CHAPTER 3 Motion in Two and Three Dimensions

Answers to Understanding the Concepts Questions

1. In the absence of gravity, the boy would simply aim directly at the coconut in order to make a direct hit. Since gravity pulls the rock down while it is in flight, the boy needs to aim higher than the coconut.

2. If the athlete behaved like a projectile once he or she has left the ground, then we know from our thinking about projectiles that there are two factors: the initial speed and the initial angle. The athlete approaches the take-off spot with a short sprint, just long enough so that his or her speed is at its maximum value. At this point the athlete has to translate the motion so that it has a vertical component, and some of the initial horizontal velocity is translated into vertical motion, with a net loss in speed. There is a trade-off here, and the human body is such that too much initial speed is lost if the athlete tries to take off at a 45° angle, even though this angle maximizes the range for a given initial speed. Thus a long jump champion leaves the take-off spot at an angle smaller than 45°, trading the reduced range associated with a smaller angle for a higher initial speed. A second factor that certainly plays a role, albeit a somewhat smaller one, is wind resistance. A following wind can add inches to the jump, and if the following wind is too strong a jump cannot qualify as a record. As our discussion shows, the most important contribution to the length of the jump is the ability to convert the highest possible initial speed to a vertical motion that gets as close as possible to a takeoff at 45°. It is known that good sprinters generally make good long-jumpers, but not all sprinters hold long-jump records.

3. A strong wind blowing in the direction of motion of the athlete can increase the speed of the athlete relative to the ground. Such increase, which may result in an improvement of, say, an extra inch in the long jump or 1/10 of a second shorter in the 100-m dash, provides an unfair advantage and renders the world-record meaningless.

4. If the motion is entirely linear, any nonzero acceleration leads to a changing speed. In fact any component of the acceleration parallel to the velocity will change the velocity's magnitude. Only if the acceleration is entirely perpendicular to the motion does the speed remain unchanged, although the direction of the velocity must change. We studied a version of this case in some detail: uniform circular motion.

5. No. In order to reach the point directly across from the river, your boast must have no velocity component *along* the flow of the river current. Since the river flows at 2 mi/h you must row the boat in such a way that its velocity component along the riverbank is 2 mi/h upstream, relative to the water, so as to cancel the velocity of the river flow. But since 2 mi/h is how fast you can row the boat relative to the water, that would mean that you do not have any velocity component that is perpendicular to the river bank. Your boat would just end up motionless in water, relative to the bank. To be able to cross the river you must land somewhere downstream and walk to the destination.

6. The upper trajectory is more subject to effects due to wind, both because it is higher and because the flight time is longer for that trajectory. The lower trajectory has a shorter flight time, and this may allow the pass to reach the receiver before defenders can react properly. This trajectory is superior when the receiver is between the passer and a defender. However, the higher trajectory may be better in that the ball may be beyond the reach of intermediate defenders. The upper trajectory is superior when the receiver is beyond the defender, so that this is the trajectory normally chosen for long passes.

7. The range, $R = v_0^2 \sin(2\theta_0)/g$, is proportional to v_0^2. So as v_0 doubles R increases by a factor of 4.

8. To an observer in the moving elevator, the velocity of the ball is zero at the instant it reaches the top of its path. This means that the ball and the elevator must have the same velocity relative to the ground, i.e., ball moves at the same upward velocity as the elevator at that instant, relative to the ground.

9. The released ball has the horizontal component of the car's velocity at the moment the ball is released. The ball retains that horizontal component. When the car moves with a constant velocity, it "tracks" the ball, so that the ball comes back down into the car. If the car slows down, it will fall behind the horizontal position of the ball, and the ball will land ahead of the car.

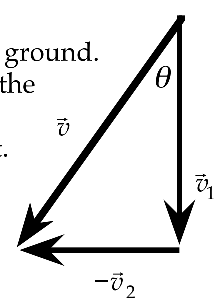
10. $a = v^2/R = (2\pi R/v)^2/R = 4\pi^2 R/T^2$. The ratio in question is then

$$\frac{a_{\text{earth}}}{a_{\text{moon}}} = \frac{4\pi^2 R_{\text{earth-sun}}/T_{\text{earth}}^{\ 2}}{4\pi^2 R_{\text{moon-earth}}/T_{\text{moon}}^{\ 2}} = \left(\frac{R_{\text{earth-sun}}}{R_{\text{moon-earth}}}\right)\left(\frac{T_{\text{moon}}}{T_{\text{earth}}}\right)^2 = 400\left(\frac{27.3\text{ days}}{365.25\text{ days}}\right)^2 = 2.23.$$

11. No. As the phonograph turns, the stylus picks up the sound signal by making contact with the grooves, which form spirals. As the stylus rides along the spiraling grooves its distance to the center of the record continuously changes, so essentially the motion of the contact point between the record and the stylus is a near-circular one with a slowly varying radius (r). Even though the time T it takes for the record to complete each spin is the same, the liner speed v at which the contact point moves, $v \approx 2\pi r/T$, is not a constant, as r changes.

12. There are an infinite number of velocities that give the same maximum height. The relation $h = v_0^2 \sin^2(\theta_0)/2g$ can be satisfied for a whole range of values of v_0 provided the angle is adjusted properly in the range 0° to 90°. The same argument applies to the time of flight.

13. Suppose that the rain falls at a speed v_1 and you run at a speed v_2, both relative to the ground. Then the velocity $\vec{v}$ of the rain with respect to you, $\vec{v} = \vec{v}_1 - \vec{v}_2$, is at an angle θ from the vertical, such that $\tan\theta = v_2/v_1$. For the most protection, then, you need to tilt your umbrella forward at that angle θ relative to the ground. See the diagram to the right.



14. Let's say you toss the ball at the moment that you are moving due west. You throw it straight up in your own frame, so the ball has a component of horizontal motion to the west equal to your own at the moment you toss it. The ball continues to move due west in a frame of reference fixed to Earth, and this motion takes it obliquely away from the center of the merry-go-round. However, you are moving in a circle, and so change your direction continuously. To you, the ball will appear to move in part away from the center of the merry-go-round and in part to fall behind you. This makes for a rather complicated curving motion that you describe as accelerating. To an observer fixed to Earth it is evident that this "acceleration" is just a manifestation of the fact that you are in an accelerating frame.

15. As the ball leaves your hand it has both an upward initial velocity and a horizontal initial velocity (measured relative to the ground) that is equal to the tangential velocity of the merry-go-round where you stand. If you throw the ball very high it is going to stay in the air for an extended period of time, and with its horizontal velocity it will land outside the edge of the merry-go-round. If you throw the ball just above your head it will land quickly, and during its short flight your tangential velocity has not changed by much, meaning that both you and the ball have about the same horizontal velocity — so the ball will land on yourself, or just next to you.

16. The projectile is fired from the North Pole so it has no initial motion due to Earth's rotation. As it is airborne, Earth turns from east to west, so to a person on Earth the projectile is moving both southward and westward. In general, the projectile will not land on the particular spot on the equator, unless it so happens that its time of flight is a multiple of 24 hours, as Earth turns around every 24 hours.

17. In Question 16, the projectile is fired from the North Pole and so has no initial motion associated with Earth's rotation. Here there is an initial motion that is provided by the motion of a point in Washington, D.C.

18. No. The platform is turning from the perspective of a bystander outside the platform, and to that bystander the motorcycle is not moving in a circle. Rather, it is moving outward from the center of the platform while at the same time turning together with the platform. So it appears to be moving in a spiral trajectory.

19. (a) This evidence supports the claim. While the two bullets have different horizontal velocities their vertical motions are still identical (as they are dropped from the same height with the same zero initial vertical velocity), so they hit the ground at the same time.
(b) and (c) These two statements tell us how the angle of launch affects the subsequent motion of the projectile. But since the value of this angle affects the horizontal and vertical motions simultaneously, neither statement offer any direct support for the independence of the motions in these two directions.

20. True. As a vector quantity the velocity is specified by both its magnitude and its direction. Any change in either its magnitude or direction constitutes a change in the velocity, and that requires an acceleration. For example, as you step on the accelerator of your car, which is moving in a straight line, you are changing the magnitude of its velocity. As your car make a turn in a circular path while its speed remains unchanged, only the direction of motion of the car is changing but not its speed. In either case, the car is undergoing acceleration.

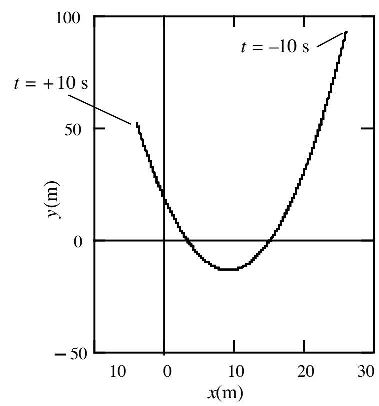
21. No. An ideal projectile with a parabolic path requires that the acceleration be uniform (usually equal to $\vec{g}$, downward). In the case of a missile traveling over a long range, the curvature of the earth as well as the altitude of the missile at various segments of its flight would have to be considered and **g** is no longer uniform over the entire range of the flight. Also, the high speed of the missile causes a significant air resistance, which is absent in an ideal projectile motion. And, most importantly, the missile is powered by its own rocket engine that provides it with its own acceleration, which can be variable and totally different from $\vec{g}$.

22. As the platform turns, you are turning with it together, so if you simply aim at the center of the platform (where the center of the table is) the ball would veer off its intended course — if you are moving to the right relative to the ground due to the rotation, for example, the ball would fly to the right relative to the center of the platform (which is not moving relative to the ground). So you need to compensate for this by projecting the ball not only inward (i.e., toward the center of the platform), but also sidewise, in the opposite direction of the rotation of the platform.

Solutions to Problems

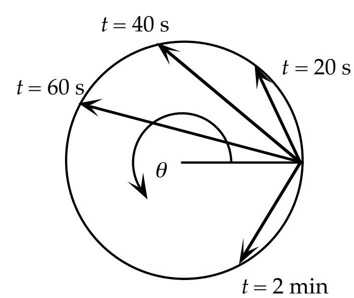
1. The position of the first turn is $\vec{r}_1 = [(21\cos 45°)\hat{i} + (21\sin 45°)\hat{j}]$ km $= \boxed{(15\hat{i} + 15\hat{j})\text{ km}}$.
The position of the second turn is $\vec{r}_2 = \vec{r}_1 + 15\hat{i}$ km $= \boxed{(30\hat{i} + 15\hat{j})\text{ km}}$.
The final position is $\vec{r} = \vec{r}_2 + 28\hat{j}$ km $= (30\hat{i} + 43\hat{j})$ km.
The magnitude of the total displacement is $r = [(30\text{ km})^2 + (43\text{ km})^2]^{1/2} = 52$ km;
the angle is found from $\tan\theta = 43\text{ km}/30\text{ km}$; thus $\theta = 55°$, or $\vec{r} = \boxed{52\text{ km, }55°\text{ N of E}}$.

2. Given $\vec{r} = (c_1 - c_2 t)\,\hat{i} + (d_1 + d_2 t + d_3 t^2)\,\hat{j}$
$= [11\text{ m} - (1.5\text{m/s})t]\,\hat{i} +$
$[-12\text{ m} + (-2.0\text{ m/s})t + (0.85\text{ m/s}^2)t^2]\,\hat{j}$.
To find the times to pass through $x = 0$, we solve
$x = 0 = 11\text{ m} - (1.5\text{ m/s})t$ to get $t = 7.3$ s.
Setting $x = y$ gives us
$11\text{ m} - (1.5\text{ m/s})t = (-12\text{ m}) - (2.0\text{ m/s})t + (0.85\text{ m/s}^2)t^2$,
from which we get $t_1 = \boxed{5.5\text{ s}}$ and $t_2 = \boxed{4.9\text{ s}}$.
To find the locations, we substitute these values into the expression for $\vec{r}$:
$t = 5.5$ s: $\vec{r} = \boxed{(2.8\hat{i} + 2.7\hat{j})\text{ m}}$; $t = -4.9$ s: $\vec{r} = \boxed{(18\hat{i} + 18\hat{j})\text{ m}}$.

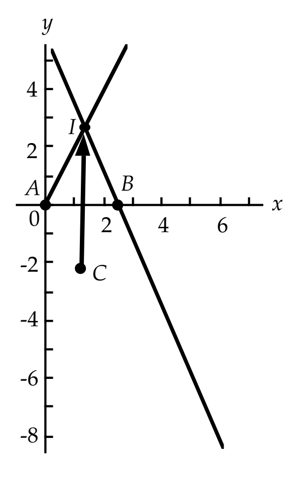


3. $\vec{r}_A = \boxed{0}$; $\vec{r}_B = \boxed{(25\text{ m})\hat{i}}$; $\vec{r}_C = \boxed{(25\hat{i} + 35\hat{j})\text{ m}}$; $\vec{r}_D = \boxed{(35\text{ m})\,\hat{j}}$.

4. The speed is $v = (27.0\text{ km/h})(10^3\text{ m/km})(1\text{ h}/3600\text{ s})$
$= 7.50\text{ m/s}$ (direction CCW)
The circumference is $2\pi R = 2\pi(172\text{ m}) = 1080$ m.
The time for a complete lap is
$T = 2\pi R/v = 1080\text{ m}/(7.50\text{ m/s}) = 144$ s.
To find the angle from the initial position:
$\theta = (360°/T)t = (360°/144\text{ s})t$.
Thus $\theta_{20} = 50°$; $\theta_{40} = 100°$; $\theta_{60} = 150°$; $\theta_{120} = 300°$.

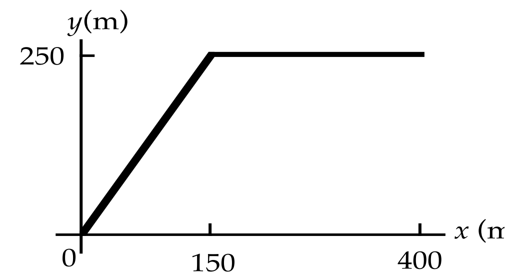


5. For the first line:
$y_1 = (4.0\text{ km}/2.0\text{ km})x_1 + 0 = 2.0\,x_1$;
for the second line:
$y_2 = [(-8\text{ km} - 0)/(6.0\text{ km} - 2.5\text{ km})]x_2 +$
$[(-8\text{ km} - 0)/(6.0\text{ km} - 2.5\text{ km})](0 - 2.5\text{ km})$
$= -2.3\,x_2 + 5.7$ km.
The intersection of the two lines occurs when $x_1 = x_2 = x_I$ and $y_1 = y_2 = y_I$:
$2.0\,x_I = -2.3x_I + 5.7$ km,
which gives $x_I = 1.3$ km and $y_I = 2.7$ km or
$\vec{r}_I = \boxed{(1.3\hat{i} + 2.7\hat{j})\text{ km}}$.
Given that $\vec{r}_C = (1.2\text{ km})\hat{i} - (2.2\text{ km})\hat{j}$, then
$\vec{r}_I - \vec{r}_C = \boxed{(0.1\hat{i} + 4.9\hat{j})\text{ km}}$.



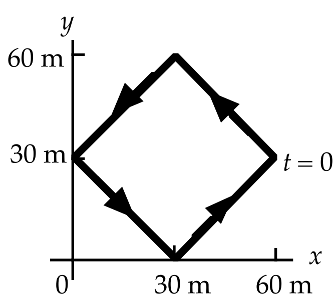
6. Given $\vec{r} = [(-6 + 4t^2)\hat{i} + (-4 + 3t)\hat{j}\,]$ m, the particle will cross the y-axis when
   $x = -6 + 4t^2 = 0$, which gives $t = \boxed{1.2\text{ s}}$.
   The particle will cross the x-axis when $y = -4 + 3t = 0$, which gives $t = \boxed{1.3\text{ s}}$.
   To relate y to x, we must eliminate t between the two components: $x = 5 - 2t^2$, $y = -4 + 3t$.
   Thus $t = (y + 4)/3$ and $x = -6 + 4[(y + 4)/3]^2$, so $\boxed{9(x + 6) + 4(y + 4)^2 = 0}$.
   To have the particle at the origin at $t = 0$, $\boxed{\text{move the } x\text{-axis 6 m to the left and the } y\text{-axis 4 m up}}$,
   then $\vec{r} = (4t^2)\hat{i} + 3t\hat{j}$.

$\boxed{7}$. The speed is $v = (25.2\ \text{km/h})(10^3\ \text{m/km})(1\ \text{h}/3600\ \text{s}) = 7.00\ \text{m/s}$.
   We find the orientation of the first segment from
   $\tan\theta = y_1/x_1 = (250\ \text{m})/(150\ \text{m}) = 1.67$; $\theta = 59°$.
   For the first segment, the velocity components are
   $v_{x1} = v\cos\theta = (7.00\ \text{m/s})\cos 59° = 3.60\ \text{m/s}$.
   $v_{y1} = v\sin\theta = (7.00\ \text{m/s})\sin 59° = 6.00\ \text{m/s}$.
   The time when the turning point is reached is
   $t_1 = x_1/v_{x1} = (150\ \text{m})/(3.60\ \text{m/s}) = 41.6\ \text{s}$.
   For the second segment, the velocity components are
   $v_{x2} = v = 7.00\ \text{m/s}$; $v_{y2} = 0$.
   The elapsed time for the second segment is
   $\Delta t = \Delta x/v_{x2} = (400\ \text{m} - 150\ \text{m})/(7.00\ \text{m/s}) = 35.7\ \text{s}$, so $t_2 =$
   $41.6\ \text{s} + 35.7\ \text{s} = 77.3\ \text{s}$.
   The position vector is



$$\vec{r} = \boxed{(3.60\ \text{m/s})t\hat{i} + (6.00\ \text{m/s})t\hat{j},\quad 0 \le t \le 41.6\ \text{s}};$$
$$\vec{r} = [150\ \text{m} + (7.00\ \text{m/s})(t - 41.6\ \text{s})]\hat{i} + (250\ \text{m})\hat{j}$$
$$= \boxed{[-141\text{m} + (7.00\ \text{m/s})t]\hat{i} + (250\ \text{m})\hat{j},\quad 41.6\ \text{s} \le t \le 77.3\ \text{s}}.$$

8. (a)



   (b) The lot is a square with sides $L = 30\sqrt{2}$ m. The area is $A = L^2 = (30\sqrt{2}\ \text{m})^2 = \boxed{1800\ \text{m}^2}$.

9. Given $\vec{r} = (4\ \text{m})\cos(\pi t/T)\hat{i} - (4\ \text{m})\sin(\pi t/T)\hat{j}$;
   $\vec{r}_{T/3} = (4\ \text{m})\cos(\pi T/3T)\hat{i} - (4\ \text{m})\sin(\pi T/3T)\hat{j} = \boxed{(2.0\hat{i} - 3.5\hat{j})\ \text{m}}$;
   $d = [(2.0\ \text{m})^2 + (3.5\ \text{m})^2]^{1/2} = \boxed{4.0\ \text{m}}$.
   $\vec{r}_{T/2} = \boxed{-(4.0\ \text{m})\hat{j}}$; $d = \boxed{4.0\ \text{m}}$.
   $\vec{r}_{2T} = \boxed{(4.0\ \text{m})\hat{i}}$; $d = \boxed{4.0\ \text{m}}$.
   The angle is found from $\tan\theta = y/x = -4\sin(\pi t/T)/[4\cos(\pi t/T)] = -\tan(\pi t/T)$, so $\theta(t) = \boxed{-\pi t/T}$.
   The particle is traveling CW around a circle.

10. Given $\vec{r}_c = (h_0 - \frac{1}{2}gt^2)\,\hat{j}$ and $\vec{r}_p = (-L + ut)\hat{i} + (h_0\, ut/L - \frac{1}{2}gt^2)\,\hat{j}$, the collision must occur if $\vec{r}_p = \vec{r}_c$ at some time. Because $x_c = 0$ always, the time when the projectile has $x_p = 0$ is $t = L/u$. At this time the y positions are

$y_c = h_0 - \frac{1}{2}g(L/u)^2$ and $y_p = h_0[u(L/u)/L] - \frac{1}{2}g(L/u)^2 = h_0 - \frac{1}{2}g(L/u)^2 = y_c$. A collision occurs.

$\vec{r}_{cp} = \vec{r}_c - \vec{r}_p = (L - ut)\hat{i} + [h_0(1 - ut/L)]\,\hat{j} = \boxed{(L - ut)[\hat{i} + (h_0/L)\hat{j}]}$, which is 0 when $t = L/u$.

11. We assume the velocities are horizontal and in the same direction. From the definition of acceleration, we have

$a_{\text{av}} = |\Delta\vec{v}\,|/\Delta t = |\,37\text{ m/s} - 45\text{ m/s}\,|/2.0\text{ s} = \boxed{4.0\text{ m/s}^2}$.

The direction of the acceleration will be opposite to the direction of the velocities.

12. We can combine $x(t)$ and $y(t)$ as $\vec{r} = [(1.5\text{ m/s})t + (-0.5\text{ m/s}^2)\,t^2]\hat{i} + [6\text{ m} - (3\text{ m/s})t + (1.5\text{ m/s}^2)\,t^2]\,\hat{j}$.

The velocity is the time rate of change of $\vec{r}$:

$\vec{v} = \text{d}\vec{r}/\text{d}t = [1.5\text{ m/s} - (1.0\text{ m/s}^2)t]\,\hat{i} + [-3\text{ m/s} + (3.0\text{ m/s}^2)t]\,\hat{j}$.

The acceleration is the time rate of change of $\vec{v}$: $\vec{a} = \text{d}\vec{v}/\text{d}t = (-1.0\text{ m/s}^2)\,\hat{i} + (3.0\text{ m/s}^2)\,\hat{j}$.

For the velocity components to be equal,

$v_x = v_y$, or $1.5\text{ m/s} - (1.0\text{ m/s}^2)t = -3\text{ m/s} + (3\text{ m/s}^2)t$, which gives $t = \boxed{1.1\text{ s}}$.

13. From the definition of acceleration, we have

$\vec{a} = \text{d}\vec{v}/\text{d}t = \text{d}\{(2.2\text{ m/s})\,\hat{i} + (3.7\text{ m/s}^2)t\,\hat{j} + [3.3\text{ m/s} - (1.2\text{ m/s}^3)t^2]\hat{k}\,\}/\text{d}t$

$= \boxed{(3.7\text{ m/s}^2)\hat{j} - (2.4\text{ m/s}^2)t\hat{k}}$.

14. From the definition of velocity, we have

$v_x = \text{d}x/\text{d}t = \text{d}[A\cos(\omega t)]/\text{d}t = \boxed{-A\omega\,\sin(\omega t)}$;

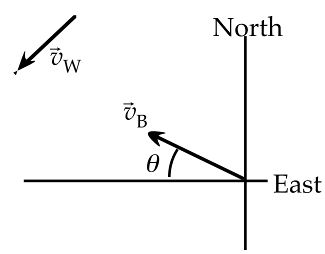
$v_y = \text{d}y/\text{d}t = \text{d}[A\sin(\omega t)]/\text{d}t = \boxed{+A\omega\,\cos(\omega t)}$.

From the definition of acceleration, we have

$a_x = \text{d}v_x/\text{d}t = \text{d}[-A\omega\sin(\omega t)]/\text{d}t = \boxed{-A\omega^2\,\cos(\omega t)}$;

$a_y = \text{d}v_y/\text{d}t = \text{d}[+A\omega\cos(\omega t)]/\text{d}t = \boxed{-A\omega^2\,\sin(\omega t)}$.

15. With the origin at Malibu and the x-axis east and the y-axis north, the position of the whale is



$\vec{r}_W = [-(5.0\text{ km})\cos 45° - (7.0\text{ km/h})(\cos 45°)t]\,\hat{i}\; +$

$[(5.0\text{ km})\sin 45° - (7.0\text{ km/h})(\sin 45°)t]\,\hat{j}$

$= [-(3.5\text{ km}) - (4.9\text{ km/h})t]\,\hat{i} + [(3.5\text{ km}) - (4.9\text{ km/h})t]\,\hat{j}$

and the position of the boat is

$\vec{r}_B = -[(30\text{ km/h})(\cos\theta)t]\,\hat{i} + [(30\text{ km/h})(\sin\theta)t]\,\hat{j}$,

where θ is the angle north of west that the boat is heading.

At the interception, $\vec{r}_W = \vec{r}_B$, or

$[-3.5\text{ km} - (4.9\text{ km/h})t]\,\hat{i} + [(3.5\text{ km}) - (4.9\text{ km/h})t]\,\hat{j} = -[(30\text{ km/h})(\cos\theta)t]\,\hat{i} + [(30\text{ km/h})(\sin\theta)t]\,\hat{j}$.

From this we get two equations relating θ and t:

$-3.5\text{ km} - (4.9\text{ km/h})t = -(30\text{ km/h})(\cos\theta)t$ and $3.5\text{ km} - (4.9\text{ km/h})t = (30\text{ km/h})(\sin\theta)t$.

When these are solved, we get $\theta = 31.6°$; $t = 0.17\text{ h} = 10\text{ min}$.

The velocity of the boat is $\vec{v}_B = (-25.5\hat{i} + 15.7\hat{j})\text{ km/h} = \boxed{30\text{ km/h},\ \ 31.6°\text{ north of west}}$,

and the interception position is $\vec{r} = \boxed{(-4.3\hat{i} + 2.7\hat{j})\text{ km}}$.

16. If we let x be the horizontal distance of the swimmer from tower of height h (= 3 m), then the length s of the rope from the swimmer to the lifeguard satisfies $s^2 = x^2 + h^2$. Take the time derivative of both sides:

$2s\,\mathrm{d}s/\mathrm{d}t = 2x\,\mathrm{d}x/\mathrm{d}t + 2h\,\mathrm{d}h/\mathrm{d}t = 2x\,\mathrm{d}x/\mathrm{d}t$, as h does not change. But $v_{\text{rope}} = \mathrm{d}s/\mathrm{d}t$ is the speed of the rope being pulled and $v = \mathrm{d}x/\mathrm{d}t$ is the speed of the swimmer, so

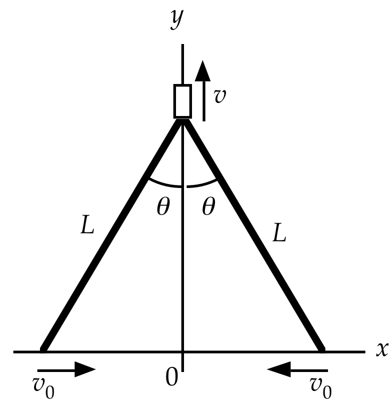
$$v = (s/x)\,v_{\text{rope}} = v_{\text{rope}}\,(x^2 + h^2)^{1/2}/x = (1\text{ m/s})\,(x^2 + h^2)^{1/2}/x.$$

Therefore:

(a) when $x = 4$ m, $v = \boxed{1.25\text{ m/s}}$;

(b) when $x = 3$ m, $v = \boxed{1.41\text{ m/s}}$.

17. We choose the coordinate system shown in the diagram, with the origin directly below the joint midway between the tips. From the symmetry of the motion, we need to consider only the motion of the right hand tip.



(a) The position of the tip is

$$x = x_0 - v_0 t = L\sin\theta - v_0 t = 0.5L - v_0 t\ .$$

Because the length of the arm is constant, the location of the joint is

$$y = (L^2 - x^2)^{1/2} = [L^2 - (0.5L - v_0 t)^2]^{1/2}.$$

For the joint $x = 0$, so its velocity is

$$\vec{v} = (\mathrm{d}y/\mathrm{d}t)\,\hat{j} = \tfrac{1}{2}\,(0.5L - v_0 t)2v_0\,\hat{j}\ /[L^2 - (0.5L - v_0 t)^2]^{1/2}$$

$$= [0.5(0.015\text{ m}) - (0.030\text{ m/s})t](0.03\text{ m/s})\ \hat{j}\ /\{(0.015\text{ m})^2 - [0.5(0.015\text{ m}) - (0.030\text{ m/s})t]^2\}^{1/2}$$

$$= \boxed{[(0.000225\text{ m/s}^2) - (0.00090\text{ m}^2/\text{s}^2)t]\hat{j}\,/[(0.0169\text{ m}^2) + (0.00045\text{ m}^2/\text{s})t - (0.00090\text{ m}^2/\text{s}^2)t^2]^{1/2}}.$$

(b) When $t = 0$, $x = 0.5L$, so we have

$$\vec{v} = [(0.00225\text{ m}^2/\text{s}) - 0]\,\hat{j}\ /[(0.0169\text{ m}^2) - 0 - 0]^{1/2} = \boxed{(0.017\text{ m/s})\,\hat{j}}.$$

When $x = 0$, $0.5L - v_0 t = 0$, so we have

$$\vec{v} = \boxed{0}.$$

Note that, if the tips could keep moving, this is where the motion of the joint would reverse.

18. By differentiating the position vectors $\vec{r}_c = (h_0 - \tfrac{1}{2}gt^2)\,\hat{j}$ and $\vec{r}_p = (-L + ut)\,\hat{i} + (h_0 ut/L - \tfrac{1}{2}gt^2)\,\hat{j}$ with respect to time, we get

$$\vec{v}_c = -gt\,\hat{j}\ ;\ \ \vec{v}_p = u\hat{i} + (h_0 u/L - gt)\,\hat{j}\,.$$

The relative velocity is $\vec{v}_p - \vec{v}_c = \boxed{u\hat{i} + (h_0 u/L)\hat{j}}$ with a magnitude of $|\,\vec{v}_p - \vec{v}_c\,| = \boxed{u[1 + (h_0/L)^2]^{1/2}}$.

The angle with respect to the x-axis is found from $\tan\theta = v_y/v_x = (h_0 u/L)/u = h_0/L$.

The angle $\theta = \boxed{\tan^{-1}(h_0/L)}$ is constant and is the original aim of the projectile.

19. By differentiating the position vector $\vec{r} = [4\cos(\pi t/T)\,\hat{i} - 4\sin(\pi t/T)\,\hat{j}\ ]$ m with respect to time, we get

$$\vec{v} = \boxed{[(4\pi/T)\sin(\pi t/T)\hat{i} - (4\pi/T)\cos(\pi t/T)\hat{j}]\text{ m/s}}\,.$$

The angle with respect to the x-axis is found from

$\tan\phi = v_y/v_x = [-(4\pi/T)\cos(\pi t/T)]/[-(4\pi/T)\sin(\pi t/T)] = \cot(\pi t/T) = \tan(\pi/2 - \pi t/T)$, so

$\phi = \boxed{\pi/2 - \pi t/T}$ and $\phi - \theta = \pi/2 - \pi t/T - (-\pi t/T) = \pi/2$; $\vec{r}$ and $\vec{v}$ are perpendicular.

20. We are given $\vec{v}_0 = (30\text{ m/s})\hat{j}$ and $\vec{a} = [3.5\text{ m/s}^2 + (0.7\text{ m/s}^5)t^3]\hat{i} + [2\text{ m/s}^2 - (0.3\text{ m/s}^4)t^2]\hat{j}$. To find the velocity we must integrate: $\int_{\vec{v}_0}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}\,dt$. This gives

$$\vec{v} - \vec{v}_0 = [(3.5\text{ m/s}^2)t + \tfrac{1}{4}(0.7\text{ m/s}^5)t^4]\hat{i} + [(2\text{ m/s}^2)t - \tfrac{1}{3}(0.3\text{ m/s}^4)t^3]\hat{j}\ ,\ \text{or}$$
$$\vec{v} = [(3.5\text{ m/s}^2)t + (0.18\text{ m/s}^5)t^4]\hat{i} + [30\text{ m/s} + (2\text{ m/s}^2)t - (0.1\text{ m/s}^4)t^3]\hat{j}.$$

To find the position we must integrate: $\int_0^{\vec{r}} d\vec{r} = \int_0^t \vec{v}\,dt$. This gives

$$\vec{r} = [(1.8\text{ m/s}^2)t^2 + (0.036\text{ m/s}^5)t^5]\hat{i} + [(30\text{ m/s})t + (1.0\text{ m/s}^2)t^2 - (0.025\text{ m/s}^4)t^4]\hat{j}.$$

At $t = 30$ s,

$$\vec{r} = \boxed{(8.8\times10^5\text{ m/s})\hat{i} - (1.8\times10^{-4}\text{ m/s})\hat{j}};$$
$$\vec{v} = \boxed{(1.5\times10^5\text{ m/s})\hat{i} - (2.6\times10^3\text{ m/s})\hat{j}}.$$

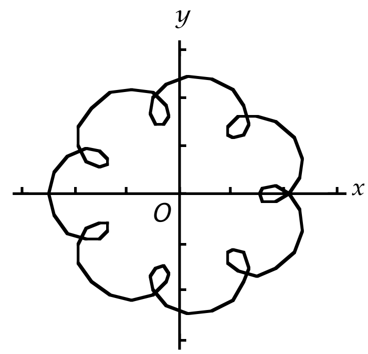
21. Because $\vec{v} = d\vec{r}/dt$, by differentiating the given position vectors,

$\vec{r}_1 = R\cos(2\pi ft)\hat{i} + R\sin(2\pi ft)\hat{j}$ and

$\vec{r}_2 = 4R\cos(\pi ft/4)\hat{i} + 4R\sin(\pi ft/4)\hat{j}$, we get

$\vec{v}_1 = -R(2\pi f)\sin(2\pi ft)\hat{i} + R(2\pi f)\cos(2\pi ft)\hat{j}$ and

$\vec{v}_2 = -4R(\pi f/4)\sin(\pi ft/4)\hat{i} + 4R(\pi f/4)\cos(\pi ft/4)\hat{j}$.



From $|\vec{r}_1| = [R^2\cos^2(2\pi ft) + R^2\sin^2(2\pi ft)]^{1/2} = R$, the first planet is always on the circle of radius R.

Similarly $|\vec{r}_2| = [(4R)^2\cos^2(\pi ft/4) + (4R)^2\sin^2(\pi ft/4)]^{1/2} = 4R$, the second planet is always on the circle of radius $4R$.

We get the speed from

$|\vec{v}_1| = R(2\pi f)[\sin^2(2\pi ft) + \cos^2(2\pi ft)]^{1/2} = R(2\pi f)$ = a constant.

Similarly $|\vec{v}_2| = 4R(\pi f/4)[\sin^2(\pi ft/4) + \cos^2(\pi ft/4)]^{1/2}$

$= R\pi f$ = a constant.

The relative position is $\vec{r}_2 - \vec{r}_1 = \boxed{[4R\cos(\pi ft/4) - R\cos(2\pi ft)]\hat{i} + [4R\sin(\pi ft/4) - R\sin(2\pi ft)]\hat{j}}$.

22. From the solution to Problem 21, $\vec{v}_1 = -R(2\pi f)\sin(2\pi ft)\hat{i} + R(2\pi f)\cos(2\pi ft)\hat{j}$.

We differentiate with respect to time to get

$$\vec{a}_1 = \boxed{-4R(\pi f)^2[\cos(2\pi ft)\hat{i} + \sin(2\pi ft)\hat{j}] = -4\pi^2 f^2\vec{r}_1}.$$

From $\vec{v}_2 = -R(\pi f)\sin(\pi ft/4)\hat{i} + R(\pi f)\cos(\pi ft/4)\hat{j}$ we differentiate to get

$$\vec{a}_2 = \boxed{-R[(\pi f)^2/4][\cos(\pi ft/4)\hat{i} + \sin(\pi ft/4)\hat{j}] = -(\pi^2 f^2/16)\vec{r}_2}.$$
$$\vec{a}_2 - \vec{a}_1 = \boxed{-R(\pi f)^2\{[\cos(\pi ft/4)/4 - 4\cos(2\pi ft)]\hat{i} + [\sin(\pi ft/4)/4 - 4\sin(2\pi ft)]\hat{j}\}}.$$

$\boxed{23}$. We are given $h = H - ut - (u/B)e^{-Bt}$.

Because the exponent $-Bt$ must be dimensionless, $[B] = [t]^{-1} = \boxed{[T^{-1}]}$.

We get the velocity from $v = dh/dt = -u - (u/B)e^{-Bt}(-B) = u[-1 + e^{-Bt}]$.

At $t = 0$, $v = v_0 = \boxed{0}$; as $t \to \infty$, $v \to \boxed{-u}$.

We get the acceleration from $a = dv/dt = u[e^{-Bt}(-B)] = -Bu\,e^{-Bt}$.

At $t = 0$, $a = a_0 = \boxed{-Bu}$; as $t \to \infty$, $a \to \boxed{0}$.

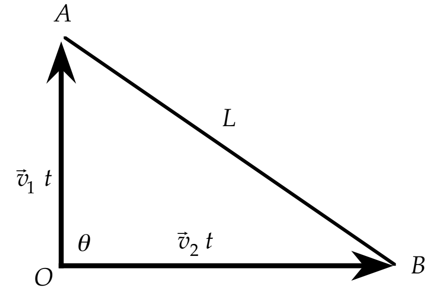
24. The two vehicles start at point O. After a time t they reach points A and B, respectively, with A and B separated by a distance L. In the triangle OAB the cosine theorem gives

$$L^2 = (v_1\, t)^2 + (v_2\, t)^2 - 2(v_1\, t)\,(v_2\, t)\cos\theta,$$

where $L = 50$ km, $v_1 = 30$ km/h, $v_2 = 40$ km/h, and $t = 1$ h. Solve for cos θ to yield

$$\cos\theta = [\,(v_1\, t)^2 + (v_2\, t)^2 - L^2]/(2v_1 v_2\, t^2) = 0, \text{ so}$$

$$\theta = \boxed{90°}.$$



25. We find the launch velocity from

$$v_L = v_0 + a_L t = 0 + (24\text{ m/s}^2)(0.5\text{ s}) = \boxed{12\text{ m/s}\quad\text{(horizontal)}}.$$

At the highest point, the velocity will be zero:

$$v^2 = v_L^2 + 2ah;$$

$$0 = (12\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)h, \quad \text{which gives}$$

$$h = \boxed{7.4\text{ m}}.$$

26. If we select a coordinate system with the origin at the hospital, x south and y up, the given data are

$$\vec{v}_0 = 600\hat{i}\text{ km/h} = [(600\text{ km/h})(10^3\text{ m/km})/(3.6\times10^3\text{ s/h})]\hat{i} = (166.7\text{ m/s})\hat{i}\,;\ \vec{a} = -(4.00\text{ m/s}^2)\,\hat{j}\,;\ t = 15\text{ s}.$$

(a) After 15 s, the displacement of the airplane is

$$x = v_0 t = (166.7\text{ m/s})(15\text{ s}) = 2500\text{ m} \quad\text{and}$$

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = 0 + \tfrac{1}{2}(-4.00\text{ m/s}^2)(15\text{ s})^2 = -450\text{ m}.$$

$$\Delta\vec{r} = \boxed{(2500\hat{i} - 450\hat{j})\text{ m}} = 2540\text{ m, } 10.2°\text{ below the horizontal.}$$

(b) The velocity of the airplane is $v_x = 166.7$ m/s; $v_y = v_{0y} + a_y t = 0 + (-4.00\text{ m/s}^2)(15\text{ s}) = -60$ m/s.

$$\vec{v} = \boxed{(167\hat{i} - 60\hat{j})\text{ m/s}} = 177\text{ m/s, } 19.8°\text{ below the horizontal.}$$

(c) Because $y_0 = 7.50\times10^3$ m, the airplane position with respect to the hospital is

$$\vec{r} = \boxed{(2500\hat{i} + 7.05\times10^3\,\hat{j})\text{ m}}.$$

27. For the first rock, $y_1 = y_{01} + v_{01}t_1 + \tfrac{1}{2}a_1 t_1^2 = 0 + (21\text{ m/s})t_1 + \tfrac{1}{2}(-9.8\text{ m/s}^2)t_1^2$.
For the second rock, $y_2 = y_{02} + v_{02}t_2 + \tfrac{1}{2}a_2 t_2^2 = 0 + (21\text{ m/s})(t_1 - 3\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_1 - 3\text{ s})^2$.

(a) The rocks will meet when $y_1 = y_2$;

$$21t_1 - 4.9t_1^2 = 21t_1 - 63 - 4.9t_1^2 + 29.4t_1 - 44.2, \text{ which gives } t_1 = \boxed{3.64\text{ s}}.$$

(b) The height at which they meet is $y_1 = (21\text{ m/s})(3.64\text{ s}) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(3.64\text{ s})^2 = \boxed{11.5\text{ m}} = y_2$.

(c) The velocities are $v_1 = v_{01} + at_1 = 21\text{ m/s} + (-9.8\text{ m/s}^2)(3.64\text{ s}) = \boxed{-14.7\text{ m/s}\ \ \text{(down)}}$ and

$$v_2 = v_{02} + a(t_1 - 3\text{ s}) = 21\text{ m/s} + (-9.8\text{ m/s}^2)(3.64\text{ s} - 3.00\text{ s}) = \boxed{+14.7\text{ m/s}\quad\text{(up)}}.$$

28. We choose a coordinate system with the origin at the floor directly below the initial position of the gymnast, with x horizontal and y vertical. The horizontal motion will have constant velocity:

$$x = x_0 + v_{0x}t = 0 + (3.0\text{ m/s})t\,; \qquad x = (3.0\text{ m/s})t\,.$$

The vertical motion is

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = 2.3\text{ m} + (7.8\text{ m/s})t + \tfrac{1}{2}(-9.8\text{ m/s}^2)t^2.$$

At the highest point, the vertical velocity will be zero:

$$v_y^2 = v_{0y}^2 + 2a_y(h - y_0);$$

$$0 = (7.8\text{m/s})^2 + 2(-9.8\text{ m/s}^2)(h - 2.3\text{ m}), \quad \text{which gives } h = \boxed{5.4\text{ m above the floor}}.$$

29. (a) v = 65 mph = (65 mi/h)(5280 ft/mi)(1 h/3600 s) = 95.3 ft/s. The time t it takes to cover a distance of 41 ft with this speed is t = 41 ft/(95.3 ft/s) = $\boxed{0.43 \text{ s}}$.
   (b) $x = \frac{1}{2}at^2$; $a = 2x/t^2$ = 2(6 in)(1 ft/ 12 in.)/(0.43 s)^2 = $\boxed{5.4 \text{ ft/s}^2, \text{ to the left}}$.

30. (a) Because the ball is released on the ship, the man will see the ball fall vertically and land at the foot of the mast. From the shore, the initial horizontal speed of the cannonball is the same as the speed of the ship, $v_{0x} = v_0$ = (8 km/h)(10^3 m/km)/(3600 s/h) = 2.22 m/s;
   both the ship and the cannonball will travel the same horizontal distance and the cannonball will land at the $\boxed{\text{foot of the mast}}$.
   (b) For the vertical motion, $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$:
   $0 = 6.5 \text{ m} + 0 + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$, which gives $t = \boxed{1.2 \text{ s}}$.
   (c) With respect to the shore the horizontal movement is $x = v_{0x}t$ = (2.22 m/s)(1.15 s) = $\boxed{2.6 \text{ m}}$.
   The vertical fall is $\boxed{6.5 \text{ m}}$.

31. We choose a coordinate system with the origin at the takeoff point, with x horizontal and y vertical. The horizontal motion will have constant velocity. We find the time required for the jump from
   $x = x_0 + v_{0x}t$;
   9.5 m = 0 + (9.0 m/s)t, which gives t = 1.06 s.
   Because this is the time to return to the ground, the vertical motion is
   $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$;
   $0 = 0 + v_{0y}(1.06 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(1.06 \text{ s})^2$, which gives $v_{0y} = \boxed{5.2 \text{ m/s}}$.

32. We will use a coordinate system with the origin at the base of the building, x horizontal and y up.
   (a) The time of fall can be found from the vertical motion: $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$. From its release at y_0 = 10 m, the ball hits the ground when y = 0:
   $0 = 10 \text{ m} + 0 + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$, which gives t = 1.43 s.
   For the horizontal motion, $x = v_{0x}t$: 8.0 m = v_{0x}(1.43 s), which gives $v_{0x} = v_0 = \boxed{5.6 \text{ m/s}}$.
   (b) If the ball is thrown up at an angle, the initial velocity terms will change.
   Thus $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$ becomes $0 = 10 \text{ m} + v_0 \sin 29° \, t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$ and
   $x = v_{0x}t$ becomes $8.0 \text{ m} = v_0 (\cos 29°)t$.
   Solving these equations simultaneously, we get $v_0 = \boxed{5.4 \text{ m/s}}$ and
   (c) $t = \boxed{1.7 \text{ s}}$.

$\boxed{33}$. We choose a coordinate system with the origin at the release point, with x horizontal and y vertical. The horizontal motion will have constant velocity:
   $v_{0x} = v_0 \cos\theta$ = (225 m/s) cos 34° = 187 m/s.
   At the highest point, the vertical velocity will be zero, so the speed is
   $v = v_{0x} = \boxed{187 \text{ m/s}}$.

34. The magnitude of the velocity is $v = (v_x^2 + v_y^2)^{1/2}$. Because v_x is constant, v will be maximum when v_y is maximum, which occurs $\boxed{\text{at the initial release and just as the projectile hits the ground}}$.
   Thus $v_y = \pm v_0 \sin\theta_0$; $v_x = v_0 \cos\theta_0$, or $\vec{v}_{\text{max}} = \boxed{v_0(\cos\theta_0 \, \hat{i} \pm \sin\theta_0 \, \hat{j})}$ with magnitude v_0.
   The speed will be minimum when $|v_y| = 0$, which occurs $\boxed{\text{at the highest point}}$: $\vec{v}_{\text{min}} = \boxed{v_0 \cos\theta_0 \, \hat{i}}$.

35. The horizontal motion will have constant velocity, v_{0x}.
   We find the vertical velocity from
   $v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 0 = v_{0y}^2$, so $v_y = -v_{0y}$.
   Because the angle is determined by the direction of the velocity, $\tan\theta = v_y/v_{0x}$, the projectile will make an angle $\boxed{25° \text{ below the horizontal}}$ when it lands.

36. With the origin at the release point, x horizontal and y up, the time of flight is found from
    $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ with $y = 0$.
    So $0 = 0 + v_0 \sin\theta_0\, t + \frac{1}{2}(-g)t^2$, which gives $t_1 = 0$ (start) and $t_2 = (2v_0 \sin\theta_0)/g$.
    The range is then $R = v_{0x}t_2 = (2v_0^2 \sin\theta_0 \cos\theta_0)/g = v_0^2 \sin(2\theta_0)/g$.
    The maximum height occurs when $v_y = 0 = v_0 \sin\theta_0 + (-g)t^2$, which gives $t_3 = (v_0 \sin\theta_0)/g \ \ (= \frac{1}{2}t_2)$.
    The maximum height is found from $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$:
    $h = 0 + v_0 \sin\theta_0\, t_3 + \frac{1}{2}(-g)t_3^2 = (v_0 \sin\theta_0)^2/g - \frac{1}{2}(v_0 \sin\theta_0)^2/g = \frac{1}{2}v_0^2 \sin^2\theta_0/g$.
    If $R = 2h$: $(2v_0^2 \sin\theta_0 \cos\theta_0)/g = 2(v_0^2 \sin^2\theta_0)/2g$, which gives $\tan\theta_0 = 2$; $\theta_0 = \boxed{63°}$.

37. (a) $R = v_0^2 \sin(2\theta_0)/g$; $h = \frac{1}{2}(v_0^2 \sin^2\theta_0)/g$. Thus
    $R/h = 2\sin(2\theta_0)/\sin^2\theta_0 = 2(2\sin\theta_0\cos\theta_0)/\sin^2\theta_0 = 4\cot\theta_0$.
    (b) The angle at which the range is maximum is found from $\mathrm{d}R/\mathrm{d}\theta_0 = 0$:
    $\mathrm{d}R/\mathrm{d}\theta_0 = (v_0^2/g)\cos(2\theta_0)\,(2) = 0$; $\cos(2\theta_0) = 0$; $2\theta_0 = \frac{1}{2}\pi$; $\theta_0 = \frac{1}{4}\pi = 45°$.
    Thus $R = 4h \cot 45° = 4h$; $h = \frac{1}{4}R$.

38. Let's assume that the grasshopper takes off at the optimum angle of $\theta_0 = 45°$. For each jump $R = v_0 \cos\theta_0\, T$ and $T = 2\, v_0 \sin\theta_0\, /g$, which gives
    $T = (2R\tan\theta_0/g)^{1/2} = [2(0.65\text{ m})\tan 45°\ /9.8\text{ m/s}^2]^{1/2} = 0.364$ s.
    The total number of jumps it can make in 1 h is $N = 3600\text{ s}/0.364\text{ s} = 9884$, and the total distance covered is $NR = 9884\,(0.65\text{ m}) = 6400\text{ m} = \boxed{6.4\text{ km}}$.

39. Set $R = 2v_0^2 \sin\theta_0 \cos\theta_0\ /g = h = \frac{1}{2}v_0^2 \sin\theta_0/g$ to obtain $\tan\theta_0 = 4$, so $\theta_0 = \boxed{76°}$.
    R is proportional to
    $\sin(2\theta_0) = \sin(2 \times 76°) = \sin 152° = \sin(180° - 152°) = \sin 28° = \sin(2 \times 14°) = \sin(2\theta_0')$, so
    $\theta_0' = \boxed{14°}$ will yield the same range.

40. Set the origin of an xy coordinate system at the release point of the stone. The coordinates of the coconut are $(x_1, y_1) = (4\text{ m}, 3\text{ m})$. To get the coconut (x_1, y_1) must be on the trajectory of the projectile, so
    $y_1 = (\tan\theta_0)x_1 - (\frac{1}{2}g/v_0^2\cos^2\theta_0)x_1^2$. Use the identity $1/\cos^2\theta_0 = 1 + \tan^2\theta_0$ to rewrite this as
    $(\frac{1}{2}gx_1^2/v_0^2)\tan^2\theta_0 - x_1\tan\theta_0 + \frac{1}{2}gx_1^2/v_0^2 + y_1 = 0$.
    Plug in $g = 9.8\text{ m/s}^2$, $x_1 = 4$ m, $y_1 = 3$ m, and $v_0 = 20$ m/s: $0.196\tan^2\theta_0 - 4\tan\theta_0 + 3.196 = 0$, which gives $\tan\theta_0 = 9.37$ or 0.833. Thus $\theta_0 = \boxed{87°}$ or $\boxed{40°}$.

41. (a) Assume that the potato is launched at the optimum angle of 45°. Set $R = 2v_0^2 \sin\theta_0 \cos\theta_0\ /g$ to obtain

$$v_0 = \sqrt{\frac{Rg}{2\sin\theta_0\cos\theta_0}} = \sqrt{\frac{(120\text{m})(9.8\text{m/s}^2)}{2\sin 45^\circ \cos 45^\circ}} = 34\text{ m/s}.$$

(b)

$$h = \frac{v_0^2\sin^2\theta_0}{2g} = \frac{(34.3\text{ m/s})^2\sin^2 45^\circ}{2(9.8\text{m/s}^2)} = 30\text{ m}.$$

(c) To reach maximum height set $\theta_0 = 90°$, so

$$h_{\text{max}} = \frac{v_0^2\sin^2\theta_0}{2g} = \frac{(34.3\text{ m/s})^2\sin^2 90^\circ}{2(9.8\text{m/s}^2)} = 60\text{ m}.$$

42. We will use a coordinate system with the origin at the base of the tower, x horizontal and y up, so $y_0 = H$, the height of the tower.
   (a) For the horizontal motion: $x = v_{0x}t = v_0 \cos 60° \, t$;
   $15 \text{ m} = v_0(0.50)(6.5 \text{ s})$, which gives $v_0 = \boxed{4.6 \text{ m/s}}$.
   (b) For the vertical motion: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$;
   $0 = H + (4.6 \text{ m/s})(\sin 60°)(6.5 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(6.5 \text{ s})^2$, which gives $H = 181 \text{ m} \approx \boxed{0.18 \text{ km}}$.
   (c) To find the velocity components:
   $v_x = v_{0x} = (4.6 \text{ m/s}) \cos 60° = 2.3 \text{ m/s}$ and
   $v_y = v_{0y} + a_y t = (4.6 \text{ m/s}) \sin 60° + (-9.8 \text{ m/s}^2)(6.5 \text{ s}) = -60 \text{ m/s}$.
   The speed is $v = (v_x^2 + v_y^2)^{1/2} = [(2.3 \text{ m/s})^2 + (-60 \text{ m/s})^2]^{1/2} = \boxed{60 \text{ m/s}}$.

43. The initial speed of the arrows can be found from the maximum range, which occurs when $\theta_0 = 45°$:
   $R = v_0^2 \sin(2\theta_0)/g$; $R_{\text{max}} = v_0^2/g = 350 \text{ m}$; $v_0^2 = (9.80 \text{ m/s}^2)(350 \text{ m})$, which gives $v_0 = 58.6 \text{ m/s}$.
   At the launch angle of 55°, the range is
   $R = (v_0^2/g) \sin[2(55°)] = (350 \text{ m})(0.939) = \boxed{329 \text{ m}}$.

44. (a) At the highest point, the vertical velocity $v_y = 0$.
   The speed will be $v_x = v_{0x} = (28 \text{ m/s}) \cos 50° = \boxed{18 \text{ m/s}}$.
   (b) At the highest point: $v_y = v_{0y} + a_y t_1$; $0 = (28 \text{ m/s}) \sin 50° + (-9.8 \text{ m/s}^2)t_1$, which gives $t_1 = 2.2 \text{ s}$.
   The height is $h = y_0 + v_{0y}t_1 + \frac{1}{2}a_y t_1^2 = 0 + (28 \text{ m/s}) \sin 50° (2.2 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.2 \text{ s})^2 = \boxed{24 \text{ m}}$.
   (c) Because the time to fall 24 m is the same as the time to rise 24 m: $t_2 = t_1 = 2.2$ s. The speed will be
   $|v_y| = |v_{0y} + a_y t_2| = |[0 + (-9.8 \text{ m/s}^2)(2.2 \text{ s})]| = \boxed{21 \text{ m/s}}$, which is $(28 \text{ m/s}) \sin 50°$.

45. (a) The ball passes the goal posts when it has traveled the horizontal distance of 35 m:
   $x = v_{0x}t$; $35 \text{ m} = (30 \text{ m/s}) \cos 32° \, t$, which gives $t = \boxed{1.4 \text{ s}}$.
   (b) To see if the kick is successful, we must find the height of the ball at this time:
   $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + (30 \text{ m/s}) \sin 32° (1.38 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(1.38 \text{ s})^2 = 12.6 \text{ m}$.
   So $\boxed{\text{yes}}$, the kick is successful and clears the bar by $12.6 \text{ m} - 4.0 \text{ m} = \boxed{8.6 \text{ m}}$.

46. The horizontal range on earth is given by $R = v_0^2 \sin(2\theta_0)/g$, whereas on a planet X it is
   $R_x = v_0^2 \sin(2\theta_0)/g_x$.
   Because we assume the same v_0 and θ_0, $R_x/R = g/g_x$, or $R_x = (g/g_x)R$.
   On the Moon $R_{\text{Moon}} = [g/(g/6)]R = 6(210 \text{ yd}) = \boxed{1260 \text{ yd}}$.

47. The horizontal range is given by $R = v_0^2 \sin(2\theta_0)/g$. Because the maximum range occurs for $\theta_0 = 45°$,
   $R_{\text{max}} = v_0^2 \sin[2(45°)]/g = v_0^2/g$.
   With the same v_0 at some other launch angle θ_0, $R/R_{\text{max}} = \sin(2\theta_0)$.
   When $R/R_{\text{max}} = \frac{1}{2}$, we get $2\theta_0 = 30°$ and $150°$, or
   $\theta_0 = \boxed{15° \text{ and } 75°}$.
   The two angles for the same range arise because the horizontal range is the product of the horizontal speed and the time of flight. For the smaller angle, the horizontal speed is greater but the time of flight is less. For the larger angle, the horizontal speed is less but the time of flight is greater.
   For $R = 0$, $\sin(2\theta_0) = 0$, which gives $2\theta_0 = 0$ and $180°$, or
   $\theta_0 = \boxed{0° \text{ and } 90°}$.

48. (a) The time to fall is found from $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$;
$0 = 20\text{ m} + 0 + \frac{1}{2}(-9.8\text{ m/s}^2)t_1{}^2$, which gives $t_1 = 2.02$ s.
Because it travels 50 m horizontally: $x = v_{0x}t$; $50\text{ m} = v_0(2.02\text{ s})$, which gives $v_0 = \boxed{24.7\text{ m/s}}$.
(b) If the ball is thrown at an angle, the same analysis, with the components of v_0, gives:
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$; $\quad 0 = 20\text{ m} + v_0 \sin 45°\ t_2 + \frac{1}{2}(-9.8\text{ m/s}^2)t_2{}^2$ and
$x = v_{0x}t$; $\quad 50\text{ m} = v_0 \cos 45°\ t_2$.
When these equations are solved simultaneously, we get $t_2 = 3.8$ s and $v_0 = \boxed{18.7\text{ m/s at } 45°}$.
(c) The horizontal component $v_x = v_{0x} = v_0 \cos 45° = \boxed{13.2\text{ m/s}}$.
Note that this is not the horizontal speed needed in part (a) because the time of flight is longer.

49. The radius of the circular path is $R = R_E + h = 6.37 \times 10^3\text{ km} + 220\text{ km} = 6.59 \times 10^3$ km.
(a) The speed is $v = 2\pi R/T = 2\pi(6.59 \times 10^6\text{ m})/[(89\text{ min})(60\text{ s/min})] = \boxed{7.8 \times 10^3\text{ m/s}} = 26 \times 10^3$ km/h.
(b) The acceleration is $a = v^2/R = (7.75 \times 10^3\text{ m/s})^2/(6.59 \times 10^6\text{ m}) = \boxed{9.1\text{ m/s}^2 \text{ toward Earth's center}}$.

50. $a = R\omega^2 = (3.84 \times 10^8\text{ m})\{2\pi/[(28\text{ d})(24\text{ h/d})(3600\text{ s/h})]\}^2 = 2.59 \times 10^{-3}\text{ m/s}^2 = \boxed{2.6 \times 10^{-4}\ g}$.

51. The speed is $v = d/t = (200\text{ m})/25.5\text{ s} = 7.84$ m/s.
The centripetal acceleration while she is running the curve is
$a = v^2/r = (7.84\text{ m/s})^2/25\text{ m} = \boxed{2.5\text{ m/s}^2}$.

52. (a) Because the period is the time to rotate 2π rad, $a = r\omega^2 = (1.2\text{ m})[2\pi/(1.8\text{ s})]^2 = \boxed{15\text{ m/s}^2}$.
(b) Because $a = v^2/r$, $v^2 = ar = (56\text{ m/s}^2)(1.2\text{ m})$, which gives $v = \boxed{8.2\text{ m/s}}$.

53. Because $a = r\omega^2$, $\omega^2 = a/r = (3.3\text{ m/s}^2)/(7.5\text{ m})$, which gives $\omega = 0.66$ rad/s.
Because $\omega = 2\pi/T$, $T = 2\pi/\omega = (2\pi\text{ rad})/(0.66\text{ rad/s}) = \boxed{9.5\text{ s}}$.

54. The angular speed of the flywheel is
$\omega = (4000\text{ rev/min})(2\pi\text{ rad/rev})/(60\text{ s/min}) = 419$ rad/s.
The centripetal acceleration of a point on the rim is
$a = r\omega^2 = (0.10\text{ m})(419\text{ rad/s})^2 = 1.75 \times 10^4\text{ m/s}^2 = \boxed{1790g}$.

55. The speed is $v = (65\text{ mi/h})(1.61 \times 10^3\text{ m/mi})(1\text{ h}/3600\text{ s}) = 29.1$ m/s.
The restriction is
$a = v^2/R \le 0.1g$, so $R \ge v^2/0.1g = (29.1\text{ m/s})^2/[0.1(9.8\text{ m/s}^2)] = \boxed{0.86\text{ km}}$.

56. We write $a = kT^\alpha R^\beta$. Then $[a] = [T]^\alpha[R]^\beta$; $[LT^{-2}] = [T]^\alpha[L]^\beta$, which gives $\beta = 1$, $\alpha = -2$.
Thus $a = \boxed{kR/T^2}$. From the detailed analysis
$a = R\omega^2 = R(2\pi)^2/T^2 = (2\pi)^2 R/T^2$.

57. The angular speed can be found from $\omega = v/r = (18\text{ m/s})/0.35\text{ m} = 51.4$ rad/s.
(a) The angle of rotation is
$\theta = \omega t = (51.4\text{ rad/s})(0.1)\text{ s} = 5.14\text{ rad} = 295°$.
Then $(x, y) = (r\cos\theta, r\sin\theta) = [(0.35\text{ m})\cos 295°, (0.35\text{ m})\sin 295°] = \boxed{(0.15\text{ m}, -0.32\text{ m})}$.
(b) The magnitude of the acceleration is
$v^2/r = (18\text{ m/s})^2/(0.35\text{ m}) = 9.3 \times 10^2\text{ m/s}^2$
and its direction is centripetal. At $t = 0$ s, the acceleration is
$\vec{a}_0 = \boxed{-(9.3 \times 10^2\text{m/s}^2)\hat{i}}$.
(c) At $\theta = 90°$, the acceleration is
$\vec{a} = \boxed{-(9.3 \times 10^2\text{m/s}^2)\hat{j}}$.

58. Because $v = r\omega$, $r = v/\omega = (7.8 \text{ km/s})/[2\pi/(87 \text{ min})(60 \text{ s/min})] = 6.6 \times 10^3$ km.
The centripetal acceleration is $a_r = v^2/r = (7.8 \times 10^3 \text{ m/s})^2/(6.6 \times 10^6 \text{ m}) = 9.4 \text{ m/s}^2$.
If we ignore the small change in orbital speed, so that the magnitude of the centripetal acceleration does not change significantly, then
$$a = (a_r^2 + a_d^2)^{1/2} = [(9.4 \text{ m/s}^2)^2 + (6 \text{ m/s}^2)^2]^{1/2} = 11 \text{ m/s}^2.$$
The direction is found from $\tan\theta = a_d/a_r = (6 \text{ m/s}^2)/(9.4 \text{ m/s}^2) = 0.64$, which gives $\theta = 33°$.
Thus $a = \boxed{11 \text{ m/s}^2, \text{ 33° back from the radial direction toward Earth}}$.

59. The initial (and constant) magnitude of the acceleration is $a = v_0^2/R$. The speed after 10 kicks is
$$v = v_0 + 10(v_0/100) = 1.10\, v_0.$$
Because $a = v^2/R_2$, $R_2 = v^2/a = v^2/(v_0^2/R) = (v/v_0)^2R = (1.10)^2R = \boxed{1.21R}$.

60. The tangential acceleration is the rate of change of the tangential speed of the automobile. Because it is constant, we can relate the speed to the distance traveled around the circle: $v^2 = v_0^2 + 2a_ts$.
For the first lap: $(30.0 \text{ m/s})^2 = 0 + 2a_t[2\pi(1.00 \times 10^3 \text{ m})]$, which gives $a_t = 7.16 \times 10^{-2} \text{ m/s}^2$.
One-eighth of a lap means an angle of $360°/8 = 45°$ and a distance of $2\pi(1.00 \times 10^3 \text{ m})/8 = 0.785 \times 10^3$ m.
The speed at this point is found from
$$v^2 = 0 + 2a_ts = 2(7.16 \times 10^{-2} \text{ m/s}^2)(0.785 \times 10^3 \text{ m}),$$
which gives a velocity of 10.6 m/s tangent to the circle.
The position is $\vec{r} = (1.00 \text{ km})\cos 45°\,\hat{i} + (1.00 \text{ km})\sin 45°\,\hat{j} = \boxed{(0.707\hat{i} + 0.707)\hat{j} \text{ km}}$.
The velocity is $\vec{v} = -(10.6 \text{ m/s})\sin 45°\,\hat{i} + (10.6 \text{ m/s})\cos 45°\,\hat{j} = \boxed{(-7.49\hat{i} + 7.49)\hat{j} \text{ m/s}}$.

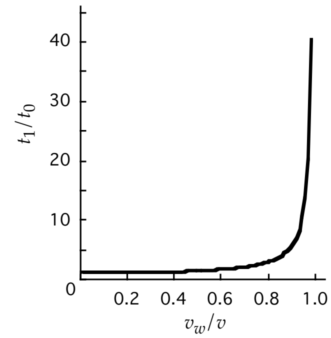
61. If we use a coordinate system with x east and y north, the velocity of the boat with respect to the water is $\vec{v}_B = \vec{v}_A - \vec{u}$, where
$\vec{v}_A = 15\hat{i}$ km/h is the velocity of the boat with respect to the land and
$\vec{u} = 5\hat{j}$ km/h is the velocity of the Gulf Stream with respect to the land. Thus
$\vec{v}_B = (15\hat{i} - 5\hat{j})$ km/h = $\boxed{\text{15.8 km/h, 18° south of east.}}$

62. (a) Because the speeds are with respect to the ground, we have
$\vec{v}_{\text{leader}} = \vec{v}_{\text{cyclist}} + \vec{u}$, where $\vec{u}$ is the velocity of the leader with respect to the cyclist.
Because all velocities are up the hill, we have
$v_{\text{leader}} = v_{\text{cyclist}} + u$;
$24 \text{ km/h} = 21 \text{ km/h} + u$, which gives $u = \boxed{3 \text{ km/h}}$.
(b) Because the elapsed time is with respect to the ground, the leader had been traveling at 24 km/h for 30 s before the cyclist reached the message board:
$x = v_{\text{leader}}t = (24 \text{ km/h})(30/3600 \text{ h}) = \boxed{0.20 \text{ km}}$.

63. If we use a coordinate system with x in the direction of travel and y up, the velocity of the rain with respect to the car is $\vec{v}_B = \vec{v}_A - \vec{u}$, where $\vec{v}_A = -v_A\hat{j}$ is the velocity of the rain with respect to the ground and $\vec{u} = 80\hat{i}$ km/h is the velocity of the car with respect to the ground.
If θ is the angle on the window with respect to the vertical, then
$$-v_B\cos\theta\,\hat{i} - v_B\sin\theta\,\hat{j} = (-v_A\hat{j} - 80\hat{i}) \text{ km/h}.$$
Equating the two components, we get $v_A = v_B\cos\theta$ and $80 \text{ km/h} = v_B\sin\theta$. Dividing the two equations, we get
$$v_A/(80 \text{ km/h}) = \cot\theta = \cot 58°, \text{ or } v_A = \boxed{50 \text{ km/h}}.$$

64. (a) With no wind, the speed in each direction will be v, so the time is $t_0 = 2L/v$.
    (b) With a north wind, her speed while cycling north will be $v - v_w$ and returning will be $v + v_w$. Thus her total time will be

$$t_1 = \frac{L}{v - v_w} + \frac{L}{v + v_w} = \frac{L(v + v_w + v - v_w)}{(v^2 - v_w^2)} = \frac{2Lv}{(v^2 - v_w^2)} = \frac{t_0}{1 - (v_w/v)^2}.$$



    (c) If $v_w << v$, then

$$t_1 = \frac{2Lv}{v^2\left[1 - (v_w/v)^2\right]} \simeq \frac{2L}{v}[1 + (v_w/v)^2] = t_0[1 + (v_w/v)^2].$$

    (d) At $v_w = v$, the time becomes infinite because the cyclist will have zero velocity with respect to the ground on the first part of her trip and will never move.

65. (a) $h_{\max} = \frac{1}{2}v_0{}^2/g$,

$$v_0 = \sqrt{2g\,h_{\max}} = \sqrt{2(32.2\ \text{ft/s}^2)(40\ \text{in.})(1\ \text{ft}/12\ \text{in.})} = 15\ \text{ft/s}.$$

    (b)

$$h = \frac{v_0{}^2 \sin^2\theta_0}{2g} = h_{\max}\sin^2\theta_0 = (40\ \text{in.})\sin^2 45^\circ = 20\ \text{in.}$$

$$R = \frac{2v_0{}^2 \sin\theta_0 \cos\theta_0}{g} = \frac{2(14.65\ \text{ft/s})^2 \sin 45^\circ \cos 45^\circ}{32.2\ \text{ft/s}^2} = 6.7\ \text{ft.}$$

66. Refer to the diagram in the solution to Problem 24. To be specific, choose the direction of motion of the truck (2) to be along the positive x-axis and that of the car (1) to be along the positive y-axis. Then $\vec{v}_1 = v_1\hat{j}$ and $\vec{v}_2 = v_2\hat{i}$. The velocity of the truck relative to the car is then

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = v_2\hat{i} - v_1\hat{j} = \boxed{(40\hat{i} - 30\hat{j})\ \text{km/h}}.$$

The magnitude of the relative velocity is

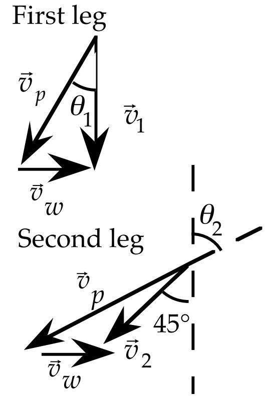
$$v_{21} = \sqrt{(40\ \text{km/h})^2 + (30\ \text{km/h})^2} = 50\ \text{km/h},$$

and $\vec{v}_{21}$ makes an angle of θ clockwise from the positive x-axis, where

$$\theta = \tan^{-1}\left(\frac{30\ \text{km/h}}{40\ \text{km/h}}\right) = 37^\circ.$$

$\boxed{67}$. During the first leg, in order to fly due south, the airplane must head southwest, as in the diagram. If $\vec{v}_p$ is the velocity of the airplane with respect to the air, $\vec{v}_1$ is the velocity of the airplane with respect to the ground, and $\vec{v}_w$ the velocity of the wind with respect to the ground, then

$$\vec{v}_1 = \vec{v}_p + \vec{v}_w.$$



Because $v_p = 900$ km/h and $v_w = 120$ km/h, from the diagram we get

$$\sin\theta_1 = (120 \text{ km/h})/(900 \text{ km/h}), \text{ or } \theta_1 = 7.7°.$$

Then $v_1 = (900 \text{ km/h}) \cos 7.7° = 892$ km/h.

The distance traveled in the first leg is $d_1 = (892 \text{ km/h})(2.0 \text{ h}) = 1784$ km.

During the second leg, the airplane must turn more toward the west, as shown.

If $\vec{v}_2$ is the new velocity of the airplane with respect to the ground, then

$$\vec{v}_2 = \vec{v}_p + \vec{v}_w.$$

The two component equations can be obtained from the diagram:

$$(900 \text{ km/h}) \sin\theta_2 - 120 \text{ km/h} = v_2 \sin 45° \quad \text{and}$$
$$(900 \text{ km/h}) \cos\theta_2 = v_2 \cos 45°.$$

Because $\sin 45° = \cos 45°$, we can write this as

$(900 \text{ km/h}) \sin\theta_2 - 120 \text{ km/h} = (900 \text{ km/h}) \cos\theta_2$, which reduces to $\sin\theta_2 = \cos\theta_2 + 0.133$.

By squaring both sides, we get $1 - \cos^2\theta_2 = \cos^2\theta_2 + 0.266\cos\theta_2 + (0.133)^2$, which has the solution $\cos\theta_2 = 0.637$ or $\theta_2 = 50.4°$. Then $v_2 = (900 \text{ km/h}) (\cos 50.4°)/0.707 = 811$ km/h.

The distance traveled during the second leg is $d_2 = (811 \text{ km/h})(3.0 \text{ h}) = 2433$ km.

(a) The average speed is (total distance)/(total time) = (1784 m + 2433 m)/(5.0 h) = $\boxed{843 \text{ km/h}}$.

(b) To find the average velocity, we must first find the displacement:

$$\vec{r} = -(2433 \text{ m})(0.707)\hat{i} + [(-2433 \text{ m})(0.707) - 1784]\hat{j} = -(1720 \text{ km})\hat{i} - (3505 \text{ km})\hat{j}.$$

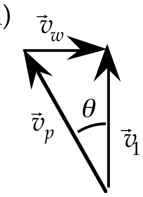
The average velocity is $\vec{v} = \vec{r}/(5 \text{ h}) = (-344\hat{i} - 701)\hat{j}$ km/h or $\boxed{781 \text{ km/h}, 26° \text{ W of S}}$.

(c) From part (b) the final position is

$$\vec{r} = \boxed{(-1720\hat{i} - 3505)\hat{j} \text{ km}}, \quad \text{or} \quad \boxed{3900 \text{ km}, 26° \text{ W of S}}.$$

68. (a) In order to fly due north, the airplane must head northwest, as in the diagram. If $\vec{v}_p$ is the velocity of the airplane with respect to the air, $\vec{v}_1$ the velocity of the airplane with respect to the ground, and $\vec{v}_w$ the velocity of the wind with respect to the ground, then $\vec{v}_1 = \vec{v}_p + \vec{v}_w$.

(a)



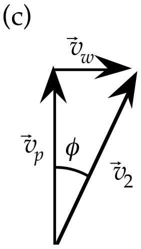
From the diagram we see that $\sin q = v_w/v_p = (85 \text{ mi/h})/(320 \text{ mi/h})$, which gives

$\theta = \boxed{15.4° \text{ W of N}}$. Because the diagram is a velocity diagram, the calculation would $\boxed{\text{not}}$ change if the distance were greater.

(b) From the diagram we also get $v_1 = v_p \cos\theta = (320 \text{ mi/h}) \cos 15.4° = 309$ mi/h.

The time of the flight is $t_1 = d/v_1 = 673 \text{ mi}/(309 \text{ mi/h}) = \boxed{2.18 \text{ h}}$.

(c)



(c) If the airplane heads north, its velocity with respect to the ground will be toward the northeast, as shown. From this diagram we see that

$\tan\phi = v_w/v_p = (85 \text{ mi/h})/(320 \text{ mi/h})$, which gives $\phi = 14.9°$.

The time until the plane turns west can be found from

$t_2 =$ (north component of distance)$/v_p = 673 \text{ mi}/(320 \text{ mi/h}) = 2.10$ h.

The distance traveled toward the east is $d_E = v_w t_2 = (85 \text{ mi/h})(2.10 \text{ h}) = 179$ mi, which is the distance the airplane must fly due west to get to St. Louis. Its speed with respect to the ground will be $v_3 = v_p - v_w = 320 \text{ mi/h} - 85 \text{ mi/h} = 235$ mi/h. The time it will take is $t_3 = d_E/v_3 = 179 \text{ mi}/(235 \text{ mi/h}) = 0.761$ h. Then the total time for the trip is

$t_2 + t_3 = 2.10 \text{ h} + 0.76 \text{ h} = \boxed{2.86 \text{ h}}$.

69. We can express the data as $r_e = 6.4 \times 10^3$ km and $r_o = 1.5 \times 10^8$ km. Because we know the periods of the two motions, we can find the corresponding speeds from $v = r\omega = r(2\pi/T)$:

$v_{\text{rot}} = r_e(2\pi/T_e) = (6.4 \times 10^3 \text{ km})[2\pi/(24 \text{ h})(3600 \text{ s/h})] = 0.465 \text{ km/s}$,

$v_{\text{orb}} = r_o(2\pi/T_o) = (1.5 \times 10^8 \text{ km})[2\pi/(365 \text{ d})(24 \text{ h})(3600 \text{ s/h})] = 30.0 \text{ km/s}$.

(a) At the nearest point, the velocities are in opposite directions, so

$v_1 = v_{\text{orb}} - v_{\text{rot}} = 30.0 \text{ km/s} - 0.465 \text{ km/s} = \boxed{29.5 \text{ km/s}}$.

(b) At the farthest point, the velocities are in the same direction, so

$v_2 = v_{\text{orb}} + v_{\text{rot}} = 30.0 \text{ km/s} + 0.465 \text{ km/s} = \boxed{30.5 \text{ km/s}}$.

(c) At the midway point, the velocities are perpendicular, so

$v_3 = (v_{\text{orb}}^2 + v_{\text{rot}}^2)^{1/2} = [(30.0 \text{ km/s})^2 + (0.465 \text{ km/s})^2]^{1/2} = \boxed{30.0 \text{ km/s}}$.

70. From the period we can find the angular speed: $\omega = 2\pi/T = 2\pi/26 \text{ s} = \pi/13 \text{ rad/s}$.

The velocity is $v = r\omega = (40 \times 10^3 \text{ m})(\pi/13 \text{ rad/s}) = \boxed{9.7 \times 10^3 \text{ m/s, tangential}}$.

The acceleration is $a = r\omega^2 = (40 \times 10^3 \text{ m})(\pi/13 \text{ rad/s})^2 = \boxed{2.3 \times 10^3 \text{ m/s}^2 \approx 230g \text{ centripetal}}$.

71. We choose a coordinate system with the origin at the tee, x horizontal and y up.

The horizontal motion is $x = v_{0x}t_1 = 155 \text{ m} = v_0 \cos 65° \, t_1$, or $v_0 t_1 = 367$ m.

The vertical motion is $y = y_0 + v_{0y} + \frac{1}{2}a_y t_1^2$; $4.0 \text{ m} = 0 + v_0 \sin 65° \, t_1 + \frac{1}{2}(-9.8 \text{ m/s}^2)t_1^2$.

(a) When these two equations are solved simultaneously, we get $t_1 = 8.2$ s and $v_0 = \boxed{45 \text{ m/s at } 65°}$.

(b) Because $v_y = 0$ at the maximum height, we can find the time to reach this height from

$v_y = v_{0y} + a_y t_2$; $0 = (44.8 \text{ m/s}) \sin 65° + (-9.8 \text{ m/s}^2)t_2$, which gives $t_2 = 4.14$ s.

(Note that this is less than $\frac{1}{2}t_1$ because of the slope of the ground.) The height above the green is

$y_{\max} = 0 + (44.8 \text{ m/s})(\sin 65°)(4.14 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(4.14 \text{ s})^2 = \boxed{84 \text{ m}}$.

72. We choose a coordinate system with the origin at home plate, x horizontal and y up.

The horizontal motion is $x = v_{0x}t$;

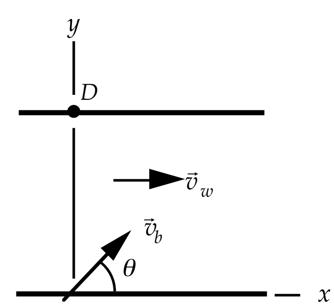
$65 \text{ m} = v_0 \cos 30° \, t$, which gives $v_0 t = 75.1$ m.

The vertical motion is $y = y_0 + v_{0y} + \frac{1}{2}a_y t_1^2$;

$1.8 = 0.9 + v_0 \sin 30° \, t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$.

When these two equations are solved simultaneously, we get $t = 2.7$ s and $v_0 = \boxed{27 \text{ m/s}}$.

73. The coordinate system is shown on the diagram. If $v_B = 10$ km/h is the speed of the boat with respect to the water, then its velocity with respect to the shore is



$\vec{v}_s = (v_B \cos\theta + v_W)\hat{i} + v_B \sin\theta \hat{j}$.

The position vector from its starting point is

$\vec{r} = \vec{v}_s t = (v_B \cos\theta + v_W)t\hat{i} + (v_B \sin\theta)t\hat{j}$, or

$\vec{r} = \boxed{(10\cos\theta + 6)t\,\hat{i} + (10\sin\theta)t\,\hat{j}}$, with r in km and t in hr.

To land at the point D, $x = 0$ or

$v_B \cos\theta + v_W = 0$;

$\cos\theta = -(6 \text{ km/h})/(10 \text{ km/h})$, which gives

$\theta = \boxed{127°}$.

The trip time is found from $y = v_B \sin\theta \, t$;

$150 \text{ m} = [(10 \times 10^3 \text{ m/h})/(3600 \text{ s/h})] \sin 127° \, t$, which gives

$t = \boxed{68 \text{ s}}$.

74. We choose a coordinate system with the origin at the base of the table, x horizontal and y up.
    (a) The time is found from the horizontal motion:
    $x = v_{0x}t$; 1.08 m = (2.50 m/s)t, which gives $t = \boxed{0.432 \text{ s}}$.
    (b) To find g we use the vertical motion:
    $y = y_0 + v_{0y}t + \frac{1}{2}at^2$; $0 = 0.86 \text{ m} + 0 + \frac{1}{2}(-g)(0.432 \text{ s})^2$, which gives $g = \boxed{9.22 \text{ m/s}^2}$.
    (c) When the mass hits the floor, the horizontal velocity is still 2.50 m/s and the vertical velocity is
    $v_y = v_{0y} + a_y t = 0 + (-9.80 \text{ m/s}^2)(0.432) = \boxed{-4.23 \text{ m/s}}$.
    The speed is $v = (v_x{}^2 + v_y{}^2)^{1/2} = [(2.50 \text{ m/s})^2 + (4.23 \text{ m/s})^2]^{1/2} = \boxed{4.91 \text{ m/s}}$.

75. We choose a coordinate system with the origin at the base of the mast, x horizontal and y up. At release the hammer is at $x = 0$, $y = 26$ m and will have the horizontal velocity of the top of the mast. The time of fall is found from
    $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$;
    $0 = 26 \text{ m} + 0 + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$, which gives $t = 2.3$ s.
    The horizontal motion is
    $x = v_{0x}t = (3.6 \text{ m/s})(2.3 \text{ s}) = 8.3$ m.
    Because this is less than half the width of the ship, the hammer $\boxed{\text{will hit the deck}}$, which is assumed to be horizontal at impact.

76. Set up an xy coordinate system originated at the position where the ball is thrown out, with the x-axis horizontal and the y-axis pointing vertically upward. Suppose that the ball hits the incline at (x_1, y_1), then x_1 and y_1 must satisfy the trajectory equation:
    $y_1 = x_1\tan\theta_0 - gx_1{}^2/2v_0{}^2\cos\theta_0{}^2$.
    Also, $y_1/x_1 = -\tan\alpha$, with $\alpha = 30°$. (Note the negative sign here, which is because $y_1 < 0$). Eliminate y_1 and solve for x_1:
    $$x_1 = \frac{2(\tan\alpha + \tan\theta_0)v_0{}^2\cos^2\theta_0}{g} = \frac{2(\tan 30° + \tan 0)(10 \text{ m/s})^2(\cos 0)^2}{9.8 \text{ m/s}^2} = 11.8 \text{ m},$$
    so the distance between the launch point and the landing point along the inclined plane is
    $L = x_1/\cos\alpha = 11.8 \text{ m}/\cos 30° = \boxed{14 \text{ m}}$.

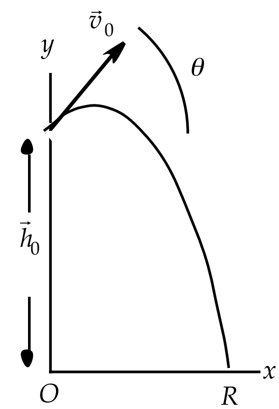
77. Because there are four balls with 0.3 s between balls, the time for a complete cycle is 1.2 s, 0.3 s of which the ball is in the hands. The time in the air for each ball is 1.2 s – 0.3 s = 0.9 s, which means that it takes 0.45 s to reach the maximum height.
    (a) Because the speed is zero at the maximum height, we can use
    $v_y = v_{0y} + a_y t$;
    $0 = v_0 + (-9.8 \text{ m/s}^2)(0.45 \text{ s})$, which gives
    $v_0 = \boxed{4.4 \text{ m/s}}$.
    (b) A ball has just been caught: $y = 0$.
    A ball is at $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$
    $= 0 + (4.41 \text{ m/s})(0.3 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(0.3 \text{ s})^2$
    $= \boxed{0.88 \text{ m going up}}$.
    A ball is at $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$
    $= 0 + (4.41 \text{ m/s})(0.6 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(0.6 \text{ s})^2$
    $= \boxed{0.88 \text{ m coming down}}$.
    A ball has just been thrown: $y = \boxed{0}$.
    (c) For 5 balls the cycle time is 1.5 s and the time in the air is 1.2 s.
    Using the analysis from part (a), the initial speed must be
    $v_0 = g(0.6 \text{ s}) = 5.9 \text{ m/s}$.
    The maximum height is reached in half the time in the air:
    $y = 0 + (5.9 \text{ m/s})(0.6 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(0.6 \text{ s})^2 = \boxed{1.8 \text{ m}}$.

78. Using the coordinate system shown in the diagram, we can write the equations of motion as

$x = v_0 \cos\theta\, t$ and

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$, or

$0 = h_0 + v_0 \sin\theta\, t + \frac{1}{2}(-g)t^2$.



The y-equation yields a quadratic equation for the time to reach the ground, t:

$t^2 - [(2v_0 \sin\theta)/g]\, t - 2h_0/g = 0.$

The solutions are

$t = +(2v_0 \sin\theta/2g) \pm \frac{1}{2}[(2v_0 \sin\theta/g)^2 + (8h_0/g)]^{1/2}.$

Because the cannonball starts at $t = 0$, the physical answer is the positive result:

$t = +(v_0/g)\,[\sin\theta + (\sin^2\theta + 2h_0 g/v_0^2)^{1/2}].$

The horizontal range R is

$$R = v_0 \left(\cos\theta\right)\left(\frac{v_0}{g}\right)\left[\sin\theta + \left(\sin^2\theta + \frac{2h_0 g}{v_0^2}\right)^{1/2}\right].$$

To find the angle θ at which R is a maximum, we take the derivative $dR/d\theta$ and set it equal to zero:

$$\frac{dR}{d\theta} = -\left(\frac{v_0^2}{g}\right)(\sin\theta)\left[\sin\theta + \left(\sin^2\theta + \frac{2h_0 g}{v_0^2}\right)^{1/2}\right] + \left(\frac{v_0^2}{g}\right)(\cos\theta)\left[\cos\theta + \frac{1}{2}\left(\sin^2\theta + \frac{2h_0 g}{v_0^2}\right)^{-1/2}\left(2\sin\theta\cos\theta\right)\right] = 0.$$

This reduces to

$$-\sin^2\theta + \cos^2\theta + \frac{-\sin^3\theta - \left(2h_0 g/v_0^2\right)\sin\theta + \sin\theta\cos^2\theta}{\sqrt{\sin^2\theta + 2h_0 g/v_0^2}} = 0.$$

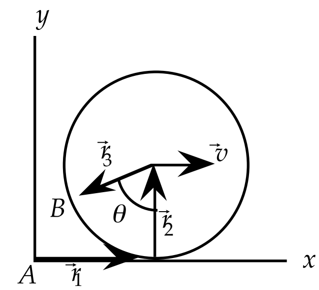
If we let $2h_0 g/v_0^2 = D$, use $\cos^2\theta = 1 - \sin^2\theta$ and let $z = \sin\theta$, we can get an equation in z:

$1 - 2z^2 + (-z^3 - Dz + z - z^3)/(z^2 + D)^{1/2} = 0.$

After some algebraic manipulation, including squaring, this reduces to

$z^2 = 1/(D + 2)$, or $\sin^2\theta = \frac{1}{2}v_0^2/(gh_0 + v_0^2)$.

79. We will use a coordinate system with the origin at the point A on the ground that was in contact at $t = 0$ with the point B on the rim, as shown in the diagram. Let the radius be R and the speed of the center be
$$V = (18 \text{ km/h})(10^3 \text{ m/km})(1 \text{ h}/3600 \text{ s}) = 5.0 \text{ m/s}.$$
Because the wheel is rolling, the length of arc from the current contact point to point B must equal the horizontal distance the center traveled:
$$\theta = r_1/R \quad \text{and}$$
$$\omega = d\theta/dt = (dr_1/dt)/R = V/R = (5.0 \text{ m/s})/(0.36 \text{ m}) = 14 \text{ rad/s}.$$
Thus the angle turned through is $\theta = \omega t$, and $V = R\omega$.
The position of B can be treated as the sum of three displacements:
$$\vec{r}_1 = Vt\hat{i}\,;\ \vec{r}_2 = R\hat{j}\,;\ \vec{r}_3 = -R\cos\theta\,\hat{i} - R\sin\theta\,\hat{j}. \text{ Thus}$$
$$\vec{r} = \vec{r}_1 + \vec{r}_2 + \vec{r}_3 = (Vt - R\sin\theta)\hat{i} + (R - R\cos\theta)\hat{j}.$$



The velocity is $\vec{v} = d\vec{r}/dt = [V - R\cos\theta\,(d\theta/dt)]\,\hat{i} + R\sin\theta\,(d\theta/dt)\,\hat{j} =$
$(V - R\omega\cos\theta)\hat{i} + R\omega\sin\theta\,\hat{j}$ or
$\vec{v} = V[(1-\cos\theta)\hat{i} + \sin\theta\,\hat{j}]$; velocity of the center + tangential velocity of the point on the rim.
The acceleration is
$\vec{a} = d\vec{v}/dt = V[\sin\theta\,(d\theta/dt)\hat{i} + \cos\theta\,(d\theta/dt)\hat{j}] = (V^2/R)[\sin\theta\,\hat{i} + \cos\theta\,\hat{j}]$; the centripetal acceleration.
When the numerical values are substituted, with $\theta = (14 \text{ rad/s})t$,
$$\vec{r} = \boxed{[(5.0 \text{ m/s})t - (0.36 \text{ m})\sin\theta]\hat{i} + [(0.36 \text{ m})(1-\cos\theta)]\hat{j}},$$
$$\vec{v} = \boxed{(5.0 \text{ m/s})[(1-\cos\theta)\hat{i} + \sin\theta)\hat{j}]},$$
$$\vec{a} = \boxed{(69 \text{ m/s}^2)(\sin\theta\,\hat{i} + \cos\theta\,\hat{j})}.$$