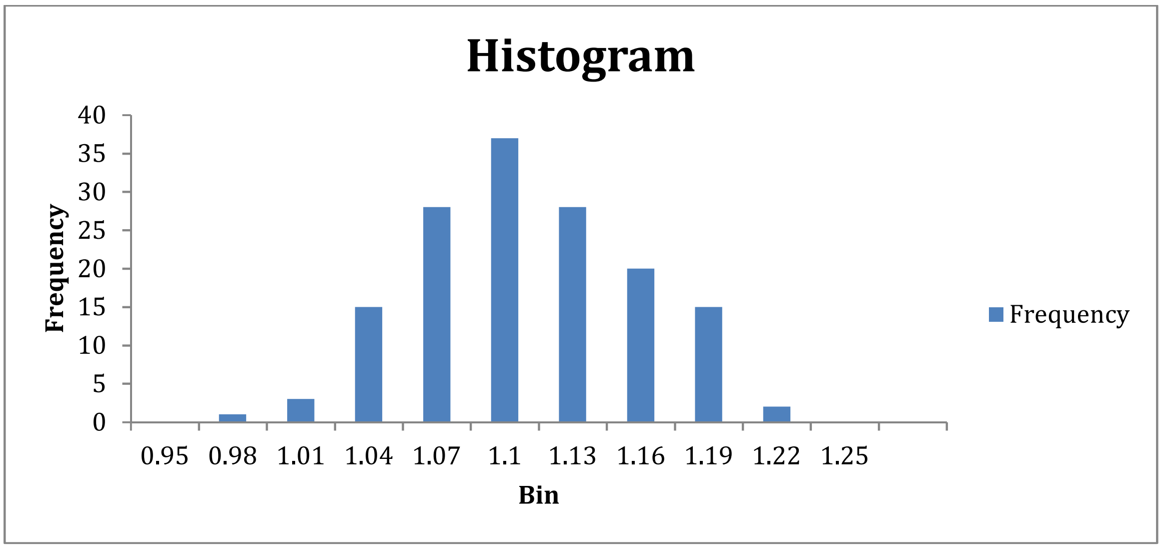
   Conclusion: The burrito weight analysis indicates a good approximation of a normal distribution with fairly consistent weights. The intervening variable is the "degree of customization for each customer."

Descriptive Statistics	
Mean	**1.100**
Standard Error	**0.004**
Median	1.100
Mode	1.090
Standard Deviation	0.048
Sample Variance	0.002
Kurtosis	-0.293
Skewness	-0.138

Bin	*Frequency*
1.25	0
1.30	3
1.35	9
1.40	16
1.45	17
1.50	34
1.55	22
1.60	23
1.65	11

Range	**0.240**
Minimum	0.960
Maximum	1.200
Sum	165.040
Count	150.000
Confidence Level (95.0 percent)	1.200

1.70	7
1.75	6
1.80	1
1.85	1
More	0



4. What recommendations for decision-making and improvement can you make to the store manager?

Recommendations for improvement include:

a. Work to ensure that food is served hot (low average score of 3.60)
b. Develop a panel to do taste testing of various existing and new products (average scores are 3.84 for food was tasty and 3.92 for value for price paid).
c. Provide incentives for repeat customers, such as discounts for people who eat there three times, six times, nine times, etc. (since likely to dine with us again average score is 3.56 and likely to recommend us to friends average score is 3.44).
d. Consider job design and work method ways to ensure that exact weighs of ingredients can be measured and assembled in the burritos. That is, how can we continuously improve our job, equipment, and process designs to reduce variability?

Any average customer survey score below 4.0 is an opportunity for improvement and should be investigated!

<u>Original Two RB Data Sets</u>

Rapido Burrito
Customer Survey Results (1st Eight Customers Only)

Customer survey responses	Customer Number							
	1	2	3	4	5	6	7	8
Menu was easy to read	4	3	5	4	5	5	5	5
Order was prepared correctly	4	4	5	3	4	5	5	5
Food was tasty	5	3	4	3	4	5	4	3
Food was served hot	4	2	3	1	5	5	3	4
Employees were courteous and polite	5	4	4	3	4	5	4	4
Restaurant was clean	5	5	4	3	4	5	4	4
Value for price paid	5	4	3	2	5	5	3	3
Overall satisfaction	4	3	4	3	4	5	4	4
Likely to dine with us again?	4	3	3	2	4	5	3	3
Likely to recommend us to friends?	4	2	3	2	4	5	3	3
How often do you eat at Sizzlegrill? First time, less than once/month, 1-3 times a month, weekly, [1,2,3,4]	3	2	1	1	4	1	2	3
What was the main ingredient: chicken, beef, pork, beans [1,2,3,4]	1	1	3	4	1	1	2	2

<u>Second Set of Data on Burrito Weights (1st 10 observations only)</u>

Weights of Burritos (Pounds) Sample Number			
1	1.43	1.40	1.84
2	1.43	1.68	1.50
3	1.34	1.29	1.62
4	1.34	1.62	1.61
5	1.66	1.46	1.57
6	1.60	1.53	1.65
7	1.35	1.31	1.46
8	1.63	1.71	1.55
9	1.47	1.50	1.59
10	1.54	1.72	1.40

Both data sets are in Rapido Burrito Case Data.xlsx

COLLIER/EVANS

OM

6 Operations + Supply Chain Management

2

Measuring Performance in Operations and Value Chains

LEARNING OUTCOMES

1. Describe the types of measures used for decision making
2. Explain the use of analytics in operations management and how internal and external measures are related
3. Explain how to design a good performance measurement system
4. Describe four models of organizational performance

Measurement

- Act of quantifying the performance of:
  - *Organizational units and goods and services*
  - *Processes, people, and other business activities*
- Provides a scorecard of performance
- Helps identify performance gaps
- Makes accomplishments visible to workforce, stock market, and other stakeholders

Exhibit 2.1 Scope of Business and Operations Performance Measurement

EXHIBIT 2.1 The Scope of Business and Operations Performance Measurement		
Performance Measurement Category	**Typical Organizational-Level Performance Measures**	**Typical Operational-Level Performance Measures**
Financial	Revenue and profit Return on assets Earnings per share	Labor and material costs Cost of quality Budget variance
Customer and market	Customer satisfaction Customer retention Market share	Customer claims and complaints Type of warranty failure/upset Sales forecast accuracy
Quality	Customer ratings of goods and services Product recalls	Defects/unit or errors/opportunity Service representative courtesy
Time	Speed Reliability	Flow processing or cycle time Percent of time meeting promised due date
Flexibility	Design flexibility Volume flexibility	Number of engineering changes Assembly-line changeover time
Innovation and learning	New product development rates Employee satisfaction Employee turnover	Number of patent applications Number of improvement suggestions implemented Percent of workers trained on statistical process control
Productivity and operational efficiency	Labor productivity Equipment utilization	Manufacturing yield Order fulfillment time
Sustainability	Environmental and regulatory compliance Product-related litigation Financial audits	Toxic waste discharge rate Workplace safety violations Percent of employees with emergency preparedness training

Customer-Satisfaction Measurement System

- Provides a company with customer ratings of specific goods and service features
- Indicates the relationship between customer ratings and a customer's likely future buying behavior

Quality

- Measures the degree to which the output of a process meets customer requirements
  - ***Goods quality****: Physical performance and characteristics of a good*
  - ***Service quality***
    - Consistently meeting or exceeding customer expectations and service-delivery system performance for services

Quality (Continued)

- *Assessed by measuring:*
  - Tangibles
  - Reliability
  - Responsiveness
  - Assurance
  - Empathy
- Affected by errors made during service encounters
  - ***Service failures/upsets****: Errors in service creation and delivery*

Time

- Performance measures
  - *Speed of performing a task*
    - Measured by **processing time** and **queue/wait time**
  - *Variability of processes*
    - Measured using standard deviation or mean absolute deviation

Flexibility

- Ability to adapt quickly and effectively to changing requirements
  - ***Goods and service design flexibility***
    - Ability to develop a wide range of customized goods or services to meet different or changing customer needs
  - ***Volume flexibility****: Ability to respond quickly to changes in the volume and type of demand*

Innovation and Learning

Innovation

- *Ability to create new and unique goods and services that delight customers and create competitive advantage*

Learning

- *Creating, acquiring, and transferring knowledge*
- *Modifying behavior of employees in response to internal and external change*

Productivity and Operational Efficiency

Productivity

- *Ratio of the output of a process to its input*

Operational efficiency

- *Ability to provide goods and services to customers with minimum waste and maximum utilization of resources*

Triple bottom line (TBL or 3BL)

- Measurement of sustainability related to:
  - *Environmental factors*
    - Energy consumption, recycling, resource conservation activities, air emissions, solid and hazardous waste rates, etc.
  - *Social factors*
    - Consumer and workplace safety, community relations, and corporate ethics and governance

Triple bottom line (TBL or 3BL) (Continued)

- *Economic factors*
  - Auditing, regulatory compliance, sanctions, donations, fines, etc.

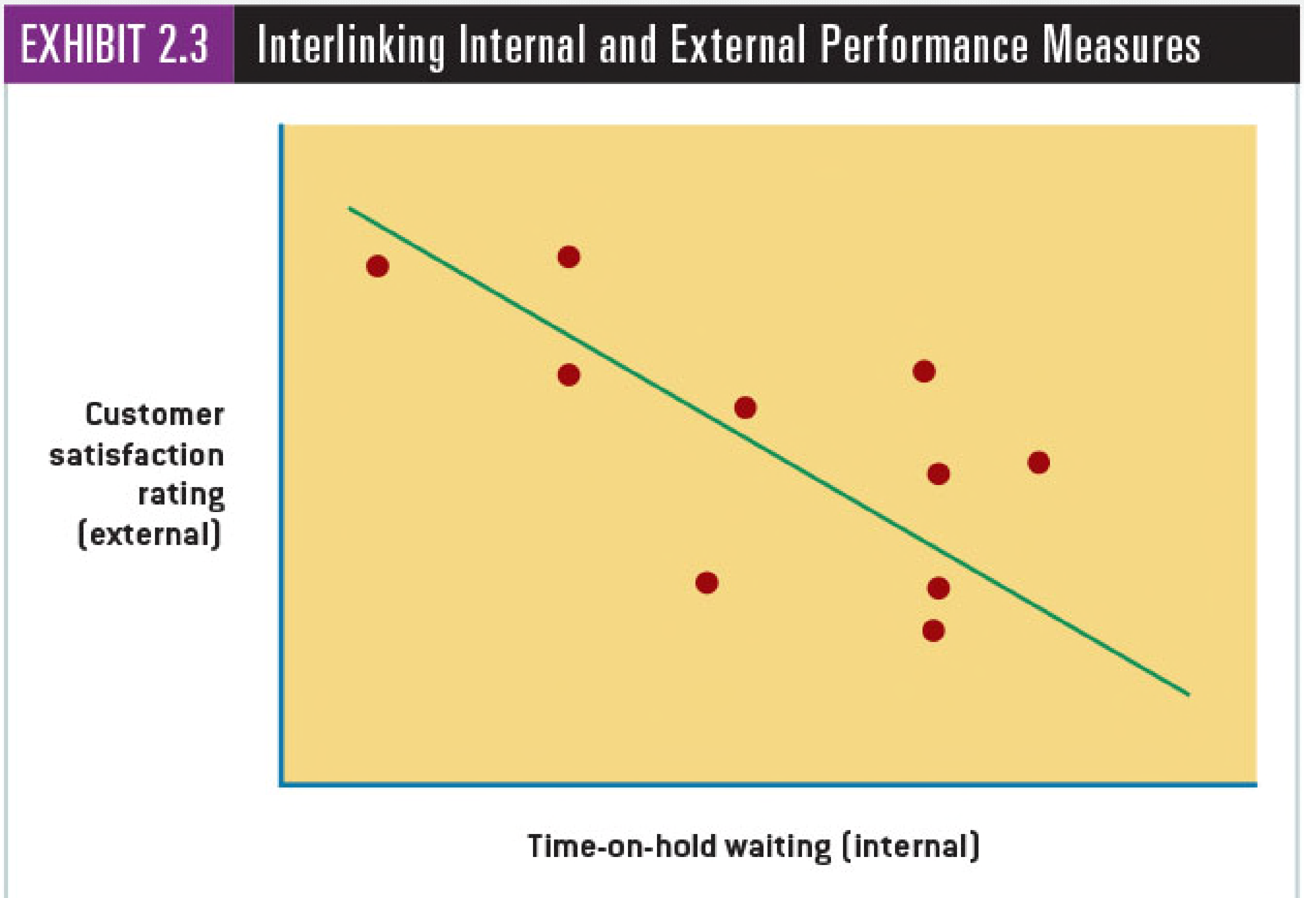
Business Analytics

- Helps operations managers analyze data effectively and make better decisions
  - *Applications*
    - Visualizing data to examine performance trends
    - Calculating basic statistical measures
    - Comparing results relative to other business units, competitors, or best-in-class benchmarks
    - Using correlation and regression analysis

Interlinking

- Quantitative modeling of cause-and-effect relationships between external and internal performance criteria
- Helps quantify performance relationships between all parts of a value chain

Exhibit 2.3 Interlinking Internal and External Performance Measures



Value of a Loyal Customer (VLC)

- Quantifies total revenues or profits each target market customer generates over a buyer's life cycle
  - *Total market value - Multiplying VLC by the absolute number of customers gained or lost*

Actionable Measures

- Provide the basis for decisions at the level at which they are applied
  - *Levels include value chain, organization, process, department, workstation, job, and service encounters*

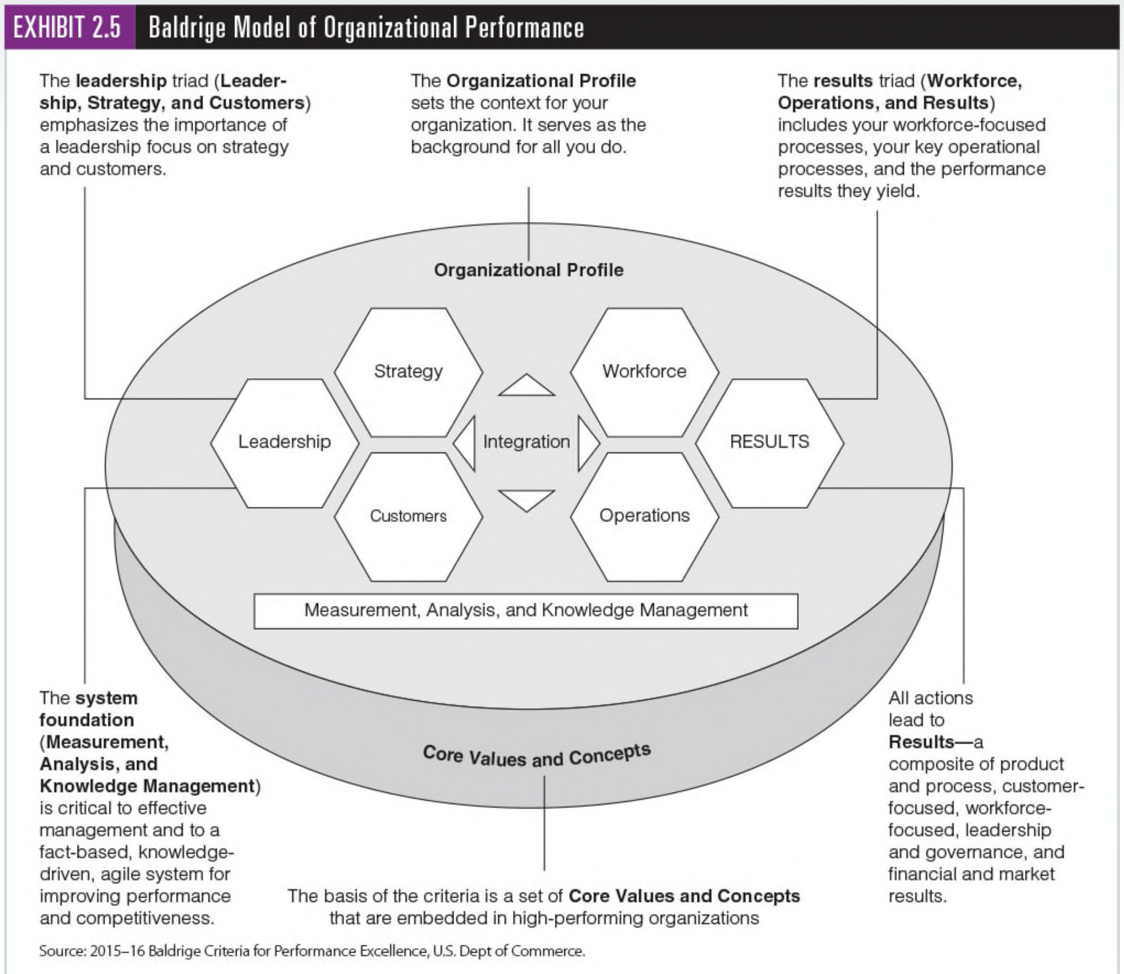
Models of Organizational Performance

Baldrige Performance Excellence framework	Balanced scorecard
Value chain model	Service-profit chain

Baldrige Performance Excellence Framework

- Helps in the process of self-assessment to understand an organization's strengths and weaknesses
  - *Self-assessment:*
    - Helps improve quality, productivity, and overall competitiveness
    - Encourages development of high-performance management practices

Exhibit 2.5 Baldrige Model of Organizational Performance



Balanced Scorecard Model

- Translates strategies into measures that uniquely communicate an organization's vision
- Performance perspectives
  - *Financial - Measures value provided to shareholders*
  - *Customer - Focuses on customer needs and satisfaction and market share and its growth*

Balanced Scorecard Model (Continued)

- *Innovation and learning*
  - Emphasizes people and infrastructure
- *Internal*
  - Focuses attention on the performance of key internal processes that drive a business

Value Chain Model

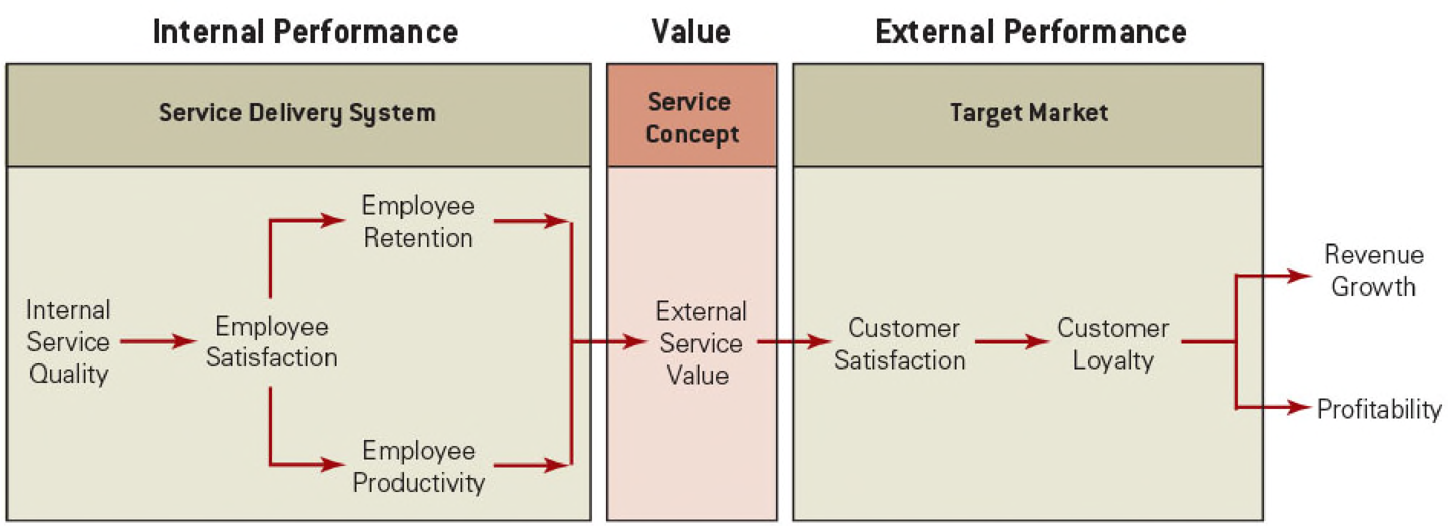
- Evaluates performance throughout the value chain by identifying measures associated with:
  - *Suppliers*
  - *Inputs*
  - *Value creation processes*
  - *Goods and service outputs and outcomes*
  - *Customers and market segments*
  - *Supporting and general management processes*

Service-Profit Chain Model

- States that employees create customer value and drive profitability through a service-delivery system
- Based on a set of cause-and-effect linkages between internal and external performance
    - *Helps define key performance measurements on which service-based firms should focus*

Exhibit 2.8 Service-Profit Chain Model

EXHIBIT 2.8 The Service-Profit Chain Model



Adapted from J. L. Heskett, T. O. Jones, G. W. Loveman, W. E. Sasser, Jr., and L. A. Schlesinger, "Putting the Service-Profit Chain to Work," Harvard Business Review, March–April 1994, pp. 164–174.

KEY TERMS

- Measurement
- Customer-satisfaction measurement system
- Quality
- Goods quality
- Service quality
- Service failures/upsets
- Processing time
- Queue/wait time
- Flexibility

KEY TERMS

- Goods and service design flexibility
- Volume flexibility
- Innovation
- Learning
- Productivity
- Operational efficiency
- Triple bottom line (TBL or 3BL)
- Interlinking
- Value of a loyal customer (VLC)
- Actionable measures

SUMMARY

- Applications of business analytics help managers with effective decisions
- Interlinking helps quantify performance relationships between all parts of a value chain
- VLC helps understand operational decisions on revenue and customer retention
- Four models of organizational performance help in designing, monitoring, and evaluating performance