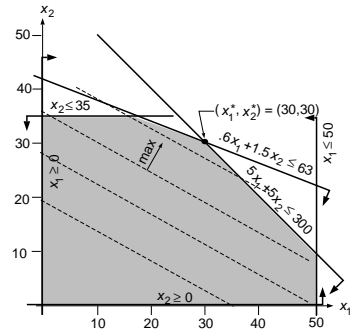


### Chapter 2 Solutions <sup>1 2</sup>

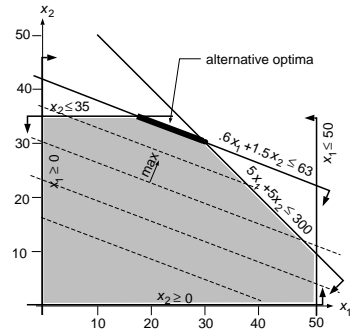
**2-1. (a)** max  $200x_1 + 350x_2$  (max total profit), s.t.  $5x_1 + 5x_2 \leq 300$  (legs),  $0.6x_1 + 1.5x_2 \leq 63$  (assembly hours),  $x_1 \leq 50$  (wood tops),  $x_2 \leq 35$  (glass tops),  $x_1 \geq 0$ ,  $x_2 \geq 0$

**(b)**  $x_1^*$ =basic=30,  $x_2^*$ =deluxe=30

**(c)**



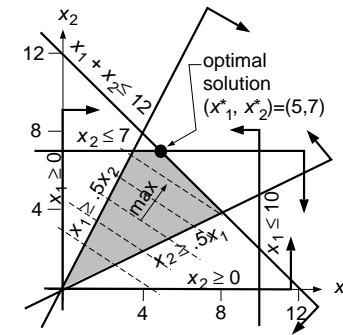
**(d)**



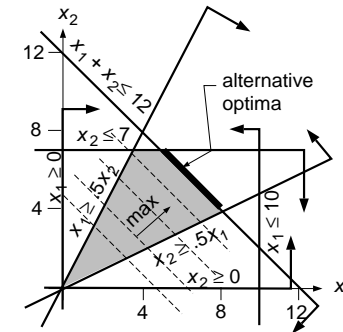
All optimal from  $\mathbf{x} = (30, 30)$  to  $\mathbf{x} = (17.5, 35)$ .

**2-2. (a)** max  $.11x_1 + .17x_2$  (max total return), s.t.  $x_1 + x_2 \leq 12$  (\$12 million investment),  $x_1 \leq 10$  (max \$10 million domestic),  $x_2 \leq 7$  (max \$7 million foreign),  $x_1 \geq .5x_2$  (domestic at least half foreign),  $x_2 \geq .5x_1$  (foreign at least half domestic),  $x_1 \geq 0, x_2 \geq 0$  **(b)**  $x_1^*$ =domestic=\$5 million,  $x_2^*$ = foreign=\$7 million

**(c)**



**(d)**



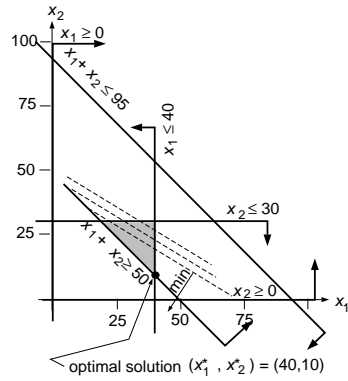
All optimal from  $\mathbf{x} = (5, 7)$  to  $\mathbf{x} = (8, 4)$ .

**2-3. (a)** min  $3x_1 + 5x_2$  (min total cost), s.t.  $x_1 + x_2 \geq 50$  (at least 50 thousand acres),  $x_1 \leq 40$  (at most 40 thousand from Squawking Eagle),  $x_2 \leq 30$  (at most 30 thousand from Crooked Creek),  $x_1 \geq 0, x_2 \geq 0$  **(b)**  $x_1^*$ =Squawking Eagle=40 thousand,  $x_2^*$ =Crooked Creek=10 thousand

<sup>1</sup>Supplement to the 2nd edition of *Optimization in Operations Research*, by Ronald L. Rardin, Pearson Higher Education, Hoboken NJ, ©2017.

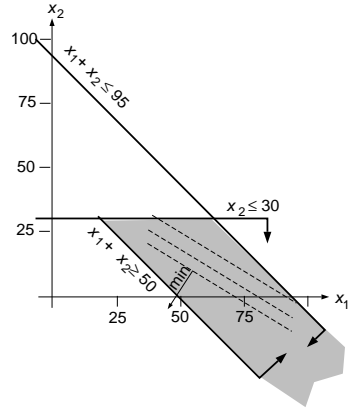
<sup>2</sup>As of September 24, 2015

(c)

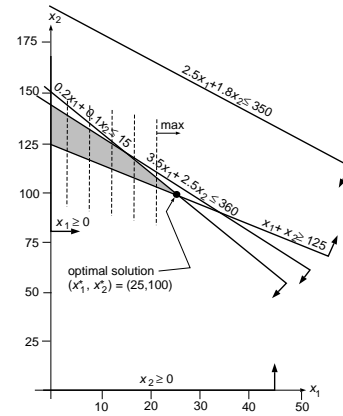


$x_1 + x_2 \geq 125$  (weight at least 125),  
 $2.5x_1 + 1.8x_2 \leq 350$  (calories at most 350),  
 $0.2x_1 + 0.1x_2 \leq 15$  (fat at most 15),  
 $3.5x_1 + 2.5x_2 \leq 360$  (sodium at most 360),  
 $x_1 \geq 0, x_2 \geq 0$  (b)  $x_1^*$ =beef=25g,  
 $x_2^*$ =chicken=100g

(d)

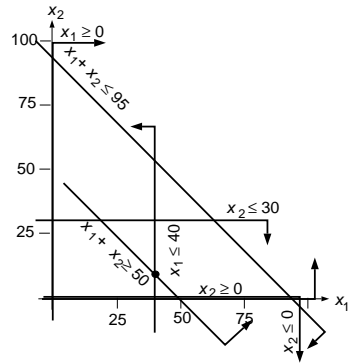


(c)



Improves forever in direction  $\Delta x_1 = 1, \Delta x_2 = -1$ .

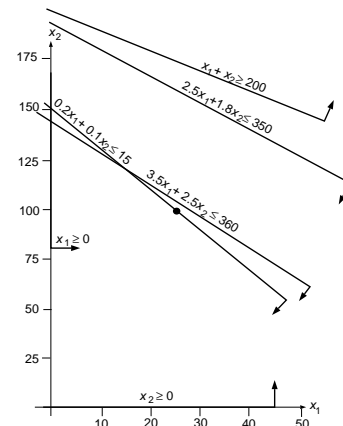
(e)



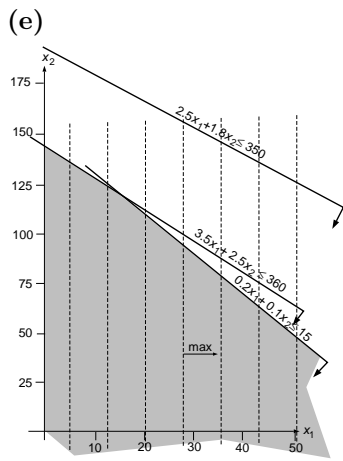
$x_2 = 0$  leaves no feasible.

2-4. (a) max  $x_1$  (max beef content), s.t.

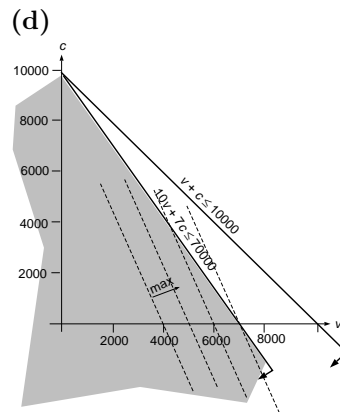
(d)



$x_1 + x_2 \geq 200$  leaves no feasible.

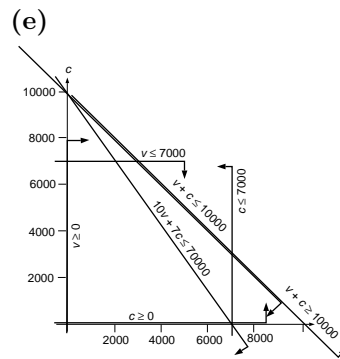


Improve forever in direction  $\Delta x_1 = 1$ ,  $\Delta x_2 = -2$ .

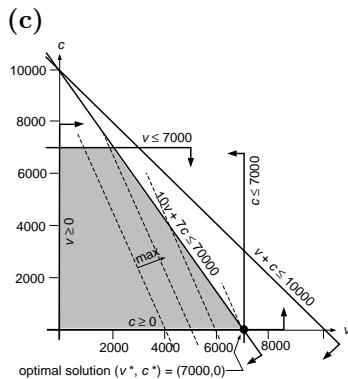


Improves forever in direction  $\Delta v = 10$ ,  $\Delta c = -7$ .

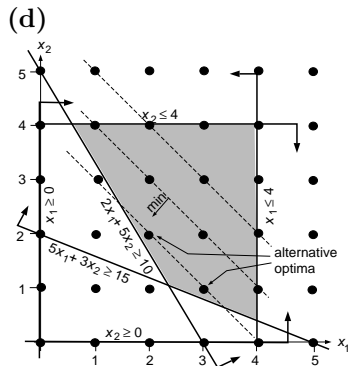
**2-5. (a)** max  $450v + 200c$  (max total profit),  
 s.t.  $10v + 7c \leq 70000$  (water at most 70000 units),  
 $v + c \leq 10000$  (total acreage 10000),  
 $v \leq 7000$  (at most 70% vegetables),  
 $c \leq 7000$  (at most 70% cotton),  
 $v \geq 0, c \geq 0$  **(b)**  
 $v^* = 7000, c^* = 0$



No solution with  $v + c = 10000$ .

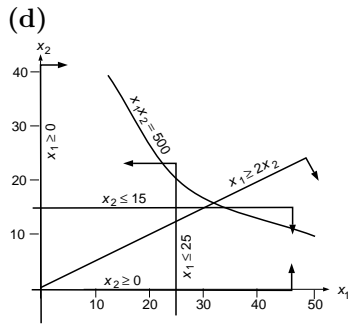
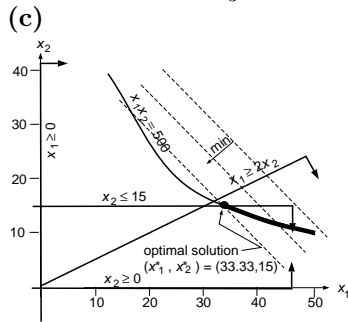


**2-6. (a)** min  $x_1 + x_2$  (min used stock), s.t.  
 $5x_1 + 3x_2 \geq 15$  (cut at least 15 long rolls),  
 $2x_1 + 5x_2 \geq 10$  (cut at least 10 short rolls),  
 $x_1 \leq 4$  (at most 4 times on pattern 1),  
 $x_2 \leq 4$  (at most 4 times on pattern 2),  
 $x_1, x_2 \geq 0$  and integer. **(b)** Partial cuts make no physical sense because all unused material is scrap. **(c)**  
 Either  $x_1^* = x_2^* = 2$ , or  $x_1^* = 3, x_2^* = 1$



(e) Both (2, 2) and (3, 1) are feasible and lie on the best contour of the objective.

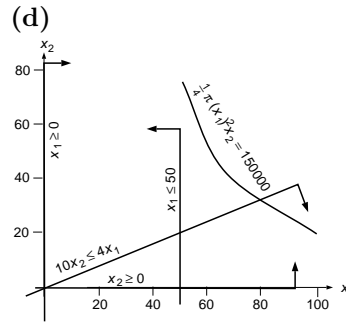
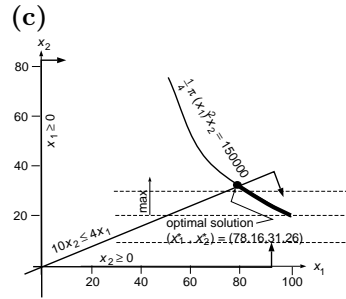
**2-7. (a)** min  $16x_1 + 16x_2$  (min total wall area), s.t.  $x_1x_2 = 500$  (500 sqft pool),  $x_1 \geq 2x_2$  (length at least twice width),  $x_2 \leq 15$  (width at most 15 ft),  $x_1 \geq 0, x_2 \geq 0$   
**(b)**  $x_1^*$ =length= $33\frac{1}{3}$  feet,  $x_2^*$ =width=15 feet



$x_1 \leq 25$  leaves no feasible.

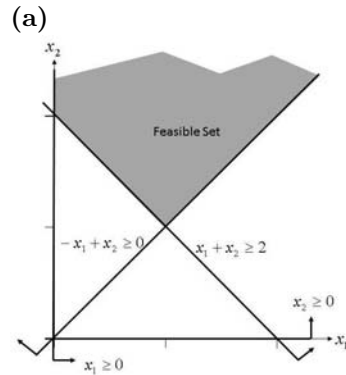
**2-8. (a)** max  $x_2$  (max number of floors), s.t.  $\pi/4(x_1)^2x_2 = 150000$  (150000 sqft floor space),  $10x_2 \leq 4x_1$  (height at most 4 times diameter),  $x_1 \geq 0, x_2 \geq 0$  **(b)**  $x_1^*$  = diameter

= 78.16 feet,  $x_2^*$  = floors = 31.26



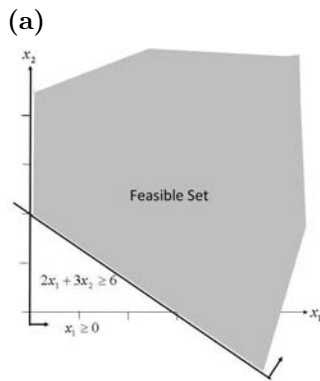
$x_1 \leq 50$  leaves no feasible.

**2-9.**



**(b)** min  $x_2$  **(c)** min  $x_1 + x_2$  **(d)** max  $x_2$  **(e)**  $x_2 \leq 1/2$

**2-10.**



(b)  $\min x_1 + x_2$  (c)  $\min x_1$  (d)  $\max x_1$  (e)  $x_1 + x_2 \leq 1$

2-11. (a)  $\min \sum_{i=3}^4 i \sum_{j=1}^2 y_{i,j}$

(b)  $\max \sum_{i=1}^4 i y_{i,3}$

(c)  $\max \sum_{i=1}^p \alpha_i y_{i,4}$

(d)  $\min \sum_{i=1}^t \delta_i y_i$

(e)  $\sum_{j=1}^4 y_{i,j} = s_i, i = 1, \dots, 3$

(f)  $\sum_{j=1}^4 a_{j,i} y_j = c_i, i = 1, \dots, 3$

2-12. (a)

$\sum_{i=1}^{17} x_{i,j,t} \leq 200, j = 1, \dots, 5; t = \dots, 7; 35$   
constraints

(b)  $\sum_{j=1}^5 \sum_{t=1}^7 x_{5,j,t} \leq 4000; 1$  constraint

(c)

$\sum_{j=1}^5 x_{i,j,t} \geq 100, i = 1, \dots, 17; t = 1, \dots, 7;$   
119 constraints

2-13. model; param m; param n; param p;  
set products := 1 .. m; set lines := 1 .. n;  
set weeks := 1 .. p; var x{i in products, j in lines, t in weeks} >= 0; subject to

# part (a)

linecap {j in lines, t in weeks}: sum {i in products} x[i,j,t] <= 200;

# part (b)

prod5lim: sum {j in lines, t in weeks} x[5,j,t] <= 4000;

# part (c)

minprodn{i in products, t in weeks}: sum {j in lines} x[i,j,t] >= 100;

#

data; param m := 17; param n := 5; param p := 7;

2-14. (a)

$\sum_{j=1}^9 x_{i,j,t} \leq p_i, i = 1, \dots, 47; t = 1, \dots, 10;$   
470 constraints

(b)  $0.25 \sum_{i=1}^{47} \sum_{j=1}^9 x_{i,j,t} \leq \sum_{i=1}^{47} x_{i,4,t}; t = 1, \dots, 5; 5$  constraints

(c)  $x_{i,1,t} \geq x_{i,j,t} i = 1, \dots, 47; j = 1, \dots, 9; t = 1, \dots, 10; 4230$  constraints

2-15. model; param m; param n; param q;  
set plots := 1 .. m; set crops := 1 .. n;  
set years := 1 .. q; param p {i in plots};  
var x{i in plots, j in crops, t in years} >= 0; subject to

# part (a)  
acrelims {i in plots, t in years}:  
sum {j in crops} x[i,j,t] <= p[i];

# part (b)  
crop4min {t in years: t <= 5}:  
0.25\* sum {i in plots, j in crops} x[i,j,t] <= sum {i in plots} x[i,4,t];

# part (c)  
beam1st {i in plots, j in crops, t in years}:  
x[i,1,t] >= x[i,j,t];

#

data; param m := 47; param n := 9; param q := 10;

2-16. (a)  $f(y_1, y_2, y_3) \triangleq (y_1)^2 y_2 / y_3,$

$g_1(y_1, y_2, y_3) \triangleq y_1 + y_2 + y_3, b_1 = 13,$

$g_2(y_1, y_2, y_3) \triangleq 2y_1 - y_2 + 9y_3, b_2 = 0,$

$g_3(y_1, y_2, y_3) \triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_3, b_4 = 0$

(b)  $f(y_1, y_2, y_3) \triangleq 13y_1 + 22y_2 + 10y_2 y_3 + 100,$

$g_1(y_1, y_2, y_3) \triangleq y_1 - y_2 + 9y_3, b_1 = -5,$

$g_2(y_1, y_2, y_3) \triangleq 8y_2 - 4y_3, b_2 = 0, g_3(y_1, y_2, y_3)$

$\triangleq y_1, b_3 = 0, g_4(y_1, y_2, y_3) \triangleq y_2, b_4 = 0,$

$g_5(y_1, y_2, y_3) \triangleq y_3, b_5 = 0,$

2-17. (a) Linear because LHS is a weighted

sum of the decision variables. (b) Linear

because both LHS and RHS are weighted

sums of the decision variables. (c) Nonlinear

because LHS has reciprocal  $1/x_9$ . (d) Linear

because LHS is a weighted sum of the decision

variables. (e) Nonlinear because LHS has

$(x_j)^2$  terms. (f) Nonlinear because LHS has

$\log(x_1)$  term, and RHS has a product of

variables. (g) Nonlinear because LHS has max operator. (h) Linear because LHS is a weighted sum of the decision variables.

**2-18.** (a) LP because the objective and all constraints are linear. (b) NLP because of the nonlinear objective function with reciprocal of  $w_2$ . (c) NLP because of the nonlinear first constraint. (d) LP because the objective and all constraints are linear.

**2-19.** (a) Continuous because fractions make sense. (b) Discrete because they either closed or not. (c) Discrete because a specific process must be used. (d) Continuous because fractions can probably be ignored.

**2-20.** (a)  $\sum_{j=1}^8 x_j = 3$  (b)  $x_1 + x_2 + x_4 + x_5 \geq 2$  (c)  $x_3 + x_8 \leq 1$  (d)  $x_4 \geq x_1$

**2-21.** (a) max  $85x_1 + 70x_2 + 62x_3 + 93x_4$  (max total score), s.t.

$700x_1 + 400x_2 + 300x_3 + 600x_4 \leq 1000$  (\$1 million available),  $x_j = 0$  or 1,  $j = 1, \dots, 4$

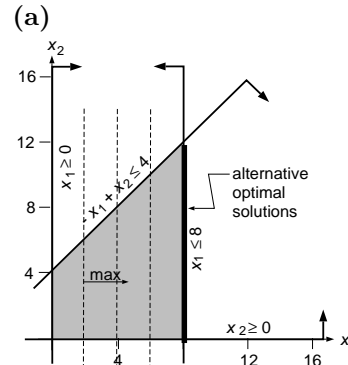
(b) Fund 2 and 4, i.e.  $x_1^* = x_3^* = 0$ ,  $x_2^* = x_4^* = 1$

**2-22.** (a) min  $43y_1 + 175y_2 + 60y_3 + 35y_4$  (min total land cost), s.t.  $y_2 + y_4 \geq 1$  (service NW),  $y_1 + y_2 + y_4 \geq 1$  (service SW),  $y_2 + y_3 \geq 1$  (service capital),  $y_1 + y_4 \geq 1$  (service NE),  $y_1 + y_2 + y_3 \geq 1$  (service SE),  $y_j = 0$  or 1,  $j = 1, \dots, 4$  (b) Build 3 and 4, i.e.  $y_1^* = y_2^* = 0$ ,  $y_3^* = y_4^* = 1$

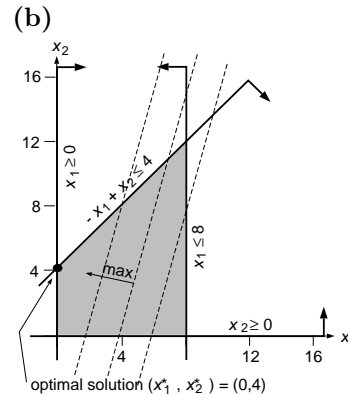
**2-23.** (a) ILP because the objective and all constraints are linear, but variables are discrete. (b) NLP because the objective is nonlinear and all variables are continuous. (c) INLP because the objective is nonlinear and variables are discrete. (d) LP because the objective and all constraints are linear, and all variables are continuous. (e) INLP because the one constraint is nonlinear, and  $z_3$  are discrete. (f) ILP because the objective and all constraints are linear, but variables  $z_1$  and  $z_3$  are discrete. (g) LP because the objective and all constraints are linear, and all variables are continuous. (h) INLP because the objective is nonlinear and  $z_3$  is discrete.

**2-24.** (a) Model (d) because LP's are generally more tractable than ILP's. (b) Model (d) because LP's are generally more tractable than NLP's. (c) Model (d) because LP's are generally more tractable than INLP's. (d) Model (f) because ILP's are generally more tractable than INLP's. (e) Model (g) because LP's are generally more tractable than ILP's.

**2-25.**

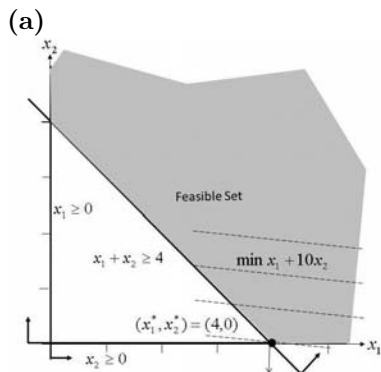


Alternative optima from  $x_1^* = 8$ ,  $x_2^* = 0$  to  $x_1^* = 8$ ,  $x_2^* = 12$

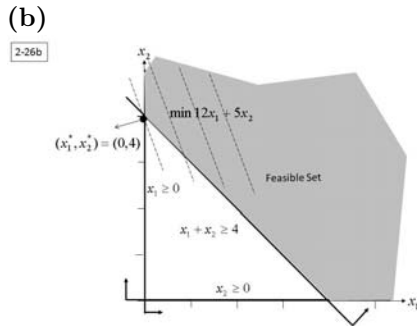


Unique optimum  $x_1^* = 0$ ,  $x_2^* = 4$  (c) Helping one can hurt the other.

**2-26.**



Unique optimum  $x_1^* = 4$ ,  $x_2^* = 0$



Unique optimum  $x_1^* = 0$ ,  $x_2^* = 4$  (c) Helping one can hurt the other.

**2-27. (a)** min  
 $.092x_4 + .112x_5 + .141x_6 + .420x_9 + .719x_{12}$   
 (min total cost),  
 s.t.  $x_4 + x_5 + x_6 + x_9 + x_{12} = 16000$  (16000m line),  
 $.279x_4 + .160x_5 + .120x_6 + .065x_9 + .039x_{12} \leq 1600$  (at most 1600 Ohms resistance),  
 $.00175x_4 + .00130x_5 + .00161x_6 + .00095x_9 + .00048x_{12} \leq 8.5$  (at most 8.5 dBell attenuation),  
 $x_4, x_5, x_6, x_9, x_{12} \geq 0$

(b) Nonzeros:  $x_5^* = 1000$ ,  $x_{12}^* = 15000$

**2-28. (a)** Pump rates are the decisions to be made.

(b)  $u_j \triangleq$  the capacity of pump  $j$ ,  $c_j \triangleq$  the pumping cost of pump  $j$

(c) min  $\sum_{j=1}^{10} c_j x_j$

(d)  $x_1 + x_4 + x_7 \leq 3000$  (well 1),

$x_2 + x_5 + x_8 \leq 2500$  (well 2),

$x_3 + x_6 + x_9 + x_{10} \leq 7000$  (well 3)

(e)  $x_j \leq u_j$ ,  $j = 1, \dots, 10$

(f)  $\sum_{j=1}^{10} x_j \geq 10000$

(g)  $x_j \geq 0$ ,  $j = 1, \dots, 10$

(h) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(i)  $x_1^* = x_2^* = x_3^* = 1100$ ,  $x_4^* = x_6^* = x_9^* = 1500$ ,  
 $x_5^* = 1400$ ,  $x_7^* = 400$ ;  $x_8^* = x_{10}^* = 0$ ,  $x_{11}^* = 1900$

**2-29. (a)** The decisions to be made are which projects to undertake.

(b)  $p_j \triangleq$  the profit for project  $j$ ,  $m_j \triangleq$  the man-days required on project  $j$ , and  $t_j \triangleq$  the CPU time required on project  $j$ .

(c) max  $\sum_{j=1}^8 p_j x_j$

(d)  $7 \leq \left( \sum_{j=1}^8 m_j x_j \right) / 240 \leq 10$

(e)  $\sum_{j=1}^8 t_j x_j \leq 1000$  (computer time),

$\sum_{j=1}^8 x_j \geq 3$  (select at least 3);

$x_3 + x_4 + x_5 + x_8 \geq 1$  (include at least 1 of director's favorites)

(f)  $x_j = 0$  or 1,  $j = 1, \dots, 8$

(g) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(h)  $x_1^* = x_3^* = x_6^* = x_7^* = 1$ , others = 0

**2-30. (a)** We must decide what quantities to move from surplus sites to fulfill each need.

(b)  $s_i \triangleq$  the supply available at  $i$ ,  $r_j \triangleq$  the quantity needed at  $j$ ,  $d_{i,j} \triangleq$  the distance from  $i$  to  $j$ .

(c) min  $\sum_{i=1}^4 \sum_{j=1}^7 d_{i,j} x_{i,j}$

(d)  $\sum_{j=1}^7 x_{i,j} = s_i$ ,  $i = 1, \dots, 4$

(e)  $\sum_{i=1}^4 x_{i,j} = r_j$ ,  $j = 1, \dots, 7$

(f)  $x_{i,j} \geq 0$ ,  $i = 1, \dots, 4$ ,  $j = 1, \dots, 7$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzeros:  $x_{1,1}^* = 81$ ,  $x_{1,2}^* = 93$ ,

$x_{1,3}^* = 166$ ,  $x_{1,5}^* = 90$ ,  $x_{1,6}^* = 85$ ,  $x_{1,7}^* = 145$ ,

$x_{2,2}^* = 301$ ,  $x_{3,1}^* = 166$ ,  $x_{3,4}^* = 105$ ,  $x_{4,3}^* = 99$

**2-31. (a)** The values to be chosen are the

coefficients in the estimating relationship.

(b)  $\min \sum_{j=1}^n (c_j - k/(1 + e^{a+bf_j}))^2$  (min total squared error)

(c) Single objective NLP because the objective is quadratic, there are no constraints, and all variables are continuous.

**2-32.** (a) The decisions to be made are where to assign each teacher.

(b)  $\min \sum_{i=1}^{22} \sum_{j=1}^{22} c_{i,j} x_{i,j}$  (min total cost),  
 $\max \sum_{i=1}^{22} \sum_{j=1}^{22} t_{i,j} x_{i,j}$  (max total teacher preference),  $\max \sum_{i=1}^{22} \sum_{j=1}^{22} s_{i,j} x_{i,j}$  (max total supervisor preference),  $\max \sum_{i=1}^{22} \sum_{j=1}^{22} p_{i,j} x_{i,j}$  (max total principal preference)

(c)  $\sum_{j=1}^{22} x_{i,j} = 1, i = 1, \dots, 22$  (each teacher  $i$ )

(d)  $\sum_{i=1}^{22} x_{i,j} = 1, j = 1, \dots, 22$  (each school  $j$ )

(e)  $x_{i,j} = 0$  or  $1, i, j = 1, \dots, 22$

(f) A multiobjective ILP because the 4 objectives and all constraints are linear, but variables are discrete.

**2-33.** (a) Each task must go to Assistant 0 or Assistant 1.

(b)  $\max 100(1 - x_1) + 80x_1 + 85(1 - x_2) + 70x_2 + 40(1 - x_3) + 90x_3 + 45(1 - x_4) + 85x_4 + 70(1 - x_5) + 80x_5 + 82(1 - x_6) + 65x_6$

(c)  $\sum_{j=1}^6 x_j = 3$

(d)  $x_5 = x_6$

(e)  $x_j = 0$  or  $1, j = 1, \dots, 6$

(f) A single objective ILP because the one objective and all constraints are linear, but variables are discrete.

(g)  $x_2^* = x_3^* = x_4^* = 1$ , others = 0

**2-34.** (a) Batch sizes are the decisions to be made.

(b)  $\min x_j/d_j, j = 1, \dots, 4$  (each burger  $j$ )

(c)  $\sum_{j=1}^4 t_j d_j/x_j \leq 60$

(d)  $0 \leq x_j \leq u_j, j = 1, \dots, 4$

(e) Multiobjective NLP because the first constraint is nonlinear and all variables are continuous.

**2-35.** (a) The issue is how many cars to move from where to where.

(b) Relatively large values can be rounded if fractional without much loss, and continuous is more tractable.

(c)  $c_{i,j} \triangleq$  the cost of moving a car from  $i$  to  $j$ ,  
 $p_j \triangleq$  the number of cars presently at  $j$ ,  $n_j \triangleq$  the number of cars required at  $j$

(d)  $\min \sum_{i=1}^5 \sum_{j=1, j \neq i}^5 c_{i,j} x_{i,j}$

(e)  $\sum_{i=1, i \neq k}^5 x_{i,k} - \sum_{j=1, j \neq k}^5 x_{k,j} = n_k - p_k$ ,  
 $k = 1, \dots, 5$  (each region  $k$ )

(f)  $x_{i,j} \geq 0, i, j = 1, \dots, 5, i \neq j$

(g) A single objective LP because the one objective and all constraints are linear, and all variables are continuous.

(h) Nonzero values:  $x_{4,2}^* = 115, x_{4,3}^* = 165, x_{5,1}^* = 85, x_{5,3}^* = 225$

**2-36.** (a) We must decide how much of what fuel to burn at each plant.

(b)  $\min \sum_{f=1}^4 \sum_{p=1}^{23} c_{f,p} x_{f,p}$

(c)  $\min \sum_{f=1}^4 s_f \sum_{p=1}^{23} x_{f,p}$

(d)  $\sum_{f=1}^4 e_f x_{f,p} \geq r_p, p = 1, \dots, 23$  (each plant  $p$ ); 23 constraints

(e)  $x_{f,p} \geq 0, f = 1, \dots, 4, p = 1, \dots, 23$ ; 92 constraints

(f) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

**2-37.** (a) The available options are to buy whole logs or green lumber.

(b) Relatively large magnitudes can be rounded without much loss, and continuous is more tractable.

(c) min

$70x_{10} + 200x_{15} + 620x_{20} + 1.55y_1 + 1.30y_2$

(d)  $100(.09)x_{10} + 240(.09)x_{15} + 400(.09)x_{20} + .10y_1 + .08y_2 \geq 2350$

(e)  $x_{10} + x_{15} + x_{20} \leq 1500$  (sawing capacity),  
 $100x_{10} + 240x_{15} + 400x_{20} + y_1 + y_2 \leq 26500$  (drying capacity)

(f)  $x_{10} \leq 50$  (size 10 log availability),

$x_{15} \leq 25$  (size 15 log availability),  $x_{20} \leq 10$  (size 20 log availability),  $y_1 \leq 5000$  (grade 1 green lumber availability)

(g)  $x_{10}, x_{15}, x_{20}, y_1, y_2 \geq 0$

(h) A single objective LP because the one



objective and all constraints are linear, and all variables are continuous.

(i)  $x_{10}^* = 50$ ,  $x_{15}^* = 25$ ,  $x_{20}^* = 5$ ,  $y_1^* = 5000$ ,  $y_2^* = 8500$

**2-38.** (a) Decisions to be made are when to schedule each film.

(b)  $\min \sum_{j=1}^{m-1} \sum_{j'=j+1}^m a_{j,j'} \sum_{t=1}^n x_{j,t} x_{j',t}$   
 (c)  $\sum_{t=1}^n x_{j,t} = 1$ ,  $j = 1, \dots, m$  (each film  $j$ )  
 (d)  $\sum_{j=1}^m x_{j,t} \leq 4$ ,  $t = 1, \dots, n$  (each time  $t$ )  
 (e)  $x_{j,t} = 0$  or  $1$ ,  $j = 1, \dots, m$ ;  $t = 1, \dots, n$   
 (f) A single objective INLP because the one objective is nonlinear, and variables are discrete. (g) model; param m; param n; set films := 1 .. m; set slots := 1 .. n; var x{j in films, t in slots} binary; param a{j in films, jp in films}; minimize totconflict: sum{j in films, jp in films: j < m and jp > j} a[j,jp]\*sum{t in slots} x[j,t]\*x[jp,t]; subject to allin{j in films}: sum{t in slots} x[j,t] = 1; max4 {t in slots}:sum{j in films} x[j,t] <= 4;

**2-39.** (a) We need to decide both which offices to open and how to service customers from them.

(b) Offices must either be opened or not.

(c)  $f_i \triangleq$  fixed cost of site  $i$ ,  $c_{i,j} \triangleq$  unit cost of audits at  $j$  from  $i$ ,  $r_j \triangleq$  required number of audits in state  $j$

(d)  $\min \sum_{i=1}^5 \sum_{j=1}^5 c_{i,j} r_j x_{i,j} + \sum_{i=1}^5 f_i y_i$

(e)  $\sum_{i=1}^5 x_{i,j} = 1$ ,  $j = 1, \dots, 5$  (each location  $j$ )

(f)  $x_{i,j} \leq y_i$ ,  $i, j = 1, \dots, 5$  (each site  $i$ , location  $j$  combination)

(g)  $x_{i,j} \geq 0$ ,  $i, j = 1, \dots, 5$ ,  $y_i = 0$  or  $1$ ,  $i = 1, \dots, 5$

(h) A single objective ILP because the one objective and all constraints are linear, but the  $y_i$  variables are discrete.

(i) Nonzeros:

$x_{2,2}^* = x_{2,4}^* = x_{3,1}^* = x_{3,3}^* = x_{5,5}^* = 1$ ,

$y_2^* = y_3^* = y_5^* = 1$  (j) model; param m; param n; set sites := 1 .. m; set

states := 1 .. n; var x{i in sites, j in states} >= 0; var y{i in sites} binary; param c {i in sites, j in states}; param f {i in sites} binary; param r {j in states}; minimize totcost: sum{i in sites, j in states} c[i,j]\*r[j]\*x[i,j] + sum{i in sites} f[i]\*y[i]; x[j,t]\*x[jp,t]; subject to doeach{j in states}: sum{i in sites} x[i,j] = 1; switch {i in sites, j in states}: x[i,j] <= y[i]; data; param m := 5; param n := 5; param f := 1 160 2 49 3 246 4 86 4 100; param r := 1 200 2 100 3 300 4 100 5 200; param c: 1 2 3 4 5 := 1 0.0 0.4 0.8 0.4 0.8 2 0.7 0.0 0.8 0.4 0.4 3 0.6 0.4 0.0 0.5 0.4 4 0.6 0.4 0.9 0.0 0.4 5 0.9 0.4 0.7 0.4 0.0 ;

**2-40.** (a)  $\max \sum_{j=1}^8 r_j x_j$ , subject to,

$\sum_{j=1}^8 x_j \leq 4$ ,  $x_1 + x_2 + x_3 \geq 2$ ,

$x_4 + x_5 + x_6 + x_7 + x_8 \geq 1$ ,

$x_2 + x_3 + x_4 + x_8 \geq 2$ ,  $x_1 \dots x_8 = 0$  or  $1$  (b)

model; param n; set games := 1 .. n; #ratings param r{j in games}; #home? param h{j in games}; #state? param s{j in games}; #cover? var x{j in games} binary; maximize totat: sum{j in games} r[j]\*x[j]; subject to capacity: sum{j in games} x[j] <= 4; home: sum{j in games} h[j]\*x[j] >= 2; away: sum{j in games} (1-h[j])\* x[j] >= 1; state: sum{j in games} s[j]\*x[j] >= 2; data; param n := 8; param r := 1 3.0 2 3.7 3 2.6 4 1.8 5 1.5 6 1.3 7 1.6 8 2.0; param h:=1 1 2 1 3 1 4 0 5 0 6 0 7 0 8 0; param s:=1 0 2 1 3 1 4 1 5 0 6 0 7 0 8 1; (c) The model is an ILP because all constraints and the objective are linear, but decision variables are binary.

**2-41.** (a) How to divide funds is the issue.

(b)  $\max \sum_{j=1}^n v_j x_j$

(c)  $\min \sum_{j=1}^n r_j x_j$

(d)  $\sum_{j=1}^n x_j = 1$

(e)  $x_j \geq \ell_j$ ,  $j = 1, \dots, n$  (each category  $j$ )

(f)  $x_j \leq u_j, j = 1, \dots, n$  (each category  $j$ )

(g) A multiobjective LP because the 2 objectives and all constraints are linear, and all variables are continuous.

$$31x_{2,4} + 18x_{3,4} \leq 7777, x_{1,1} + x_{2,1} + x_{3,1} \geq 200, \\ x_{1,2} + x_{2,2} + x_{3,2} \geq 300, x_{1,3} + x_{2,3} + x_{3,3} \geq 250, \\ x_{1,4} + x_{2,4} + x_{3,4} \geq 500, x_{j,t} \geq 0, j = 1, \dots, 3, t = 1, \dots, 4.$$

**2-42.** (a) The issue is which module goes to which site.

(b) If  $x_{i,j}x_{i',j'} = 1$  the  $i$  is at  $j$  and  $i'$  is at  $j'$ , so wire  $d_{j,j'}$  will be required. Summing over all possible location pairs captures the wire requirements for  $i$  and  $i'$ .

(c) min

$$\sum_{i=1}^{m-1} \sum_{i'=i+1}^m a_{i,i'} \sum_{j=1}^n \sum_{j'=1}^n d_{j,j'} x_{i,j} x_{i',j'}$$

(d)  $\sum_{j=1}^n x_{i,j} = 1, i = 1, \dots, m$  (each module  $i$ )

(e)  $\sum_{i=1}^m x_{i,j} \leq 1, j = 1, \dots, n$  (each site  $j$ )

(f)  $x_{i,j} = 0$  or  $1, i = 1, \dots, m, j = 1, \dots, n$

(g) Single objective INLP because the one objective is nonlinear and variables are

discrete. (h) model; param m; param n; set modules := 1 .. m; set sites := 1 .. n; var x{i in modules, j in sites} binary; param a{ i in modules, ip in modules }; param d{ j in sites, jp in sites }; minimize totdist: sum{ i in modules, ip in modules: i < m and ip > i } a[i,ip] sum{ j in sites, jp in sites : j < n and jp > j } d[j,jp]\*x[i,j]\*x[ip,jp]; subject to alli {i in modules }: sum{ j in sites } x[i,j] = 1; allj { j in sites }: sum { i in modules } x[i,j] <= 1;

**2-43.** max  $199x_1 + 229x_2 + 188x_3 + 205x_4 - 180y_1 - 224y_2 - 497y_3$ , subject to,  
 $23x_3 + 41x_4 \leq 2877y_1, 14x_1 + 29x_2 \leq 2333y_2,$   
 $11x_3 + 27x_4 \leq 3011y_3,$   
 $x_1 + x_2 + x_3 + x_4 \geq 205, y_1 + y_2 + y_3 \leq 2,$   
 $x_1, \dots, x_4 \geq 0, y_1, \dots, y_3 = 0$  or  $1$

**2-44.** max  $11x_{1,1} + 15x_{1,2} + 19x_{1,3} + 10x_{1,4} + 19x_{2,1} + 23x_{2,2} + 44x_{2,3} + 67x_{2,4} + 17x_{3,1} + 18x_{3,2} + 24x_{3,3} + 55x_{3,4}$ , subject to,  $15x_{1,1} + 24x_{2,1} + 17x_{3,1} \leq 7600, 19x_{1,2} + 26x_{2,2} + 13x_{3,2} \leq 8200, 23x_{1,3} + 18x_{2,3} + 16x_{3,3} \leq 6015,$   
 $14x_{1,4} + 33x_{2,4} + 14x_{3,4} \leq 5000, 31x_{1,1} + 26x_{2,1} + 21x_{3,1} \leq 6600, 25x_{1,2} + 28x_{2,2} + 17x_{3,2} \leq 7900,$   
 $39x_{1,3} + 22x_{2,3} + 20x_{3,2} \leq 5055, 29x_{1,4} +$