Chapter 2 Solutions

2.1 $\frac{\partial^2\psi}{\partial z^2} = \frac{1}{\upsilon^2}\frac{\partial^2\psi}{\partial t^2}$

$\frac{\partial\psi}{\partial z} = 2(z+\upsilon t)$

$\frac{\partial^2\psi}{\partial z^2} = 2$

$\frac{\partial\psi}{\partial t} = 2\upsilon(z+\upsilon t)$

$\frac{\partial^2\psi}{\partial t^2} = 2\upsilon^2$

It's a twice differentiable function of $(z-\upsilon t)$, where υ is in the negative z direction.

2.2 $\frac{\partial^2\psi}{\partial y^2} = \frac{1}{\upsilon^2}\frac{\partial^2\psi}{\partial t^2}$

$\psi(y,t) = (y-4t)^2$

$\frac{\partial\psi}{\partial y} = 2(y-4t)$

$\frac{\partial^2\psi}{\partial y^2} = 2$

$\frac{\partial\psi}{\partial t} = -8(y-4t)$

$\frac{\partial^2\psi}{\partial t^2} = 32$

Thus, $\upsilon = 4, \upsilon^2 = 16,$ and,

$$\frac{\partial^2\psi}{\partial y^2} = 2 = \frac{1}{16}\frac{\partial^2\psi}{\partial t^2}$$

The velocity is $\nu = 4$ in the positive y direction.

2.3 Starting with:

$\psi(z, t) = \frac{A}{(z-\upsilon t)^2+1}$

$\frac{\partial^2\psi}{\partial z^2} = \frac{1}{\upsilon^2}\frac{\partial^2\psi}{\partial t^2}$

$\frac{\partial\psi}{\partial z} = -2A\frac{(z-\upsilon t)}{[(z-\upsilon t)^2+1]}$

$$\frac{\partial^2\psi}{\partial z^2} = -2A\left[\frac{-2(z-\upsilon t)^2}{[(z-\upsilon t)^2+1]^3}+\frac{1}{[(z-\upsilon t)^2+1]^2}\right]$$

$$= -2A\left[\frac{-4(z-\upsilon t)^2}{[(z-\upsilon t)^2+1]^3}+\frac{(z-\upsilon t)^2+1}{[(z-\upsilon t)^2+1]^3}\right]$$

$$= 2A\frac{3(z-\upsilon t)^2-1}{[(z-\upsilon t)^2+1]^3}$$

$$\frac{\partial \psi}{\partial t} = 2A\upsilon \frac{(z-\upsilon t)}{[(z-\upsilon t)^2+1]^2}$$

$$\frac{\partial^2 \psi}{\partial t^2} = 2A\upsilon \frac{\partial}{\partial t}\left(\frac{(z-\upsilon t)}{[(z-\upsilon t)^2+1]^2}\right)$$

$$= 2A\upsilon\left[\frac{-\nu}{[(z-\upsilon t)^2+1]^2} + (z-\upsilon t)\frac{4\nu(z-\upsilon t)}{[(z-\upsilon t)^2+1]^3}\right]$$

$$= 2A\upsilon\left[\frac{-\nu[(z-\upsilon t)^2+1]}{[(z-\upsilon t)^2+1]^2} + \frac{4\nu(z-\upsilon t)^2}{[(z-\upsilon t)^2+1]^3}\right]$$

$$= 2A\upsilon^2 \frac{3(z-\upsilon t)^2-1}{[(z-\upsilon t)^2+1]^3}$$

Thus since

$$\frac{\partial^2 \psi}{\partial z^2} = \frac{1}{\upsilon^2}\frac{\partial^2 \psi}{\partial t^2}$$

The wave moves with velocity υ in the positive z direction.

2.4 $c = \nu\lambda$

$$\nu = \frac{c}{\lambda} = \frac{3\times 10^8\ m/s}{5.145\times 10^{-7}\ m} = 5.831\times 10^{14}\ Hz$$

2.5 Starting with:

$\psi(y,\ t) = A\exp[-a(by-ct)^2]$

$\psi(y,\ t) = A\exp[a(by-ct)^2] = A\exp[a(by-ct)^2]$

$$\frac{\partial \psi}{\partial t} = -\frac{2Aa}{b^2}\frac{c}{b}\left(y-\frac{c}{b}t\right)\exp[a(by-ct)^2]$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{4Aa^2}{b^4}\frac{c^2}{b^2}\left(y-\frac{c}{b}t\right)^2 \exp[a(by-ct)^2]$$

$$\frac{\partial \psi}{\partial y} = -\frac{2Aa}{b^2}\left(y-\frac{c}{b}t\right)\exp[a(by-ct)^2]$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{4Aa^2}{b^4}\left(y-\frac{c}{b}t\right)^2 \exp[a(by-ct)^2]$$

Thus $\psi(y,\ t) = A\exp[-a(by-ct)^2]$ is a solution of the wave equation with $\upsilon = c/b$ in the $+\,y$ direction.

2.6 $(0.003)\ (2.54\times 10^{-2}/580\times 10^{-9})$ = number of waves = 131, $c = \nu\lambda$,

$\lambda = c/\nu = 3\times 10^8/10^{10}$, $\lambda = 3$ cm. Waves extend 3.9 m.

2.7 $\lambda = c/\nu = 3\times 10^8/5\times 10^{14} = 6\times 10^{-7}$ m = 0.6μ m.

$\lambda = 3\times 10^8/60 = 5\times 10^6$ m = 5×10^3 km.

2.8 $\upsilon = \lambda\nu = 5\times 10^{-7}\times 6\times 10^8 = 300$ m/s.

2.9 The time between the crests is the period, so $\tau = 1/2$ s; hence $\nu = 1/\tau = 2.0$ Hz. As for the speed $\upsilon = L/t = 4.5$ m/1.5 s $= 3.0$ m/s. We now know τ, ν, and υ and must determine λ. Thus, $\lambda = \upsilon/\nu = 3.0$ m/s/2.0 Hz $= 1.5$ m.

2.10 $\upsilon = \nu\lambda = 3.5 \times 10^3$ m/s $= \nu(4.3$ m$)$; $\nu = 0.81$ kHz.

2.11 $\upsilon = \nu\lambda = 1498$ m/s $= (440$ HZ$)\ \lambda$; $\lambda = 3.40$ m.

2.12 $\upsilon = (10$ m$)/2.0$ s$) = 5.0$ m/s; $\nu = \upsilon/\lambda = (5.0$ m/s$)/(0.50$ m$) = 10$ Hz.

2.13 $\upsilon = \nu\lambda = (\omega/2\pi)\ \lambda$ and so $\omega = (2\pi/\lambda)\upsilon$.

2.14

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$
$\sin\theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$
$\cos\theta$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta - \pi/4)$	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta - \pi/2)$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$
$\sin(\theta - 3\pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\sin(\theta + \pi/2)$	0	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$

θ	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$	0
$\cos\theta$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1
$\sin(\theta - \pi/4)$	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1	$-\sqrt{2}/2$
$\sin(\theta - \pi/2)$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$	-1
$\sin(\theta - 3\pi/4)$	$\sqrt{2}/2$	1	$\sqrt{2}/2$	0	$-\sqrt{2}/2$
$\sin(\theta + \pi/2)$	-1	$-\sqrt{2}/2$	0	$\sqrt{2}/2$	1

$\sin\theta$ leads $\sin(\theta - \pi/2)$.

2.15

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
$kx = \dfrac{2\pi x}{\lambda}$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos(kx - \pi/4)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\cos(kx + 3\pi/4)$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$

2.16

t	$-\tau/2$	$-\tau/4$	0	$\tau/4$	$\tau/2$	$3\tau/4$	τ
$\omega t = (2\pi/\tau)t$	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\sin(\omega t + \pi/4)$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$
$\sin(\pi/4 - \omega t)$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$

2.17 Comparing y with Eq. (2.13) tells us that $A = 0.02$ m. Moreover, $2\pi/\lambda = 157\ \text{m}^{-1}$ and so $\lambda = 2\pi/(157\text{m}^{-1}) = 0.0400$ m. The relationship between frequency and wavelength is $\upsilon = \nu\lambda$, and so $\nu = \upsilon/\lambda = (1.2\ \text{m/s})/0.0400\ \text{m} = 30$ Hz. The period is the inverse of the frequency, and therefore $\tau = 1/\nu = 0.033$ s.

2.18 (a) $\lambda = (4.0 - 0.0)\ \text{m} = 4.0$ m

(b) $\upsilon = \nu\lambda$, so

$$\nu = \frac{\upsilon}{\lambda} = \frac{20\ \text{m/s}}{4.0\ \text{m}} = 5.0\ \text{Hz}$$

(c) $\psi(x,t) = A\sin(kx - \omega t + \varepsilon)$

From the figure, A = 0.020 m

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4.0\ \text{m}} = 0.5\pi\ \text{m}^{-1};\ \omega = 2\pi\nu = 2\pi(5.0\ \text{Hz}) = 10\pi\ \text{rad}/s$$

$$\psi(x,t) = [0.020\ \text{m}]\sin\left(\frac{\pi}{2}x - 10\pi t - \frac{\pi}{2}\right) = 0.020\cos\left(\frac{\pi}{2}x - 10\pi t\right)$$

2.19 (a) $\lambda = (30.0 - 0.0)$ cm = 30.0 cm. (c) $\upsilon = \nu\lambda$, so $\nu = \upsilon/\lambda = (100\ \text{cm/s})/(30.0\ \text{cm}) = 3.33$ Hz

2.20 (a) $\tau = (0.20 - 0.00)$ s = 0.20 s. (b) $\nu = 1/\tau = 1/(0.20\ \text{s}) = 5.00$ Hz. (c) $\upsilon = \nu\lambda$, so $\lambda = \upsilon/\nu = (40.0\ \text{cm/s})/(5.00\ \text{s}^{-1}) = 8.00$ cm.

2.21 $\psi = A \sin 2\pi(kx - \nu t)$, $\psi_1 = 4\sin 2\pi(0.2x - 3t)$. (a) $\nu = 3$, (b) $\lambda = 1/0.2$, (c) $\tau = 1/3$, (d) $A = 4$, (e) $\upsilon = 15$, (f) positive x
$\psi = A \sin(kx + \omega t)$, $\psi_2 = (1/2.5) \sin(7x + 3.5t)$. (a) $\nu = 3.5/2\pi$, (b) $\lambda = 2\pi/7$, (c) $\tau = 2\pi/3.5$, (d) $A = 1/2.5$, (e) $\upsilon = 1/2$, (f) negative x

2.22 From of Eq. (2.26) $\psi(x, t) = A \sin(kx - \omega t)$ (a) $\omega = 2\pi\nu$, so $\nu = \omega/2\pi = (20.0\ \text{rad/s})/2\pi$, (b) $k = 2\pi/\lambda$, so $\lambda = 2\pi/k = 2\pi/(6.28\ \text{rad/m}) = 1.00$ m, (c) $\nu = 1/\tau$, so $\tau = 1/\nu = 1/(10.0/\pi\ \text{Hz}) = 0.10\pi\, s$, (d) From the form of ψ, $A = 30.0$ cm, (e) $\upsilon = \omega/k = (20.0\ \text{rad/s})/(6.28\ \text{rad/m}) = 3.18$ m/s, (f) Negative sign indicates motion in $+x$ direction.

2.23 (a) 10, (b) 5.0×10^{14} Hz, (c) $\lambda = \frac{c}{\nu} = \frac{3.0\times10^{8}}{5.0\times10^{14}} = 6.0\times10^{-7}\,\text{m}$, (d) 3.0×10^{8} m/s,

(e) $\frac{1}{\nu} = \tau = 2.0\times10^{-15}\,s$, (f) $-y$ direction

2.24 $\partial^2\psi/\partial x^2 = -k^2\psi$ and $\partial^2\psi/\partial t^2 = -k^2\upsilon^2\psi$. Therefore $\partial^2\psi/\partial x^2 - (1/\upsilon^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$.

2.25 $\partial^2\psi/\partial x^2 = -k^2\psi; \partial^2\psi/\partial t^2 = -\omega^2\psi; \omega^2/\upsilon^2 = (2\pi\nu)^2/\upsilon^2 = (2\pi/\lambda)^2 = k^2$; therefore, $\partial^2\psi/\partial x^2 - (1/\upsilon^2)\partial^2\psi/\partial t^2 = (-k^2 + k^2)\psi = 0$.

2.26 $\psi(x, t) = A \cos(kx - \omega t - (\pi/2)) =$
$A\{\cos(kx - \omega t) \cos(-\pi/2) - \sin(kx - \omega t) \sin(-\pi/2)\} = A \sin(kx - \omega t)$

2.27 $\upsilon_y = -\omega A \cos(kx - \omega t + \varepsilon)$, $a_y = -\omega^2 y$. Simple harmonic motion since $a_y \propto y$.

2.28 $\tau = 2.2 \times 10^{-15}$ s; therefore $\nu = 1/\tau = 4.5 \times 10^{14}$ Hz; $\upsilon = \nu\lambda$,
$\lambda = \upsilon/\nu = 6.7 \times 10^{-7}$ m and $k = 2\pi/\lambda = 9.4 \times 10^{6}$ m^{-1}.
$\psi(x, t) = (10^3 V/m) \cos[9.4 \times 10^6 \text{m}^{-1}(x + 3 \times 10^8 (\text{m/s})t)]$. It's cosine because cos 0 = 1.

2.29 $y(x, t) = C/[2 + (x + \upsilon t)^2]$.

2.30 $\psi(0, t) = A\cos(k\upsilon t + \pi) = -A\cos(k\upsilon t) = -A\cos(\omega t)$, then
$\psi(0, \tau/2) = -A\cos(\omega\tau/2) = -A\cos(\pi) = A$,
$\psi(0, 3\tau/4) = -A\cos(3\omega\tau/4) = -A\cos(3\pi/2) = 0$.

2.31 Since $\psi(y, t) = (y - \upsilon t) A$ is only a function of $(y - \upsilon t)$, it does satisfy the conditions set down for a wave. Since $\partial^2\psi/\partial y^2 = \partial^2\psi/\partial t^2 = 0$, this function is a solution of the wave equation. However, $\psi(y, 0) = Ay$ is unbounded, so cannot represent a localized wave profile.

2.32 $k = \pi 3 \times 10^6$ m^{-1}, $\omega = \pi 9 \times 10^{14}$ Hz, $\upsilon = \omega/k = 3 \times 10^8$ m/s.

2.33 $\nu = \upsilon\lambda = \lambda/\tau$

$\lambda = \nu\tau = (2.0 \text{ m/s})(1/4 \text{ s}) = 0.5 \text{ m}$

$$\psi(z,t) = (0.020 \text{ m})\sin 2\pi\left(\frac{z}{0.50 \text{ m}} + \frac{t}{1/4 \text{ s}}\right)$$

$$\psi(z,t) = (0.020 \text{ m})\sin 2\pi\left(\frac{1.5 \text{ m}}{0.50 \text{ m}} + \frac{2.2 \text{ s}}{1/4 \text{ s}}\right)$$

$\psi(z, t) = (0.020 \text{ m}) \sin 2\pi(3.0 + 8.8)$
$\psi(z, t) = (0.020 \text{ m}) \sin 2\pi(11.8)$
$\psi(z, t) = (0.020 \text{ m}) \sin 23.6\pi$
$\psi(z, t) = (0.020 \text{ m}) (-0.9511)$
$\psi(z, t) = -0.019 \text{ m}$

2.34 $d\psi/dt = (\partial\psi/\partial x)(dx/dt) + (\partial\psi/\partial y)(dy/dt)$ and let $y = t$ whereupon
$d\psi/dt = \partial\psi/\partial x(\pm\upsilon) + \partial\psi/\partial t = 0$ and the desired result follows immediately.

2.35 $d\varphi/dt = (\partial\varphi/\partial x)(dx/dt) + \partial\varphi/\partial t = 0 = k(dx/dt) - k\upsilon$ and this is zero provided $dx/dt = \pm\upsilon$, as it should be. For the particular wave of Problem 2.32, $\dfrac{d\varphi}{dt} = \partial\varphi/\partial y(\pm\upsilon) + \partial\varphi/\partial t = \pi 3 \times 10^6(\pm\upsilon) + \pi 9 \times 10^{14} = 0$
and the speed is -3×10^8 m/s.

2.36 $-a(bx + ct)^2 = -ab^2(x + ct/b)^2 = g(x + \upsilon t)$ and so $\upsilon = c/b$ and the wave travels in the negative x-direction. Using Eq. (2.34) $\left(\partial\psi/\partial t\right)_x / \left(\partial\psi/\partial x\right)_t =$
$-[A(-2a)(bx+ct)c\exp[-a(bx+ct)^2]]/[A(-2a)(bx+ct)b\exp[-a(bx+ct)^2]] = -c/b$;
the minus sign tells us that the motion is in the negative x-direction.

2.37 $\psi(z, 0) = A\sin(kz + \varepsilon)$; $\psi(-\lambda/12, 0) = A\sin(-\pi/6 + \varepsilon) = 0.866$;
$\psi(\lambda/6, 0) = A\sin(\pi/3 + \varepsilon) = 1/2$; $\psi(\lambda/4, 0) = A\sin(\pi/2 + \varepsilon) = 0$.
$A\sin(\pi/2 + \varepsilon) = A(\sin\pi/2\cos\varepsilon + \cos\pi/2\sin\varepsilon) = A\cos\varepsilon = 0$, $\varepsilon = \pi/2$.
$A\sin(\pi/3 + \pi/2) = A\sin(5\pi/6) = 1/2$; therefore $A = 1$, hence
$\psi(z, 0) = \sin(kz + \pi/2)$.

2.38 Both (a) and (b) are waves since they are twice differentiable functions of $z - \upsilon t$ and $x + \upsilon t$, respectively. Thus for (a) $\psi = a^2(z - bt/a)^2$ and the velocity is b/a in the positive z-direction. For (b) $\psi = a^2(x + bt/a + c/a)^2$ and the velocity is b/a in the negative x-direction.

2.39 (a) $\psi(y, t) = \exp[-(ay - bt)^2]$, a traveling wave in the $+y$ direction, with speed $\upsilon = \omega/k = b/a$. (b) not a traveling wave. (c) traveling wave in the $-x$ direction, $\upsilon = a/b$, (d) traveling wave in the $+x$ direction, $\upsilon = 1$.

2.40 $\psi(x, t) = 5.0 \exp[-a(x + \sqrt{b/a}t)^2]$, the propagation direction is negative x; $\upsilon = \sqrt{b/a} = 0.6$ m/s. $\psi(x, 0) = 5.0 \exp(-25x^2)$.

2.41 $\lambda = \upsilon/\nu = 0.300$ m; 10.0 cm is a fraction of a wavelength viz. (0.100 m)/(0.300 m) = 1/3; hence $2\pi/3 = 2.09$ rad.

2.42 30° corresponds to $\lambda/12$ or $\left(\frac{1}{12}\right)\left(\frac{3\times10^8}{6\times10^{14}}\right) = 42\text{nm}$.

2.43 $\psi(x, t) = A \sin 2\pi(x/\lambda \pm t/\tau)$, $\psi = 60 \sin 2\pi(x/400 \times 10^{-9} - t/1.33 \times 10^{-15})$,
$\lambda = 400$ nm, $\upsilon = 400 \times 10^{-9}/1.33 \times 10^{-15} = 3 \times 10^8$ m/s.
$\nu = (1/1.33) \times 10^{15}$ Hz, $\tau = 1.33 \times 10^{-15}$ s.

2.44 $\exp[i\alpha]\exp[i\beta] = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta) = (\cos\alpha\cos\beta - \sin\alpha\sin\beta) +$
$i(\sin\alpha\cos\beta + \cos\alpha\sin\beta) = \cos(\alpha+\beta) + i\sin(\alpha+\beta) = \exp[i(\alpha+\beta)]$
$\psi\psi^* = A\exp[i\omega t]\,A\exp[-i\omega t] = A^2$; $\sqrt{\psi\psi^*} = A$. In terms of Euler's formula
$\psi\psi^* = A^2(\cos\omega t + i\sin\omega t)(\cos\omega t - i\sin\omega t) = A^2(\cos^2\omega t + \sin^2\omega t) = A^2$.

2.45 If $z = x + iy$, then $z^* = x - iy$ and $z - z^* = 2yi$.

2.46
$$\tilde{z}_1 = x_1 + iy_1$$
$$\tilde{z}_2 = x_2 + iy_2$$
$$\tilde{z}_1 + \tilde{z}_2 = x_1 + x_2 + iy_1 + iy_2$$
$$\text{Re}(\tilde{z}_1 + \tilde{z}_2) = x_1 + x_2$$
$$\text{Re}(\tilde{z}_1) + \text{Re}(\tilde{z}_2) = x_1 + x_2$$

2.47
$$\tilde{z}_1 = x_1 + iy_1$$
$$\tilde{z}_2 = x_2 + iy_2$$
$$\text{Re}(\tilde{z}_1) \times \text{Re}(\tilde{z}_2) = x_1x_2$$
$$\text{Re}(\tilde{z}_1 \times \tilde{z}_2) = \text{Re}(x_1x_2 + ix_1y_2 + ix_2y_1 - y_1y_2) = x_1x_2 - y_1y_2$$
Thus $\text{Re}(\tilde{z}_1) \times \text{Re}(\tilde{z}_2) \neq \text{Re}(\tilde{z}_1 \times \tilde{z}_2)$.

2.48 $\psi = A \exp i(k_xx + k_yy + k_zz)$, $k_x = k\alpha$, $k_y = k\beta$, $k_z = k\gamma$,
$|\vec{k}| = [(k\alpha)^2 + (k\beta)^2 + (k\gamma)^2]^{1/2} = k(\alpha^2 + \beta^2 + \gamma^2)^{1/2}$.

2.49 Consider Eq. (2.64), with $\partial^2\psi/\partial x^2 = \alpha^2 f''$, $\partial^2\psi/\partial y^2 = \beta^2 f''$,
$\partial^2\psi/\partial z^2 = \gamma^2 f''$, $\partial^2\psi/\partial t^2 = \upsilon^2 f''$. Then
$\nabla^2\psi - (1/\upsilon^2)\partial^2\psi/\partial t^2 = (\alpha^2 + \beta^2 + \gamma^2 - 1)f'' = 0$ whenever
$\alpha^2 + \beta^2 + \gamma^2 = 1$.

**************************<<INSERT MATTER OF 2.50 IS MISSING>>************************

2.51 Consider the function: $\psi(z, t) = A\exp[-(a^2z^2 + b^2t^2 + 2abzt)]$.
Where A, a, and b are all constants. First factor the exponent:

$$(a^2z^2 + b^2t^2 + 2abzt) = (az + bt)^2 = \frac{1}{a^2}\left(z + \frac{b}{a}t\right)^2.$$

Thus,
$$\psi(z,t) = A\exp\left[-\frac{1}{a^2}\left(z + \frac{b}{a}t\right)^2\right].$$

This is a twice differentiable function of $(z - \upsilon t)$, where $\upsilon = -b/a$, and travels in the $-z$ direction.

2.52 $\lambda = (h/m)\upsilon = 6.6 \times 10^{-34}/6(1) = 1.1 \times 10^{-34}$ m.

2.53 $\vec{k}$ can be constructed by forming a unit vector in the proper direction and multiplying it by k. The unit vector is
$[(4-0)\hat{i} + (2-0)\hat{j} + (1-0)\hat{k}]/\sqrt{4^2+2^2+1^2} = (4\hat{i} + 2\hat{j} + \hat{k})/\sqrt{21}$ and
$\vec{k} = k(4\hat{i} + 2\hat{j} + \hat{k})/\sqrt{21}$. $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, hence
$\psi(x, y, z, t) = A\sin[(4k/\sqrt{21})x + (2k/\sqrt{21})y + (k/\sqrt{21})z - \omega t]$.

2.54 $\vec{k} = (1\hat{\mathrm{i}} + 0\hat{\mathrm{j}} + 0\hat{k}), \vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{k}$, so,
$\psi = A\sin(\vec{k}\cdot\vec{r} - \omega t + \varepsilon) = A\sin(kx - \omega t + \varepsilon)$ where $k = 2\pi/\lambda$ (could use cos instead of sin).

2.55 $\psi(\vec{r}_1, t) = \psi[\vec{r}_2 - (\vec{r}_2 - \vec{r}_1), t] = \psi(\vec{k}\cdot\vec{r}_1, t) = \psi[\vec{k}\cdot\vec{r}_2 - \vec{k}\cdot(\vec{r}_2 - \vec{r}_1), t] =$
$\psi(\vec{k}\cdot\vec{r}_2, t) = \psi(\vec{r}_2, t)$ since $\vec{k}\cdot(\vec{r}_2 - \vec{r}_1) = 0$

2.56 $\psi = A\exp[i(\vec{k}\cdot\vec{r} + \omega t + \varepsilon)]$
$= A\exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$
The wave equation is:

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2\psi = -(k_x{}^2 + k_y{}^2 + k_z{}^2)A\exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2 A\exp\left[i\left(k_x x + k_y y + k_z z + \omega t + \varepsilon\right)\right]$$

where

$$|k| = \sqrt{k_x{}^2 + k_y{}^2 + k_z{}^2}$$

$$k^2 = k_x{}^2 + k_y{}^2 + k_z{}^2$$

then,

$$\nabla^2\psi = -k^2 A\exp[i(k_x x + k_y y + k_z z + \omega t + \varepsilon)]$$

This means that ψ is a solution of the wave equation if $v^2 = \omega^2/k^2 \rightarrow v = \omega/k$.

2.57

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0
$2\sin\theta$	-2	$-\sqrt{2}$	0	$\sqrt{2}$	2	$\sqrt{2}$	0	$-\sqrt{2}$	-2	$-\sqrt{2}$	0
$3\sin\theta$	-3	$-3/\sqrt{2}$	0	$3/\sqrt{2}$	3	$3/\sqrt{2}$	0	$-3/\sqrt{2}$	-3	$-3/\sqrt{2}$	0

2.58

θ	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$\sin\theta$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0
$\sin(\theta-\pi/2)$	0	$-1/\sqrt{2}$	-1	$-1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$	0	$-1/\sqrt{2}$	-1
$\sin\theta+\sin(\theta-\pi/2)$	-1	$-\sqrt{2}$	-1	0	1	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	-1

2.59 Note that the amplitude of $\{\sin(\theta)+\sin(\theta-\pi/2)\}$ is greater than 1, while the amplitude of $\{\sin(\theta)+\sin(\theta-3\pi/4)$ is less than 1. The phase difference is $\pi/8$.

2.60

x	$-\lambda/2$	$-\lambda/4$	0	$\lambda/4$	$\lambda/2$	$3\lambda/4$	λ
kx	$-\pi$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$	2π
$\cos kx$	-1	0	1	0	-1	0	1
$\cos(kx+\pi)$	1	0	-1	0	1	0	-1
$\cos kx+\cos(kx+\pi)$	0	0	0	0	0	0	0