## Chapter 1

## INTRODUCTION

## Conceptual Questions

1. Knowledge of physics is important for a full understanding of many scientific disciplines, such as chemistry, biology, and geology. Furthermore, much of our current technology can only be understood with knowledge of the underlying laws of physics. In the search for more efficient and environmentally safe sources of energy, for example, physics is essential. Also, many people study physics for the sense of fulfillment that comes with learning about the world we inhabit.
2. Without precise definitions of words for scientific use, unambiguous communication of findings and ideas would be impossible.
3. Even when simplified models do not exactly match real conditions, they can still provide insight into the features of a physical system. Often a problem would become too complicated if one attempted to match the real conditions exactly, and an approximation can yield a result that is close enough to the exact one to still be useful.
4. (a) 3 (b) 9
5. Scientific notation eliminates the need to write many zeros in very large or small numbers, and to count them. Also, the number of significant digits is indicated unambiguously when a quantity is written this way.
6. In scientific notation the decimal point is placed after the first (leftmost) numeral. The number of digits written equals the number of significant figures.
7. Not all of the significant digits are known definitely. The last (rightmost) digit, called the least significant digit, is an estimate and is less definitely known than the others.
8. It is important to list the correct number of significant figures so that we can indicate how precisely a quantity is known and so that we do not mislead the reader by writing digits that are not at all known to be correct.
9. The kilogram, meter, and second are three of the base units used in the SI, the international system of units.
10. The international system SI uses a well-defined set of internationally agreed upon standard units and makes measurements in terms of these units, their combinations, and their powers of ten. The U.S. customary system contains units that are primarily of historical origin and are not based upon powers of ten. As a result of this international acceptance and of the ease of manipulation that comes from dealing with powers of ten, scientists around the world prefer to use SI.
11. Fathoms, kilometers, miles, and inches are units with the dimension length. Grams and kilograms are units with the dimension mass. Years, months, and seconds are units with the dimension time.
12. The first step toward successfully solving almost any physics problem is to thoroughly read the question and obtain a precise understanding of the scenario. The second step is to visualize the problem, often making a quick sketch to outline the details of the situation and the known parameters.
13. Trends in a set of data are often the most interesting aspect of the outcome of an experiment. Such trends are more apparent when data is plotted graphically rather than listed in numerical tables.
14. The statement gives a number for the speed of sound in air, but fails to indicate the units used for the measurement. Without units, the reader cannot relate the speed to one given in familiar units such as $\mathrm{km} / \mathrm{s}$.
15. After solving a problem, it is a good idea to check that the solution is reasonable and makes intuitive sense. It may also be useful to explore other possible methods of solution as a check on the validity of the first. A good student thinks of a framework of ideas and skills that she is constructing for herself. The problem solution may extend or strengthen this framework.

## Multiple-Choice Questions

1.(b) 2.(b) 3. (a) 4. (c) 5. (d) 6. (d) 7. (b) 8. (d) 9. (b) 10. (c)

## Problems

1. Strategy The new fence will be $100 \%+37 \%=137 \%$ of the height of the old fence.

Solution Find the height of the new fence. $\quad 1.37 \times 1.8 \mathrm{~m}=2.5 \mathrm{~m}$
Discussion. Long ago you were told that $37 \%$ of 1.8 is 0.37 times 1.8 .
2. Strategy There are $\frac{60 \mathrm{~s}}{1 \mathrm{~min}} \times \frac{60 \mathrm{~min}}{1 \mathrm{~h}} \times \frac{24 \mathrm{~h}}{1 \mathrm{~d}}=86,400$ seconds in one day and 24 hours in one day.

Solution Find the ratio of the number of seconds in a day to the number of hours in a day.
$\frac{86,400}{24}=\frac{24 \times 3600}{24}=3600 / 1$
3. Strategy Relate the surface area $A$ to the radius $r$ using $A=4 \pi r^{2}$.

Solution Find the ratio of the new radius to the old.

$$
\begin{aligned}
A_{1}= & 4 \pi r_{1}^{2} \text { and } A_{2}=4 \pi r_{2}^{2}=1.160 A_{1}=1.160\left(4 \pi r_{1}^{2}\right) \\
4 \pi r_{2}^{2} & =1.160\left(4 \pi r_{1}^{2}\right) \\
r_{2}^{2} & =1.160 r_{1}^{2} \\
\left(\frac{r_{2}}{r_{1}}\right)^{2} & =1.160 \\
\frac{r_{2}}{r_{1}} & =\sqrt{1.160}=1.077
\end{aligned}
$$

The radius of the balloon increases by $7.7 \%$.
Discussion. The factor of $4 \pi$ plays no part in determining the answer. The answer just comes from the proportionality of area to the square of the radius. The circumference also increases by $7.7 \%$.
4. Strategy Relate the surface area $A$ to the radius $r$ using $A=4 \pi r^{2}$.

Solution Find the ratio of the new radius to the old.

$$
\begin{aligned}
& A_{1}=4 \pi r_{1}^{2} \text { and } A_{2}=4 \pi r_{2}^{2}=2.0 A_{1}=2.0\left(4 \pi r_{1}^{2}\right) . \\
& 4 \pi r_{2}^{2}=2.0\left(4 \pi r_{1}^{2}\right) \quad \text { so } \quad r_{2}^{2}=2.0 r_{1}^{2} \\
& \left(\frac{r_{2}}{r_{1}}\right)^{2}=2.0 \quad \text { and } \quad \frac{r_{2}}{r_{1}}=\sqrt{2.0}=1.4
\end{aligned}
$$

The radius of the balloon increases by a factor of 1.4 .
5. Strategy To find the factor by which the metabolic rate of a 70 kg human exceeds that of a 5.0 kg cat use a ratio.

Solution Find the factor.
$\left(\frac{m_{\mathrm{h}}}{m_{\mathrm{c}}}\right)^{3 / 4}=\left(\frac{70}{5.0}\right)^{3 / 4}=7.2$
Discussion. Get used to using your calculator to follow the order of operations without your having to re-enter any numbers. On my calculator I type $70 \div 5={ }^{\wedge} 0.75=$.
6. Strategy To find the factor Samantha's height increased, divide her new height by her old height. Subtract 1 from this value and multiply by $100 \%$ to find the percentage increase.

Solution Find the factor.
$\frac{1.65 \mathrm{~m}}{1.50 \mathrm{~m}}=1.10$
Find the percentage.
$1.10-1=0.10$, so the percent increase is $10 \%$.
7. Strategy Recall that area has dimensions of length squared.

Solution Find the ratio of the area of the park as represented on the map to the area of the actual park.
$\frac{\text { map length }}{\text { actual length }}=\frac{1}{10,000}=10^{-4}$, so $\frac{\text { map area }}{\text { actual area }}=\left(10^{-4}\right)^{2}=10^{-8}$.
8. Strategy Let $X$ be the original value of the index. Follow what happens to it.

Solution Find the net percentage change in the index for the two days.
final value $=($ originalvalue $) \times($ first day change factor $) \times($ second day change factor $)=$

$$
=X \times(1+0.0500) \times(1-0.0500)=0.9975 X
$$

The net percentage change is $(0.9975-1) \times 100 \%=-0.25 \%$, or down $0.25 \%$.
9. Strategy Use a proportion.

Solution Find Jupiter's orbital period.

$$
T^{2} \propto R^{3}, \text { so } \frac{T_{\mathrm{J}}^{2}}{T_{\mathrm{E}}^{2}}=\frac{R_{\mathrm{J}}^{3}}{R_{\mathrm{E}}^{3}}=5.19^{3} . \text { Thus, } T_{\mathrm{J}}=5.19^{3 / 2} T_{\mathrm{E}}=11.8 \mathrm{yr} .
$$

Discussion. People since the ancient Babylonians have watched Jupiter step majestically every year from one constellation into the next of twelve lying along the ecliptic. (You should too.) In Chapter 5 we will show that Kepler's third law is a logical consequence of Newton's law of gravitation and Newton's second law of motion. Science does not necessarily answer "why" questions, but that derivation and this problem give reasons behind the motion of Jupiter in the sky.
10. Strategy The area of the circular garden is given by $A=\pi r^{2}$. Let the original and final areas be $A_{1}=\pi r_{1}^{2}$ and $A_{2}=\pi r_{2}^{2}$, respectively.

Solution Calculate the percentage increase of the area of the garden plot.

$$
\frac{\Delta A}{A} \times 100 \%=\frac{\pi r_{2}^{2}-\pi r_{1}^{2}}{\pi r_{1}^{2}} \times 100 \%=\frac{r_{2}^{2}-r_{1}^{2}}{r_{1}^{2}} \times 100 \%=\frac{1.25^{2} r_{1}^{2}-r_{1}^{2}}{r_{1}^{2}} \times 100 \%=\frac{1.25^{2}-1}{1} \times 100 \%=56 \%
$$

11. Strategy The area of a rectangular poster is given by $A=\ell w$. Let the original and final areas be $A_{1}=\ell_{1} w_{1}$ and $A_{2}=\ell_{2} w_{2}$, respectively.

Solution Calculate the percentage reduction of the area.
$A_{2}=\ell_{2} w_{2}=\left(0.800 \ell_{1}\right)\left(0.800 w_{1}\right)=0.640 \ell_{1} w_{1}=0.640 A_{1}$
$\frac{A_{1}-A_{2}}{A_{1}} \times 100 \%=\frac{A_{1}-0.640 A_{1}}{A_{1}} \times 100 \%=36.0 \%$
Discussion. Proportional reasoning is so profound that it applies to a triangular, round, star-shaped, or dragonshaped poster, as long as the final shape is geometrically similar to the original and length and width are interpreted as two perpendicular maximum distances across the poster.
12. Strategy The volume of the rectangular room is given by $V=\ell w h$. Let the original and final volumes be $V_{1}=\ell_{1} w_{1} h_{1}$ and $V_{2}=\ell_{2} w_{2} h_{2}$, respectively.

Solution Find the factor by which the volume of the room increased.
$\frac{V_{2}}{V_{1}}=\frac{\ell_{2} w_{2} h_{2}}{\ell_{1} w_{1} h_{1}}=\frac{\left(1.50 \ell_{1}\right)\left(2.00 w_{1}\right)\left(1.20 h_{1}\right)}{\ell_{1} w_{1} h_{1}}=3.60$
13. Strategy Assuming that the cross section of the artery is a circle, we use the area of a circle, $A=\pi r^{2}$.

## Solution

$A_{1}=\pi r_{1}^{2}$ and $A_{2}=\pi r_{2}^{2}=\pi\left(2.0 r_{1}\right)^{2}=4.0 \pi r_{1}^{2}$.
Form a proportion.
$\frac{A_{2}}{A_{1}}=\frac{4.0 \pi r_{1}^{2}}{\pi r_{1}^{2}}=4.0$
The cross-sectional area of the artery increases by a factor of 4.0 .
Discussion. To keep on feeling a lot (four times?) better, the patient needs to exercise and reduce risk factors.
14. (a) Strategy The diameter of the xylem vessel is one six-hundredth of the magnified image.

Solution Find the diameter of the vessel.

$$
d_{\text {actual }}=\frac{d_{\text {magnified }}}{600}=\frac{3.0 \mathrm{~cm}}{600}=5.0 \times 10^{-3} \mathrm{~cm}
$$

(b) Strategy The area of the cross section is given by $A=\pi r^{2}=\pi(d / 2)^{2}=(1 / 4) \pi d^{2}$.

Solution Find by what factor the cross-sectional area has been increased in the micrograph.

$$
\frac{A_{\text {magnified }}}{A_{\text {actual }}}=\frac{\frac{1}{4} \pi d_{\text {magnified }^{2}}^{\frac{1}{4} \pi d_{\text {actual }}{ }^{2}}=\left(\frac{3.0 \mathrm{~cm}}{5.0 \times 10^{-3} \mathrm{~cm}}\right)^{2}=600^{2}=360,000}{.}
$$

15. Strategy Use the fact that $R_{\mathrm{B}}=1.42 R_{\mathrm{A}}$.

Solution Calculate the ratio of $P_{\mathrm{B}}$ to $P_{\mathrm{A}}$.
$\frac{P_{\mathrm{B}}}{P_{\mathrm{A}}}=\frac{\frac{V^{2}}{R_{\mathrm{B}}}}{\frac{V^{2}}{R_{\mathrm{A}}}}=\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}}=\frac{R_{\mathrm{A}}}{1.42 R_{\mathrm{A}}}=\frac{1}{1.42}=0.704$
16. Strategy Recall that each digit to the right of the decimal point is significant.

Solution Comparing the significant figures of each value, we have (a) 5, (b) 4, (c) 2, (d) 2 , and (e) 3 . From fewest to greatest we have $\mathrm{c}=\mathrm{d}, \mathrm{e}, \mathrm{b}, \mathrm{a}$.
17. (a) Strategy To see what is going on, rewrite the numbers so that the power of 10 is the same for each. Then add and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Perform the operation with the appropriate number of significant figures.
$3.783 \times 10^{6} \mathrm{~kg}+1.25 \times 10^{8} \mathrm{~kg}=0.03783 \times 10^{8} \mathrm{~kg}+1.25 \times 10^{8} \mathrm{~kg}=1.29 \times 10^{8} \mathrm{~kg}$
(b) Strategy Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Perform the operation with the appropriate number of significant figures.
$\left(3.783 \times 10^{6} \mathrm{~m}\right) \div\left(3.0 \times 10^{-2} \mathrm{~s}\right)=1.3 \times 10^{8} \mathrm{~m} / \mathrm{s}$
Discussion. Notice also the units. A number of meters plus a number of meters is still a number of meters. A number of meters divided by a number of seconds is a number of $\mathrm{m} / \mathrm{s}$.
18. (a) Strategy Move the decimal point eight places to the left and multiply by $10^{8}$.

Solution Write the number in scientific notation.
$310,000,000$ people $=3.1 \times 10^{8}$ people
(b) Strategy Move the decimal point 15 places to the right and multiply by $10^{-15}$.

Solution Write the number in scientific notation.
$0.0000000000000038 \mathrm{~m}=3.8 \times 10^{-15} \mathrm{~m}$
19. (a) Strategy Rewrite the numbers so that the power of 10 is the same for each. Then subtract and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Perform the calculation using an appropriate number of significant figures.
$3.68 \times 10^{7} \mathrm{~g}-4.759 \times 10^{5} \mathrm{~g}=3.68 \times 10^{7} \mathrm{~g}-0.04759 \times 10^{7} \mathrm{~g}=3.63 \times 10^{7} \mathrm{~g}$
(b) Strategy Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Perform the calculation using an appropriate number of significant figures.
$\frac{6.497 \times 10^{4} \mathrm{~m}^{2}}{5.1037 \times 10^{2} \mathrm{~m}}=1.273 \times 10^{2} \mathrm{~m}$
20. (a) Strategy Rewrite the numbers so that the power of 10 is the same for each. Then add and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Write your answer using the appropriate number of significant figures.
$6.85 \times 10^{-5} \mathrm{~m}+2.7 \times 10^{-7} \mathrm{~m}=6.85 \times 10^{-5} \mathrm{~m}+0.027 \times 10^{-5} \mathrm{~m}=6.88 \times 10^{-5} \mathrm{~m}$
(b) Strategy Add and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Write your answer using the appropriate number of significant figures.
$702.35 \mathrm{~km}+1897.648 \mathrm{~km}=2600.00 \mathrm{~km}$
(c) Strategy Multiply and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Write your answer using the appropriate number of significant figures.
$5.0 \mathrm{~m} \times 4.3 \mathrm{~m}=22 \mathrm{~m}^{2}$
(d) Strategy Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Write your answer using the appropriate number of significant figures.
$(0.04 / \pi) \mathrm{cm}=0.01 \mathrm{~cm}$
(e) Strategy Find the quotient and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Write your answer using the appropriate number of significant figures.
$(0.040 / \pi) \mathrm{m}=0.013 \mathrm{~m}$
21. Strategy Multiply and give the answer in scientific notation with the number of significant figures determined by the number with the fewest significant figures.

Solution Solve the problem.
$(3.2 \mathrm{~m}) \times\left(4.0 \times 10^{-3} \mathrm{~m}\right) \times\left(1.3 \times 10^{-8} \mathrm{~m}\right)=1.7 \times 10^{-10} \mathrm{~m}^{3}$
Discussion. If you use your calculator to find $(3.2 \mathrm{~m})\left(10^{-3} \mathrm{~m}\right)\left(5.2 \times 10^{-8} \mathrm{~m}\right)$, what keys would you hit? On my calculator it is $3.2 \times 1$ enterexponent changesign $3 \times 5.2$ enterexponent changesign $8=$. Make sure you can make sense of where the 1 comes from.
22. Strategy Follow the rules for identifying significant figures.

## Solution

(a) All three digits are significant, so 7.68 g has 3 significant figures.
(b) The first zero is not significant, since it is used only to place the decimal point. The digits 4 and 2 are significant, as is the final zero, so 0.420 kg has 3 significant figures.
(c) The first two zeros are not significant, since they are used only to place the decimal point. The digits 7 and 3 are significant, so 0.073 m has 2 significant figures.
(d) All three digits are significant, so $7.68 \times 10^{5} \mathrm{~g}$ has 3 significant figures.
(e) The zero is significant, since it comes after the decimal point. The digits 4 and 2 are significant as well, so $4.20 \times 10^{3} \mathrm{~kg}$ has 3 significant figures.
(f) Both 7 and 3 are significant, so $7.3 \times 10^{-2} \mathrm{~m}$ has 2 significant figures.
(g) Both 2 and 3 are significant. The two zeros are significant as well, since they come after the decimal point, so $2.300 \times 10^{4}$ s has 4 significant figures.
23. Strategy Divide and give the answer with the number of significant figures determined by the number with the fewest significant figures.

Solution Solve the problem.

$$
\frac{3.21 \mathrm{~m}}{7.00 \mathrm{~ms}}=\frac{3.21 \mathrm{~m}}{7.00 \times 10^{-3} \mathrm{~s}}=459 \mathrm{~m} / \mathrm{s}
$$

Discussion. With $\mathrm{m}=\mathrm{milli}=10^{-3}$, we could say $1 / \mathrm{milli}=\mathrm{kilo}=\mathrm{k}$.
24. Strategy Convert each length to meters. Then, rewrite the numbers so that the power of 10 is the same for each. Finally, add and give the answer with the number of significant figures determined by the less precise of the two numbers.

Solution Solve the problem.
$3.08 \times 10^{-1} \mathrm{~km}+2.00 \times 10^{3} \mathrm{~cm}=3.08 \times 10^{2} \mathrm{~m}+2.00 \times 10^{1} \mathrm{~m}=3.08 \times 10^{2} \mathrm{~m}+0.200 \times 10^{2} \mathrm{~m}=3.28 \times 10^{2} \mathrm{~m}$
25. Strategy Use the rules for determining significant figures and for writing numbers in scientific notation.

Solution (a) 0.00574 kg has three significant figures, 5,7 , and 4 . The zeros are not significant, since they are used only to place the decimal point. To write this measurement in scientific notation, we move the decimal point three places to the right and multiply by $10^{-3}$. We have $5.74 \times 10^{-3} \mathrm{~kg}$.
(b) 2 m has one significant figure, 2 . This measurement is already written in scientific notation, or we could show it as $2 \times 10^{0} \mathrm{~m}$.
(c) $0.450 \times 10^{-2} \mathrm{~m}$ has three significant figures, 4,5 , and the 0 to the right of 5 . The zero is significant, since it comes after the decimal point and is not used to place the decimal point. To write this measurement in scientific notation, we move the decimal point one place to the right and multiply by $10^{-1}$.
(d) 45.0 kg has three significant figures, 4,5 , and 0 . The zero is significant, since it comes after the decimal point and is not used to place the decimal point. To write this measurement in scientific notation, we move the decimal point one place to the left and multiply by $10^{1}$.
(e) $10.09 \times 10^{4}$ s has four significant figures, 1,9 , and the two zeros. The zeros are significant, since they are between two significant figures. To write this measurement in scientific notation, we move the decimal point one place to the left and multiply by $10^{1}$.
(f) $0.09500 \times 10^{5} \mathrm{~mL}$ has four significant figures, 9,5 , and the two zeros to the right of 5 . The zeros are significant, since they come after the decimal point and are not used to place the decimal point. To write this measurement in scientific notation, we move the decimal point two places to the right and multiply by $10^{-2}$.

The results of parts (a) through (f) are shown in the table below.

|  | Measurement | Significant Figures | Scientific Notation |
| :--- | :---: | :---: | :---: |
| (a) | 0.00574 kg | 3 | $5.74 \times 10^{-3} \mathrm{~kg}$ |
| (b) | 2 m | 1 | 2 m |
| (c) | $0.450 \times 10^{-2} \mathrm{~m}$ | 3 | $4.50 \times 10^{-3} \mathrm{~m}$ |
| (d) | 45.0 kg | 3 | $4.50 \times 10^{1} \mathrm{~kg}$ |
| (e) | $10.09 \times 10^{4} \mathrm{~s}$ | 4 | $1.009 \times 10^{5} \mathrm{~s}$ |
| (f) | $0.09500 \times 10^{5} \mathrm{~mL}$ | 4 | $9.500 \times 10^{3} \mathrm{~mL}$ |

Discussion. It would be more natural to convert to scientific notation first and then count the significant digits.
26. Strategy Convert each length to scientific notation.

Solution In scientific notation, the lengths
are: (a) $1 \mu \mathrm{~m}=1 \times 10^{-6} \mathrm{~m}, \quad$ (b) $1000 \mathrm{~nm}=1 \times 10^{3} \times 10^{-9} \mathrm{~m}=1 \times 10^{-6} \mathrm{~m}$,
(c) $100000 \mathrm{pm}=1 \times 10^{5} \times 10^{-12} \mathrm{~m}=1 \times 10^{-7} \mathrm{~m}$, (d) $0.01 \mathrm{~cm}=1 \times 10^{-2} \times 10^{-2} \mathrm{~m}=1 \times 10^{-4} \mathrm{~m}$, and
(e) $0.0000000001 \mathrm{~km}=1 \times 10^{-10} \times 10^{3} \mathrm{~m}=1 \times 10^{-7} \mathrm{~m}$.

From smallest to greatest, we have $\mathrm{c}=\mathrm{e}, \mathrm{a}=\mathrm{b}, \mathrm{d}$.
27. Strategy Convert each length to meters and each time to seconds. From the inside front cover, $1 \mathrm{mi}=1609 \mathrm{~m}$.

Solution In scientific notation, we have:
(a) $55 \mathrm{mi} / \mathrm{h} \times 1609 \mathrm{~m} / \mathrm{mi} \times 1 \mathrm{~h} / 3600 \mathrm{~s}=25 \mathrm{~m} / \mathrm{s}$, (b) $82 \mathrm{~km} / \mathrm{h} \times 1 \mathrm{~h} / 3600 \mathrm{~s} \times 1000 \mathrm{~m} / \mathrm{km}=23 \mathrm{~m} / \mathrm{s}$,
(c) $33 \mathrm{~m} / \mathrm{s}, \quad$ (d) $3.0 \mathrm{~cm} / \mathrm{ms} \times 1 \mathrm{~m} / 100 \mathrm{~cm} \times 1000 \mathrm{~ms} / \mathrm{s}=30 \mathrm{~m} / \mathrm{s}$, and
(e) $1.0 \mathrm{mi} / \mathrm{min} \times 1 \mathrm{~min} / 60 \mathrm{~s} \times 1609 \mathrm{~m} / \mathrm{mi}=27 \mathrm{~m} / \mathrm{s}$.

From smallest to greatest, we have $\mathrm{b}, \mathrm{a}, \mathrm{e}, \mathrm{d}, \mathrm{c}$.
Discussion. Can you directly make sense of how one mile per minute is greater than $55 \mathrm{mi} / \mathrm{h}$ ? Three centimeters per millisecond is greater than either because a millisecond is so short a time.
28. Strategy Recall that $1 \mathrm{~kg}=1000 \mathrm{~g}$ and $100 \mathrm{~cm}=1 \mathrm{~m}$.

Solution Convert the density of body fat from $\mathrm{g} / \mathrm{cm}^{3}$ to $\mathrm{kg} / \mathrm{m}^{3}$.
$0.9 \mathrm{~g} / \mathrm{cm}^{3} \times 1 \mathrm{~kg} / 1000 \mathrm{~g} \times(100 \mathrm{~cm} / \mathrm{m})^{3}=900 \mathrm{~kg} / \mathrm{m}^{3}$
29. Strategy There are exactly 2.54 centimeters in one inch.

Solution Find the thickness of the cell membrane in inches.
$7.0 \times 10^{-9} \mathrm{~m} \times \frac{1 \text { inch }}{0.0254 \mathrm{~m}}=2.8 \times 10^{-7}$ inches
Discussion. The thickness of plastic film or the diameter of a wire can be stated in mils, where one mil is a thousandth of an inch. But no one speaks of a microinch or a nanoinch.
30. (a) Strategy There are approximately 3.785 liters per gallon from the inside front cover, and 128 fluid ounces per gallon.

Solution Find the number of fluid ounces in the bottle.

$$
\frac{128 \mathrm{fl} \mathrm{oz}}{1 \mathrm{gal}} \times \frac{1 \mathrm{gal}}{3.785 \mathrm{~L}} \times 355 \mathrm{~mL} \times \frac{1 \mathrm{~L}}{10^{3} \mathrm{~mL}}=12.0 \text { fluid ounces }
$$

(b) Strategy From part (a), we have $355 \mathrm{~mL}=12.0$ fluid ounces.

Solution Find the number of milliliters in the drink.
$16.0 \mathrm{fl} \mathrm{oz} \times \frac{355 \mathrm{~mL}}{12.0 \mathrm{fl} \mathrm{oz}}=473 \mathrm{~mL}$
31. Strategy There are exactly 2.54 centimeters in an inch and 12 inches in one foot.

Solution Convert to meters.
(a) $1595.5 \mathrm{ft} \times \frac{12 \mathrm{in}}{1 \mathrm{ft}} \times \frac{0.0254 \mathrm{~m}}{1 \mathrm{in}}=4.8631 \times 10^{2} \mathrm{~m}$
(b) $6016 \mathrm{ft} \times \frac{12 \mathrm{in}}{1 \mathrm{ft}} \times \frac{0.0254 \mathrm{~m}}{1 \mathrm{in}}=1.834 \times 10^{3} \mathrm{~m}$
32. Strategy For (a), convert milliliters to liters; then convert liters to cubic centimeters using the conversion $1 \mathrm{~L}=10^{3} \mathrm{~cm}^{3}$. For (b), convert cubic centimeters to cubic meters using the fact that $100 \mathrm{~cm}=1 \mathrm{~m}$.

Solution Convert each volume.
(a) $255 \mathrm{~mL} \times \frac{10^{-3} \mathrm{~L}}{1 \mathrm{~mL}} \times \frac{10^{3} \mathrm{~cm}}{1 \mathrm{~L}}=255 \mathrm{~cm}^{3}$
(b) $255 \mathrm{~cm}^{3} \times\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{3}=255 \mathrm{~cm}^{3} \times \frac{1 \mathrm{~m}^{3}}{10^{6} \mathrm{~cm}^{3}}=2.55 \times 10^{-4} \mathrm{~m}$
33. Strategy For (a), we can convert meters per second to miles per hour using the conversion $1 \mathrm{mi} / \mathrm{h}=0.4470 \mathrm{~m} / \mathrm{s}$. For (b), convert meters per second to centimeters per millisecond using the conversions $1 \mathrm{~m}=100 \mathrm{~cm}$ and $1 \mathrm{~s}=$ 1000 ms .
Solution Convert each speed.
(a) $80 \mathrm{~m} / \mathrm{s} \times \frac{1 \mathrm{mi} / \mathrm{h}}{0.4470 \mathrm{~m} / \mathrm{s}}=180 \mathrm{mi} / \mathrm{h}$
(b) $80 \mathrm{~m} / \mathrm{s} \times \frac{10^{2} \mathrm{~cm}}{1 \mathrm{~m}} \times \frac{1 \mathrm{~s}}{10^{3} \mathrm{~ms}}=8.0 \mathrm{~cm} / \mathrm{ms}$

Discussion. John Kennedy was President for a thousand days. Quoting the speed of the nerve impulse either in miles per hour or in meters per second should make it sound fast to you. Quoting it in centimeters per millisecond suggests that one bit of the cell responds quickly to a stimulus from an adjacent bit.
34. Strategy With the datum quoted to five significant digits, we use the exact conversions between miles and feet, feet and inches, inches and centimeters, and centimeters and kilometers.

Solution Find the length of the marathon race in miles.
$42.195 \mathrm{~km} \times \frac{100,000 \mathrm{~cm}}{1 \mathrm{~km}} \times \frac{1 \mathrm{in}}{2.54 \mathrm{~cm}} \times \frac{1 \mathrm{ft}}{12 \mathrm{in}} \times \frac{1 \mathrm{mi}}{5280 \mathrm{ft}}=26.219 \mathrm{mi}$
35. Strategy Calculate the change in the exchange rate and divide it by the original price to find the drop.

Solution Find the actual drop in the value of the dollar over the first year.
$\frac{1.27-1.45}{1.45}=\frac{-0.18}{1.45}=-0.12$
The actual drop is 0.12 or $12 \%$.
36. Strategy There are 1000 watts in one kilowatt and 100 centimeters in one meter.

Solution Convert $1.4 \mathrm{~kW} / \mathrm{m}^{2}$ to $\mathrm{W} / \mathrm{cm}^{2}$.
$\frac{1.4 \mathrm{~kW}}{1 \mathrm{~m}^{2}} \times \frac{1000 \mathrm{~W}}{1 \mathrm{~kW}} \times\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{2}=0.14 \mathrm{~W} / \mathrm{cm}^{2}$
37. Strategy Convert the radius to centimeters; then use the conversions $1 \mathrm{~L}=10^{3} \mathrm{~cm}^{3}$ and $60 \mathrm{~s}=1 \mathrm{~min}$.

Solution Find the volume rate of blood flow
volume rate of blood flow $=\pi r^{2} v=\pi(1.2 \mathrm{~cm})^{2}(18 \mathrm{~cm} / \mathrm{s}) \times \frac{1 \mathrm{~L}}{10^{3} \mathrm{~cm}^{3}} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}=4.9 \mathrm{~L} / \mathrm{min}$
Discussion. The given equation for the volume flow rate makes sense because the volume passing in one unit of time can be visualized as the volume of a cylinder with base area equal to the cross-sectional area of the vessel and length equal to the distance that one atom travels in a unit of time, representing its linear speed.
38. Strategy The distance traveled $d$ is equal to the rate of travel $v$ times the time of travel $t$. There are 1000 milliseconds in one second.
Solution Find the distance the molecule would move.
$d=v t=\frac{459 \mathrm{~m}}{1 \mathrm{~s}} \times 7.00 \mathrm{~ms} \times \frac{1 \mathrm{~s}}{1000 \mathrm{~ms}}=3.21 \mathrm{~m}$
39. Strategy There are 1000 meters in a kilometer and $1,000,000$ millimeters in a kilometer.

Solution The volume can be visualized as that of a very thin, wide ribbon of plastic film that rolls out from a press in a factory, perhaps in a couple of days. We multiply its three perpendicular dimensions to find its volume, just as if it were a brick. We find the product and express the answer in the huge unit of $\mathrm{km}^{3}$ with the appropriate number of significant figures.
$(3.2 \mathrm{~km}) \times$

$$
\times(4.0 \mathrm{~m}) \times\left(13 \times 10^{-3} \mathrm{~mm}\right) \times \frac{1 \mathrm{~km}}{1000 \mathrm{~m}} \times \frac{1 \mathrm{~km}}{1,000,000 \mathrm{~mm}}=1.7 \times 10^{-10} \mathrm{~km}^{3}
$$

40. (a) Strategy There are 12 inches in one foot and 2.54 centimeters in one inch.

Solution Find the number of square centimeters in one square foot.
$1 \mathrm{ft}^{2} \times\left(\frac{12 \mathrm{in}}{1 \mathrm{ft}}\right)^{2} \times\left(\frac{2.54 \mathrm{~cm}}{1 \mathrm{in}}\right)^{2}=929 \mathrm{~cm}^{2}$
(b) Strategy There are 100 centimeters in one meter.

Solution Find the number of square centimeters in one square meter.
$1 \mathrm{~m}^{2} \times\left(\frac{100 \mathrm{~cm}}{1 \mathrm{~m}}\right)^{2}=1 \times 10^{4} \mathrm{~cm}^{2}$
(c) Strategy Divide one square meter by one square foot. Estimate the quotient.

Solution Find the approximate number of square feet in one square meter. Ten thousand divided by one thousand is ten, so ten thousand divided by nine hundred is a bit larger than ten.
$\frac{1 \mathrm{~m}^{2}}{1 \mathrm{ft}^{2}}=\frac{10,000 \mathrm{~cm}^{2}}{929 \mathrm{~cm}^{2}} \approx 11$
41. (a) Strategy There are 12 inches in one foot, 2.54 centimeters in one inch, and 60 seconds in one minute.

Solution Express the snail's speed in feet per second.
$\frac{5.0 \mathrm{~cm}}{1 \mathrm{~min}} \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}} \times \frac{1 \mathrm{in}}{2.54 \mathrm{~cm}} \times \frac{1 \mathrm{ft}}{12 \mathrm{in}}=2.7 \times 10^{-3} \mathrm{ft} / \mathrm{s}$
(b) Strategy There are 5280 feet in one mile, 12 inches in one foot, 2.54 centimeters in one inch, and 60 minutes in one hour.
Solution Express the snail's speed in miles per hour.

$$
\frac{5.0 \mathrm{~cm}}{1 \mathrm{~min}} \times \frac{60 \mathrm{~min}}{1 \mathrm{~h}} \times \frac{1 \mathrm{in}}{2.54 \mathrm{~cm}} \times \frac{1 \mathrm{ft}}{12 \mathrm{in}} \times \frac{1 \mathrm{mi}}{5280 \mathrm{ft}}=1.9 \times 10^{-3} \mathrm{mi} / \mathrm{h}
$$

Discussion. It is natural to use a small unit for a low speed. Continents can drift at a few $\mathrm{cm} / \mathrm{yr}$.
42. Strategy A micrometer is $10^{-6} \mathrm{~m}$ and a millimeter is $10^{-3} \mathrm{~m}$; therefore, a micrometer is $10^{-6} / 10^{-3}=10^{-3} \mathrm{~mm}$.

Solution Find the area in square millimeters.
$150 \mu \mathrm{~m}^{2} \times\left(\frac{10^{-3} \mathrm{~mm}}{1 \mu \mathrm{~m}}\right)^{2}=1.5 \times 10^{-4} \mathrm{~mm}^{2}$
43. Strategy Replace each quantity in $U=m g h$ with its SI base units.

Solution Find the combination of SI base units that are equivalent to joules.
$U=m g h \quad$ implies $\quad \mathrm{J}=\mathrm{kg} \times \mathrm{m} / \mathrm{s}^{2} \times \mathrm{m}=\mathrm{kg} \cdot \mathrm{m}^{2} \cdot \mathrm{~s}^{-2}$
44. (a) Strategy Replace each quantity in $m a$ and $k x$ with its dimensions.

Solution Show that the dimensions of $m a$ and $k x$ are equivalent.
$m a$ has dimensions $[\mathrm{M}] \times \frac{[\mathrm{L}]}{[\mathrm{T}]^{2}}$ and $k x$ has dimensions $\frac{[\mathrm{M}]}{[\mathrm{T}]^{2}} \times[\mathrm{L}]=[\mathrm{M}] \times \frac{[\mathrm{L}]}{[\mathrm{T}]^{2}}$.
Since $[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}=[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}$, the dimensions are equivalent.
(b) Strategy Use the results of part (a).

Solution Since $F_{n e t}=m a$ and $F=-k x$, the dimensions of the force unit are $[\mathrm{M}][\mathrm{L}][\mathrm{T}]^{-2}$.
45. Strategy Replace each quantity in $T^{2}=4 \pi^{2} r^{3} /(G M)$ with its dimensions.

Solution Show that the equation is dimensionally correct.
$T^{2}$ has dimensions $[\mathrm{T}]^{2}$ and $\frac{4 \pi^{2} r^{3}}{G M}$ has dimensions $\frac{[\mathrm{L}]^{3}}{\frac{[\mathrm{~L}]^{3}}{[\mathrm{M}][\mathrm{T}]^{2}} \times[\mathrm{M}]}=\frac{[\mathrm{L}]^{3}}{[\mathrm{M}]} \times \frac{[\mathrm{M}][\mathrm{T}]^{2}}{[\mathrm{~L}]^{3}}=[\mathrm{T}]^{2}$.
Since $[\mathrm{T}]^{2}=[\mathrm{T}]^{2}$, the equation is dimensionally correct.
Discussion. It might seem more natural to you to write the test in terms of units, like this:
$\mathrm{s}^{2}$ is supposed to be equal to $\frac{\mathrm{m}^{3}}{\left(\mathrm{~m}^{3} / \mathrm{kg} \cdot \mathrm{s}^{2}\right) \mathrm{kg}}=\frac{\mathrm{m}^{3}}{\mathrm{~kg}} \frac{\mathrm{~kg} \cdot \mathrm{~s}^{2}}{\mathrm{~m}^{3}}=\mathrm{s}^{2}$ and it is. The test in terms of dimensions is
stated in more general terms. Someone might insist on measuring the period of a planet in years or the comparative masses of astronomical objects as multiples of some large mass that is actually unknown, but those choices still leave the equation dimensionally correct.
46. Strategy Determine the SI unit of momentum using a process of elimination.

Solution Find the SI unit of momentum.
$K=\frac{p^{2}}{2 m}$ has units of $\frac{\mathrm{kg} \cdot \mathrm{m}^{2}}{\mathrm{~s}^{2}}$. Since the SI unit for $m$ is kg , the SI unit for $p^{2}$ is $\frac{\mathrm{kg}^{2} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}}$. Taking the square root, we find that the SI unit for momentum is $\mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-1}$.
47. (a) Strategy Replace each quantity (except for $V$ ) in $F_{\mathrm{B}}=\rho g V$ with its dimensions.

Solution Find the dimensions of $V$.
$V=\frac{F_{\mathrm{B}}}{\rho g}$ has dimensions $\frac{\left[\mathrm{MLT}^{-2}\right]}{\left[\mathrm{ML}^{-3}\right] \times\left[\mathrm{LT}^{-2}\right]}=\left[\mathrm{L}^{3}\right]$.
(b) Strategy and Solution Since velocity has dimensions $\left[\mathrm{LT}^{-1}\right]$ and volume has dimensions $\left[\mathrm{L}^{3}\right]$, the correct interpretation of $V$ is that is represents volume.
48. (a) Strategy $a$ has dimensions $\frac{[\mathrm{L}]}{[\mathrm{T}]}$; $v$ has dimensions $\frac{[\mathrm{L}]}{[\mathrm{T}]}$; $r$ has dimension [L]. We assume that $a$ depends just on $v$ and $r$.

Solution Let $a \propto v^{\mathrm{p}} r^{\mathrm{q}}$ where $p$ and $q$ are unknown exponents. The units in the equation, $\mathrm{m} \cdot \mathrm{s}^{-2}=(\mathrm{m} / \mathrm{s})^{\mathrm{p}} \mathrm{m}^{\mathrm{q}}$ must agree across the equality sign, so we have $\mathrm{m}^{1}=m^{\mathrm{p}+\mathrm{q}}$ and $\mathrm{s}^{-2}=\mathrm{s}^{-\mathrm{p}}$. Then $p=2$ and $1=2+q$ so $q=-1$. The equation must be $a=K \frac{v^{2}}{r}$, where $K$ is a dimensionless constant.
(b) Strategy Divide the new acceleration by the old, and use the fact that the new speed is 1.100 times the old.

Solution Find the percent increase in the radial acceleration.
$\frac{a_{2}}{a_{1}}=\frac{K \frac{v_{2}^{2}}{r}}{K \frac{v_{1}^{2}}{r}}=\left(\frac{v_{2}}{v_{1}}\right)^{2}=\left(\frac{1.100 v_{1}}{v_{1}}\right)^{2}=1.100^{2}=1.210$
$1.210-1=0.210$, so the radial acceleration increases by $21.0 \%$.
49. Strategy Approximate the distance from your eyes to a book held at your normal reading distance.

Solution The normal reading distance is about 30 to 40 cm , so the approximate distance from your eyes to a book you are reading is 30 to 40 cm .
50. Strategy Estimate the length, width, and height of your textbook. Then use $V=\ell w h$ to estimate its volume.

Solution Find the approximate volume of your physics textbook in $\mathrm{cm}^{3}$.
The length, width, and height of your physics textbook are approximately $30 \mathrm{~cm}, 20 \mathrm{~cm}$, and 4.0 cm , respectively. $V=\ell w h=(30 \mathrm{~cm})(20 \mathrm{~cm})(4.0 \mathrm{~cm})=2400 \mathrm{~cm}^{3}$ or on the order of $10^{3} \mathrm{~cm}^{3}$.
51. Strategy and Solution The mass of the lower leg is about 5 kg and that of the upper leg is about 7 kg , so an order of magnitude estimate of the mass of a person's leg is $10 \mathrm{~kg}=10^{1} \mathrm{~kg}$.
52. Strategy and Solution A normal heart rate is about 70 beats per minute and a person may live for about 70 years, so the heart beats about $\frac{70 \text { beats }}{1 \mathrm{~min}} \times \frac{70 \mathrm{yr}}{\text { lifetime }} \times \frac{5.26 \times 10^{5} \mathrm{~min}}{1 \mathrm{yr}}=2.6 \times 10^{9}$ times per lifetime, or about $3 \times 10^{9}$ heartbeats.
53. Strategy One story is about 3 m high.

Solution Find the order of magnitude of the height in meters of a 40 -story building.
$(3 \mathrm{~m})(40) \sim 100 \mathrm{~m}=10^{2} \mathrm{~m}$
54. Strategy The area of skin is the area of the sides of the cylinder approximating the human torso plus 2 times the area of each arm. The surface of a cylinder, including the ends, is $2 \pi r h+2 \pi r^{2}$ (see Appendix A.6).

Solution Estimate the surface area of skin covering a human body.

$$
\begin{aligned}
A_{\text {skin }} & \approx A_{\mathrm{t}}+2 A_{\mathrm{a}}=2 \pi r_{\mathrm{t}} h_{\mathrm{t}}+2 \pi r_{\mathrm{t}}^{2}+2 \times 2 \pi r_{\mathrm{a}} h_{\mathrm{a}}+2 \times 2 \pi r_{\mathrm{a}}^{2} \\
& =2 \pi(0.15 \mathrm{~m})(2.0 \mathrm{~m})+2 \pi(0.15 \mathrm{~m})^{2}+2 \times 2 \pi(0.050 \mathrm{~m})(1.0 \mathrm{~m})+2 \times 2 \pi(0.050 \mathrm{~m})^{2}=2.7 \mathrm{~m}^{2}
\end{aligned}
$$

The contributions of the ends of the cylinders to the total area are comparatively small, so for an estimate we could ignore them. Then the estimate would be $2.5 \mathrm{~m}^{2}$, still on the order of $1 \mathrm{~m}^{2}=10^{0} \mathrm{~m}^{2}$.
55. Strategy The plot of temperature versus elapsed time is shown. Use the graph to answer the questions.


## Solution

(a) By inspection of the graph, it appears that the temperature at noon was $101.8^{\circ} \mathrm{F}$.
(b) Estimate the slope of the line.

$$
m=\frac{102.6^{\circ} \mathrm{F}-100.0^{\circ} \mathrm{F}}{1: 00 \text { P.M. }-10: 00 \text { A.M. }}=\frac{2.6^{\circ} \mathrm{F}}{3 \mathrm{~h}}=0.90^{\circ} \mathrm{F} / \mathrm{h}
$$

(c) After the next twelve hours, the temperature would, according to the trend, be approximately $T=\left(0.9^{\circ} \mathrm{F} / \mathrm{h}\right)(12 \mathrm{~h})+102.5^{\circ} \mathrm{F}=113^{\circ} \mathrm{F}$.
The patient would be dead before the temperature reached so high a value. So, the answer is no.
56. Strategy Use the two temperatures and their corresponding times to find the rate of temperature change with respect to time (the slope of the graph of temperature vs. time). Then, write the linear equation for the temperature with respect to time and find the temperature at $3: 35$ P.M.

Solution Find the rate of temperature change.
$m=\frac{\Delta T}{\Delta t}=\frac{101.0^{\circ} \mathrm{F}-97.0^{\circ} \mathrm{F}}{4.0 \mathrm{~h}}=1.0^{\circ} \mathrm{F} / \mathrm{h}$
Use the slope-intercept form of a graph of temperature vs. time to find the temperature at 3:35 P.M.
$T=m t+T_{0}=\left(1.0^{\circ} \mathrm{F} / \mathrm{h}\right)(3.5 \mathrm{~h})+101.0^{\circ} \mathrm{F}=104.5^{\circ} \mathrm{F}$
57. Strategy Put the equation that describes the line in slope-intercept form, $y=m x+b$, with $v$ replacing $y$ and $t$ replacing $x$.
$a t=v-v_{0} \quad$ becomes $\quad v=a t+v_{0}$

## Solution

(a) $v$ is the dependent variable and $t$ is the independent variable, so $a$ is the slope of the line.
(b) The slope-intercept form is $y=m x+b$. Find the vertical-axis intercept. $v \leftrightarrow y, t \leftrightarrow x, a \leftrightarrow m$, so $v_{0} \leftrightarrow b$.
Thus, $+v_{0}$ is the vertical-axis intercept of the line.
Discussion. A graph shows the data as individual points, and shows a model describing all of them as a line. The most important thing about a graph is its shape. In this case the straightness of a line fitting the data points convincingly indicates that the acceleration is constant. The scatter of the points away from the best-fit line can give an estimate of the experimental uncertainty. Plus, as the problem notes, the slope and vertical-axis intercept of the line give values for the acceleration and original velocity.
58. (a) Strategy Modeled on $y=m x+b$, the equation of the speed versus time graph is given by $v=a t+v_{0}$, where $a=6.0 \mathrm{~m} / \mathrm{s}^{2}$ and $v_{0}=3.0 \mathrm{~m} / \mathrm{s}$.

Solution Find the change in speed.

$$
\begin{aligned}
v_{2} & =a t_{2}+v_{0} \\
-\left(v_{1}\right. & \left.=a t_{1}+v_{0}\right) \\
\hline v_{2}-v_{1} & =a\left(t_{2}-t_{1}\right) \\
v_{2}-v_{1} & =\left(6.0 \mathrm{~m} / \mathrm{s}^{2}\right)(6.0 \mathrm{~s}-4.0 \mathrm{~s})=12 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(b) Strategy Use the equation found in part (a).

Solution Find the speed when the elapsed time is equal to 5.0 seconds.

$$
v=\left(6.0 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})+3.0 \mathrm{~m} / \mathrm{s}=33 \mathrm{~m} / \mathrm{s}
$$

59. (a) Strategy Refer to the figure. Use the definition of the slope of a line and the fact that the vertical axis intercept is the distance $(x)$ value corresponding to $t=0$.

Solution Compute the slope.
$\frac{\Delta x}{\Delta t}=\frac{17.0 \mathrm{~km}-3.0 \mathrm{~km}}{9.0 \mathrm{~h}-0.0 \mathrm{~h}}=1.6 \mathrm{~km} / \mathrm{h}$.
When $t=0, x=3.0 \mathrm{~km}$; therefore, the vertical axis intercept is 3.0 km .
(b) Strategy and Solution The physical significance of the slope of the graph is that it represents the speed of the object. The physical significance of the vertical axis intercept is that it represents the starting position of the object (position at time zero).
60. Strategy To determine values for $c$ and $A_{0}$ from the experimental data, graph $A$ versus $B^{3}$.

Solution To graph $A$ versus $B^{3}$, graph $A$ on the vertical axis and $B^{3}$ on the horizontal axis. Then the slope is the experimental value of $c$ and the vertical-axis intercept is the experimental value for $A_{0}$. Both can then be compared with theoretical or separately-measured values for $c$ and $A_{0}$.
61. Strategy Use the slope-intercept form, $y=m x+b$.

Solution Since $x$ is on the vertical axis, it corresponds to $y$. Since $t^{4}$ is on the horizontal axis, it corresponds to $x$ (in $y=m x+b)$. So, the equation for $x$ as a function of $t$ is $x=\left(25 \mathrm{~m} / \mathrm{s}^{4}\right) t^{4}+3 \mathrm{~m}$.

Discussion. A graph of $x$ versus $t$ from the same data would curve strongly upward as a "quartic parabola." Your eye can judge the straightness of a straight line and how well it fits points around it. But a quartic parabola is much less familiar. That is why we make the particular choice to use the fourth power of every $t$ value as the horizontal-axis coordinate of a data point.
62. Strategy Use graphing rules 3,5 , and 7 under Graphing Data in Section 1.9 Graphs.

## Solution

(a) To obtain a linear graph, the students should plot $v$ versus $r^{2}$, where $v$ is the dependent variable and $r^{2}$ is the independent variable.
(b) The students should measure the slope of the best-fit line obtained from the graph of the data; set the value of the slope equal to $2 g\left(\rho-\rho_{\mathrm{f}}\right) /(9 \eta)$; and solve for $\eta$.
63. (a) Strategy Plot the decay rate on the vertical axis and the time on the horizontal axis.

Solution The plot is shown.

(b) Strategy Plot the natural logarithm of the decay rate on the vertical axis and the time on the horizontal axis.

Solution. We find the natural logarithm of each decay rate. The first is $\ln (405)=6.00$. The others are

| shown: Time $(\mathrm{min})$ | 0 | 15 | 30 | 45 | 60 | 75 | 90 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\ln ($ decay rate in Bq$)$ | 6.00 | 5.47 | 4.94 | 4.50 | 4.00 | 3.47 | 2.94 |

The plot is shown.
Presentation of the data in this form-with the natural logarithm of the decay rate-is useful because the graph is linear, demonstrating that the decay rate decreases exponentially. That is, the data points fit a function like $R=R_{0} e-\lambda t$ where $R_{0}$ and $\lambda$ are constants. In this case $\ln (R)=-\lambda t+\ln \left(R_{0}\right)$, which has the form $y=$ $m x+b$.

64. (a) Strategy Make an order-of-magnitude estimate. Assume 8 seconds per breath.

Solution Estimate the number of breaths you take in one year.
breaths per year $=\frac{1 \text { breath }}{8 \mathrm{~s}} \times \frac{3.156 \times 10^{7} \mathrm{~s}}{1 \mathrm{yr}}=4 \times 10^{6}$ breaths $/ \mathrm{y} \sim 10^{7}$ breaths/y We choose to write the order of magnitude as $10^{7}$ breaths instead of $10^{6}$ because our answer differs from $10^{7}$ by a factor of only 2.5 while differing from $10^{6}$ by a factor of 4 .
(b) Strategy Assume 0.5 L per breath.

Solution Estimate the volume of air you breathe in during one year.

$$
\text { volume }=4 \times 10^{6} \text { breaths } \times \frac{0.5 \mathrm{~L}}{1 \text { breath }}=2 \times 10^{6} \mathrm{~L} \times \frac{10^{-3} \mathrm{~m}^{3}}{1 \mathrm{~L}}=2000 \mathrm{~m}^{3} \sim 10^{3} \mathrm{~m}^{3}
$$

65. Strategy Replace $v, r, \omega$, and $m$ with their dimensions. Then use dimensional analysis to determine how $v$ depends upon some or all of the other quantities.

Solution $v, r, \omega$, and $m$ have dimensions $\frac{[\mathrm{L}]}{[\mathrm{T}]},[\mathrm{L}], \frac{1}{[\mathrm{~T}]}$, and [M], respectively. Let $v=r^{\mathrm{e}} \omega^{f} m^{\mathrm{g}}$, where $e, f$, and $g$ are unknown exponents. Then $\frac{[\mathrm{L}]}{[\mathrm{T}]}=[\mathrm{L}]^{e} \frac{1}{[\mathrm{~T}]^{f}}[\mathrm{M}]^{g}$ The lefthand side is $[\mathrm{L}]^{1}[\mathrm{~T}]^{-1}[\mathrm{M}]^{0}$ so we require $e=1,-f=-1$, and $g=0$. Thus $f=1$. We have $v=r^{1} \omega^{1}$ and $v$ does not depend upon $m$. With no dimensionless constant involved in the relation, $v$ is equal to the product of $\omega$ and $r$, or $v=\omega r$.

Discussion. The argument could be stated in terms of units. We need a combination of m and $\mathrm{s}^{-1}$ and kg that will equal $\mathrm{m} / \mathrm{s}$, and the only combination is $(\mathrm{m})\left(\mathrm{s}^{-1}\right)$.
66. Strategy (Answers will vary.) We use San Francisco, California, for the city. The population of San Francisco is approximately 750,000 . Assume that there is one automobile for every two residents of San Francisco, that an average automobile needs three repairs or services per year, and that the average shop can service 10 automobiles per day.

Solution Estimate the number of automobile repair shops in San Francisco.
If an automobile needs three repairs or services per year, then it needs $\frac{3 \text { repairs }}{\text { auto } \cdot \mathrm{yr}} \times \frac{1 \mathrm{yr}}{365 \mathrm{~d}} \approx \frac{0.01 \text { repairs }}{\text { auto } \cdot \mathrm{d}}$.
If there is one auto for every two residents, then there are $\frac{1 \text { auto }}{2 \text { residents }} \times 750,000$ residents $\approx 4 \times 10^{5}$ autos.
If a shop requires one day to service 10 autos, then the number of shops-days per repair is
1 shop $\times \frac{1 \mathrm{~d}}{10 \text { repairs }}=\frac{0.1 \mathrm{shop} \cdot \mathrm{d}}{\text { repair }}$.
The estimated number of auto shops is $4 \times 10^{5}$ autos $\times \frac{0.01 \text { repairs }}{\text { auto } \cdot \mathrm{d}} \times \frac{0.1 \text { shop } \cdot \mathrm{d}}{\text { repair }}=400$ shops .
Checking the phone directory, we find that there are approximately 463 automobile repair and service shops in San Francisco. The estimate is off by $\frac{400-463}{400} \times 100 \%=-16 \%$. The estimate was $16 \%$ too low, but in the ball park!
67. (a) Strategy Plot the weights and ages on a weight versus age graph.

Solution See the graph.

(b) Strategy Find the slope of the average-gain line shown, between age 0 and age 5 months.

Solution Find the slope. $m=\frac{13.6 \mathrm{lb}-6.6 \mathrm{lb}}{5.0 \mathrm{mo}-0.0 \mathrm{mo}}=\frac{7.0 \mathrm{lb}}{5.0 \mathrm{mo}}=1.4 \mathrm{lb} / \mathrm{mo}$
(c) Strategy Find the slope of the average-gain line between age 5 and age 10 months.

Solution Find the slope. $\quad m=\frac{17.5 \mathrm{lb}-13.6 \mathrm{lb}}{10.0 \mathrm{mo}-5.0 \mathrm{mo}}=\frac{3.9 \mathrm{lb}}{5.0 \mathrm{mo}}=0.78 \mathrm{lb} / \mathrm{mo}$
(d) Strategy Write a linear equation for the weight of the baby as a function of time. The slope is that found in part (b), $1.4 \mathrm{lb} / \mathrm{mo}$. The intercept is the weight of the baby at birth.

Solution Find the extrapolated weight of the child at age 12 years.
$W=(1.4 \mathrm{lb} / \mathrm{mo})(144 \mathrm{mo})+6.6 \mathrm{lb}=210 \mathrm{lb}$
68. Strategy For parts (a) through (d), perform the calculations.

Solution
(a) $186.300+0.0030=186.303$
(b) $186.300-0.0030=186.297$
(c) $186.300 \times 0.0030=0.56$
(d) $186.300 / 0.0030=62,000$
(e) Strategy For cases (a) and (b), the percent error is given by $\frac{0.0030}{\text { Actual Value }} \times 100 \%$.

Solution Find the percent error. Case (a): $\frac{0.0030}{186.303} \times 100 \%=0.0016 \%$
Case (b): $\frac{0.0030}{186.297} \times 100 \%=0.0016 \%$
For case (c), ignoring 0.0030 causes you to multiply by zero and get a zero result. For case (d), ignoring 0.0030 causes you to divide by zero and get an infinite answer.
(f) Strategy Make a rule about neglecting small values using the results obtained above.

## Solution

You can neglect small values when they are added to or subtracted from sufficiently large values. The term "sufficiently large" is determined by the number of significant figures required.
69. Strategy There are on the order of $10^{3}$ hairs in a one-square-inch area of a typical human head. An order-ofmagnitude estimate of the area of the average human scalp is $10^{2}$ square inches.

Solution Calculate the estimate. $10^{3}$ hairs $/ \mathrm{in}^{2} \times 10^{2}$ in $^{2}=10^{5}$ hairs
Discussion. "On the order of $10^{5}$ " includes all the numbers between about $3 \times 10^{4}$ and about $3 \times 10^{5}$. Our answer applies to very many people, if not to the bald.
70. Strategy Use the metric prefixes $\mathrm{n}\left(10^{-9}\right), \mu\left(10^{-6}\right), \mathrm{m}\left(10^{-3}\right)$, or $\mathrm{M}\left(10^{6}\right)$.

## Solution

(a) M (or mega) is equal to $10^{6}$, so $6 \times 10^{6} \mathrm{~m}=6 \mathrm{Mm}$.
(b) There are approximately 3.28 feet in one meter, so $6 \mathrm{ft} \times \frac{1 \mathrm{~m}}{3.28 \mathrm{ft}}=2 \mathrm{~m}$.
(c) $\mu$ (or micro) is equal to $10^{-6}$, so $10^{-6} \mathrm{~m}=1 \mu \mathrm{~m}$.
(d) n (or nano) is equal to $10^{-9}$, so $3 \times 10^{-9} \mathrm{~m}=3 \mathrm{~nm}$.
(e) n (or nano) is equal to $10^{-9}$, so $3 \times 10^{-10} \mathrm{~m}=0.3 \mathrm{~nm}$.
71. Strategy The volume of the spherical virus is given by $V_{\text {virus }}=(4 / 3) \pi r_{\text {virus }}{ }^{3}$. The volume of viral particles is one billionth the volume of the saliva.

Solution Calculate the number of viruses that have landed on you.

$$
\text { number of viral particles }=\frac{10^{-9} V_{\text {saliva }}}{V_{\text {virus }}}=\frac{0.010 \mathrm{~cm}^{3}}{10^{9}\left(\frac{4}{3} \pi\right)\left(\frac{85 \mathrm{~nm}}{2}\right)^{3}\left(\frac{10^{-7} \mathrm{~cm}}{1 \mathrm{~nm}}\right)^{3}} \sim 10^{4} \text { viruses }
$$

72. Strategy The circumference of a viroid is approximately 300 times 0.35 nm . The diameter is given by $C=\pi d$, or $d=C / \pi$.
Solution Find the diameter of the viroid in the required units.
(a) $d=\frac{300(0.35 \mathrm{~nm})}{\pi} \times \frac{10^{-9} \mathrm{~m}}{1 \mathrm{~nm}}=3.3 \times 10^{-8} \mathrm{~m}$
(b) $d=\frac{300(0.35 \mathrm{~nm})}{\pi} \times \frac{10^{-3} \mu \mathrm{~m}}{1 \mathrm{~nm}}=3.3 \times 10^{-2} \mu \mathrm{~m}$
(c) $d=\frac{300(0.35 \mathrm{~nm})}{\pi} \times \frac{10^{-7} \mathrm{~cm}}{1 \mathrm{~nm}} \times \frac{1 \mathrm{in}}{2.54 \mathrm{~cm}}=1.3 \times 10^{-6} \mathrm{in}$
73. (a) Strategy There are 3.28 feet in one meter.

Solution Find the length in meters of the largest recorded blue whale. $1.10 \times 10^{2} \mathrm{ft} \times \frac{1 \mathrm{~m}}{3.28 \mathrm{ft}}=33.5 \mathrm{~m}$
(b) Strategy Divide the length of the largest recorded blue whale by the length of a double-decker London bus.

Solution Find the length of the blue whale in double-decker-bus lengths.
$\frac{1.10 \times 10^{2} \mathrm{ft}}{8.0 \frac{\mathrm{~m}}{\text { bus length }}} \times \frac{1 \mathrm{~m}}{3.28 \mathrm{ft}}=4.2$ bus lengths
Discussion. The picture in the textbook is quite accurate.
74. Strategy The volume of the blue whale can be found by dividing the mass of the whale by its average density.

Solution Find the volume of the blue whale in cubic meters. From $\rho=m / V$ we have

$$
V=\frac{m}{\rho}=\frac{1.9 \times 10^{5} \mathrm{~kg}}{0.85 \mathrm{~g} / \mathrm{cm}^{3}} \times \frac{1000 \mathrm{~g}}{1 \mathrm{~kg}} \times\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{3}=2.2 \times 10^{2} \mathrm{~m}^{3}
$$

75. Strategy Modeling the capillaries as completely filled with blood, the total volume of blood is given by the crosssectional area of the blood vessel times the length.

Solution Estimate the total volume of blood in the human body.
$V=\pi r^{2} l=\pi\left(4 \times 10^{-6} \mathrm{~m}\right)^{2}\left(10^{8} \mathrm{~m}\right)=0.005 \mathrm{~m}^{3}=5 \mathrm{~L}$
In reality, blood flow through the capillaries is regulated, so they are not always full of blood. On the other hand, we've neglected the additional blood found in the larger vessels (arteries, arterioles, veins, and venules).
76. Strategy The shape of a sheet of paper (when not deformed) is a rectangular prism. The volume of a rectangular prism is equal to the product of its length, width, and height (or thickness).

Solution Find the volume of a sheet of paper in cubic meters.
$27.95 \mathrm{~cm} \times 8.5 \mathrm{in} \times 0.10 \mathrm{~mm} \times \frac{1 \mathrm{~m}}{100 \mathrm{~cm}} \times \frac{0.0254 \mathrm{~m}}{1 \mathrm{in}} \times \frac{1 \mathrm{~m}}{1000 \mathrm{~mm}}=6.0 \times 10^{-6} \mathrm{~m}^{3}$
77. Strategy If $v$ is the speed of the molecule, then $v \propto \sqrt{T}$ where $T$ is the temperature.

Solution Form a proportion. $\frac{v_{\text {cold }}}{v_{\text {warm }}}=\frac{\sqrt{T_{\text {cold }}}}{\sqrt{T_{\text {warm }}}}$
Find $v_{\text {cold }} . \quad v_{\text {cold }}=v_{\text {warm }} \sqrt{\frac{T_{\text {cold }}}{T_{\text {warm }}}}=(475 \mathrm{~m} / \mathrm{s}) \sqrt{\frac{250.0 \mathrm{~K}}{300.0 \mathrm{~K}}}=434 \mathrm{~m} / \mathrm{s}$
Discussion. When we study thermodynamics, we will do a lot with the idea that a thermometer is like a speedometer for molecules.
78. Strategy Use dimensional analysis to convert from furlongs per fortnight to the required units.

Solution (a) Convert to $\mu \mathrm{m} / \mathrm{s}$.

$$
\frac{1 \text { furlong }}{1 \text { fortnight }} \times \frac{220 \mathrm{yd}}{1 \text { furlong }} \times \frac{1 \text { fortnight }}{14 \text { days }} \times \frac{1 \text { day }}{86,400 \mathrm{~s}} \times \frac{3 \mathrm{ft}}{1 \mathrm{yd}} \times \frac{1 \mathrm{~m}}{3.28 \mathrm{ft}} \times \frac{1,000,000 \mu \mathrm{~m}}{1 \mathrm{~m}}=166 \mu \mathrm{~m} / \mathrm{s}
$$

(b) Convert to $\mathrm{km} /$ day. $\frac{1 \text { furlong }}{1 \text { fortnight }} \times \frac{220 \mathrm{yd}}{1 \text { furlong }} \times \frac{1 \text { fortnight }}{14 \text { days }} \times \frac{3 \mathrm{ft}}{1 \mathrm{yd}} \times \frac{1 \mathrm{~m}}{3.28 \mathrm{ft}} \times \frac{1 \mathrm{~km}}{1000 \mathrm{~m}}=0.0144 \mathrm{~km} / \mathrm{day}$
79. Strategy There are 2.54 cm in one inch and 3600 seconds in one hour.

Solution Find the conversion factor for changing meters per second to miles per hour. The conversion equation
is $\frac{1 \mathrm{~m}}{1 \mathrm{~s}} \times \frac{100 \mathrm{~cm}}{1 \mathrm{~m}} \times \frac{1 \mathrm{in}}{2.54 \mathrm{~cm}} \times \frac{1 \mathrm{ft}}{12 \mathrm{in}} \times \frac{1 \mathrm{mi}}{5280 \mathrm{ft}} \times \frac{3600 \mathrm{~s}}{1 \mathrm{~h}}=2.24 \mathrm{mi} / \mathrm{h}=1 \mathrm{~m} / \mathrm{s}$
or the conversion factor is $(2.24 \mathrm{mi} / \mathrm{h}) /(1 \mathrm{~m} / \mathrm{s}))=1$
So, for a quick, approximate conversion, multiply by 2 .
80. (a) Strategy There are $20,000=2 \times 10^{4}$ quarter dollars in $\$ 5000$. The mass of a quarter-dollar coin is about 6 grams, or $6 \times 10^{-3}$ kilograms.

Solution Estimate the mass of the coins as $2 \times 10^{4}$ coins $\times 6 \times 10^{-3} \mathrm{~kg} /$ coin $=1.2 \times 10^{2} \mathrm{~kg}$, or $\sim 100 \mathrm{~kg}$.
(b) Strategy There are $\$ 1,000,000 / \$ 20=50,000$ twenty-dollar bills in $\$ 1,000,000$. The mass of a twenty-dollar bill is about 1 gram, or $10^{-3}$ kilograms.

Solution Estimate the mass of the bills as 50,000 bills $\times 10^{-3} \mathrm{~kg} / \mathrm{bill}=50 \mathrm{~kg}$, or $\sim 100 \mathrm{~kg}$.
81. Strategy The SI base unit for mass is kg. Replace each quantity in $W=m g$ with its SI base units.

Solution The SI unit for weight is $\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^{2}}=\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^{2}}$
Discussion. In everyday life one meets combination units such as kilometers per hour, person-hours, and even dollars per person-hour. No doubt gloomy research has been done on how many people understand quantities involving these units. We may be glossing over the difficulty of forming a mental picture of a kilogram-meter-per-second-squared, when we call it a newton and feel one newton as the gravitational force on a particular apple.
82. Strategy It is given that $T^{2} \propto r^{3}$. Divide the period of Mars by that of Venus.

Solution Compare the period of Mars to that of Venus.
$\frac{T_{\text {Mars }}^{2}}{T_{\text {Venus }}^{2}}=\frac{r_{\text {Mars }}^{3}}{r_{\text {Venus }}^{3}}$, so $T_{\text {Mars }}^{2} \approx\left(\frac{r_{\text {Mars }}}{r_{\text {Venus }}}\right)^{3} T_{\text {Venus }}^{2}$, or $T_{\text {Mars }} \approx\left(\frac{2 r_{\text {Venus }}}{r_{\text {Venus }}}\right)^{3 / 2} T_{\text {Venus }}=2^{3 / 2} T_{\text {Venus }} \approx 2.8 T_{\text {Venus }}$.
83. Strategy $\$ 59,000,000,000$ has a precision of 1 billion dollars; $\$ 100$ has a precision of 100 dollars, so his net worth is the same to two significant figure.

Solution Find the net worth. $\$ 59,000,000,000-\$ 100=\$ 59,000,000,000$
84. Strategy Solution methods will vary, but the order of magnitude answer should not. One example follows: In a car on the interstate highway you can drive a thousand kilometers in two days, but that does not get you around the circumference of the Earth by a very large angle. The circumference of the Earth is about forty thousand kilometers. The radius of the Earth is on the order of $10^{7} \mathrm{~m}$. The area of a sphere is $4 \pi r^{2}$, or on the order of $10^{1} \cdot r^{2}$. The average depth of the oceans is about $4 \times 10^{3} \mathrm{~m}$. The oceans cover about seven tenths of the Earth's surface.

Solution Calculate an order-of-magnitude estimate of the volume of water contained in Earth's oceans.
The surface area of the Earth is on the order of $10^{1} \cdot\left(10^{7} \mathrm{~m}\right)^{2}=10^{15} \mathrm{~m}^{2}$; therefore, the volume of water in the oceans is about area $\times$ depth $\sim 0.7\left(10^{15} \mathrm{~m}^{2}\right)\left(4 \times 10^{3} \mathrm{~m}\right) \approx 3 \times 10^{18} \mathrm{~m}^{3} \sim 10^{18} \mathrm{~m}^{3}$.
85. (a) Strategy There are 7.0 leagues in one step and 4.8 kilometers in one league.

Solution Find your speed in kilometers per hour.

$$
\frac{120 \text { steps }}{1 \mathrm{~min}} \times \frac{7.0 \text { leagues }}{1 \text { step }} \times \frac{4.8 \mathrm{~km}}{1 \text { league }} \times \frac{60 \mathrm{~min}}{1 \mathrm{~h}}=2.4 \times 10^{5} \mathrm{~km} / \mathrm{h}
$$

(b) Strategy The circumference of the earth is approximately $40,000 \mathrm{~km}$. The time it takes to march around the Earth is found by dividing the distance by the speed.

Solution Find the time of travel. $40,000 \mathrm{~km} \times \frac{1 \mathrm{~h}}{2.4 \times 10^{5} \mathrm{~km}} \times \frac{60 \mathrm{~min}}{1 \mathrm{~h}}=10 \mathrm{~min}$
Discussion. You could use the radius of the Earth as listed on the inside back cover. But we happen to know that a meter was originally defined to make the circumference of the planet (around meridians of longitude) forty thousand kilometers.
86. Strategy Use conversion factors from the inside cover of the book.

Solution (a) $\frac{12.5 \mathrm{US} \mathrm{gal}}{1} \times \frac{3.785 \mathrm{~L}}{\mathrm{US} \mathrm{gal}} \times \frac{10^{3} \mathrm{~mL}}{\mathrm{~L}} \times \frac{0.06102 \mathrm{in}^{3}}{\mathrm{~mL}}=2890 \mathrm{in}^{3}$
(b) $\frac{2887 \mathrm{in}^{3}}{1} \times\left(\frac{1 \text { cubit }}{18 \text { in }}\right)^{3}=0.495$ cubic cubits
87. Strategy The weight is proportional to the planet mass and inversely proportional to the square of the radius, so $W \propto m / r^{2}$. Thus, for Earth and Jupiter, we have $W_{\mathrm{E}}=K m_{\mathrm{E}} / r_{\mathrm{E}}^{2}$ and $W_{\mathrm{J}}=K m_{\mathrm{J}} / r_{\mathrm{J}}^{2}$. with some constant $K$.

Solution Form a proportion. $\frac{W_{\mathrm{J}}}{W_{\mathrm{E}}}=\frac{m_{\mathrm{J}} / r_{\mathrm{J}}^{2}}{m_{\mathrm{E}} / r_{\mathrm{E}}^{2}}=\frac{m_{\mathrm{J}}}{m_{\mathrm{E}}}\left(\frac{r_{\mathrm{E}}}{r_{\mathrm{J}}}\right)^{2}=\frac{320 m_{\mathrm{E}}}{m_{\mathrm{E}}}\left(\frac{r_{\mathrm{E}}}{11 r_{\mathrm{E}}}\right)^{2}=\frac{320}{121}$
On Jupiter, the apple would weigh $\frac{320}{121}(1.0 \mathrm{~N})=2.6 \mathrm{~N}$.
88. Strategy Replace each quantity in $v=K \lambda^{p} g^{q}$ by its units. Then, use the relationships between $p$ and $q$ to determine their values.

Solution Find the values of $p$ and $q$.
In units, $\frac{\mathrm{m}}{\mathrm{s}}=\mathrm{m}^{p} \cdot \frac{\mathrm{~m}^{q}}{\mathrm{~s}^{2 q}}=\frac{\mathrm{m}^{p+q}}{\mathrm{~s}^{2 q}}$.
So, we have the following restrictions on $p$ and $q: p+q=1$ and $2 q=1$.
Solve for $q$ and $p$.

$$
\begin{array}{rlrl}
2 q & =1 & p+q & =1 \\
q=\frac{1}{2} & p+\frac{1}{2} & =1 \\
p & =\frac{1}{2}
\end{array}
$$

Thus, $v=K \lambda^{1 / 2} g^{1 / 2}=K \sqrt{\lambda g}$.
89. Strategy Since there are about $3 \times 10^{8}$ people in the U.S., a reasonable estimate of the number of automobiles is $1.5 \times 10^{8}$. There are 365 days per year. A reasonable estimate for the average volume of gasoline used per day per car is greater than 1 gal , but less than 10 gal ; for an order-of-magnitude estimate, let's guess 2 gallons per day.
Solution Calculate the estimate. $1.5 \times 10^{8}$ cars $\times 365$ days $\times 2 \frac{\mathrm{gal}}{\mathrm{car} \cdot \text { day }} \sim 10^{11} \mathrm{gal}$
Discussion. Not everyone uses the tilde symbol $\sim$ for "is on the order of," but we find it convenient.
90. Strategy The order of magnitude of the volume of water required to fill a bathtub is $10^{1} \mathrm{ft}^{3}$. The order of magnitude of the number of cups in a cubic foot is $10^{2}$. Then $10^{1} \mathrm{ft}^{3} \times 10^{2} \mathrm{cups} / \mathrm{ft}^{3}=10^{3} \mathrm{cups}$
Solution Another chain of logic: Two cups make a pint, two pints make a quart, a quart is roughly a liter, a liter is one thousandth of a cubic meter, and two tenths of a cubic meter is plenty of water for a bathtub that will not overflow when you get in. Find the order of magnitude of the number of cups of water required to fill a bathtub.

$$
0.2 \mathrm{~m}^{3} \frac{1000 \mathrm{~L}}{1 \mathrm{~m}^{3}} \frac{\text { about } 1 \text { quart }}{1 \mathrm{~L}} \frac{2 \text { pints }}{1 \text { quart }} \frac{2 \text { cups }}{1 \text { pint }}=800 \text { cups } \sim 10^{3} \mathrm{cups}
$$

91. (a) Strategy Inspect the units of $G, c$, and $h$ and generate three simultaneous equations from how the units (or dimensions) must agree across the equality sign when these constants are used to compute a time.
Solution Do not assume that $p, q$, and $r$ are integers, but they are dimensionless numbers in
$t_{\text {Planck }}=c^{p} G^{q} h^{r}$, requiring in terms of units $\mathrm{s}^{1}=\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{p}\left(\frac{\mathrm{~m}^{3}}{\mathrm{~kg}^{1} \mathrm{~s}^{2}}\right)^{q}\left(\frac{\mathrm{~kg}^{1} \mathrm{~m}^{2}}{\mathrm{~s}^{1}}\right)^{r}$ Then the base units must separately appear to the same power on both sides of the equation. For seconds, $1=-p-2 q-r$. For meters, $0=p+3 q+2 r$. For kilograms, $0=0-q+r$. Substitute $r=q$ from the last equation into the first two to get down to two equations in two unknowns: $1=-p-3 q$ and $0=p+5 q$. Now substitute $p=-5 q$ from the second equation into the first: $1=+5 q-3 q$. Then $q=+1 / 2$. If you tried to solve simultaneous equations with determinants, you would have to do much more work to find the other two unknowns. The method of substitution makes all the answers after the first come out easily, just as when the cat has kittens. From above, $p=-5 q=-5 / 2$ and $r=q=+1 / 2$. We find that the only combination of $G, c$, and $h$ that has the dimensions of time is $\sqrt{\sqrt{\frac{h G}{c^{5}}}}$.
(b) Strategy Substitute the values of the constants into the expression found in part (a).

Solution Find the time in seconds. $\sqrt{\frac{h G}{c^{5}}}=\sqrt{\frac{\left(6.6 \times 10^{-34} \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}}\right)\left(6.7 \times 10^{-11} \frac{\mathrm{~m}^{3}}{\mathrm{~kg} \cdot \mathrm{~s}^{2}}\right)}{\left(3.0 \times 10^{8} \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{5}}}=1.3 \times 10^{-43 \mathrm{~s}}$
92. Strategy The dimensions of $L, g$, and $m$ are length, length per time squared, and mass, respectively. The period has units of time, so $T$ cannot depend upon $m$. (There are no other quantities with units of mass with which to cancel the units of $m$.) Use a combination of $L$ and $g$.
Solution In $T=L^{p} g^{q}$ we require for the units to agree $\mathrm{s}=\mathrm{m}^{p}\left(\mathrm{~m} / \mathrm{s}^{2}\right)^{q}$ so separately $1=-2 q$ and $0=p+q$. Then $q=-1 / 2$ and $p=-q=+1 / 2$. The square root of $L / g$ does indeed have dimensions of time, so
$T=C \sqrt{\frac{L}{g}}$, where $C$ is a constant of proportionality.
93. Strategy The dimensions of $k$ and $m$ are mass per time squared and mass, respectively. Dividing either quantity by the other will eliminate the mass dimension, so the equation giving frequency must involve the quotient of $m$ and $k$.

Solution The square root of $k / m$ has dimensions of inverse time, which is correct for frequency.
So, $f=C \sqrt{k / m}$. where $C$ is some dimensionless constant. We set up a proportion to find $k$.
$f_{1}=C \sqrt{\frac{k}{m_{1}}}$, so $f_{1}^{2}=C^{2} \frac{k}{m_{1}}$, or $k=\frac{m_{1} f_{1}^{2}}{C^{2}}$.
Find the frequency of the chair with the $75-\mathrm{kg}$ astronaut.

$$
f_{2}=C \sqrt{\frac{k}{m_{2}}}=C \sqrt{\frac{m_{1} f_{1}^{2}}{C^{2} m_{2}}}=f_{1} \sqrt{\frac{m_{1}}{m_{2}}}=\left(0.50 \mathrm{~s}^{-1}\right) \sqrt{\frac{62 \mathrm{~kg}+10.0 \mathrm{~kg}}{75 \mathrm{~kg}+10.0 \mathrm{~kg}}}=0.46 \mathrm{~s}^{-1}
$$

Discussion. We will study this bouncing of a mass on a spring in a chapter on vibration. Here we see that dimensional analysis and proportional reasoning are more powerful than you might imagine.
94. Strategy Approach it as a unit-conversion problem.

Solution (a) The length of the road section, expressed as a multiple of the distance between reflectors, is 2.20 mile $\left(\frac{5280 \mathrm{ft}}{1 \text { mile }}\right) \frac{1 \mathrm{yd}}{3 \mathrm{ft}}\left(\frac{1 \text { space between reflectors }}{17.6 \mathrm{yd}}\right)=220$ spaces . We may need an extra reflector to furnish both ends, so we requisition 221 reflectors.
(b) As in part (a), $3.54 \mathrm{~km}=3.54 \mathrm{~km}\left(\frac{1000}{\mathrm{k}}\right) \frac{1 \text { space between }}{16 \mathrm{~m}}=221$ spaces between, so we order 222 reflectors in case adjoining road sections have none.

Within the strict limit of doing routing calculations that people have thought of in advance, U.S. customary units (or "former British" units) are fine. The reflectors are just $1 / 100$ mile apart. In real life SI units are better. Professionals built the Titanic. Noah thought about what he was doing while building the ark.
95. Strategy and solution. The doctor prescribed $3 / 4$ milliliter for each dose. The pharmacist printed $3 / 4$ teaspoon for each dose, which is larger by 4.9 times. The most common unit-conversion mistake comes from ignoring the units.
96. Strategy. Approach it as a unit-conversion problem.

Solution. The captain (pilot) intended to dive from 1450 m to $1500 \mathrm{ft}(1 \mathrm{~m} / 3.28 \mathrm{ft})=457 \mathrm{~m}$, a downward vertical distance of $1450 \mathrm{~m}-457 \mathrm{~m}=990 \mathrm{~m}$. The first officer (copilot) made the most common unit-conversion mistake: ignoring the units.
97. (a) Strategy Plot the data on a graph with mass on the vertical axis and time on the horizontal axis. Then, draw a best-fit smooth curve.

Solution See the graph.

(b) Strategy Estimate by eye the value of the total mass that the graph appears to be approaching asymptotically.

Solution The graph appears to be approaching asymptotically a maximum value close to 100 g , so the carrying capacity is about 100 g .
(c) Strategy Plot the data on a graph with the natural logarithm of $\mathrm{m} / m_{0}$ on the vertical axis and time on the horizontal axis. Draw a line through the points and find its slope to estimate the intrinsic growth rate.

Solution We find for the starting point $m / m_{0}=1$ and $\ln \left(m / m_{0}\right)=0$. For the first nonzero point,
$m / m_{0}=5.9 / 3.2=1.84$ and $\ln \left(m / m_{0}\right)=0.612$. We
 compute similarly

| Time, h | 0 | 2 | 4 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\ln \left(m / m_{0}\right)$ | 0 | 0.612 | 1.22 | 1.79 | 2.28 |

From these we plot the graph. Only the zeroth point and the next three lie close to a single straight line, so we omit the point for 8 hours. From the plot of $\ln \frac{m}{m_{0}}$ vs. $t$, the slope $r$ appears to be

$$
r=\frac{1.83-0.0}{6.0 \mathrm{~s}-0.0 \mathrm{~h}}=\frac{1.83}{6.0 \mathrm{~h}}=0.305 \mathrm{~h}^{-1} \text {. }
$$

Discussion. Many politicians and economists behave as if our national motto was "In Growth we trust." They put their faith in promoting "steady growth" by some nice percentage per year. They try to sell the idea to other societies, not understanding that this exponential increase is always unsustainable. An equation like $y=A t^{2}$ or $y=B t^{5}$ does not describe exponential growth-those equations are associated with some constant factor of increase in $y$ whenever $t$ increases by a constant factor. In the proportionality described by $y=B t^{5}$, for example, $y$ increases by 32 times whenever $t$ doubles. The exponential function $m=m_{0} e^{r t}$ is infinitely more sinister. It describes $m$ doubling again and again, without limit, whenever $t$ changes by a certain fixed step. From $\ln 2=0.693$, the doubling time $T_{d}$ for the yeast cells with unlimited resources is given by $r T_{d}=0.693$ so $T_{d}=0.693 /\left(0.305 \mathrm{~h}^{-1}\right)=2.27$ hours. As time regularly and inexorably ticks on as $2.27 \mathrm{~h}, 4.54 \mathrm{~h}, 6.81 \mathrm{~h}, 9.09 \mathrm{~h}$, $\ldots$ the population of yeast cells counts up in the pattern $1,2,4,8,16,32,64,128,256,512,1024,2048,4096$, $8192, \ldots$. In every doubling time the population uses up as much resources as they have in all previous history. The numbers tabulated in the problem start their increase at "only" $30.5 \%$ per hour or $0.5 \%$ per minute. But the numbers show that this exponential growth could not continue even as long as six or eight hours. Again, "steady growth" is exponential increase and is unsustainable.

## Chapter 2 <br> MOTION ALONG A LINE

## Conceptual Questions

1. Distance traveled is a scalar quantity equal to the total length of the path taken in moving from one point to another. Displacement is a vector quantity directed from the initial point towards the final point with a magnitude equal to the straight line distance between the two points. The magnitude of the displacement has no direction, and is always less than or equal to the total distance traveled.
2. The velocity of an object is a vector quantity equal to the displacement of the object per unit time interval. The speed of an object is a scalar quantity equal to the distance traveled by the object per unit time interval.
3. The area under the curve of a $v_{x}$ versus time graph is equal to the $x$-component of the displacement.
4. The slope of a line tangent to a curve on a $v_{x}$ versus time graph is equal to the $x$-component of the acceleration at the time corresponding to the point where the tangent line touches the curve.
5. The area under the curve of an $a_{x}$ versus time graph is equal to the change in the $x$-component of the velocity.
6. The slope of a line tangent to a curve on a graph plotting the $x$-component of position versus time is equal to the $x$ component of the instantaneous velocity at the time corresponding to the point where the tangent line touches the curve.
7. The average velocity of an object is defined as the ratio of the displacement of the object during an interval of time to the length of the time interval. The instantaneous velocity of an object is obtained from the average velocity by using a time interval that approaches zero. An object can have different average velocities for different time intervals. However, the average velocity for one particular time interval has a unique value.
8. Yes, the instantaneous velocity of an object can be zero while the acceleration is nonzero. When you toss a ball straight up in the air, its acceleration is directed downward, with a magnitude of $g$, the entire time it's in the air. Its velocity is zero at the highest point of its path, however. At previous times its acceleration is associated with its upward velocity decreasing. At later times the same acceleration describes the downward velocity increasing in magnitude. At the apex of the motion, the same acceleration describes the object's changing direction.
9. (a) $a_{x}>0$ and $v_{x}<0$ means you are moving south and slowing down.
(b) $a_{x}=0$ and $v_{x}<0$ means you are moving south at a constant speed.
(c) $a_{x}<0$ and $v_{x}=0$ means you are momentarily at rest as you change your direction of motion from north to south.
(d) $a_{x}<0$ and $v_{x}<0$ means you are moving south and speeding up.
(e) As can be seen from our answers above, it is not a good idea to use the term "negative acceleration" to mean slowing down. In parts (c) and (d), the acceleration is negative, but the bicycle is speeding up. Also, in part (a), the acceleration is positive, but the bicycle is slowing down.
10. At the highest point of the coin's motion, it is momentarily at rest, so its velocity is zero. Throughout the coin's motion, its acceleration is downward, with magnitude called $g$, the local free-fall acceleration or the local gravitational field. (We ignore air resistance for a coin much denser than air, not shaped like a parachute or potato chip, and moving slowly compared to sound.)
11. The balls cross paths at a height above $h / 2$. From the top of its flight, the first ball moves relatively slowly and for the same time interval as the second ball, which has higher speeds in its motion from the moment it is released until it meets the first ball. The higher speed of the second ball means it moves over a greater distance than the first ball in the section of motion we consider.
12. At the highest point of the coin's motion, it is momentarily at rest, so its velocity is zero. Throughout the coin's motion, its acceleration is downward, with magnitude called $g$, the local free-fall acceleration or the local gravitational field. (We ignore air resistance for a coin much denser than air, not shaped like a potato chip, and moving slowly compared to sound.)

## Multiple Choice Questions

1. (c) 2 2. (d) 3. (a) 4. (b) 5. (c) 6. (a) 7. (b) 8. (a) 9. (a)
2. (c) 11. (a) 12. (a) 13. (d) 14. (c)
3. (d)
4. (a) 17. (b) 18. (a) 19. (d) 20. (c) 21. + 22. $+x$ 23. $-x$
5. not changing
6. 26. $+x$
1.     - 
2. 0
3. $-x$ 30. decreasing

## Problems

1. Strategy Let east be the $+x$-direction.

Solution Draw a vector diagram; then compute the sum of the three displacements.
The vector diagram:


The sum of the three displacements is $(32 \mathrm{~cm}+48 \mathrm{~cm}-64 \mathrm{~cm})$ east $=16 \mathrm{~cm}$ east .
Discussion. We would get the same physical answer if we chose west as the positive direction.
2. Strategy Let the positive direction be to the right. Make a vector diagram with the location of the stone as the starting point.

Solution The vector diagram:
Stone


Add the displacements.
$\Delta \overrightarrow{\mathbf{r}}=4.0 \mathrm{~m}$ right +1.0 m left +6.5 m right +8.3 m left
$=4.0 \mathrm{~m}$ right -1.0 m right +6.5 m right -8.3 m right $=1.2 \mathrm{~m}$ right
The squirrel's total displacement from his starting point is 1.2 m to the right of the starting point.
3. Strategy Let east be the $+x$-direction.

Solution Compute the displacements; then find the total distance traveled.
(a) The runner's displacement from his starting point is

$$
\Delta \overrightarrow{\mathbf{r}}=\overrightarrow{\mathbf{r}}_{\mathrm{f}}-\overrightarrow{\mathbf{r}}_{\mathrm{i}}=20 \mathrm{~m} \text { west }-60 \mathrm{~m} \text { east }=20 \mathrm{~m} \text { west }+60 \mathrm{~m} \text { west }=80 \mathrm{~m} \text { west or }-80 \mathrm{~m} .
$$

(b) Since the runner is located 20 m west of the milestone, his displacement from the milestone is

$$
20 \mathrm{~m} \text { west or }-20 \mathrm{~m} \text {. }
$$

(c) The runner's displacement from his starting point is

$$
\Delta \overrightarrow{\mathbf{r}}=\overrightarrow{\mathbf{r}}_{\mathrm{f}}-\overrightarrow{\mathbf{r}}_{\mathrm{i}}=140 \mathrm{~m} \text { east }-60 \mathrm{~m} \text { east }=80 \mathrm{~m} \text { east or }+80 \mathrm{~m} .
$$

(d) The runner first jogs $60 \mathrm{~m}+20 \mathrm{~m}=80 \mathrm{~m}$; then he jogs $20 \mathrm{~m}+140 \mathrm{~m}=160 \mathrm{~m}$. The total distance traveled is
$80 \mathrm{~m}+160 \mathrm{~m}=240 \mathrm{~m}$.
4. Strategy Let south be the $+x$-direction.

Solution Draw vector diagrams for each situation; then find the displacements of the car.

(a)

$x_{3}-x_{1}=12 \mathrm{~km}-20 \mathrm{~km}=-8 \mathrm{~km}$
The displacement of the car between 3 P.M. and 6 P.M. is 8 km north of its position at 3 P.M.
(b)

$x_{1}+x_{2}=20 \mathrm{~km}+96 \mathrm{~km}=116 \mathrm{~km}$
The displacement of the car from the starting point to the location at 4 P.M. is 116 km south of the starting point.
(c)

$x_{3}-\left(x_{1}+x_{2}\right)=12 \mathrm{~km}-116 \mathrm{~km}=-104 \mathrm{~km}$
The displacement of the car between 4 P.M. and 6 P.M. is 104 km north of its position at 4 P.M.
5. Strategy Use the definition of average velocity.

Solution Find the average velocity of the train.
$\overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{10 \mathrm{~km} \text { east }-3 \mathrm{~km} \text { east }}{3: 28-3: 14}=\frac{7 \mathrm{~km} \text { east }}{14 \mathrm{~min}} \times \frac{60 \mathrm{~min}}{1 \mathrm{~h}}=30 \mathrm{~km} / \mathrm{h}$ east
Discussion. The values of $\overrightarrow{\mathbf{r}}$ depend on the choice of origin, but the values of $\Delta \overrightarrow{\mathbf{r}}$ and $\overrightarrow{\mathbf{v}}$ do not.
6. Strategy Use the definition of average velocity.

Solution Find the average velocity of the cyclist in meters per second.

$$
\overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{10.0 \mathrm{~km} \text { east }}{11 \mathrm{~min} 40 \mathrm{~s}}=\frac{10.0 \times 10^{3} \mathrm{~m} \text { east }}{700 \mathrm{~s}}=14.3 \mathrm{~m} / \mathrm{s} \text { east }
$$

7. Strategy Since the swift flies in a single direction, use the definition of average speed and the fact that it flies due north. The conversion factor is $1 \mathrm{mi} / \mathrm{h}=0.4470 \mathrm{~m} / \mathrm{s}$.

Solution Find the average velocity of the swift.
$v_{\mathrm{av}}=\frac{\Delta r}{\Delta t}=\frac{3.2 \times 10^{3} \mathrm{~m}}{32.8 \mathrm{~s}}=98 \mathrm{~m} / \mathrm{s}$ and $v_{\mathrm{av}}=\frac{\Delta r}{\Delta t}=\frac{3.2 \times 10^{3} \mathrm{~m}}{32.8 \mathrm{~s}} \times \frac{1 \mathrm{mi} / \mathrm{h}}{0.4470 \mathrm{~m} / \mathrm{s}}=220 \mathrm{mi} / \mathrm{h}$, so $\overrightarrow{\mathbf{v}}_{\mathrm{av}}=98 \mathrm{~m} / \mathrm{s}(220 \mathrm{mi} / \mathrm{h})$ due north.

Discussion. We could have used the conversion factors $1 \mathrm{mi}=1609 \mathrm{~m}$ and $1 \mathrm{~h}=3600 \mathrm{~s}$.
8. Strategy Jason never changes direction, so the direction of the average velocity is due west. Find the average speed by dividing the total distance traveled by the total time.

Solution The distance traveled during each leg of the trip is given by $\Delta x=v_{\mathrm{av}} \Delta t$.

$$
v_{\mathrm{av}}=\frac{(35.0 \mathrm{mi} / \mathrm{h})(0.500 \mathrm{~h})+(60.0 \mathrm{mi} / \mathrm{h})(2.00 \mathrm{~h})+(25.0 \mathrm{mi} / \mathrm{h})(10.0 / 60.0) \mathrm{h}}{0.500 \mathrm{~h}+2.00 \mathrm{~h}+(10.0 / 60.0) \mathrm{h}}=53.1 \mathrm{mi} / \mathrm{h}
$$

So, the average velocity is $53.1 \mathrm{mi} / \mathrm{h}$ due west.
9. Strategy When the Boxster catches the Scion, the displacement of the Boxster will be $\Delta r+186 \mathrm{~m}$ and the displacement for the Scion will be $\Delta r$. For both cars the $\Delta t$ will be the same.


The steeper line represents the motion of the Boxster. The line that starts out on top is the Scion. On the graph it appears that the faster car overtakes in about 32 s . We can find a precise value algebraically:

Solution Find the time it takes for the Boxster to catch the Scion.
$v_{\mathrm{av}}=\frac{\Delta r_{\mathrm{car}}}{\Delta t}$, so for the Boxster, $v_{\mathrm{av}, \mathrm{B}}=\frac{\Delta r+186 \mathrm{~m}}{\Delta t}$ or $\Delta r=v_{\mathrm{av}, \mathrm{B}} \Delta t-186 \mathrm{~m}$.
For the Scion,
$v_{\mathrm{av}, \mathrm{S}}=\frac{\Delta r}{\Delta t}$, so $\Delta r=v_{\mathrm{av}, \mathrm{S}} \Delta t$.
Equate the two expressions for $\Delta r$.

$$
\begin{aligned}
v_{\mathrm{av}, \mathrm{~S}} \Delta t & =v_{\mathrm{av}, \mathrm{~B}} \Delta t-186 \mathrm{~m} \\
\left(v_{\mathrm{av}, \mathrm{~S}}-v_{\mathrm{av}, \mathrm{~B}}\right) \Delta t & =-186 \mathrm{~m} \\
\Delta t & =\frac{186 \mathrm{~m}}{v_{\mathrm{av}, \mathrm{~B}}-v_{\mathrm{av}, \mathrm{~S}}}=\frac{186 \mathrm{~m}}{24.4 \mathrm{~m} / \mathrm{s}-18.6 \mathrm{~m} / \mathrm{s}}=\frac{186 \mathrm{~m}}{5.8 \mathrm{~m} / \mathrm{s}}=32 \mathrm{~s}
\end{aligned}
$$

Discussion. The graph clearly shows the situation of the faster car overtaking the slower car, as the intersection of the two lines. The cars have the same position then. The graph complements the precise calculation we have done.
10. Strategy Use the area under the curve to find the displacement of the car.

Solution The displacement of the car is given by the area under the $v_{x}$ vs. $t$ curve. Under the curve, there are 16 squares and each square represents $(5 \mathrm{~m} / \mathrm{s})(2 \mathrm{~s})=10 \mathrm{~m}$. Therefore, the car moves $16(10 \mathrm{~m})=160 \mathrm{~m}$.
Counting squares at 2 -s intervals gives the motion diagram

the same scale as the vertical position axis of the graph. It is to be read vertically upward. It shows positions at $t=$ $0,2 \mathrm{~s}, 4 \mathrm{~s}, 6, \mathrm{~s}, 8 \mathrm{~s}, 10 \mathrm{~s}$, and, with the black box, at 12,14 , and 16 s .
11. (a) Strategy Use the area between the $v_{y}$ vs. $t$ curve and the $x$-axis to find the displacement of the elevator.

Solution From $t=0 \mathrm{~s}$ to $t=10 \mathrm{~s}$, there are 8 squares. From $t=14 \mathrm{~s}$ to $t=20 \mathrm{~s}$, there are 4 squares. Each square represents $(1 \mathrm{~m} / \mathrm{s})(2 \mathrm{~s})=2 \mathrm{~m}$. The displacement from $t=14 \mathrm{~s}$ to $t=20 \mathrm{~s}$ is negative $\left(v_{y}<0\right)$. So the total displacement is $\Delta y=8(2 \mathrm{~m})+(-4)(2 \mathrm{~m})=8 \mathrm{~m}$, and the elevator is 8 m above its starting point.
(b) Strategy Use the slope of the curve to determine when the elevator reaches its highest point.

Solution The vertical velocity is positive for $t=0 \mathrm{~s}$ to $t=10 \mathrm{~s}$. It is negative for $t=14 \mathrm{~s}$ to $t=20 \mathrm{~s}$. It is zero for $t$ $=10 \mathrm{~s}$ to $t=14 \mathrm{~s}$. So the elevator reaches its highest location at $t=10 \mathrm{~s}$ and remains there until $t=14 \mathrm{~s}$ before it goes down. Thus, the elevator is at its highest location from $t=10 \mathrm{~s}$ to $t=14 \mathrm{~s}$.

## (c) Strategy and Solution.

The elevator starts from rest at the origin at time zero. It gains upward speed for 2 s , moves steadily upward at 2 $\mathrm{m} / \mathrm{s}$ for 6 s , and slows to a stop in another 2 s . It stays at height 16 m above its starting point for 4 s , then gains speed moving down for 2 s , coasts steadily down at $2 \mathrm{~m} / \mathrm{s}$ for 2 s , and comes to a stop at elevation 8 m in a final 2-s time interval.
(d)

12. Strategy and Solution Look at the slope or steepness of the line in each $1-\mathrm{s}$ time interval. 4 to 5 s has the biggest (and only) downward slope. 2 to 3 s has zero slope. 0 to 1,1 to 2 , and 3 to 4 s have gentle upward slopes and 5 to 6 s has the steepest upward slope.
13. Strategy and Solution Look at the steepness of the line in each 1 -s interval without regard for whether it is going up or down. The line from 2 to 3 s is flat. 0 to 1,1 to 2 , and 3 to 4 s have gentle slopes. 5 to 6 s is steeper and 4 to 5 s is still closer to vertical.

Discussion. The answer to this question is just like the answer to question 12, about slope as a signed number, except for the placement in the ranking of the one down-sloping section ( 4 to 5 s ). Instead of being placed as the smallest actual slope, it is now placed with the biggest absolute value of slope.
14. Strategy and Solution Look at the slope or steepness of the line in each $1-\mathrm{s}$ time interval. 5 to 6 s has the steepest upward slope. 0 to 1,1 to 2 , and 3 to 4 s have gentle upward slopes. 2 to 3 s has zero slope. 4 to 5 s has the biggest (and only) downward slope.
15. Strategy Use the graph to answer the question. What color is George Washington's white horse?

Solution It starts at $x=0$ at $t=0$ and at $t=3 \mathrm{~s}$ is at $x=20 \mathrm{~m}$, so it has moved 20 m .
16. Strategy Set up ratios of speeds to distances.

Solution Use $v=\Delta x / \Delta t$, write it as $v / \Delta x=1 / \Delta t$, apply it to each object, set the expression for the baseball equal to that for the softball, and find the speed of the baseball $v$.
$\frac{v}{60.5 \mathrm{ft}}=\frac{65.0 \mathrm{mi} / \mathrm{h}}{43.0 \mathrm{ft}}$, so $v=\frac{65.0 \mathrm{mi} / \mathrm{h}}{43.0 \mathrm{ft}}(60.5 \mathrm{ft})=91.5 \mathrm{mi} / \mathrm{h}$.
17. Strategy Use vector subtraction to find the change in velocity.

Solution Find the change in velocity of the scooter.
$\Delta \overrightarrow{\mathbf{v}}=\overrightarrow{\mathbf{v}}_{\mathrm{f}}-\overrightarrow{\mathbf{v}}_{\mathrm{i}}=15 \mathrm{~m} / \mathrm{s}$ west $-12 \mathrm{~m} / \mathrm{s}$ east $=15 \mathrm{~m} / \mathrm{s}$ west $-(-12 \mathrm{~m} / \mathrm{s}$ west $)=27 \mathrm{~m} / \mathrm{s}$ west
Discussion. Thinking about a vector change, or a change that is larger than the original value, is a mental step that a normal person puts some effort into understanding.
18. Strategy Determine the maximum time allowed to complete the run.

Solution Massimo must take no more than $\Delta t=\Delta x / v=1000 \mathrm{~m} /(4.0 \mathrm{~m} / \mathrm{s})=250 \mathrm{~s}$ to complete the run. Since he ran the first 900 m is 250 s , he cannot pass the test because he would have to run the last 100 m in 0 s .
19. Strategy Use the area under the curve to find the displacement of the skateboard.

Solution The displacement of the skateboard is given by the area under the $v$ vs. $t$ curve. Under the curve for $t=3.00 \mathrm{~s}$ to $t=8.00 \mathrm{~s}$, there are 16.5 squares and each square represents $(1.0 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})=1.0 \mathrm{~m}$; so the board moves 16.5 m . Counting squares at $1-\mathrm{s}$ intervals from 3 s to 8 s gives the motion diagram

the graph, with the same scale as the vertical position axis of the graph. It is to be read vertically upward.
20. Strategy Use the definition of average velocity.

Solution Find the average velocities of the skater.
(a) $v_{\mathrm{av}, x}=\frac{\Delta x}{\Delta t}=\frac{6.0 \mathrm{~m}-0}{4.0 \mathrm{~s}}=1.5 \mathrm{~m} / \mathrm{s}$
(b) $v_{\mathrm{av}, x}=\frac{6.0 \mathrm{~m}-0}{5.0 \mathrm{~s}}=1.2 \mathrm{~m} / \mathrm{s}$
21. Strategy The slope of the $x$ vs. $t$ curve is equal to $v_{x}$. Use the definition of instantaneous velocity.

Solution Compute the instantaneous velocity at $t=2.0 \mathrm{~s}$ as the slope of the straight segment going through this point. $v_{x}=\frac{6.0 \mathrm{~m}-4.0 \mathrm{~m}}{3.0 \mathrm{~s}-1.0 \mathrm{~s}}=1.0 \mathrm{~m} / \mathrm{s}$
Discussion. We made a free choice of any two points along the line segment passing through ( $2 \mathrm{~s}, 5 \mathrm{~m}$ ).
22. Strategy The slope of each segment (between changes in the slope) of the graph is equal to the speed during that time period.
Solution Find the speed during each time period.
Then, plot $v_{x}$ as a function of time.
$0<t<1 \mathrm{~s}: v_{x}=\frac{4 \mathrm{~m}}{1 \mathrm{~s}}=4 \mathrm{~m} / \mathrm{s}$
$1<t<3 \mathrm{~s}: v_{x}=\frac{6 \mathrm{~m}-4 \mathrm{~m}}{3 \mathrm{~s}-1 \mathrm{~s}}=1 \mathrm{~m} / \mathrm{s}$
$3<t<5 \mathrm{~s}: v_{x}=\frac{6 \mathrm{~m}-6 \mathrm{~m}}{5 \mathrm{~s}-3 \mathrm{~s}}=0 \mathrm{~m} / \mathrm{s}$
$5<t<6 \mathrm{~s}: v_{x}=\frac{1 \mathrm{~m}-6 \mathrm{~m}}{6 \mathrm{~s}-5 \mathrm{~s}}=-5 \mathrm{~m} / \mathrm{s}$

$6<t<8 \mathrm{~s}: v_{x}=\frac{0 \mathrm{~m}-1 \mathrm{~m}}{8 \mathrm{~s}-6 \mathrm{~s}}=-0.5 \mathrm{~m} / \mathrm{s}$
23. (a) Strategy Let the positive direction be to the right. Draw a diagram.

Solution Find the chipmunk's total displacement.

$80 \mathrm{~cm}-30 \mathrm{~cm}+90 \mathrm{~cm}-310 \mathrm{~cm}=-170 \mathrm{~cm}$
The total displacement is 170 cm to the left.
(b) Strategy The average speed is found by the dividing the total distance traveled by the elapsed time.

Solution Find the total distance traveled. $80 \mathrm{~cm}+30 \mathrm{~cm}+90 \mathrm{~cm}+310 \mathrm{~cm}=510 \mathrm{~cm}$
Find the average speed. $\frac{510 \mathrm{~cm}}{18 \mathrm{~s}}=28 \mathrm{~cm} / \mathrm{s}$
(c) Strategy The average velocity is found by dividing the displacement by the elapsed time.

Solution Find the average velocity.
$\overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{170 \mathrm{~cm} \text { to the left }}{18 \mathrm{~s}}=9.4 \mathrm{~cm} / \mathrm{s}$ to the left
24. Strategy The average speed is found by dividing the total distance traveled by the elapsed time.
(a) Solution Find the average speed for the first $10-\mathrm{km}$ segment of the race.
$v_{1}=\frac{10.0 \times 10^{3} \mathrm{~m}}{0.5689 \mathrm{~h}} \frac{1 \mathrm{~h}}{3600 \mathrm{~s}}=\frac{10.0 \times 10^{3} \mathrm{~m}}{2048 \mathrm{~s}}=4.88 \mathrm{~m} / \mathrm{s}$
(b) Solution Find the average speed for the entire race.
$v_{\mathrm{av}}=\frac{42,195 \mathrm{~m}}{2.3939 \mathrm{~h}} \frac{1 \mathrm{~h}}{3600 \mathrm{~s}}=\frac{42,195 \mathrm{~m}}{8618 \mathrm{~s}}=4.90 \mathrm{~m} / \mathrm{s}$
25. Strategy Use the definition of average velocity. Find the time spent by each runner in completing her portion of the race.

Solution The times for each runner are given by $\Delta t=\Delta r / v$ as $300.0 \mathrm{~m} \div 7.30 \mathrm{~m} / \mathrm{s}=41.1 \mathrm{~s}, 300.0 \mathrm{~m} \div 7.20 \mathrm{~m} / \mathrm{s}=41.7 \mathrm{~s}$, and $100.0 \mathrm{~m} \div 7.80 \mathrm{~m} / \mathrm{s}=12.8 \mathrm{~s}$. The net displacement of the baton is 100.0 m to the north, so the average velocity of the baton for the entire race is $\overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{100.0 \mathrm{~m} \text { north }}{41.1 \mathrm{~s}+41.7 \mathrm{~s}+12.8 \mathrm{~s}}=1.05 \mathrm{~m} / \mathrm{s}$ to the north.

Discussion. Note the contrast in adding net displacement and total time. Time is not a vector.
26. Strategy. Use the definition of instantaneous speed as the magnitude of the slope of a tangent to a position-versus-time graph.
Solution. Pick out by eye the steepest slope of the graph in Figure 2.8 as at about 46 s . A tangent line drawn here passes through about ( $34 \mathrm{~s}, 15 \mathrm{~km}$ ) and ( $46 \mathrm{~s},-10 \mathrm{~m} / \mathrm{s}$ ) and ( $56 \mathrm{~s},-30 \mathrm{~m} / \mathrm{s}$ ). We can choose any pair of these points to find its slope. For best precision we choose the two farthest apart:

$$
\left|\overrightarrow{\mathbf{v}}_{\mathrm{av}}\right|=\frac{|\Delta \overrightarrow{\mathbf{r}}|}{\Delta t}=\frac{|-30 \mathrm{~km}-15 \mathrm{~km}|}{56 \min -34 \min }=2.05 \frac{\mathrm{~km}}{\min }=2.05 \frac{\mathrm{~km}}{\min } \frac{60 \mathrm{~min}}{1 \mathrm{~h}}=1.2 \times 10^{2} \mathrm{~km} / \mathrm{h} .
$$

27. Strategy Use $\Delta v=a \Delta t$ and solve for $\Delta t$.

## Solution

$$
\Delta t=\frac{\Delta v}{a}=\frac{22 \mathrm{~m} / \mathrm{s}-0}{1.7 \mathrm{~m} / \mathrm{s}^{2}}=13 \mathrm{~s}
$$

28. Strategy Use the definition of average acceleration.

## Solution

$$
\begin{aligned}
\overrightarrow{\mathbf{a}}_{\mathrm{av}} & =\frac{\Delta \overrightarrow{\mathbf{v}}}{\Delta t}=\frac{\overrightarrow{\mathbf{v}}_{\mathrm{f}}-\overrightarrow{\mathbf{v}}_{\mathrm{i}}}{\Delta t} \\
& =\frac{0-28 \mathrm{~m} / \mathrm{s} \text { in the direction of the car's travel }}{4.0 \mathrm{~s}}=7.0 \mathrm{~m} / \mathrm{s}^{2} \text { in the direction opposite the car's velocity }
\end{aligned}
$$

29. Strategy Refer to the graph. The absolute value of the slope of a $v$ versus $t$ graph is equal to the magnitude of the acceleration. The steeper the slope up or down, the larger the magnitude.

Solution We see directly the steepest slope for the time interval between 5 and 6 s , then between 0 and 1 s , then between 1 and 3 s , then between 6 and 8 s , and zero slope between 3 and 5 s . For the instants of time that this problem asks about, the ranking in order of the magnitude of the acceleration, from largest to smallest, is

$$
5.5 \mathrm{~s}, 0.5 \mathrm{~s}, 1.5 \mathrm{~s}=2.5 \mathrm{~s}, 3.5 \mathrm{~s}=4.5 \mathrm{~s} \text {. }
$$

Discussion. We can calculate the magnitude of the acceleration at each instant by computing the magnitude of the slope of the tangent there, which is the slope of the segment of the graph passing through the point. This is easy to do by counting boxes, because each box is 1 s by $1 \mathrm{~m} / \mathrm{s}$.

| $t(\mathrm{~s})$ | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\|a\|\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | 4 | 1 | 1 | 0 | 0 | $\|-5\|=5$ |

Our ranking is confirmed.
30. Strategy The acceleration is equal to the value of the slope of the $v$ versus $t$ graph.

Solution Between 8 and 10 s , for example, we have $a=\Delta v / \Delta t=(0-2 \mathrm{~m} / \mathrm{s}) / 2 \mathrm{~s}=-1 \mathrm{~m} / \mathrm{s}^{2}$. The sketch of the acceleration of the elevator is shown.
$a_{y}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$

31. Strategy Use the definition of average acceleration.

Solution Find the average acceleration of the airplane.

$$
a_{\mathrm{av}}=\frac{\Delta v}{\Delta t}=\frac{(35 \mathrm{~m} / \mathrm{s}-0)}{8.0 \mathrm{~s}}=4.4 \mathrm{~m} / \mathrm{s}^{2} ; \text { thus } \overrightarrow{\mathbf{a}}_{\mathrm{av}}=4.4 \mathrm{~m} / \mathrm{s}^{2} \text { forward } .
$$

32. Strategy Use the definitions of instantaneous acceleration, displacement, and average velocity.

## Solution

(a) We draw a line tangent to the point at $(14.0 \mathrm{~s}, 55.0 \mathrm{~m} / \mathrm{s})$.


It looks as if the tangent line passes through the point $(7.0 \mathrm{~s}, 45.0 \mathrm{~m} / \mathrm{s})$. Its slope is then $\overrightarrow{\mathbf{a}}_{x}=\frac{\Delta \overrightarrow{\mathbf{v}}_{x}}{\Delta t}=\frac{(55.0 \mathrm{~m} / \mathrm{s}-45.0 \mathrm{~m} / \mathrm{s}) \text { in the }+x \text {-direction }}{14.0 \mathrm{~s}-7.0 \mathrm{~s}}=1.4 \mathrm{~m} / \mathrm{s}^{2}$ in the $+x$-direction
(b) The area under the $v_{x}$ vs. $t$ curve from $t=12.0 \mathrm{~s}$ to $t=16.0 \mathrm{~s}$ represents the displacement of the body. Each grid square represents $(10.0 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})=1.0 \times 10^{1} \mathrm{~m}$, and there are approximately 22 squares under the curve for $t=12.0 \mathrm{~s}$ to $t=16.0 \mathrm{~s}$, so the car travels 220 m in the $+x$-direction.
(c) $\quad \overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{220 \mathrm{~m} \text { in the }+x \text {-direction }}{4.0 \mathrm{~s}}=55 \mathrm{~m} / \mathrm{s}$ in the $+x$-direction
33. Strategy Follow the directions and note the contrast between average and instantaneous.

## Solution


(a) The average acceleration over the zero-to-eight-second time interval is
$a_{a v g}=\frac{\Delta v}{\Delta t}=\frac{0-24 \mathrm{~m} / \mathrm{s}}{8 \mathrm{~s}-0}=-3.0 \mathrm{~m} / \mathrm{s}^{2}$ so $\overrightarrow{\mathbf{a}}_{a v g}=3.0 \mathrm{~m} / \mathrm{s}^{2}$ away from the deer
(b) We draw by eye a tangent line at $t=2 \mathrm{~s}$ and find its slope from two well-separated points:
$a=\frac{\Delta v}{\Delta t} \approx \frac{0-20 \mathrm{~m} / \mathrm{s}}{5 \mathrm{~s}-0}=-4.0 \mathrm{~m} / \mathrm{s}^{2}$ so $\overrightarrow{\mathbf{a}} \approx 4.0 \mathrm{~m} / \mathrm{s}^{2}$ away from the deer
Discussion One way to estimate the uncertainty is to do an operation a few or several times, using different methods if possible, and see how much scatter shows up; or just to apply critical thinking in general. This applies to laboratory measurements and to sketching a tangent line. Here, if $(0,20 \mathrm{~m} / \mathrm{s})$ and $(5 \mathrm{~s}, 0)$ are on the straight tangent line, the vertical-axis coordinate at $t=2 \mathrm{~s}$ should be $20-4(2)=12 \mathrm{~m} / \mathrm{s}$. This agrees with the data table, so we have some confidence that answer (b) is accurate to two digits.
34. Strategy Consider (say) an $800-\mathrm{m}$ race. For the motion diagram, the dots will be closer together at the beginning and at the end of the race than in the middle, to reflect the positive and negative accelerations-speeding up and slowing down. The dots can be evenly spaced for the middle of the race as the runner can be running with approximately constant speed. Use the definitions of displacement, velocity, and acceleration to sketch the graphs. (In a $100-\mathrm{m}$ race, sprinters are taught to speed up all the way and to break the tape when they are moving faster than ever before, so there might be no constant-speed portion in the diagram or graphs.)
Solution Sketch the motion diagram.

- • - • • • • •

Sketch the graph of $x(t)$.
${ }^{x}$


Sketch the graph of $v_{x}(t)$.


Sketch the graph of $a_{x}(t)$.

35. Strategy Use the definitions of position, velocity, and acceleration.

Solution Sketch graphs of $x(t), v_{x}(t)$, and $a_{x}(t)$. Describe the motion in words.
(a) Since the distance between the dots is increasing to the right, the motion of the object is to the right with increasing speed, and the acceleration is positive and to the right.



(b) Since the distance between the dots is decreasing to the right, the motion of the object is to the right with decreasing speed, and the acceleration is negative and to the left.
${ }^{x}$



(c) Since the distance between the dots is the same and the motion is from right to left, the motion of the object is to the left with constant speed, and the acceleration is zero.



(d) The distance between the dots decreases from left to right; then increases from right to left. The object moves to the right with decreasing speed, turns around at point 4, and then moves to the left with increasing speed. Since the speed decreases when the object moves to the right and increases when it moves to the left, the acceleration is negative and to the left.



36. Strategy The magnitude of the acceleration is the absolute value of the slope of the graph at $t=7.0 \mathrm{~s}$.

Solution We choose two well-separated points along the straight section of graph that passes through the $t=7 \mathrm{~s}$ point.
$a_{x}=\left|\frac{\Delta v_{x}}{\Delta t}\right|=\left|\frac{0-20.0 \mathrm{~m} / \mathrm{s}}{12.0 \mathrm{~s}-4.0 \mathrm{~s}}\right|=2.5 \mathrm{~m} / \mathrm{s}^{2}$

37. (a) Strategy The acceleration $a_{x}$ is equal to the slope of the $v_{x}$ versus $t$ graph. Use the definition of average acceleration.
Solution $\quad a_{\mathrm{av}, x}=\frac{\Delta v_{x}}{\Delta t}=\frac{14 \mathrm{~m} / \mathrm{s}-4 \mathrm{~m} / \mathrm{s}}{11 \mathrm{~s}-6 \mathrm{~s}}=2.0 \mathrm{~m} / \mathrm{s}^{2}$
(b) Strategy For constant acceleration, $v_{\mathrm{av}, x}=\left(v_{\mathrm{f}}+v_{\mathrm{i}}\right) / 2$.

Solution $v_{\mathrm{av}, x}=\frac{14 \mathrm{~m} / \mathrm{s}+4 \mathrm{~m} / \mathrm{s}}{2}=9.0 \mathrm{~m} / \mathrm{s}$
(c) Strategy $v_{\mathrm{av}, x}=\Delta x / \Delta t$ and $\Delta x$ is the area under the graph in the figure. Find the area.

Solution Each square represents $(1.0 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})=1.0 \mathrm{~m}$ and there are 195 squares under the graph. So,
$v_{\mathrm{av}, x}=\frac{195(1.0 \mathrm{~m})}{20.0 \mathrm{~s}}=9.8 \mathrm{~m} / \mathrm{s}$.
(d) Strategy At $t=10 \mathrm{~s}, v_{x}=12 \mathrm{~m} / \mathrm{s}$, and at $t=15 \mathrm{~s}, v_{x}=14 \mathrm{~m} / \mathrm{s}$.

Solution $\Delta v_{x}=14 \mathrm{~m} / \mathrm{s}-12 \mathrm{~m} / \mathrm{s}=2.0 \mathrm{~m} / \mathrm{s}$
(e) Strategy The area under the $v_{x}$ vs. $t$ graph for $t=10 \mathrm{~s}$ to $t=15 \mathrm{~s}$ represents the displacement of the car.

Solution Each square represents 1.0 m . There are 69 of these squares, so the car has traveled $(1.0 \mathrm{~m})(69)=69 \mathrm{~m}$.
Discussion For the section of motion from 6 to 11 s , the graph of $v$ versus $t$ is a single straight line, so $a$ is constant, so a constant-acceleration equation like $v_{\mathrm{av}, x}=\left(v_{\mathrm{f}}+v_{\mathrm{i}}\right) / 2$. is true. In part (e), for the section of motion from 10 to 15 s , the acceleration does not have a single constant value, so we cannot compute the correct value for $\Delta x$ from $\Delta x=\left(v_{\mathrm{f}}+v_{\mathrm{i}}\right) t / 2$. [Try it and see how the answer is tempting but wrong. That is also the reason that the calculation in part (b) does not give the answer to part (c).] On the other hand, we do not really have to count squares here. We can take the 10 -to- 15 s section of motion apart into two constant- $a$ sections: from 10 s to 11 s with $\Delta x=\left(v_{\mathrm{f}}+v_{\mathrm{i}}\right) t / 2 .=(13 \mathrm{~m} / \mathrm{s})(1 \mathrm{~s})=13 \mathrm{~m}$ and from 11 to 15 s with $\Delta x=v x \Delta t=(14 \mathrm{~m} / \mathrm{s})(4 \mathrm{~s})=$ 56 m . Then the net displacement is $56+13 \mathrm{~m}=69 \mathrm{~m}$.
38. Strategy The acceleration $a_{x}$ is equal to the slope of the $v_{x}$ versus $t$ graph.

Solution Sketch a graph of $a_{x}(t)$ for the car. The constant slope of the curve from $t=6 \mathrm{~s}$ to $t=11 \mathrm{~s}$ is
$a_{\mathrm{av}, x}=\frac{\Delta v_{x}}{\Delta t}=\frac{14 \mathrm{~m} / \mathrm{s}-4 \mathrm{~m} / \mathrm{s}}{11 \mathrm{~s}-6 \mathrm{~s}}=2.0 \mathrm{~m} / \mathrm{s}^{2}$.

39. Strategy The acceleration $a_{x}$ is equal to the slope of the $v_{x}$ versus $t$ graph. The displacement is equal to the area under the curve.

## Solution

(a) $a_{x}$ is the slope of the graph at $t=11 \mathrm{~s}$.

$$
a_{x}=\frac{\Delta v_{x}}{\Delta t}=\frac{10.0 \mathrm{~m} / \mathrm{s}-30.0 \mathrm{~m} / \mathrm{s}}{12.0 \mathrm{~s}-10.0 \mathrm{~s}}=-10 \mathrm{~m} / \mathrm{s}^{2}
$$

(b) Since $v_{x}$ is staying constant, $a_{x}=0$ at $t=3 \mathrm{~s}$.
(c) Sketch the acceleration using the $v_{x}$ versus $t$ graph.

(d) The area under the $v_{x}$ vs. $t$ curve from $t=12 \mathrm{~s}$ to $t=14 \mathrm{~s}$ represents the displacement of the body. Each square represents $(10.0 \mathrm{~m} / \mathrm{s})(1.0 \mathrm{~s})=1.0 \times 10^{1} \mathrm{~m}$, and there is $1 / 2$ square under the curve for $t=12 \mathrm{~s}$ to $t=14 \mathrm{~s}$, so the car travels 5.0 m .
40. Strategy Use the idea of instantaneous acceleration as the slope of a tangent to a $v$-versus- $t$ curve. Use the idea of displacement as area on a $v$-versus- $t$ curve.

Solution. (a) The tangent line drawn at $t=2 \mathrm{~s}$ in Figure 2.15 appears to go through ( $0.5 \mathrm{~s},-5 \mathrm{~m} / \mathrm{s}$ ) and
$(3.5 \mathrm{~s},-2 \mathrm{~m} / \mathrm{s})$. Its slope is then $a=\frac{-2-(-5)}{3.5-0.5} \frac{\mathrm{~m} / \mathrm{s}}{\mathrm{s}}=+1.0 \mathrm{~m} / \mathrm{s}^{2}$
(b) Method one: Think about cutting the shape bounded by the graph line and the axes out of plywood, turning it so that the velocity axis is horizontal, and homing in on a point on the $v$ axis where the shape could be balanced, with as much weight of higher speeds on one side as of lower speeds on the other side. We estimate in this way that the average velocity is about $-2 \mathrm{~m} / \mathrm{s}$.

Method two: There are about -9.7 rectangles of area between the graph line and the $t$ axis and between $t=0$ and $t=10 \mathrm{~s}$. Each rectangle is $(2 \mathrm{~s})(1 \mathrm{~m} / \mathrm{s})=2 \mathrm{~m}$, so the stopping distance is about $(-9.7)(2 \mathrm{~m})=-19 \mathrm{~m}$. Then the average velocity in the process is $v_{\text {avg }}=\Delta x / \Delta t=-19 \mathrm{~m} / 10 \mathrm{~s}=-1.9 \mathrm{~m} / \mathrm{s}$, We think of this estimate as more precise than that from method one, so we say the average speed is $1.9 \mathrm{~m} / \mathrm{s}$.
41. Strategy Since the time intervals are the same, a greater change in distance between each successive pairs of dots indicates a greater magnitude acceleration. If there is no change in distance between dots, the acceleration is zero.

Solution The distance between dots in (b) and (c) is constant; therefore, the accelerations are zero. The distance between dots in (a) and (d) is increasing, indicating a positive acceleration. The increase is greater for (a) than for (d); therefore, (a) represents a greater magnitude acceleration than (d). Ranking the motion diagrams in order of the magnitude of the acceleration, from greatest to lest, we have (a), (d), (b) = (c).

Discussion. Start believing that a lot of information is packed into a motion diagram. Use a ruler to demonstrate what we state about the changes in distance between the dots.
42. (a) Strategy Plot the data on a $v$ versus $t$ graph. Draw a best-fit line.

(b) Strategy The slope of the graph gives the acceleration.

Solution The points appear to lie quite close to the single straight line drawn for comparison, so yes, it is plausible that the acceleration is constant. Compute the magnitude of the acceleration.
$a=\frac{\Delta v}{\Delta t}=\frac{7.2 \mathrm{~m} / \mathrm{s}-0}{3.0 \mathrm{~s}-0}=2.4 \mathrm{~m} / \mathrm{s}^{2}$
The acceleration is $2.4 \mathrm{~m} / \mathrm{s}^{2}$ in the direction of motion.
43. Strategy Use equations for motion with constant acceleration.

Solution (a) With $v_{\mathrm{ix}}=200 \mathrm{~m} / \mathrm{s}, v_{\mathrm{fx}}=700 \mathrm{~m} / \mathrm{s}, a_{\mathrm{x}}=9.0 g=882 \mathrm{~m} / \mathrm{s}^{2}$ we want $\Delta t$. We choose $v_{f x}-v_{i x}=a_{x} \Delta t$ to find $\Delta t=\frac{v_{f x}-v_{i x}}{a_{x}}=\frac{700 \mathrm{~m} / \mathrm{s}-200 \mathrm{~m} / \mathrm{s}}{88.2 \mathrm{~m} / \mathrm{s}^{2}}=5.7 \mathrm{~s}$
(b) With the same data we can find $\Delta x$ from $v_{f x}^{2}-v_{i x}^{2}=2 a_{x} \Delta x$ thus:

$$
\Delta x=\frac{v_{f x}^{2}-v_{i x}^{2}}{2 a_{x}}=\frac{(700 \mathrm{~m} / \mathrm{s})^{2}-(200 \mathrm{~m} / \mathrm{s})^{2}}{2\left(88.2 \mathrm{~m} / \mathrm{s}^{2}\right)}=2.6 \mathrm{~km}
$$

44. (a) Strategy The graph will be a line with a slope of $1.20 \mathrm{~m} / \mathrm{s}^{2}$.

Solution $v_{x}=0$ when $t=0$. The graph is shown.
(b) Strategy Use Eq. (2-12).

Solution Find the distance the train traveled.

$$
\begin{aligned}
\Delta x & =v_{\mathrm{i} x} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2}=(0) \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2}=\frac{1}{2} a_{x}(\Delta t)^{2} \\
& =\frac{1}{2}\left(1.20 \mathrm{~m} / \mathrm{s}^{2}\right)(12.0 \mathrm{~s})^{2}=86.4 \mathrm{~m}
\end{aligned}
$$


(c) Strategy Use Eq. (2-13).

Solution Find the final speed of the train.

$$
v_{\mathrm{f} x}-v_{\mathrm{i} x}=v_{\mathrm{f} x}-0=a_{x} \Delta t, \text { so } v_{\mathrm{f} x}=a_{x} \Delta t=\left(1.20 \mathrm{~m} / \mathrm{s}^{2}\right)(12.0 \mathrm{~s})=14.4 \mathrm{~m} / \mathrm{s} .
$$

(d) Strategy Refer to Figure 2.17, which shows various motion diagrams.

Solution The motion diagram is shown.

45. Strategy Relate the acceleration, speed, and distance using Eq. (2-13). Let southwest be the positive direction.

Solution Find the constant acceleration required to stop the airplane. The acceleration must be opposite to the direction of motion of the airplane, so the direction of the acceleration is ( - southwest) = northeast. Use the acceleration for the slope of the $v_{x}(t)$ curve.

$$
v_{\mathrm{f} x}^{2}-v_{\mathrm{i} x}^{2}=2 a_{x} \Delta x \text {, so } a_{x}=\frac{v_{\mathrm{f} x}^{2}-v_{\mathrm{i} x}^{2}}{2 \Delta x}=\frac{0-(55 \mathrm{~m} / \mathrm{s})^{2}}{2\left(1.0 \times 10^{3} \mathrm{~m}\right)}=-1.5 \mathrm{~m} / \mathrm{s}^{2} .
$$

Thus, the acceleration is $1.5 \mathrm{~m} / \mathrm{s}^{2}$ northeast. The slope is $-1.5 \mathrm{~m} / \mathrm{s}^{2}$. The $v_{x}$-intercept is $55 \mathrm{~m} / \mathrm{s}$. Sketch the
graph.


Discussion. Do a check: find the area under the curve as the area of a triangle and see if it comes out convincingly close to 1000 m . Note the terminology: A graph line is called a curve whether it is curved or straight or has corners. The "shape" of a graph is not a shape like a square or triangle. Instead, the shape of a graph can be described as flat, as straight and sloping up or down, as curved and starting from a positive or negative value with a positive or negative slope and concave up or concave down, or as a line with corners and sections that can individually be described.
46. (a) Strategy Between 0 to 2 s the velocity is $-24.0 \mathrm{~m} / \mathrm{s}$, and between 11 and 12 s the velocity is $-6.0 \mathrm{~m} / \mathrm{s}$. Since the acceleration is constant between 2 and 11 s , draw a straight line between the two horizontal lines of constant speed.

Solution Draw the graph.

(b) Strategy Let north be the $+x$-direction. Use Eq. (2-9) to find the acceleration of the train between 2 and 11 s . Before 2 s and after 11 s the acceleration is zero.

## Solution

$v_{\mathrm{f} x}-v_{\mathrm{i} x}=a_{x} \Delta t$, so $a_{x}=\left(v_{\mathrm{f} x}-v_{\mathrm{i} x}\right) / \Delta t=(-6.00 \mathrm{~m} / \mathrm{s}-[-24.0 \mathrm{~m} / \mathrm{s}]) /(9.00 \mathrm{~s})=+2.00 \mathrm{~m} / \mathrm{s}^{2}$.
The acceleration is $2.00 \mathrm{~m} / \mathrm{s}^{2}$ north.
(c) Strategy We can use Eq. (2-12).

Solution Find the displacement of the train.
$\Delta x=v_{\mathrm{ix}} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2}=(-24.0 \mathrm{~m} / \mathrm{s})(9.00 \mathrm{~s})+\frac{1}{2}\left(+2.00 \mathrm{~m} / \mathrm{s}^{2}\right)(9.00 \mathrm{~s})^{2}=-135 \mathrm{~m}$, meaning 135 m south
47. (a) Strategy For motion in a straight line, the magnitude of a constant acceleration is equal to the change in speed divided by the time elapsed.

Solution Find how long the airplane accelerated.
$a=\frac{\Delta v}{\Delta t}$, so $\Delta t=\frac{\Delta v}{a}=\frac{46.0 \mathrm{~m} / \mathrm{s}-0}{5.00 \mathrm{~m} / \mathrm{s}^{2}}=9.20 \mathrm{~s}$.
(b) Strategy Use Eq. (2-13).

Solution Find the distance the plane traveled along the runway.
$v_{\mathrm{f} x}^{2}-v_{\mathrm{i} x}^{2}=v_{\mathrm{f} x}^{2}-0=2 a_{x} \Delta x$, so $\Delta x=\frac{v_{\mathrm{f} x}^{2}}{2 a_{x}}=\frac{(46.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(5.00 \mathrm{~m} / \mathrm{s}^{2}\right)}=212 \mathrm{~m}$.
48. Strategy Use Eq. (2-9) for (a) and Eq. (2-11) for (b). Let the "initial" point be at clock reading 10 s and the "final" point be at 12 s .

## Solution

(a) Find the speed at 12.0 s .

$$
v_{\mathrm{f} x}-v_{\mathrm{i} x}=a_{x} \Delta t, \text { so } v_{\mathrm{f} x}=a_{x} \Delta t+v_{\mathrm{i} x}=\left(2.0 \mathrm{~m} / \mathrm{s}^{2}\right)(12.0 \mathrm{~s}-10.0 \mathrm{~s})+1.0 \mathrm{~m} / \mathrm{s}=4.0 \mathrm{~m} / \mathrm{s}+1.0 \mathrm{~m} / \mathrm{s}=5.0 \mathrm{~m} / \mathrm{s} .
$$

(b) Find the distance traveled between $t=10.0 \mathrm{~s}$ and $t=12.0 \mathrm{~s}$.
$\Delta x=\frac{1}{2}\left(v_{\mathrm{f} x}+v_{\mathrm{i} x}\right) \Delta t=\frac{1}{2}(5.0 \mathrm{~m} / \mathrm{s}+1.0 \mathrm{~m} / \mathrm{s})(12.0 \mathrm{~s}-10.0 \mathrm{~s})=6.0 \mathrm{~m}$
49. Strategy Find the time it takes for the car to collide with the tractor (assuming it does) by setting the distance the car travels equal to that of the tractor plus the distance between them and solving for $t$. Use Eq. (2-12). Then, use the standard relationships for constant acceleration to answer the remaining questions.

Solution Solve for the time in $\Delta x$ for car $=\Delta x$ for tractor +25 m
$(27.0 \mathrm{~m} / \mathrm{s}) \Delta t+\frac{1}{2}\left(-7.00 \mathrm{~m} / \mathrm{s}^{2}\right)(\Delta t)^{2}=(10.0 \mathrm{~m} / \mathrm{s}) \Delta t+25.0 \mathrm{~m}$, so $0=\left(3.50 \mathrm{~m} / \mathrm{s}^{2}\right)(\Delta t)^{2}-(17.0 \mathrm{~m} / \mathrm{s}) \Delta t+25.0 \mathrm{~m}$ and solving for $t$ we get an imaginary value for the time. Therefore, you won't hit the tractor.
Find the distance the car requires to stop.
$\Delta x_{\mathrm{c}}=\left(v_{\mathrm{fc}}^{2}-v_{\mathrm{ic}}^{2}\right) /(2 a)=\left[0-(27.0 \mathrm{~m} / \mathrm{s})^{2}\right] /\left[2\left(-7.00 \mathrm{~m} / \mathrm{s}^{2}\right)\right]=52.1 \mathrm{~m}$. Since the acceleration of the car is constant, the average speed of the car as it attempts to stop is $v_{\mathrm{c}, \mathrm{av}}=\left(v_{\mathrm{fc}}+v_{\mathrm{ic}}\right) / 2=(0+27.0 \mathrm{~m} / \mathrm{s}) / 2=13.5 \mathrm{~m} / \mathrm{s}$. Thus, the time required for the car to stop is $\Delta t=\Delta x_{\mathrm{c}} / v_{\mathrm{c}, \text { av }}=(52.1 \mathrm{~m}) /(13.5 \mathrm{~m} / \mathrm{s})=3.86 \mathrm{~s}$. The distance the tractor travels in this time is $\Delta x_{\mathrm{t}}=v_{\mathrm{t}} \Delta t=(10.0 \mathrm{~m} / \mathrm{s})(3.86 \mathrm{~s})=38.6 \mathrm{~m}$. Now, $38.6 \mathrm{~m}+25.0 \mathrm{~m}=63.6 \mathrm{~m}$, which is $63.6 \mathrm{~m}-52.1 \mathrm{~m}=11.5 \mathrm{~m}$ beyond the stopping point of the car.

Discussion. You might give a convincing solution by assuming the car does not hit the tractor, finding the time interval and distance required to stop, and finding where the tractor is when the car comes to rest. Our method is good for variety and works even though the tractor is moving faster than the car during the last bit of the car's motion. The 11.5 m is not the minimum distance between the two, which occurs slightly earlier. Note that the same equation 2-12 describes the car with one set of numbers. The same equation containing an acceleration term describes the tractor with another set of numbers, even though the tractor is moving at constant speed. The set of equations for constant acceleration are all you need to describe constant-velocity motion, by substituting $a=0$.
50. (a) Strategy Use Eq. (2-13) to find the constant acceleration of the sneeze.

Solution Find the acceleration of the sneeze as it moves the initial 0.25 cm .
$v_{\mathrm{f} x}^{2}-v_{\mathrm{i} x}^{2}=2 a_{x} \Delta x$, so $a_{x}=\frac{v_{\mathrm{f} x}^{2}-v_{\mathrm{i} x}^{2}}{2 \Delta x}=\frac{(44 \mathrm{~m} / \mathrm{s})^{2}-0}{2(0.0025 \mathrm{~m})}=3.9 \times 10^{5} \mathrm{~m} / \mathrm{s}^{2}$.
(b) Strategy Use Eq. (2-11) to find the time to travel the initial 0.25 cm ; then use Eq. (2-12) to find the time to travel the remaining 1.75 cm .

Solution Find the time to travel the initial 0.25 cm .

$$
\Delta x=\frac{1}{2}\left(v_{\mathrm{f} x}+v_{\mathrm{i} x}\right) \Delta t, \text { so } \Delta t=\frac{2 \Delta x}{v_{\mathrm{f} x}+v_{\mathrm{i} x}}=\frac{2(0.0025 \mathrm{~cm})}{44 \mathrm{~m} / \mathrm{s}+0}=1.1 \times 10^{-4} \mathrm{~s}=0.11 \times 10^{-3} \mathrm{~s}=0.11 \mathrm{~ms} .
$$

Find the time to travel the remaining 1.75 cm .

$$
\Delta x=v_{i x} \Delta t+0, \text { so } \Delta t=\frac{\Delta x}{v_{i x}}=\frac{0.0175 \mathrm{~cm}}{44 \mathrm{~m} / \mathrm{s}}=4.0 \times 10^{-4} \mathrm{~s}=0.40 \mathrm{~ms}
$$

The time the sneeze takes to travel the $2.0-\mathrm{cm}$ distance in the nose is $0.11 \mathrm{~ms}+0.40 \mathrm{~ms}=0.51 \mathrm{~ms}$.
(c) Strategy Use the information in parts (a) and (b) to graph $v_{x}(t)$.

Solution Sketch the graph.

51. Strategy Use Eq. (2-13).

Solution Find the distance the train travels in stopping.
$\Delta x=\frac{v_{\mathrm{f} x}{ }^{2}-v_{\mathrm{i} x}{ }^{2}}{2 a_{x}}=\frac{0-(26.8 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-1.52 \mathrm{~m} / \mathrm{s}^{2}\right)}=236 \mathrm{~m}$
$236 \mathrm{~m}>184 \mathrm{~m}$, so the answer is no; it takes 236 m for the train to stop.
52. (a) Strategy Use Eq. (2-13) and Newton's second law.

Solution Find the final speed of the electrons.
$v_{\mathrm{f} x}^{2}-v_{\mathrm{i} x}^{2}=v_{\mathrm{f} x}^{2}-0=2 a_{x} \Delta x=2 \Delta x$, so
$v_{\mathrm{f} x}= \pm \sqrt{2 a_{x} \Delta x}=\sqrt{2\left(7.03 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}\right)(0.020 \mathrm{~m})}=1.7 \times 10^{6} \mathrm{~m} / \mathrm{s}$. (Speed is always positive.)
(b) Strategy Use Eq. (2-12).

Solution Find the time it takes the electrons to travel the length of the tube.
$x=0$ to $2.0 \mathrm{~cm}: \Delta x_{1}=v_{\mathrm{i} x} \Delta t_{1}+\frac{1}{2} a_{x}\left(\Delta t_{1}\right)^{2}=0+\frac{1}{2} a_{x}\left(\Delta t_{1}\right)^{2}$, so $\left(\Delta t_{1}\right)^{2}=\frac{2 \Delta x_{1}}{a_{x}}$ or $\Delta t_{1}=+\sqrt{\frac{2 \Delta x_{1}}{a_{x}}}$.
$x=2.0 \mathrm{~cm}$ to $47 \mathrm{~cm}: \Delta x_{2}=v_{\mathrm{f} x} \Delta t_{2}$, so $\Delta t_{2}=\frac{\Delta x_{2}}{v_{\mathrm{f} x}}$.
Find the total time.
$\Delta t=\Delta t_{1}+\Delta t_{2}=\sqrt{\frac{2 \Delta x_{1}}{a_{x}}}+\frac{\Delta x_{2}}{v_{\mathrm{f} x}}=\sqrt{\frac{2(0.020 \mathrm{~m})}{7.03 \times 10^{13} \mathrm{~m} / \mathrm{s}^{2}}}+\frac{0.45 \mathrm{~m}}{1.677 \times 10^{6} \mathrm{~m} / \mathrm{s}}=290 \mathrm{~ns}$
53. Strategy Refer to the figure. Analyze graphically and algebraically. Each square represents $(10 \mathrm{~m} / \mathrm{s})(1 \mathrm{~s})=10 \mathrm{~m}$. Count squares to determine the distance traveled at each time.

Solution Sketch the motion diagram and describe the motion in words.


The $x$-coordinates are determined by choosing $x=0$ at $t=0$. At $t=9.0 \mathrm{~s}$ and $x=220 \mathrm{~m}$, the object has its maximum speed of $40 \mathrm{~m} / \mathrm{s}$. From $t=9.0 \mathrm{~s}$ to $t=13.0 \mathrm{~s}$, the speed of the object decreases at a constant rate until it reaches zero at $x=300 \mathrm{~m}$.

Graphical analysis:
The displacement of the object is given by the area under the $v_{x}$ vs. $t$ curve between $t=9.0 \mathrm{~s}$ and $t=13.0 \mathrm{~s}$. The area is a triangle, $A=\frac{1}{2} b h$.
$\Delta x=\frac{1}{2}(13.0 \mathrm{~s}-9.0 \mathrm{~s})(40 \mathrm{~m} / \mathrm{s})=80 \mathrm{~m}$
Algebraic solution:
Use the definition of average velocity.
$\Delta x=v_{\mathrm{av}, x} \Delta t=\frac{v_{\mathrm{i} x}+v_{\mathrm{f} x}}{2} \Delta t=\frac{40 \mathrm{~m} / \mathrm{s}+0}{2}(13.0 \mathrm{~s}-9.0 \mathrm{~s})=80 \mathrm{~m}$
The object goes 80 m .
Discussion The graph shows that the object has different accelerations over the intervals from 0 to 5 s , from 5 to 9 s , and from 9 to 13 s . If the problem had asked about displacement over a time interval from 3 to 12 s , for the algebraic solution we would have been required to figure out the answer in steps, adding distances for three different parts of this motion, because the acceleration does not have a single value between 3 s and 12 s .
54. Strategy Refer to the figure. Each square represents $(10 \mathrm{~m} / \mathrm{s})(1 \mathrm{~s})=10 \mathrm{~m}$. Count squares to determine the distance traveled at each time.

Solution Sketch the motion diagram and describe the motion in words.


The $x$-coordinates are determined by choosing $x=0$ at $t=0$. At $t=5.0 \mathrm{~s}$ and $x=100 \mathrm{~m}$, the object's speed is $20 \mathrm{~m} / \mathrm{s}$. From $t=5.0 \mathrm{~s}$ to $t=9.0 \mathrm{~s}$, the speed of the object increases at a constant rate until it reaches $40 \mathrm{~m} / \mathrm{s}$ at $x=220 \mathrm{~m}$.

Find the slope of the graph to find the acceleration.
$a_{\mathrm{av}, x}=\frac{40 \mathrm{~m} / \mathrm{s}-20 \mathrm{~m} / \mathrm{s}}{9.0 \mathrm{~s}-5.0 \mathrm{~s}}=5.0 \mathrm{~m} / \mathrm{s}^{2}$
The acceleration is $5.0 \mathrm{~m} / \mathrm{s}^{2}$ in the $+x$-direction.
55. (a) Strategy The graph will be a line with a slope of $-1.40 \mathrm{~m} / \mathrm{s}^{2}$.

Solution $v_{x}=22 \mathrm{~m} / \mathrm{s}$ when $t=0$.

(b) Strategy Since the train slows down, the acceleration is negative. Use Eq. (2-9).

Solution $v_{\mathrm{f} x}-v_{\mathrm{i} x}=a_{x} \Delta t$, so $v_{\mathrm{f} x}=v_{\mathrm{i} x}+a_{x} \Delta t=22 \mathrm{~m} / \mathrm{s}+\left(-1.4 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~s})=11 \mathrm{~m} / \mathrm{s}$.
(c) Strategy Use Eq. (2-12) to find the distance the train traveled up the incline.

Solution $\quad x_{\mathrm{f}}=x_{\mathrm{i}}+v_{\mathrm{i} x} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2}=0+(22 \mathrm{~m} / \mathrm{s})(8.0 \mathrm{~s})+\frac{1}{2}\left(-1.4 \mathrm{~m} / \mathrm{s}^{2}\right)(8.0 \mathrm{~s})^{2}=130 \mathrm{~m}$
(d) Strategy Refer to Figure 2.17, which shows various motion diagrams.

Solution The motion diagram is shown.

56. Strategy Use Eq. (2-12).

Solution Solve for $\Delta t$ using the quadratic formula.
$\Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}$, so $\frac{1}{2} a_{y}(\Delta t)^{2}+v_{\mathrm{i} y} \Delta t-\Delta y=0$.
$\Delta t=\frac{-v_{\mathrm{i} y} \pm \sqrt{v_{\mathrm{i} y}^{2}+2 a_{y} \Delta y}}{a_{y}}=\frac{-3.00 \mathrm{~m} / \mathrm{s} \pm \sqrt{(3.00 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-78.4 \mathrm{~m})}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=-3.71 \mathrm{~s}$ or 4.32 s
Since $\Delta t>0$, the brick lands on the ground 4.32 s after it is thrown from the roof.
57. Strategy Use Eq. (2-13).

Solution Find the final speed of the penny.
$v_{\mathrm{fy}}{ }^{2}-v_{\mathrm{i} y}^{2}=v_{\mathrm{fy}}{ }^{2}-0=-2 g \Delta y$, so $v_{\mathrm{f} y}= \pm \sqrt{-2 g \Delta y}=-\sqrt{-2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0 \mathrm{~m}-369 \mathrm{~m})}=-85.0 \mathrm{~m} / \mathrm{s}$.
Therefore, $\overrightarrow{\mathbf{v}}=85.0 \mathrm{~m} / \mathrm{s}$ down .
Discussion This is much less than the speed of a rifle bullet. If it hits a person on the head, I do not think it would necessarily cause serious harm. If you learned in a different physics course that the free-fall acceleration is $9.81 \mathrm{~m} / \mathrm{s}^{2}$, you used the value for the coast of Normandy (at sea level, $45^{\circ}$ latitude). But $g$ is $9.80 \mathrm{~m} / \mathrm{s}^{2}$ in New York, across most of the lower forty-eight United States, in southern Europe and China, and in great tracts of southern South America, Africa, and Australia.
58. Strategy If we ignore air resistance, the golf ball is in free fall. Use Eq. (2-12).

## Solution

(a) Find the time it takes the golf ball to fall 12.0 m .
$v_{\mathrm{i} y}=0$, so $\Delta y=-\frac{1}{2} g(\Delta t)^{2}$ and $\Delta t=\sqrt{-\frac{2 \Delta y}{g}}=\sqrt{\frac{-2(0-12.0 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=1.56 \mathrm{~s}$.
(b) Find how far the golf ball would fall in $2 \sqrt{\frac{-2(0-12.0 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}=3.13 \mathrm{~s}$.
$\Delta y=-\frac{1}{2} g(\Delta t)^{2}=-\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.13 \mathrm{~s})^{2}=-48.0 \mathrm{~m}$, so the golf ball would fall through 48.0 m .
59. Strategy The final speed is zero for the upward part of his free-fall flight. Use Eq. (2-13).

Solution Find the initial speed.
$v_{\mathrm{f} y}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=0-v_{\mathrm{i} y}{ }^{2}=-2 g \Delta y$, so $v_{\mathrm{i} y}=\sqrt{2 g \Delta y}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.3 \mathrm{~m}-0)}=5.0 \mathrm{~m} / \mathrm{s}$.
60. Strategy The acceleration of the camera is given by $v_{y 1} / \Delta t_{1}$, where $v_{y 1}=3.3 \mathrm{~m} / \mathrm{s}$ and $\Delta t_{1}=2.0 \mathrm{~s}$. Use Eq. (2-12).

Solution After 4.0 s , the camera has fallen
$\Delta y=\frac{1}{2} a_{y}(\Delta t)^{2}=\frac{1}{2}\left(\frac{v_{y 1}}{\Delta t_{1}}\right)(\Delta t)^{2}=\frac{3.3 \mathrm{~m} / \mathrm{s}}{2(2.0 \mathrm{~s})}(4.0 \mathrm{~s})^{2}=13 \mathrm{~m}$.
61. Strategy Use Eq. (2-12) to find the time it takes for the coin to reach the water. Then, find the time it takes the sound to reach Glenda's ear. Add these two times. Let $h=7.00 \mathrm{~m}$.

Solution Find the time elapsed between the release of the coin and the hearing of the splash.
$h=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}=0+\frac{1}{2} g\left(\Delta t_{1}\right)^{2}$, so $\Delta t_{1}=\sqrt{\frac{2 h}{g}}$. Then $h=v_{\mathrm{s}} \Delta t_{2}$, so $\Delta t_{2}=\frac{h}{v_{\mathrm{s}}}$.
Therefore, the time elapsed is $\Delta t=\Delta t_{1}+\Delta t_{2}=\sqrt{\frac{2 h}{g}}+\frac{h}{v_{\mathrm{s}}}=\sqrt{\frac{2(7.00 \mathrm{~m})}{9.80 \mathrm{~m} / \mathrm{s}^{2}}}+\frac{7.00 \mathrm{~m}}{343 \mathrm{~m} / \mathrm{s}}=1.22 \mathrm{~s}$.
Discussion Note the contrast: Sound is a wave, not an object with mass. So sound can behave as if it had infinite acceleration, immediately traveling off from its source at full speed. If you knew the time for a somewhat deeper well, could you find the depth? Suppose the time is 2.20 s and try it. Figure out how to get 22.3 m .
62. (a) Strategy The stone is instantaneously at rest at its maximum height. Use Eq. (2-13).

Solution Find the maximum height of the stone.

$$
v_{\mathrm{f} y}^{2}-v_{\mathrm{i} y}^{2}=0-v_{\mathrm{i} y}^{2}=-2 g \Delta y=-2 g\left(y_{\mathrm{f}}-y_{\mathrm{i}}\right), \text { so } y_{\mathrm{f}}=y_{\mathrm{i}}+\frac{v_{\mathrm{i} y}^{2}}{2 g}=1.50 \mathrm{~m}+\frac{(19.6 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=21.1 \mathrm{~m}
$$

(b) Strategy The stone is instantaneously at rest at its maximum height of 21.1 m . Use Eq. (2-12).

Solution Method one: Going up $21.1 \mathrm{~m}-1.50 \mathrm{~m}=19.6 \mathrm{~m}$ (to rest) takes the same time as falling down 19.6 m (from rest), so $\Delta y_{\text {up }}=\frac{1}{2} g\left(\Delta t_{\text {up }}\right)^{2}$, or $\Delta t_{\text {up }}=\sqrt{2(19.6 \mathrm{~m}) /\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=2.00 \mathrm{~s}$. Falling 21.1 m from rest takes $\Delta t_{\text {down }}=\sqrt{2(21.1 \mathrm{~m}) /\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=2.08 \mathrm{~s}$. The total time elapsed is $2.00 \mathrm{~s}+2.08 \mathrm{~s}=4.08 \mathrm{~s}$.

Method two: The velocity clearly reverses sign between the upward part of the flight and the downward part, but the acceleration is the same: $9.80 \mathrm{~m} / \mathrm{s}^{2}$ down. So we can describe the whole flight with one constant-acceleration equation. We use $\Delta y=v_{\mathrm{yi}} t-(1 / 2) g t^{2}$, or $-1.50 \mathrm{~m}=19.6 \mathrm{~m} / \mathrm{s} t-4.9 \mathrm{~m} / \mathrm{s}^{2} t^{2}$. With the requirement $t>0$ we transpose all the terms to the right-hand side and solve with the quadratic formula:

$$
\Delta t=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}=\frac{-(-19.6 \mathrm{~m} / \mathrm{s})+\sqrt{(-19.6 \mathrm{~m} / \mathrm{s})^{2}-4\left(+4.90 \mathrm{~m} / \mathrm{s}^{2}\right)(-1.50 \mathrm{~m})}}{+9.80 \mathrm{~m} / \mathrm{s}^{2}}=4.08 \mathrm{~s}
$$

63. Strategy Use Eqs. (2-12), (2-13), and (2-9). Let the $+y$-direction be down.

Solution (a) Without air resistance, the lead ball falls $\Delta y=\frac{1}{2} g(\Delta t)^{2}=\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})^{2}=44 \mathrm{~m}$.
(b) The lead ball is initially at rest. Find the speed of the ball after it has fallen 2.5 m .

$$
v_{\mathrm{f} y}^{2}-0=2 g \Delta y, \text { so } v_{\mathrm{f} y}=\sqrt{2 g \Delta y}=\sqrt{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.5 \mathrm{~m})}=7.0 \mathrm{~m} / \mathrm{s} \text {. }
$$

(c) After 3.0 s , the lead ball is falling at a speed of $v_{y}=0+g \Delta t=\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})=29 \mathrm{~m} / \mathrm{s}$.
(d) Find the change in height of the ball when $\Delta t=2.42 \mathrm{~s}$.

$$
\Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} g(\Delta t)^{2}=(4.80 \mathrm{~m} / \mathrm{s})(2.42 \mathrm{~s})+\frac{1}{2}\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(2.42 \mathrm{~s})^{2}=17.1 \mathrm{~m}
$$

The ball will be 17.1 m below the top of the tower.
Discussion Do it yourself with the choice of the $y$ direction pointing up. Prove that the physical meaning of all your answers is the same. Remember that a square root can be either positive or negative. The mass of the ball and the height of the tower are unnecessary for the solution.
64. Strategy Use Eq. (2-13). When the balloonist lets go of the sandbag, it is moving upward at $10.0 \mathrm{~m} / \mathrm{s}$.

Solution Find the sandbag's speed when it hits the ground.

$$
v_{\mathrm{f} y}^{2}-v_{\mathrm{i} y}^{2}=-2 g \Delta y, \text { so } v_{\mathrm{f} y}=\sqrt{v_{\mathrm{i} y}^{2}-2 g \Delta y}=\sqrt{(10.0 \mathrm{~m} / \mathrm{s})^{2}-2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-40.8 \mathrm{~m})}=30.0 \mathrm{~m} / \mathrm{s} .
$$

65. (a) Strategy Use Eq. (2-12) and the quadratic formula to find the time it takes the rock to reach Lois. Then, use Eq. (2-12) again to find Superman's required constant acceleration.

Solution Solve for $\Delta t$ using the quadratic formula.

$$
\begin{aligned}
& \Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}, \text { so } \frac{1}{2} a_{y}(\Delta t)^{2}+v_{\mathrm{i} y} \Delta t-\Delta y=0 \\
& \Delta t=\frac{-v_{\mathrm{i} y} \pm \sqrt{v_{\mathrm{i} y}^{2}+2 a_{y} \Delta y}}{a_{y}}=\frac{-(-2.8 \mathrm{~m} / \mathrm{s}) \pm \sqrt{(-2.8 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-14.0 \mathrm{~m})}}{-9.80 \mathrm{~m} / \mathrm{s}^{2}}=-2.00 \mathrm{~s} \text { or } 1.4286 \mathrm{~s}
\end{aligned}
$$

Since $\Delta t>0$, it takes 1.43 s for the rock to reach Lois. Find Superman's required acceleration.
$\Delta x=v_{\mathrm{i} x} \Delta t+\frac{1}{2} a_{x}(\Delta t)^{2}=0+\frac{1}{2} a_{x}(\Delta t)^{2}=\frac{1}{2} a_{x}(\Delta t)^{2}$, so $a_{x}=\frac{2 \Delta x}{(\Delta t)^{2}}=\frac{2(120 \mathrm{~m})}{(1.43 \mathrm{~s})^{2}}=120 \mathrm{~m} / \mathrm{s}^{2}$.
The Man of Steel must accelerate at $120 \mathrm{~m} / \mathrm{s}^{2}$ toward Lois to intercept the rock just touching her hair, and bat it away (the rock, not the hair) to save her.
(b) Strategy Use Eq. (2-9).

Solution Find Superman's speed when he reaches Lois.
$v_{x}=0+a_{x} \Delta t=\left(120 \mathrm{~m} / \mathrm{s}^{2}\right)(1.43 \mathrm{~s})=170 \mathrm{~m} / \mathrm{s}$
Discussion The quadratic formula gets the units right. Is that surprising? The negative answer it gives actually has physical meaning: if Lois had thrown the rock up at the right high speed just 2.00 s before the start of the process we analyzed, the villain would not have had to touch the rock. It would have passed him moving down at $2.8 \mathrm{~m} / \mathrm{s}$ and then reproduced precisely the same motion we considered.
66. Strategy The average speed of the flower pot as it passes the student's window is very nearly equal to its instantaneous speed, so $v_{\mathrm{av}, y}=\Delta y / \Delta t \approx v_{y}$.

Solution Determine the distance the flower pot fell to reach the speed $v_{y}$.
$v_{y}^{2}=2 g h$, so $h=\frac{v_{y}^{2}}{2 g} \approx \frac{1}{2 g}\left(\frac{\Delta y}{\Delta t}\right)^{2}=\frac{(1.0 \mathrm{~m})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(0.051 \mathrm{~s})^{2}}=19.6 \mathrm{~m}$.
$\frac{19.6 \mathrm{~m}}{4.0 \frac{\mathrm{~m}}{\text { floor }}}=4.9$ floors, so the pot fell from the 4 th floor +4.9 floors $=9$ th floor.
67. Strategy The unknown time for the stone to fall distance $d$ at constant acceleration, together with the unknown time for sound to return at constant speed, must add to 3.20 s . This gives an equation we hope to solve.

Solution The time for the sound of the rock hitting the bottom to reach you is $\Delta t_{\text {sound }}=d / v_{\mathrm{s}}=d /(343 \mathrm{~m} / \mathrm{s})$ The time for the rock, starting from rest, to fall to the bottom appears in
$d=\frac{1}{2} g\left(\Delta t_{r}\right)^{2}$ or $\Delta t_{r}=\sqrt{2 d / g}=\left(2 /\left[9.80 \mathrm{~m} / \mathrm{s}^{2}\right]\right)^{1 / 2} \sqrt{d}=(0.4518 \mathrm{~s} / \sqrt{\mathrm{m}}) \sqrt{d}$
The two times add to $3.20 \mathrm{~s}: d /(343 \mathrm{~m} / \mathrm{s})+\left(0.4518 \mathrm{~s} / \mathrm{m}^{1 / 2}\right) d^{1 / 2}=3.20 \mathrm{~s}$
$(0.002915 \mathrm{~s} / \mathrm{m})(\sqrt{d})^{2}+\left(0.4518 \mathrm{~s} / \mathrm{m}^{1 / 2}\right) \sqrt{d}-3.20 \mathrm{~s}=0 \quad$ Because $d=(\sqrt{d})^{2}$ we have a quadratic equation in the square root of $d$, which we can solve and then square the answer to find $d$ itself. We want the smaller of the two values, as the larger corresponds to your thowing the stone upward so cleverly that it takes 3.20 s to reach the bottom of the well.
$\sqrt{d}=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}=\frac{-0.4518 \pm \sqrt{0.4518^{2}-4(0.002915)(-3.20)}}{2(0.002915)} \frac{\mathrm{s} / \mathrm{m}^{1 / 2}}{\mathrm{~s} / \mathrm{m}}=\frac{-0.4518+0.4914}{2(0.002915)} \mathrm{m}^{1 / 2}=6.79 \mathrm{~m}^{1 / 2}$
So $d=6.79^{2} \mathrm{~m}=46.0 \mathrm{~m}$
68. (a) Strategy Use Eq. (2-12).

Solution Find the rocket's altitude when the engine fails.

$$
\Delta y=0+\frac{1}{2} a\left(\Delta t_{1}\right)^{2}=\frac{1}{2}\left(20.0 \mathrm{~m} / \mathrm{s}^{2}\right)(50.0 \mathrm{~s})^{2}=25.0 \mathrm{~km}
$$

(b) Strategy Let $v_{\mathrm{i} y}=$ the speed when the engine fails $=a \Delta t_{1}$; Then $v_{y}=v_{\mathrm{i} y}-g \Delta t=0$ at maximum height.

Solution Find the time elapsed from the engine failure to maximum height.
$0=v_{\mathrm{i} y}-g \Delta t=a \Delta t_{1}-g \Delta t$, so $\Delta t=\frac{a}{g} \Delta t_{1}=\frac{20.0 \mathrm{~m} / \mathrm{s}^{2}}{9.80 \mathrm{~m} / \mathrm{s}^{2}}(50.0 \mathrm{~s})=102 \mathrm{~s}$.
The time to maximum height from lift off is $\Delta t+\Delta t_{1}=102 \mathrm{~s}+50.0 \mathrm{~s}=152 \mathrm{~s}$.
(c) Strategy Use Eq. (2-12).

Solution Find the maximum height reached by the rocket.

$$
\begin{aligned}
y_{\mathrm{f}} & =y_{\mathrm{i}}+v_{\mathrm{i} y} \Delta t-\frac{1}{2} g(\Delta t)^{2}=y_{\mathrm{i}}+\left(a \Delta t_{1}\right)\left(\frac{a}{g} \Delta t_{1}\right)-\frac{1}{2} g\left(\frac{a}{g} \Delta t_{1}\right)^{2}=y_{\mathrm{i}}+\frac{a^{2}\left(\Delta t_{1}\right)^{2}}{g}-\frac{a^{2}\left(\Delta t_{1}\right)^{2}}{2 g}=y_{\mathrm{i}}+\frac{a^{2}\left(\Delta t_{1}\right)^{2}}{2 g} \\
& =25.0 \mathrm{~km}+\frac{\left(20.0 \mathrm{~m} / \mathrm{s}^{2}\right)^{2}(50.0 \mathrm{~s})^{2}}{2(9.80 \mathrm{~N} / \mathrm{kg})}=76.0 \mathrm{~km}
\end{aligned}
$$

(d) Strategy Use Eq. (2-13). For the downward motion $v_{i y}=0$ at the maximum height.

Solution Find the final velocity.
$v_{\mathrm{fy}}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=v_{\mathrm{fy}}{ }^{2}-0=2 a_{y} \Delta y=-2 g \Delta y$, so $v_{\mathrm{f} y}=\sqrt{-2 g \Delta y}=\sqrt{-2(9.80 \mathrm{~N} / \mathrm{kg})\left(0-76.0 \times 10^{3} \mathrm{~m}\right)}=1220 \mathrm{~m} / \mathrm{s}$.
Thus, $\overrightarrow{\mathbf{v}}=1220 \mathrm{~m} / \mathrm{s}$ downward.
69. Strategy Each car has traveled the same distance $\Delta x$ in the same time $\Delta t$ when they meet.

Solution Using Eq. (2-12), we have $\Delta x$ for police $=\Delta x$ for speeder $\Delta x=v_{\mathrm{i}} \Delta t+\frac{1}{2} a(\Delta t)^{2}=0+\frac{1}{2} a(\Delta t)^{2}=v \Delta t$, so $\Delta t=\frac{2 v}{a}$. The speed of the police car is $v_{\mathrm{p}}=a \Delta t=a(2 v / a)=2 v$.

Discussion If it is a little hard for you to distinguish the knowns $a$ and $v$ from the unknown final speed of the police car, or to see just what we have done, then solve this problem: The speeder goes past at $34 \mathrm{~m} / \mathrm{s}$ and the police car accelerates at $6.0 \mathrm{~m} / \mathrm{s}^{2}$. Find the police car's speed at the overtake point. Then go back and solve the symbolic problem with precisely the same steps.
70. Strategy $v_{1 f x}{ }^{2}-v_{1 i x}^{2}=2 a_{1} d_{1}$, where $a_{1}=10.0 \mathrm{ft} / \mathrm{s}^{2}$ and $d_{1}$ is the distance from the start to the point of no return. $v_{2 \mathrm{fx}}{ }^{2}-v_{2 \mathrm{ix}}{ }^{2}=2 a_{2} d_{2}$, where $a_{2}=-7.00 \mathrm{ft} / \mathrm{s}^{2}$ and $d_{2}$ is the distance from the point of no return to the end of the runway. The initial speed $v_{1 \mathrm{i} x}$ and the final speed $v_{2 \mathrm{f} x}$ are zero. The speed at the point of no return is $v_{1 f x}=v_{2 \mathrm{i} x}$. Let $v_{1 \mathrm{f} x}=v_{2 \mathrm{ix}}=v$ for simplicity. Also, $d=d_{1}+d_{2}$ is the length of the runway.


Solution From the strategy, we have $v^{2}=2 a_{1} d_{1}$ and $-v^{2}=2 a_{2} d_{2}=2 a_{2}\left(d-d_{1}\right)$.
Eliminate $v^{2}$. by substitution.

$$
\begin{aligned}
& 2 a_{1} d_{1}=2 a_{2}\left(d_{1}-d\right) \\
& a_{1} d_{1}=a_{2} d_{1}-a_{2} d \\
& \left(a_{1}-a_{2}\right) d_{1}=-a_{2} d \\
& d_{1}=\frac{a_{2}}{a_{2}-a_{1}} d=\frac{-7.00 \mathrm{ft} / \mathrm{s}^{2}}{-7.00 \mathrm{ft} / \mathrm{s}^{2}-10.0 \mathrm{ft} / \mathrm{s}^{2}}(1.50 \mathrm{mi})\left(\frac{5280 \mathrm{ft}}{1 \mathrm{mi}}\right) \\
& \quad=3260 \mathrm{ft} \text { from the starting end of the runway }
\end{aligned}
$$

Find the time to $d_{1}$ using Eq. (2-12).

$$
d_{1}=\frac{1}{2} a_{1}(\Delta t)^{2}, \text { so } \Delta t=\sqrt{\frac{2 d_{1}}{a_{1}}}=\sqrt{\frac{2(3260 \mathrm{ft})}{10.0 \mathrm{ft} / \mathrm{s}^{2}}}=25.5 \mathrm{~s} .
$$

71. Strategy Suppose inspection of the videotape, camera, and dachshund show that the second witness is thoroughly credible. Use the second witness's information to determine the average speed of the flowerpot as it passed the 18 th-story window. Then, use this speed to determine the height above the window from which the flowerpot fell.

Solution The average speed of the flowerpot as it passed the window was
$v_{\mathrm{av}}=\frac{\Delta y}{\Delta t}=\frac{1.5 \mathrm{~m}}{0.044 \mathrm{~s}}=34 \mathrm{~m} / \mathrm{s}$.
This is very nearly the speed of the flowerpot as it passed the middle of the window. Assuming the flowerpot started at rest, use Eq. (2-13) to find the distance above the window from which the flowerpot fell.
$v_{\mathrm{fy} y}{ }^{2}-v_{\mathrm{i} y}{ }^{2}=v_{\mathrm{fy} y}{ }^{2}-0=2 a_{y} \Delta y=-2 g \Delta y$, so $\Delta y=-\frac{v_{\mathrm{f} y}{ }^{2}}{2 g}=-\frac{(-34 \mathrm{~m} / \mathrm{s})^{2}}{2\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)}=-59 \mathrm{~m}$.
Thus, the flowerpot fell from about 59 m above the middle of the 18 th-story window; that is, it fell from $59 \mathrm{~m}+75 \mathrm{~m}-(1.5 \mathrm{~m}) / 2=133 \mathrm{~m}$ above the ground. The 24th story window is only 94 m above the ground; so no, the flowerpot could not have fallen with zero initial velocity from the 24th-story window. It either fell from 133 m high or, if it came from a lower location (such as the 24th floor), it was thrown downward. The first witness is not credible.
72. Strategy Analyze the graph to answer each question about the motion of the elevator.

Solution (a) The area under the curve represents the change in velocity. Each space along the $t$-axis represents 2 s . $A_{1}=\frac{1}{2} b h=\frac{1}{2}(8 \mathrm{~s})\left(0.2 \mathrm{~m} / \mathrm{s}^{2}\right)=0.8 \mathrm{~m} / \mathrm{s}$ The elevator accelerates $\left(a_{y}>0\right)$ to $0.8 \mathrm{~m} / \mathrm{s}$ during the first 8 s . Then, the elevator travels at $0.8 \mathrm{~m} / \mathrm{s}\left(a_{y}=0\right)$ for the next $8 \mathrm{~s} . \quad A_{2}=\frac{1}{2} b h=\frac{1}{2}(4 \mathrm{~s})\left(-0.4 \mathrm{~m} / \mathrm{s}^{2}\right)=-0.8 \mathrm{~m} / \mathrm{s}$
The elevator slows down $\left(a_{y}<0\right)$ until it comes to rest. Then, it sits for the next 4 s . So, the passenger has gone to a higher floor.
(b) Sketch the graph of $v_{y}$ vs. $t$ by plotting points at two-second intervals with $v_{y}$ determined from the $a_{y}$ vs. $t$ graph. Each rectangle represents $(2 \mathrm{~s})\left(0.2 \mathrm{~m} / \mathrm{s}^{2}\right)=0.4 \mathrm{~m} / \mathrm{s}$.

| $t(\mathrm{~s})$ | $v_{y}$ | $t(\mathrm{~s})$ | $v_{y}$ |
| :--- | :--- | :--- | :--- |
| 0 | The elevator is at rest, so $v_{y}=0$. | 12 | $0.8 \mathrm{~m} / \mathrm{s}$ |
| 2 | $1 / 4 \times(0.4 \mathrm{~m} / \mathrm{s})=0.1 \mathrm{~m} / \mathrm{s}$ | 14 | $0.8 \mathrm{~m} / \mathrm{s}$ |
| 4 | $1(0.4 \mathrm{~m} / \mathrm{s})=0.4 \mathrm{~m} / \mathrm{s}$ | 16 | $0.8 \mathrm{~m} / \mathrm{s}$ |
| 6 | $(13 / 4)(0.4 \mathrm{~m} / \mathrm{s})=0.7 \mathrm{~m} / \mathrm{s}$ | 18 | $-1(0.4 \mathrm{~m} / \mathrm{s})+0.8 \mathrm{~m} / \mathrm{s}=0.4 \mathrm{~m} / \mathrm{s}$ |
| 8 | $2(0.4 \mathrm{~m} / \mathrm{s})=0.8 \mathrm{~m} / \mathrm{s}$ | 20 | $-1(0.4 \mathrm{~m} / \mathrm{s})+0.4 \mathrm{~m} / \mathrm{s}=0$ |
| 10 | $0.8 \mathrm{~m} / \mathrm{s}$, since $a_{y}=0$. | 22 | 0, since $a_{y}=0$ |


${ }^{t}$ Each vertical space represents $0.1 \mathrm{~m} / \mathrm{s}$ and each horizontal space is 2 s .
(c) The graph from part (b) shows that the velocity is nonnegative for $t \geq 0$, so the position of the elevator is always increasing in height until it stops.
Break the region under the graph from part (b) into six sections (blocks of time):

| Section 1: $(0,4 \mathrm{~s})$ | Section 2: $(4 \mathrm{~s}, 8 \mathrm{~s})$ | Section 3: $(8 \mathrm{~s}, 16 \mathrm{~s})$ |
| :--- | :--- | :--- |
| Section 4: $(16 \mathrm{~s}, 18 \mathrm{~s})$ | Section 5: $(18 \mathrm{~s}, 20 \mathrm{~s})$ | Section 6: $(20 \mathrm{~s}, 24 \mathrm{~s})$ |

Find the approximate distance the elevator travels in each section by counting boxes under the curve, each of which represents displacement $(0.1 \mathrm{~m} / \mathrm{s})(2 \mathrm{~s})=0.2 \mathrm{~m}$.

| Section 1: | Section 2: | Section 3: | Section 4: | Section 5: | Section 6: |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $3(0.2 \mathrm{~m})=$ | $13(0.2 \mathrm{~m})=$ | $32(0.2 \mathrm{~m})=$ | $6.5(0.2 \mathrm{~m})=$ | $1.5(0.2 \mathrm{~m})=$ | $0(0.2 \mathrm{~m})=0$ |
| 0.6 m | 2.6 m | 6.4 m | 1.3 m | 0.3 m |  |

Adding up the increments gives the position at the end of each time interval like this:

| $t, \mathrm{~s}$ | 0 | 4 | 8 | 16 | 18 | 20 | 24 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $x, \mathrm{~m}$ | 0 | 0.6 | 3.2 | 9.6 | 10.9 | 11.2 | 11.2 |

Plot points and draw a smooth curve.

73. (a) Strategy Use the definition of average speed.

Solution Find the average speed of the swimmer.
$v_{\mathrm{av}}=\frac{\Delta r}{\Delta t}=\frac{1500 \mathrm{~m}}{14 \mathrm{~min} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}+53 \mathrm{~s}}=1.7 \mathrm{~m} / \mathrm{s}$
(b) Strategy and Solution The swimmer pushes off from each end of the pool to raise his average speed.
74. (a) Strategy Use Eqs. (2-11) and (2-9) since the acceleration is constant.

Solution Find the distance traveled.
$\Delta x=v_{\mathrm{av}, x} \Delta t=\frac{v_{\mathrm{f} x}+v_{\mathrm{i} x}}{2} \Delta t=\frac{27.3 \mathrm{~m} / \mathrm{s}+17.4 \mathrm{~m} / \mathrm{s}}{2}(10.0 \mathrm{~s})=224 \mathrm{~m}$.
(b) Strategy Use the definition of average acceleration.

Solution Find the magnitude of the acceleration.
$a=\frac{\Delta v}{\Delta t}=\frac{27.3 \mathrm{~m} / \mathrm{s}-17.4 \mathrm{~m} / \mathrm{s}}{10.0 \mathrm{~s}}=0.99 \mathrm{~m} / \mathrm{s}^{2}$
75. Strategy Use the definition of average acceleration.

Solution Compute the trout's average acceleration.

$$
a_{\mathrm{av}}=\frac{\Delta v}{\Delta t}=\frac{2 \mathrm{~m} / \mathrm{s}}{0.05 \mathrm{~s}}=40 \mathrm{~m} / \mathrm{s}^{2}, \text { so } \overrightarrow{\mathbf{a}}_{\mathrm{av}}=40 \mathrm{~m} / \mathrm{s}^{2} \text { in the direction of motion } .
$$

76. (a) Strategy Use the definition of average acceleration.

Solution Find the magnitude of the acceleration. $a=\frac{\Delta v}{\Delta t}=\frac{24 \mathrm{~m} / \mathrm{s}}{2.0 \mathrm{~s}}=12 \mathrm{~m} / \mathrm{s}^{2}$
(b) Strategy Relate the distance traveled, acceleration, and time using Eq. (2-12).

Solution Find the distance traveled. $\Delta x=0+\frac{1}{2} a(\Delta t)^{2}=\frac{1}{2}\left(12 \mathrm{~m} / \mathrm{s}^{2}\right)(2.0 \mathrm{~s})^{2}=24 \mathrm{~m}$
(c) Strategy Use the definition of average acceleration.

Solution The magnitude of the acceleration of the runner is $a=\frac{\Delta v}{\Delta t}=\frac{6.0 \mathrm{~m} / \mathrm{s}}{2.0 \mathrm{~s}}=3.0 \mathrm{~m} / \mathrm{s}^{2}$.
Find $a_{\mathrm{c}} / a_{\mathrm{r}} . \quad \frac{a_{\mathrm{c}}}{a_{\mathrm{r}}}=\frac{12 \mathrm{~m} / \mathrm{s}^{2}}{3.0 \mathrm{~m} / \mathrm{s}^{2}}=4.0$
77. Strategy Use the definitions of average velocity and average acceleration.

Solution (a) $\overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{y}}}{\Delta t}=\frac{160 \times 10^{3} \mathrm{~m} \mathrm{up}}{(8.0 \mathrm{~min})\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)}=330 \mathrm{~m} / \mathrm{s}$ up
(b) $\overrightarrow{\mathbf{a}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{v}}}{\Delta t}=\frac{7600 \mathrm{~m} / \mathrm{s} \mathrm{up}-0}{(8.0 \mathrm{~min})\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right)}=16 \mathrm{~m} / \mathrm{s}^{2} \mathrm{up}$

Discussion. If we made the additional assumption that the acceleration is constant, we could say that the average velocity is the velocity halfway through the motion in time, at the four-minute mark. But there are a couple of reasons that the acceleration might not be constant: The rocket body is always losing mass as it throws its exhaust out of its back end, and air resistance drops to a very low value at 160 km up.
78. Strategy Initially, the slope (and acceleration) is $+1.0 \mathrm{~m} / \mathrm{s}^{2}$, corresponding to the streetcar speeding up. Then, the slope (and acceleration) is zero for the streetcar moving with constant speed. Finally, the slope (and acceleration) is $-2.0 \mathrm{~m} / \mathrm{s}^{2}$, corresponding to the streetcar slowing to a stop. For the duration of the trip, use the standard relationships for constant acceleration motion, Eqs. (2-9) through (2-13).

Solution Sketch the graph of $v_{x}(t)$.


Compute the duration of the trip. Refer to the figure at right.
Find $v_{x 1}$, the speed after 10.0 s .

$\Delta v_{x 1}=a_{x 1} \Delta t_{1}=\left(1.0 \mathrm{~m} / \mathrm{s}^{2}\right)(10.0 \mathrm{~s})=10 \mathrm{~m} / \mathrm{s}$
Find $\Delta x_{1}, \Delta x_{3}$, and $\Delta t_{3}$.
$\Delta x_{1}=\frac{1}{2} a_{x 1}\left(\Delta t_{1}\right)^{2}=\frac{1}{2}\left(1.0 \mathrm{~m} / \mathrm{s}^{2}\right)(10.0 \mathrm{~s})^{2}=50 \mathrm{~m}$
$\Delta x_{3}=\frac{v_{x 3}^{2}-v_{\mathrm{i} x 3^{2}}^{2}}{2 a_{3}}=\frac{0-v_{x 1}^{2}}{2 a_{3}}=\frac{-(10 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-2.0 \mathrm{~m} / \mathrm{s}^{2}\right)}=25 \mathrm{~m}$
$\Delta x_{3}=v_{\mathrm{av}, x} \Delta t_{3}$, so $\Delta t_{3}=\frac{\Delta x_{3}}{v_{\mathrm{av} 3}}=\frac{25 \mathrm{~m}}{(10 \mathrm{~m} / \mathrm{s}) / 2}=5.0 \mathrm{~s}$.
Find $\Delta x_{2}$.

$$
\Delta x_{2}=v_{x 2} \Delta t_{2}=v_{x 1} \Delta t_{2}, \text { so } \Delta t_{2}=\frac{\Delta x_{2}}{v_{x 1}}
$$

Find the total time.
$\Delta t=\Delta t_{1}+\Delta t_{2}+\Delta t_{3}=\Delta t_{1}+\frac{\Delta x_{2}}{v_{x 1}}+\Delta t_{3}=10.0 \mathrm{~s}+\frac{0.60 \times 10^{3} \mathrm{~m}-50 \mathrm{~m}-25 \mathrm{~m}}{10 \mathrm{~m} / \mathrm{s}}+5.0 \mathrm{~s}=68 \mathrm{~s}$
79. (a) Strategy Use Eq. (2-13).

Solution Find the initial velocity of the stone by taking the "final" point at the window.

$$
v_{\mathrm{f} y}^{2}-v_{\mathrm{i} y}^{2}=2 a_{y} \Delta y, \text { so } v_{\mathrm{i} y}= \pm \sqrt{v_{\mathrm{f} y}^{2}-2 a_{y} \Delta y}= \pm \sqrt{(-25.0 \mathrm{~m} / \mathrm{s})^{2}-2\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(-16.0 \mathrm{~m})}= \pm 17.6 \mathrm{~m} / \mathrm{s}
$$

Since the stone was thrown vertically downward, its initial velocity was $17.6 \mathrm{~m} / \mathrm{s}$ downward .
(b) Strategy Use Eq. (2-12).

Solution Find the change in height of the stone, now taking the final point just before it hits the ground.
$\Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}=(-17.647 \mathrm{~m} / \mathrm{s})(3.00 \mathrm{~s})+\frac{1}{2}\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.00 \mathrm{~s})^{2}=-97.0 \mathrm{~m}$
The height of the building is 97.0 m .
80. Strategy The direction of the acceleration is opposite the direction of motion. Use Eq. (2-16).

## Solution

(a) Find the magnitude of the acceleration.
$v_{\mathrm{f} x}^{2}-v_{\mathrm{ix}}^{2}=2 a_{x} \Delta x$, so $a_{x}=\frac{v_{\mathrm{f} x}^{2}-v_{\mathrm{ix}}^{2}}{2 \Delta x}=\frac{0-(29 \mathrm{~m} / \mathrm{s})^{2}}{2(1.0 \mathrm{~m})}=-420 \mathrm{~m} / \mathrm{s}^{2}$.
So, $\overrightarrow{\mathbf{a}}=420 \mathrm{~m} / \mathrm{s}^{2}$ opposite the direction of motion.
(b) Find the magnitude of the acceleration.
$a_{x}=\frac{v_{\mathrm{f} x}{ }^{2}-v_{\mathrm{i} x}^{2}}{2 \Delta x}=\frac{0-(29 \mathrm{~m} / \mathrm{s})^{2}}{2(0.100 \mathrm{~m})}=-4200 \mathrm{~m} / \mathrm{s}^{2}$
So, $\overrightarrow{\mathbf{a}}=4200 \mathrm{~m} / \mathrm{s}^{2}$ opposite the direction of motion.
81. Strategy Use the fact that distance traveled equals average rate times time elapsed.

## Solution

(a) Marcella must run the whole distance in $t=\frac{1000 \mathrm{~m}}{3.33 \mathrm{~m} / \mathrm{s}}=300 \mathrm{~s}$. She ran the first 500 m in $t_{1}=\frac{500 \mathrm{~m}}{4.20 \mathrm{~m} / \mathrm{s}}=119 \mathrm{~s}$, so she must run the last 500 m in $t_{2}=t-t_{1}=\frac{1000 \mathrm{~m}}{3.33 \mathrm{~m} / \mathrm{s}}-\frac{500 \mathrm{~m}}{4.20 \mathrm{~m} / \mathrm{s}}=181 \mathrm{~s}$.
(b) Marcella's average speed for the last 500 m must be $v=\frac{d}{t_{2}}=\frac{500 \mathrm{~m}}{181 \mathrm{~s}}=2.76 \mathrm{~m} / \mathrm{s}$.

Discussion Note that 3.33 is not halfway between 2.76 and 4.20. She ran with the latter two speeds for equal distances but not for equal times.
82. Strategy Use the definitions of displacement, average velocity, and average acceleration.

## Solution

$$
\begin{aligned}
& \Delta \overrightarrow{\mathbf{r}}=\overrightarrow{\mathbf{r}}_{\mathrm{f}}-\overrightarrow{\mathbf{r}}_{\mathrm{i}}=185 \mathrm{mi} \text { north }-126 \mathrm{mi} \text { north }=59 \mathrm{mi} \text { north } \\
& \overrightarrow{\mathbf{v}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{r}}}{\Delta t}=\frac{59 \mathrm{mi} \text { north }}{37 \mathrm{~min}}\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)=96 \mathrm{mi} / \mathrm{h} \text { north } \\
& \overrightarrow{\mathbf{a}}_{\mathrm{av}}=\frac{\Delta \overrightarrow{\mathbf{v}}}{\Delta t}=\frac{105.0 \mathrm{mi} / \mathrm{h} \text { north }-112.0 \mathrm{mi} / \mathrm{h} \text { north }}{37 \mathrm{~min}}=\frac{-7.0 \mathrm{mi} / \mathrm{h} \text { north }}{37 \mathrm{~min}}\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)=11 \mathrm{mi} / \mathrm{h}^{2} \text { south }
\end{aligned}
$$

83. Strategy Use the graph to answer the questions. The slope of the graph represents the acceleration of the ball.

## Solution

(a) The ball reaches its maximum height the first time $v_{y}=0$, or at $t=0.30 \mathrm{~s}$.
(b) The time it takes for the ball to make the transition from its extreme negative velocity to its greatest positive velocity is the time that the ball is in contact with the floor.
$0.65 \mathrm{~s}-0.60 \mathrm{~s}=0.05 \mathrm{~s}$
(c) Using Eq. (2-12) and the definition of average acceleration, we find that the maximum height of the ball is at the end of its first upward flight and is
$\Delta y=v_{\mathrm{i} y} \Delta t+\frac{1}{2} a_{y}(\Delta t)^{2}=v_{\mathrm{i} y} \Delta t+\frac{\Delta v_{y}}{2 \Delta t}(\Delta t)^{2}=(3.0 \mathrm{~m} / \mathrm{s})(0.30 \mathrm{~s})+\frac{0-3.0 \mathrm{~m} / \mathrm{s}}{2}(0.30 \mathrm{~s})=0.45 \mathrm{~m}$.
(d) $a_{y}=\frac{\Delta v_{y}}{\Delta t}=\frac{-3.0 \mathrm{~m} / \mathrm{s}-3.0 \mathrm{~m} / \mathrm{s}}{0.60 \mathrm{~s}}=-10 \mathrm{~m} / \mathrm{s}^{2}$, so the acceleration is $10 \mathrm{~m} / \mathrm{s}^{2} \mathrm{down}$.
(e) $a_{\mathrm{av}}=\frac{\Delta v}{\Delta t}=\frac{3.0 \mathrm{~m} / \mathrm{s}-(-3.0 \mathrm{~m} / \mathrm{s})}{0.05 \mathrm{~s}}=120 \mathrm{~m} / \mathrm{s}^{2}$, so the acceleration is $120 \mathrm{~m} / \mathrm{s}^{2} \mathrm{up}$.
84. (a) Strategy and Solution The intersection of the two curves indicates when the motorcycle and the police car are moving at the same speed. According to the graph, they are moving at the same speed at $t=11 \mathrm{~s}$.
(b) Strategy and Solution The displacement of each vehicle is represented by the area under each curve. The answer is no; the area under the police car curve is less than the area under the motorcycle curve.
85. Strategy Analyze the graph to answer each question about the motion of the engine. The $x$-component of the engine's velocity is represented by the slope.

## Solution

(a) $a_{x}<0$ when the engine is moving in the $+x$-direction and slowing down, when it is moving in the $-x$-direction and speeding up, and when it is momentarily at rest and changing direction from positive to negative $x$. So at $t_{3}$ and $t_{4} a_{x}<0$.
(b) $a_{x}=0$ when the engine's speed is constant. This includes the cast of speed zero and constant. So,
at $t_{0}, t_{2}, t_{5}$, and $t_{7} a_{x}=0$.
(c) $a_{x}>0$ when the engine is moving in the $+x$-direction and speeding up and when it is moving in the $-x$-direction and slowing down. So, at $\begin{aligned} & t_{1} \text { and } t_{6} \\ & a_{x}>0\end{aligned}$
(d) $v_{x}=0$ when the slope of the graph is zero. So, at $t_{0}, t_{3}$, and $t_{7} v_{x}=0$.
(e) The speed is decreasing when $a_{x}$ and $v_{x}$ have opposite directions. So, at $t_{6}$ the speed is decreasing.

Discussion Is this surprisingly tough? Just below the given graph of $x$ versus $t$ sketch a graph of its slope $v$ versus $t$. Whenever the $x$ versus $t$ graph line is curving, the engine possesses nonzero acceleration. When the graph line is "concave upward $\cup, " a$ is positive and when $x$ versus $t$ is "concave downward $\cap$ " the acceleration is negative.
86. Strategy and Solution As the speed of the glider at the gate we compute its average speed as it passes through.
$v_{\mathrm{f}}=v_{\text {gate, av }}=\frac{8.0 \mathrm{~cm}}{0.333 \mathrm{~s}}=24 \mathrm{~cm} / \mathrm{s}$ This speed when the glider is halfway through the gate in time is very close to its speed when it is halfway through the gate in distance, when it is shown in the picture. Next, since the acceleration is constant, the displacement of the glider equals the average speed times the time of travel: $v_{\mathrm{av}}=v_{\mathrm{f}} / 2=12 \mathrm{~cm} / \mathrm{s}$. Find $\Delta t$ for the motion from release to halfway through the gate.
$\Delta x=v_{\mathrm{av}} \Delta t$, so $\Delta t=\frac{\Delta x}{v_{\mathrm{av}}}$.
Find $a_{\mathrm{av}}$ from its definition; $a_{\mathrm{av}}=a$.
$a=\frac{\Delta v}{\Delta t}=\frac{\Delta v}{\Delta x / v_{\mathrm{av}}}=\frac{v_{\mathrm{av}} \Delta v}{\Delta x}=\frac{(12 \mathrm{~cm} / \mathrm{s})(24 \mathrm{~cm} / \mathrm{s}-0)}{96 \mathrm{~cm}}=3.0 \mathrm{~cm} / \mathrm{s}^{2}$
So, $\overrightarrow{\mathbf{a}}=3.0 \mathrm{~cm} / \mathrm{s}^{2}$ parallel to the velocity.
87. (a) Strategy Use the definition of average speed.

Solution For crossing a synapse $v_{\mathrm{av}}=\frac{\Delta x}{\Delta t}=\frac{100 \times 10^{-9} \mathrm{~m}}{0.10 \times 10^{-3} \mathrm{~s}}=1.0 \mathrm{~mm} / \mathrm{s}$
(b) Strategy Find the time it takes the pain signal to travel the length of a $1.0-\mathrm{m}$ long neuron. Then, add the times of travel across synapses and neurons.

Solution $\quad t_{\mathrm{n}}=\frac{x}{v}=\frac{1.0 \mathrm{~m}}{100 \mathrm{~m} / \mathrm{s}}=10 \mathrm{~ms}$
Find the total time to reach the brain.
$t_{\mathrm{n}}+t_{\mathrm{syn}}+t_{\mathrm{n}}+t_{\mathrm{syn}}=2 t_{\mathrm{n}}+2 t_{\mathrm{syn}}=2\left(t_{\mathrm{n}}+t_{\mathrm{syn}}\right)=2(10 \mathrm{~ms}+0.10 \mathrm{~ms})=20 \mathrm{~ms}$
(c) Strategy Use the definition of average speed.

Solution $v_{\mathrm{av}}=\frac{\Delta x}{\Delta t}=\frac{2.0 \mathrm{~m}+2\left(100 \times 10^{-9} \mathrm{~m}\right)}{20.2 \times 10^{-3} \mathrm{~s}}=99 \mathrm{~m} / \mathrm{s}$

## Chapter 1

## Section 1.3

## 1. Proportions

The rate $P$ at which a star emits energy as electromagnetic waves is proportional to the fourth power of its absolute temperature $T$. Suppose a star's absolute temperature increases by $10.0 \%$. By what percentage would the rate $P$ increase?

## 2. Proportions

a) The power output (i.e. the rate at which electrical energy is generated) of a wind turbine is proportional to the wind speed cubed. If the wind speed doubles, by what factor does the power output increase?
b) (Use of mathematics) Suppose you own a cylindrical water tank with radius $r$, height $h$, and can hold a certain volume of water. You one day decide to build a new tank with double the radius and half the height of the original tank. Find the capacity of the new tank as a ratio of the original tank?
c) (Use of mathematics) Suppose you decide to paint the outer surface of your new water tank mentioned in problem (b), and find out that at your local hardware store a litre of paint costs $\$ 30.00$. Estimate how much it will cost you to paint your new tank, given that $r=0.5 \mathrm{~m}$ and $h=3 \mathrm{~m}$.

## 3. Ratio Reasoning

A piece of oak made into a board has a density of $900 \mathrm{~kg} / \mathrm{m}^{3}$. An interpretation of this number is that 900 kg of wood are contained in each $1 \mathrm{~m}^{3}$ of wood volume.

Answer the following questions for this situation using ratio reasoning. DO NOT use the definition of density in your calculations.
a) What is the meaning of the number 900 ?
b) What is the meaning of the number $1 / 900=0.11$
c) What volume would 300 kg of oak occupy?
d) How much would $7 \mathrm{~m}^{3}$ of oak weigh?

## 4. Geometry and Scaling

The strength of a beam used in construction scales as the square of the diameter and the weight of the beam scales as the cube of the diameter.

Find the strength to weight ratio if the diameter of the beam is:
a) Doubled
b) Tripled
c) Halved
d) decreased by 3 times

## 5. Geometry of a Circle

The radii and circumferences of several circles are measured and the best-fit line to the data is shown below.

a) What is the diameter of a circle with a circumference of 10 cm ?
b) What is the circumference of a circle with a diameter of 2 cm ?
c) If the radius of a hula-hoop is 0.5 m larger than that of a bowling ball show how to use the graph to find the difference in their circumferences.

## Section 1.4

## 6. Significant Figures

Write the result of each calculation below with the correct number of significant figures.
a) $(12.65 \mathrm{~m}) /(0.0322 \mathrm{~s})=$
b) $77.860 \mathrm{~kg}+40.722 \mathrm{~kg}=$
c) $0.00294 \mathrm{~m} / \mathrm{s}-0.002883 \mathrm{~m} / \mathrm{s}=$

## 7. Significant Figures

How many significant figure does the following numbers have?
a) 0.02566 $\qquad$
b) 256.60
c) $25.660 \times 10^{3}$ $\qquad$
d) $2.566 \times 10^{-3}$ $\qquad$
e) 2.5660 $\qquad$
f) $2.566 \times 10^{3}$

## 8. Significant Figures

Use the rules of significant figures to complete the following calculations. Assume all digits shown were measured.
a) $10.0+0.45$
b) $10.00+0.45$
c) $10.000+0.45$
d) $10.0+0.450$

## Section 1.5

9. Units

Rank these mass measurements in increasing order:
a) 0.11 kg
b) 57 g
c) $7.2 \times 10^{7} \mu \mathrm{~g}$
d) $6.4 \times 10^{4} \mathrm{mg}$
e) 2493 cg

## 10. Unit Conversions

The highway speed limit in Ontario, Canada is $110 \mathrm{~km} / \mathrm{hr}$. Joe drives across the border from the U.S. to Ontario, Canada. Upon seeing the speed limit signs, he gets very excited, forgets all he learnt about units in his freshmen physics class, and starts driving at 110 mph . He is eventually stopped by a police cruiser. Calculate the cost of Joe's speeding ticket (in Canadian Dollars). Speeding ticket costs in Ontario, Canada are given below.

## Cost of a Speeding Ticket in Ontario (In Canadian Dollars)

1-15km/hr. over

- $\$ 3.00$ for each $\mathrm{km} / \mathrm{hr}$ over the speed limit


## 16-29km/hr. over

- $\quad \$ 4.50$ for each km/hr. over the speed limit

30-49km/over

- $\quad \$ 7.50$ for each $\mathrm{km} / \mathrm{hr}$. over the speed limit

More than 50km/hr. over

- $\$ 10.50$ for each $\mathrm{km} / \mathrm{hr}$. over the speed limit
- 30 Day suspension of drivers license


## 11.Units

A Chronograph watch is a wristwatch containing a built-in stopwatch. Some chronograph watches from the 1940's and 1950's contained a printed scale on the watch dial called a telemeter to measure phenomena related to the speed of sound in air.

For example, the chronograph with a telemeter scale could be used to determine how far a thunderstorm is away from you by noting the time between a flash of lightning and the accompanying thunder.

If the speed of sound in air is approximately 700 mph how many seconds will elapse on the watch if a storm is 1 km away? (This number is the "scale factor" of the telemeter scale.)

## Section 1.6

## 12. Dimensions

An object with mass $m$ moving in a circle of radius $r$ with speed $v$ has a linear momentum of magnitude $m v$, an angular momentum of magnitude $m v r$, a kinetic energy $\frac{1}{2} m v^{2}$, and a rest energy $m c^{2}$ (where $c$ is the speed of light).
a) Is it dimensionally possible to add the kinetic energy and the rest energy? Explain. b) Is it dimensionally possible to add the linear momentum and the angular momentum magnitudes? Explain.

## 13. Estimation

The world population is about $7 \times 10^{9}$ people. Make a rough estimate of the average number children born per second in the world by following these steps:
a) Estimate the average number of births per woman over her entire lifetime.
b) Estimate the average life expectancy of a woman worldwide.
c) Use these to estimate the average number of births per year, then convert to births per second.

## 14. Dimensional Analysis

Why is it that you can always add two quantities with the same units, but cannot always add two quantities with the same dimensions?

## 15. Dimensional Analysis

The period of oscillation of a pendulum is given as $T=\frac{2 \pi l^{n}}{g^{1 / 2}}$. Where $\boldsymbol{T}$ has unit of time, I units of distance, and $\boldsymbol{g}$ units of acceleration. Use Dimensional Analysis to find the exponent $n$.

## 16. Dimensional Analysis

The electric field surrounding an electric charge depends on the distance, $d$, between the charge and the observation point. Use dimensional analysis to determine the constant, $C$, in the following expressions if the units of the electric field are $\mathrm{kg} \mathrm{m} /(\mathrm{A} \mathrm{s})$.

| Charge Shape / Picture | Formula for the Electric Field | Units of C |
| :--- | :---: | :--- |
| a) Line charge | $\mathrm{C} / \mathrm{x}$ |  |
| b) Outside a Sphere of <br> Charge | $\mathrm{C} / \mathrm{x}^{2}$ |  |
| c) Electric Dipole | $\mathrm{C} / \mathrm{x}^{3}$ |  |
| d) Infinite Sheet of <br> Charge | C |  |

## Section 1.8

## 17. Estimation

You're throwing a pizza party and expect 20 people to attend. You estimate that each person will eat 4 slices of pizza. Cost of a large pizza at your local pizzeria is $\$ 12.50$. Given that a large pizza is cut into 12 pieces, how much is the pizza going to cost you?

## Section 1.9

## 18. Graphing

The table shows measurements of the speed $v$ of a falling stone at different times $t$.

| $v(\mathrm{~m} / \mathrm{s})$ | $t(\mathrm{~s})$ |
| :---: | :---: |
| 3.22 | 0.10 |
| 4.24 | 0.20 |
| 4.99 | 0.30 |
| 6.56 | 0.50 |
| 7.96 | 0.60 |

(a) Graph these data with $v$ on the vertical axis.
(b) Draw a best-fit line and find its slope (including units).
(c) The expected relationship between the variables is $v=a t+v_{0}$. What are the values of $a$ and $v_{0}$ ?

## 19. Catalog of Graphs

Complete the table making the two columns of information consistent, i.e., the two entries in each row must be consistent. The first case is given as an example.

## Description <br> Graph

| example: <br> Positive, increasing slope |  |
| :---: | :---: |
| a) |  |
| b) <br> negative, constant slope with positive value |  |
| c) |  |
| d) |  |

## 20. Graphing

A baseball is thrown upward and a record of its vertical position versus time is shown on the graph below. (Only the upward portion of the motion is shown.) $y=0$ is chosen to be the ground.
a) Describe in words how the vertical position of the ball changes with time.
b) Describe in words how the steepness of the vertical position graph changes in time.
c) What are the units of the slope of a tangent line to this graph?
d) What is the significance of the y-intercept of the graph?


## Chapter $[$

## Section $\quad .1$

## 1. Displacement

A sailboat travels west for 3.0 km , then south for 6.0 km south, then $45^{\circ}$ north of east for 7.0 km . Draw vector arrows on the grid below to illustrate the sum of the three displacements. Based on your sketch, what is the approximate magnitude and direction of the total displacement? One grid represents 1 km .


## 2. Position and Displacement

Motion diagrams for several objects in horizontal 1-D motion are shown below. Use the given scales to determine the distance traveled and the displacement for each case. Explain why or why not these numbers are different.


## Section $\square .2$

## 3. Speed and Velocity

The graph below shows the position of a train on an east-west track as a function of time. The $+x$ direction is east. [this is from Fig 3.11 in text]


Time ( $t$ )
(a) Rank the instants $t_{1}, t_{2}, t_{3}$ in order of decreasing instantaneous speed. Explain your reasoning.
(b) During which of the time intervals ( $0<t<t_{1}, t_{1}<t<t_{2}, t_{2}<t<t_{3}$, and $t<t_{3}$ ), if any, is the train's speed increasing? During which time intervals is the train's speed decreasing? Explain.
(c) During which of the time intervals, if any, is the train moving west? Explain.
4. Consider the velocity vs. time Graph shown below of a car moving along a horizontal road. The positive $x$-direction is to the right, and asume that the car starts at $x=0$ at time $t=0 \mathrm{~s}$. For the questions a) thru e) find the correct answer(s) from the list below.


a) During which segment(s) does the car speed up while moving to the right?
b) During which segement (s) does the car speed up while moving to the left?
c) During which segment(s) does the car slow down while moving to the right?
d) During which segement (s) does the car slow down while moving to the left?
e) During which segment dies the car travel at a constant velocity?.
f) At which point $(A, B, C, D, E, F$, or $G)$ is the car furthest from the origin $(x=0)$ ?

## 5. Velocity

For each of the following cases construct the missing graph(s) and give a written description of the motion.

6. Given below are two velocity-vs-time graphs. Draw the a-vs-t and x-vs-t graphs, given that at time $t=0$ the object was at $x=0$.
a)
b)



## 7. Velocity - Average Velocity and Motion Diagrams

Partial motion diagrams for objects in 1-2D motion are shown below. Draw vectors for the initial velocity, final velocity, and acceleration on each diagram.


## Section 2.3

Problems $\square$ to $1 \square$ : Representations of Motion. [each SC]
A train of mass $2 \times 10^{5} \mathrm{~kg}$ is moving on a straight track, braking to slow down as it approaches a station. The $x$-axis points south. The graph below shows the velocity $v_{x}$ as a function of time.


ロ.(a) Is the velocity north or south? Explain.
(b) Draw velocity vectors for the train at $t=0,5 \mathrm{~s}$, and 10 s .
․ (a) Is the acceleration constant? Explain.
(b) Find the magnitude of the acceleration. Is the acceleration north or south? Explain.
(c) What is the net force acting on the train?
10.(a) Is the displacement between $t=5 \mathrm{~s}$ and $\mathrm{t}=10 \mathrm{~s}$ north or south? Explain.
(b) Show on the graph how you would calculate the displacement of the train between $t=5 \mathrm{~s}$ and $\mathrm{t}=10 \mathrm{~s}$. (No need to calculate the value; just represent the quantity visually.)
(c) Make a qualitative motion diagram for the train (i.e. a series of dots showing the train's position every 5 s). It's not necessary to calculate numerical values; just illustrate qualitatively what the motion is like.

## 1. Graphical Representations of Motion

The graph below shows $v_{x}(t)$ for an airplane that is speeding up.

(a) Is the acceleration constant? Explain.
(b) Write an expression for the area of the shaded triangle above $v_{\mathrm{ix}}$ in terms of quantities specified on the graph.
(c) Write an expression for the area of the shaded rectangle above $v_{i x}$ in terms of quantities specified on the graph.
(d) Write an expression for the sum of the areas of the triangle and rectangle. What quantity does the total area represent?

## Section $\square$

## Problems CD to D : Motion Diagrams

For each of the motions below sketch a motion diagram as described in section You may use a dot to represent the object in motion. First, establish a coordinate axis for each motion. Draw arrows to scale above each dot to represent the velocity vector. Indicate the acceleration direction for each constant-acceleration segment of the motion. (SC)
7. A car is travelling at a constant speed of 35 mph when the driver sees a stop sign. The driver takes 5 seconds to start applying brakes and the car eventually comes to a stop at the stop sign.

0. In the $100-\mathrm{m}$ dash, a runner begins from rest and accelerates at constant rate until the $60-\mathrm{m}$ mark and then maintains her maximum speed until crossing the finish line.


ㄴ. You get into an elevator at the ground floor of a 50 story building and press the button for the $50^{\text {th }}$ floor. The elevator accelerates for 10 seconds, maintains the maximum speed for 60 seconds and then comes to a stop at the top floor.
$\square$

Draw position-vs-time and Velocity-vs-time graphs for the motions represented by the following motion diagrams.




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## ㄴ. Graphical Representations of Motion

You get on to an elevator on the $50^{\text {th }}$ floor and press the button for the ground floor. The elevator speeds up until it reaches the maximum speed, and then maintains this speed for most of the decent. Finally it slows down and stop on the ground floor.
a. Draw the motion diagram for the elevator.
b. Draw velocity-vs-time graph and position-vs-time graph for the elevator.

## Section $\square$ ㅁ

## १. Kinematic Equations - Constant Force

Construct a physical situation involving 1-D horizontal motion of a point mass with a constant acceleration that is consistent with the following kinematic equations:

Case A:

$$
625 \mathrm{~m}=1 / 2(500 \mathrm{~N} / 10 \mathrm{~kg})(5 \mathrm{sec})^{2}
$$

Case B:

$$
2600 \mathrm{~m}=50 \mathrm{~m}+5 \mathrm{~m} / \mathrm{s}(10 \mathrm{~s})+1 / 2(500 \mathrm{~N} / 10 \mathrm{~kg})(10 \mathrm{sec})^{2}
$$

Case C:
$(20 \mathrm{~m} / \mathrm{s})^{2}=2(1000 \mathrm{~N} / 100 \mathrm{~kg})(20 \mathrm{~m})$

## १. Kinematic Equations - Constant Force

State the kinematic equation that would be used if given the following information:

Case A:

$$
\mathrm{M}, \mathrm{~F}_{\text {net }}, \otimes \mathrm{X}, \otimes \mathrm{t}, \mathrm{v}_{0}
$$

Case B:

$$
\mathrm{M}, \mathrm{~F}_{\mathrm{net}}, \otimes \mathrm{X}, \mathrm{v}_{\mathrm{o}}, \mathrm{v}_{\mathrm{f}}
$$

Case C:
$M, F_{\text {net }}, v_{o}, v_{t}, \otimes t$

## ㅁ. Kinematics

A car travelling at a constant speed of $v_{i}$ can come to a stop in a minimum distance $d$ in time $t$, given that the car has a maximum deceleration a when brakes are fully engaged.


Now suppose the same car (with the same maximum deceleration a) is traveling at an initial speed of $2 v_{i}$, and has a minimum stopping distance $d^{\prime}$ in time $t^{\prime}$. Using kinematic equations, find
a. The ratio $\frac{t^{\prime}}{t}$
b. The ratio $\frac{d^{\prime}}{d}$

