2

KINEMATICS IN ONE DIMENSION

Conceptual Questions

2.1. It was a typical summer day on the interstate. I started 10 mi east of town and drove for 20 min at 30 mph west to town. My stop for gas took 10 min. Then I headed back east at 60 mph before I encountered a construction zone. Traffic was at a standstill for 10 min and then I was able to move forward (east) at 30 mph until I got to my destination 30 mi east of town.

2.2. With a slow start out of the blocks, a super sprinter reached top speed in about 5 s, having gone only 30 m. He was still able to finish his 100 m in only just over 9 s by running a world record pace for the rest of the race.

2.3. The baseball team is warming up. The pitcher (who is 50 feet from home plate) lobs the ball at 100 ft/s to the second baseman who is 100 ft from home plate. The second baseman then fires the ball at 200 ft/s to the catcher at home plate.

2.4. **(a)** At $t = 1$ s, the slope of the line for object A is greater than that for object B. Therefore, object A's speed is greater. (Both are positive slopes.)
(b) No, the speeds are never the same. Each has a constant speed (constant slope) and A's speed is always greater.

2.5. **(a)** A's speed is greater at $t = 1$ s. The slope of the tangent to B's curve at $t = 1$ s is smaller than the slope of A's line.
(b) A and B have the same speed at just about $t = 3$ s. At that time, the slope of the tangent to the curve representing B's motion is equal to the slope of the line representing A.

2.6. **(a)** B. The object is still moving, but the magnitude of the slope of the position-versus-time curve is smaller than at D.
(b) D. The slope is greatest at D.
(c) At points A, C, and E the slope of the curve is zero, so the object is not moving.
(d) At point D the slope is negative, so the object is moving to the left.

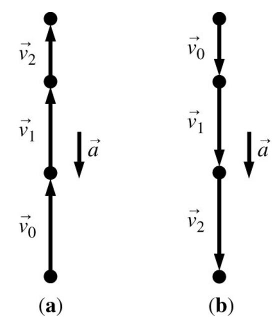
2.7. **(a)** The slope of the position-versus-time graph is greatest at D, so the object is moving fastest at this point.
(b) The slope is negative at points C, D, and E, meaning the object is moving to the left at these points.
(c) At point C the slope is increasing in magnitude (getting more negative), meaning that the object is speeding up to the left.
(d) At point B the object is not moving since the slope is zero. Before point B, the slope is positive, while after B it is negative, so the object is turning around at B.

2.8. **(a)** The positions of the third dots of both motion diagrams are the same, as are the sixth dots of both, so cars A and B are at the same locations at the time corresponding to dot 3 and again at that of dot 6.
(b) The spacing of dots 4 and 5 in both diagrams is the same, so the cars are traveling at the same speeds between times corresponding to dots 4 and 5.

2.9. No, though you have the same position along the road, his velocity is greater because he is passing you. If his velocity were not greater, then he would remain even with the front of your car.

2.10. Yes. The acceleration vector will point west when the bicycle is slowing down while traveling east.

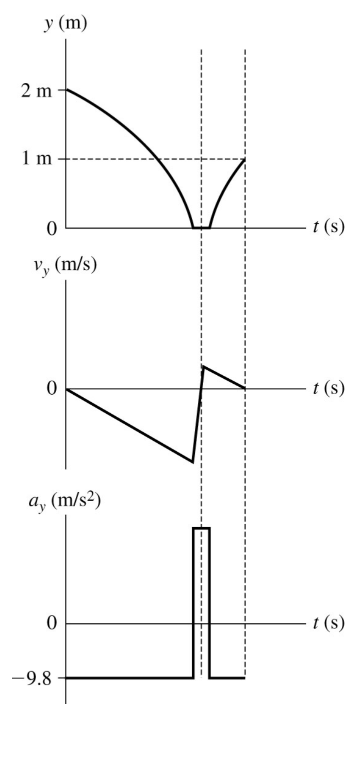
2.11. **(a)** As a ball tossed upward moves upward, its vertical velocity is positive, while its vertical acceleration is negative, opposite the velocity, causing the ball to slow down.
(b) The same ball on its way down has downward (negative) velocity. The downward negative acceleration is pointing in the same direction as the velocity, causing the speed to increase.



2.12. For all three of these situations the acceleration is equal to g in the downward direction. The magnitude and direction of the velocity of the ball do not matter. Gravity pulls down at constant acceleration. (Air friction is ignored.)

2.13. **(a)** The magnitude of the acceleration while in free fall is equal to g at all times, independent of the initial velocity. The acceleration only tells how the velocity is changing.
(b) The magnitude of the acceleration is still g because the rock is still in free fall. The speed is increasing at the same rate each instant, that is, by the same Δv each second.

2.14. The ball remains in contact with the floor for a small but noticeable amount of time. It is in free fall when not in contact with the floor. When it hits the floor, it is accelerated very rapidly in the upward direction as it bounces.

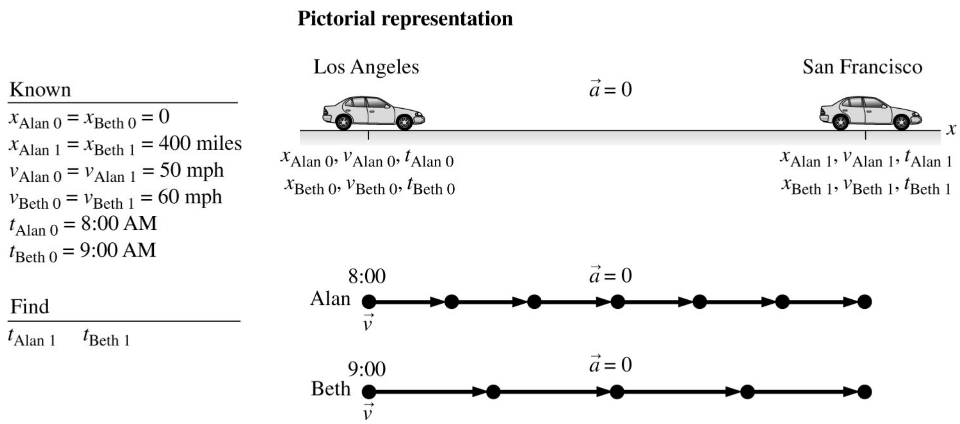


Exercises and Problems

Section 2.1 Uniform Motion

2.1. Model: Cars will be treated by the particle model.

Visualize:



Solve: Beth and Alan are moving at a constant speed, so we can calculate the time of arrival as follows:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_1 - x_0}{t_1 - t_0} \Rightarrow t_1 = t_0 + \frac{x_1 - x_0}{v}$$

Using the known values identified in the pictorial representation, we find:

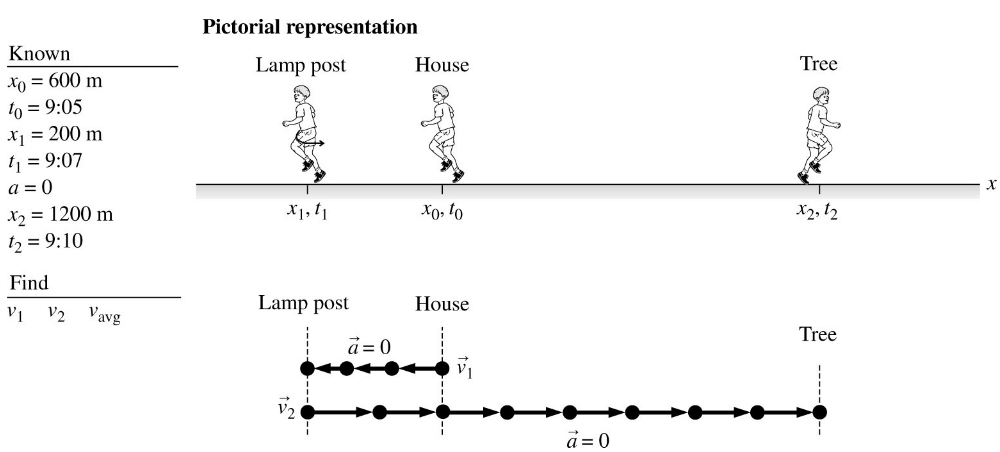
$$t_{\text{Alan }1} = t_{\text{Alan }0} + \frac{x_{\text{Alan }1} - x_{\text{Alan }0}}{v} = 8{:}00\ \text{AM} + \frac{400\ \text{mile}}{50\ \text{miles/hour}} = 8{:}00\ \text{AM} + 8\ \text{hr} = 4{:}00\ \text{PM}$$

$$t_{\text{Beth }1} = t_{\text{Beth }0} + \frac{x_{\text{Beth }1} - x_{\text{Beth }0}}{v} = 9{:}00\ \text{AM} + \frac{400\ \text{mile}}{60\ \text{miles/hour}} = 9{:}00\ \text{AM} + 6.67\ \text{hr} = 3{:}40\ \text{PM}$$

(a) Beth arrives first.
(b) Beth has to wait $t_{\text{Alan }1} - t_{\text{Beth }1} = 20\ \text{minutes}$ for Alan.
Assess: Times of the order of 7 or 8 hours are reasonable in the present problem.

2.2. Model: We will consider Larry to be a particle.

Visualize:



Solve: Since Larry's speed is constant, we can use the following equation to calculate the velocities:

$$v_s = \frac{s_f - s_i}{t_f - t_i}$$

(a) For the interval from the house to the lamppost:

$$v_1 = \frac{200\text{ m} - 600\text{ m}}{9:07 - 9:05} = -200\text{ m/min}$$

For the interval from the lamppost to the tree:

$$v_2 = \frac{1200\text{ m} - 200\text{ m}}{9:10 - 9:07} = +333\text{ m/min}$$

(b) For the average velocity for the entire run:

$$v_{avg} = \frac{1200\text{ m} - 600\text{ m}}{9:10 - 9:05} = +120\text{ m/min}$$

2.3. Solve: (a) The time for each segment is $\Delta t_1 = 50\text{ mi}/40\text{ mph} = 5/4\text{ hr}$ and $\Delta t_2 = 50\text{ mi}/60\text{ mph} = 5/6\text{hr}$. The average speed to the house is

$$\frac{100\text{ mi}}{5/6\text{ h} + 5/4\text{ h}} = 48\text{ mph}$$

(b) Julie drives the distance Δx_1 in time Δt_1 at 40 mph. She then drives the distance Δx_2 in time Δt_2 at 60 mph. She spends the same amount of time at each speed, thus

$$\Delta t_1 = \Delta t_2 \Rightarrow \Delta x_1/40\text{ mph} = \Delta x_2/60\text{ mph} \Rightarrow \Delta x_1 = (2/3)\Delta x_2$$

But $\Delta x_1 + \Delta x_2 = 100$ miles, so $(2/3)\Delta x_2 + \Delta x_2 = 100$ miles. This means $\Delta x_2 = 60$ miles and $\Delta x_1 = 40$ miles. Thus, the times spent at each speed are $\Delta t_1 = 40\text{ mi}/40\text{ mph} = 1.00\text{ h}$ and $\Delta t_2 = 60\text{ mi}/60\text{ mph} = 1.00\text{ h}$. The total time for her return trip is $\Delta t_1 + \Delta t_2 = 2.00\text{ h}$. So, her average speed is $100\text{ mi}/2\text{ h} = 50\text{ mph}$.

2.4. Model: The jogger is a particle.
Solve: The slope of the position-versus-time graph at every point gives the velocity at that point. The slope at $t = 10\text{ s}$ is

$$v = \frac{\Delta s}{\Delta t} = \frac{50\text{ m} - 25\text{ m}}{20\text{ s}} = 1.25\text{ m/s}$$

The slope at $t = 25\text{ s}$ is

$$v = \frac{50\text{ m} - 50\text{ m}}{10\text{ s}} = 0.0\text{ m/s}$$

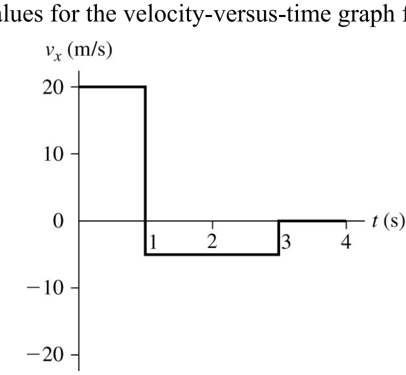
The slope at $t = 35\text{ s}$ is

$$v = \frac{0\text{ m} - 50\text{ m}}{10\text{ s}} = -5.0\text{ m/s}$$

Section 2.2 Instantaneous Velocity

Section 2.3 Finding Position from Velocity

2.5. Solve: (a) We can obtain the values for the velocity-versus-time graph from the equation $v = \Delta s/\Delta t$.



(b) There is only one turning point. At $t = 1$ s the velocity changes from $+20$ m/s to -5 m/s, thus reversing the direction of motion. At $t = 3$ s, there is an abrupt change in motion from -5 m/s to rest, but there is no reversal in motion.

2.6. Visualize: Please refer to Figure EX2.6. The particle starts at $x_0 = 10$ m at $t_0 = 0$. Its velocity is initially in the $-x$ direction. The speed decreases as time increases during the first second, is zero at $t = 1$ s, and then increases after the particle reverses direction.

Solve: (a) The particle reverses direction at $t = 1$ s, when v_x changes sign.

(b) Using the equation $x_f = x_0 +$ area of the velocity graph between t_1 and t_f,

$$x_{2s} = 10 \text{ m} - (\text{area of triangle between 0 s and 1 s}) + (\text{area of triangle between 1 s and 2 s})$$
$$= 10 \text{ m} - \frac{1}{2}(4 \text{ m/s})(1 \text{ s}) + \frac{1}{2}(4 \text{ m/s})(1 \text{ s}) = 10 \text{ m}$$
$$x_{3s} = 10 \text{ m} + \text{area of trapazoid between 2 s and 3 s}$$
$$= 10 \text{ m} + \frac{1}{2}(4 \text{ m/s} + 8 \text{ m/s})(3 \text{ s} - 2 \text{ s}) = 16 \text{ m}$$
$$x_{4s} = x_{3s} + \text{area between 3 s and 4 s}$$
$$= 16 \text{ m} + \frac{1}{2}(8 \text{ m/s} + 12 \text{ m/s})(1 \text{ s}) = 26 \text{ m}$$

2.7. Model: The graph shows the assumption that the blood isn't moving for the first 0.1 s nor at the end of the beat.

Visualize: The graph is a graph of velocity vs. time, so the displacement is the area under the graph—that is, the area of the triangle. The velocity of the blood increases quickly and decreases a bit more slowly.

Solve: Call the distance traveled Δy. The area of a triangle is $\frac{1}{2}BH$.

$$\Delta y = \frac{1}{2}BH = \frac{1}{2}(0.20 \text{ s})(0.80 \text{ m/s}) = 8.0 \text{ cm}$$

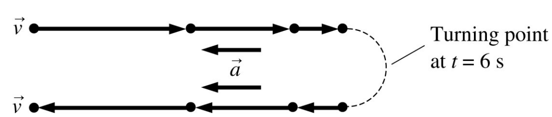
Assess: This distance seems reasonable for one beat.

2.8. Solve: (a) We can calculate the position of the particle at every instant with the equation

$$x_f = x_i + \text{area under the velocity-versus-time graph between } t_i \text{ and } t_f$$

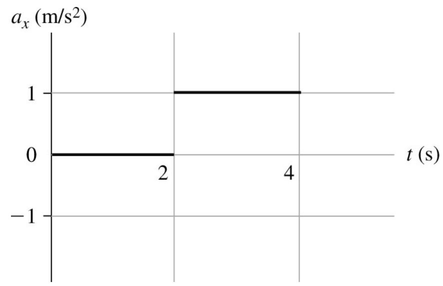
The particle starts from the origin at $t = 0$ s, so $x_i = 0$ m. Notice that the each square of the grid in Figure EX2.8 has "area" $(5 \text{ m/s}) \times (2 \text{ s}) = 10$ m. We can find the area under the curve, and thus determine x, by counting squares. You can see that x = 35 m at $t = 4$ s because there are 3.5 squares under the curve. In addition, $x = 35$ m at t = 8 s because the 5 m represented by the half square between 4 and 6 s is cancelled by the –5 m represented by the half square between 6 and 8 s. Areas beneath the axis are negative areas. The particle passes through $x = 35$ m at $t = 4$ s and again at $t = 8$ s.

(b) The particle moves to the right for $0 \text{ s} \geq t \geq 6$ s, where the velocity is positive. It reaches a turning point at $x = 40$ m at $t = 6$ s. The motion is to the left for $t > 6$ s. This is shown in the motion diagram below.



Section 2.4 Motion with Constant Acceleration

2.9. Visualize: The object has a constant velocity for 2 s and then speeds up between $t = 2$ and $t = 4$.
Solve: A constant velocity from $t = 0$ s to $t = 2$ s means zero acceleration. On the other hand, a linear increase in velocity between $t = 2$ s and $t = 4$ s implies a constant positive acceleration which is the slope of the velocity line.



2.10. Visualize: The graph is a graph of velocity vs. time, so the acceleration is the slope of the graph.
Solve: When the blood is speeding up the acceleration is

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{0.80 \text{ m/s}}{0.05\text{s}} = 16 \text{ m/s}^2$$

When the blood is slowing down the acceleration is

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-0.80 \text{ m/s}}{0.15 \text{ s}} = -5.3 \text{ m/s}^2$$

Assess: 16 m/s^2 is an impressive but reasonable acceleration.

2.11. Solve: (a) At $t = 2.0$ s, the position of the particle is

$$\begin{aligned} x_{2\text{ s}} &= 2.0 \text{ m} + \text{area under velocity graph from } t = 0 \text{ s to } t = 2.0 \text{ s} \\ &= 2.0 \text{ m} + \frac{1}{2}(4.0 \text{ m/s})(2.0 \text{ s}) = 6.0 \text{ m} \end{aligned}$$

(b) From the graph itself at $t = 2.0$ s, $v = 4$ m/s.
(c) The acceleration is

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{fx} - v_{ix}}{\Delta t} = \frac{6 \text{ m/s} - 0 \text{ m/s}}{3 \text{ s}} = 2 \text{ m/s}^2$$

2.12. Solve: (a) Using the equation

$$x_f = x_i + \text{area under the velocity-versus-time graph between } t_i \text{ and } t_f$$

we have

$$\begin{aligned} x(\text{at } t = 1 \text{ s}) &= x(\text{at } t = 0 \text{ s}) + \text{area between } t = 0 \text{ s and } t = 1 \text{ s} \\ &= 2.0 \text{ m} + (4 \text{ m/s})(1 \text{ s}) = 6 \text{ m} \end{aligned}$$

Reading from the velocity-versus-time graph, $v_x(\text{at } t = 1 \text{ s}) = 4$ m/s. Also, $a_x = \text{slope} = \Delta v/\Delta t = 0 \text{ m/s}^2$.
(b) $x(\text{at } t = 3.0 \text{ s}) = x(\text{at } t = 0 \text{ s}) +$ area between $t = 0$ s and $t = 3$ s

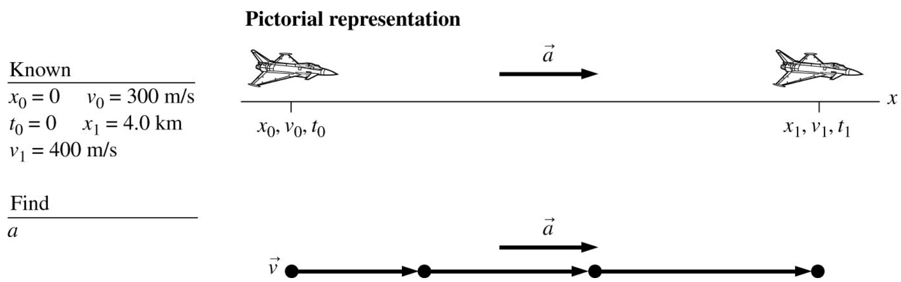
$$= 2.0 \text{ m} + 4 \text{ m/s} \times 2 \text{ s} + 2 \text{ m/s} \times 1 \text{ s} + (1/2) \times 2 \text{ m/s} \times 1 \text{ s} = 13.0 \text{ m}$$

Reading from the graph, $v_x(t = 3 \text{ s}) = 2$ m/s. The acceleration is

$$a_x(t = 3 \text{ s}) = \text{slope} = \frac{v_x(\text{at } t = 4 \text{ s}) - v_x(\text{at } t = 2 \text{ s})}{2 \text{ s}} = -2 \text{ m/s}^2$$

2.13. Model: Represent the jet plane as a particle.

Visualize:



Solve: Since we don't know the time of acceleration, we will use

$$v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$\Rightarrow a = \frac{v_1^2 - v_0^2}{2x_1} = \frac{(400\text{ m/s})^2 - (300\text{ m/s})^2}{2(4000\text{ m})} = 8.75\text{ m/s}^2 \approx 8.8\text{ m/s}^2$$

The acceleration of the jet is not quite equal to g, the acceleration due to gravity; this seems reasonable for a jet.

2.14. Model: Model the air as a particle.
Visualize: Use the definition of acceleration and then convert units.
Solve:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{150\text{ km/h}}{0.50\text{ s}}\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 83\text{ m/s}^2$$

Assess: 83 m/s^2 is a remarkable acceleration.

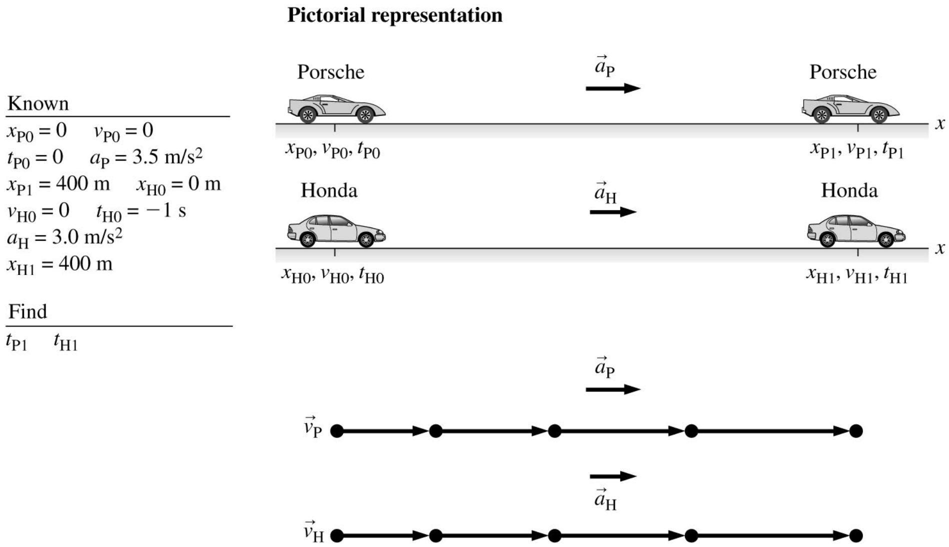
2.15. Model: We are using the particle model for the skater and the kinematics model of motion under constant acceleration.
Solve: Since we don't know the time of acceleration we will use

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

$$\Rightarrow a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.0\text{ m/s})^2 - (8.0\text{ m/s})^2}{2(5.0\text{ m})} = -2.8\text{ m/s}^2$$

Assess: A deceleration of 2.8 m/s^2 is reasonable.

2.16. Model: We are assuming both cars are particles.
Visualize:



Solve: The Porsche's time to finish the race is determined from the position equation

$$x_{P1} = x_{P0} + v_{P0}(t_{P1} - t_{P0}) + \frac{1}{2}a_P(t_{P1} - t_{P0})^2$$

$$\Rightarrow 400 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(3.5 \text{ m/s}^2)(t_{P1} - 0 \text{ s})^2 \Rightarrow t_{P1} = 15.1 \text{ s}$$

The Honda's time to finish the race is obtained from Honda's position equation as

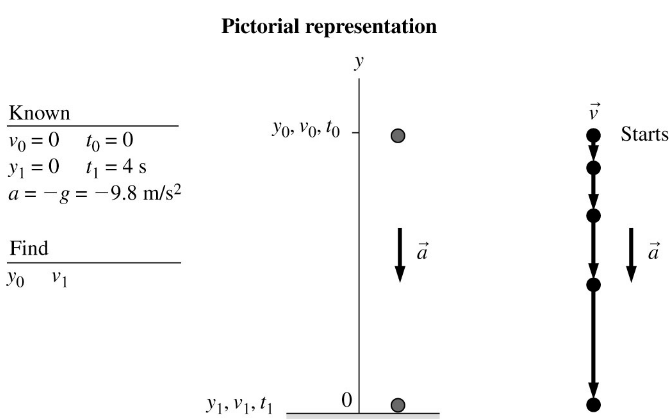
$$x_{H1} = x_{H0} + v_{H0}(t_{H1} - t_{H0}) + \frac{1}{2}a_{H0}(t_{H1} - t_{H0})^2$$

$$400 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(3.0 \text{ m/s}^2)(t_{H1} + 1 \text{ s})^2 \Rightarrow t_{H1} = 15.3 \text{ s}$$

So, the Porsche wins.
Assess: The numbers are contrived for the Porsche to win, but the time to go 400 m seems reasonable.

Section 2.5 Free Fall

2.17. Model: Represent the spherical drop of molten metal as a particle.
Visualize:



Solve: **(a)** The shot is in free fall, so we can use free fall kinematics with $a = -g$. The height must be such that the shot takes 4 s to fall, so we choose $t_1 = 4$ s. Then,

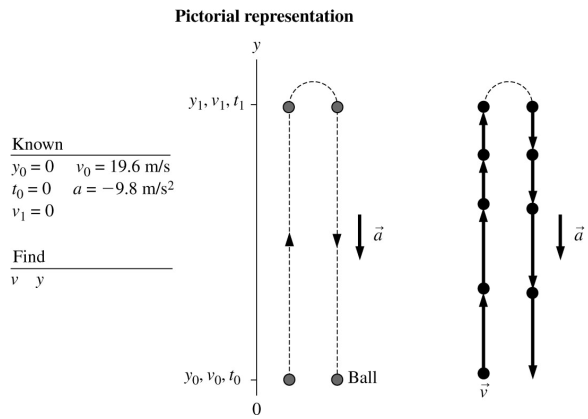
$$y_1 = y_0 + v_0(t_1 - t_0) - \frac{1}{2}g(t_1 - t_0)^2 \Rightarrow y_0 = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8\ \text{m/s}^2)(4\ \text{s})^2 = 78.4\ \text{m}$$

(b) The impact velocity is $v_1 = v_0 - g(t_1 - t_0) = -gt_1 = -39.2$ m/s.

Assess: Note the minus sign. The question asked for *velocity*, not speed, and the *y*-component of $\vec{v}$ is negative because the vector points downward.

2.18. Model: Assume the ball undergoes free-fall acceleration and that the ball is a particle.
Visualize:

(a)



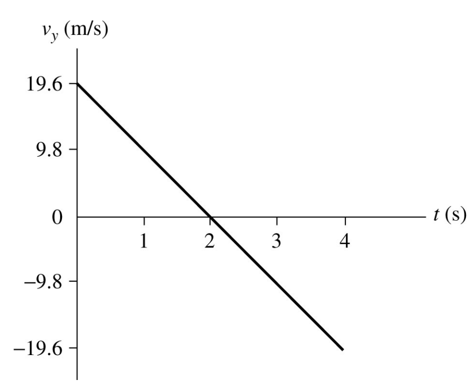
Solve: **(a)** We will use the kinematic equations

$$v = v_0 + a(t - t_0) \quad \text{and} \quad y = y_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

as follows:

$$v(\text{at } t = 1.0\ \text{s}) = 19.6\ \text{m/s} + (-9.8\ \text{m/s}^2)(1.0\ \text{s} - 0\ \text{s}) = 9.8\ \text{m/s}$$
$$y(\text{at } t = 1.0\ \text{s}) = 0\ \text{m} + (19.6\ \text{m/s})(1.0\ \text{s} - 0\ \text{s}) + 1/2(-9.8\ \text{m/s}^2)(1.0\ \text{s} - 0\ \text{s})^2 = 14.7\ \text{m}$$
$$v(\text{at } t = 2.0\ \text{s}) = 19.6\ \text{m/s} + (-9.8\ \text{m/s}^2)(2.0\ \text{s} - 0\ \text{s}) = 0\ \text{m/s}$$
$$y(\text{at } t = 2.0\ \text{s}) = 0\ \text{m} + (19.6\ \text{m/s})(2.0\ \text{s} - 0\ \text{s}) + 1/2(-9.8\ \text{m/s}^2)(2.0\ \text{s} - 0\ \text{s})^2 = 19.6\ \text{m}$$
$$v(\text{at } t = 3.0\ \text{s}) = 19.6\ \text{m/s} + (-9.8\ \text{m/s}^2)(3\ \text{s} - 0\ \text{s}) = -9.8\ \text{m/s}$$
$$y(\text{at } t = 3.0\ \text{s}) = 0\ \text{m} + (19.6\ \text{m/s})(3.0\ \text{s} - 0\ \text{s}) + 1/2(-9.8\ \text{m/s}^2)(3.0\ \text{s} - 0\ \text{s})^2 = 14.7\ \text{m}$$
$$v(\text{at } t = 4.0\ \text{s}) = 19.6\ \text{m/s} + (-9.8\ \text{m/s}^2)(4.0\ \text{s} - 0\ \text{s}) = -19.6\ \text{m/s}$$
$$y(\text{at } t = 4.0\ \text{s}) = 0\ \text{m} + (19.6\ \text{m/s})(4.0\ \text{s} - 0\ \text{s}) + 1/2(-9.8\ \text{m/s}^2)(4.0\ \text{s} - 0\ \text{s})^2 = 0\ \text{m}$$

(b)



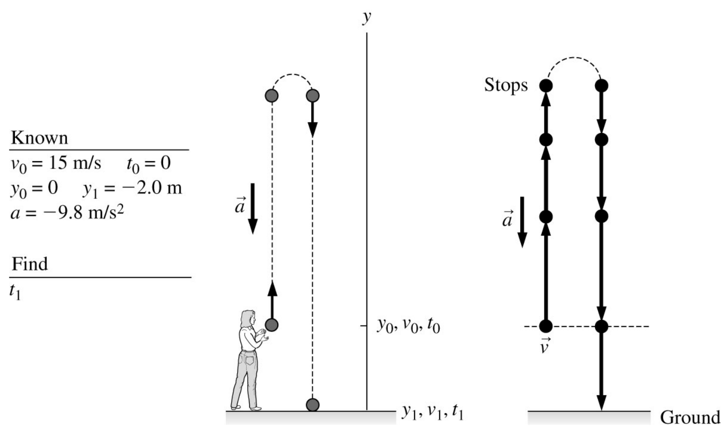
Assess: **(a)** A downward acceleration of 9.8 m/s^2 on a particle that has been given an initial upward velocity of $+19.6\text{ m/s}$ will reduce its speed to 9.8 m/s after 1 s and then to zero after 2 s. The answers obtained in this solution are consistent with the above logic.

(b) Velocity changes linearly with a negative uniform acceleration of 9.8 m/s^2. The position is symmetrical in time around the highest point which occurs at $t = 2$ s.

2.19. Model: We model the ball as a particle.

Visualize:

Pictorial representation



Solve: Once the ball leaves the student's hand, the ball is in free fall and its acceleration is equal to the free-fall acceleration g that always acts vertically downward toward the center of the earth. According to the constant-acceleration kinematic equations of motion

$$y_1 = y_0 + v_0 \Delta t + \frac{1}{2} a \Delta t^2$$

Substituting the known values

$$-2\text{ m} = 0\text{ m} + (15\text{ m/s})t_1 + (1/2)(-9.8\text{ m/s}^2)t_1^2$$

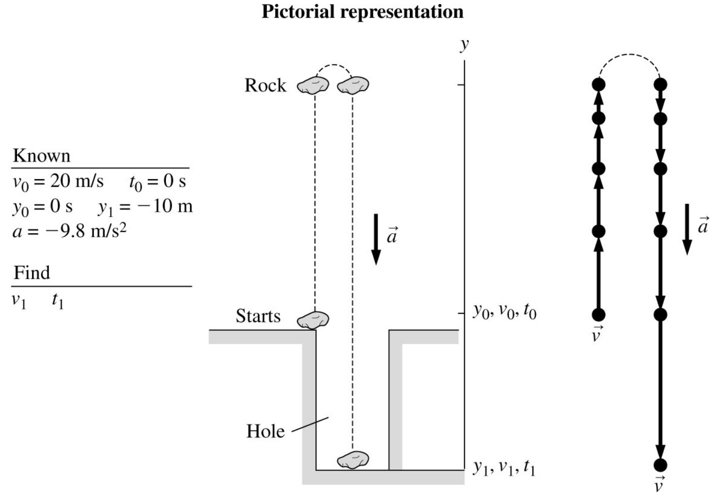
One solution of this quadratic equation is $t_1 = 3.2$ s. The other root of this equation yields a negative value for t_1, which is not valid for this problem.

Assess: A time of 3.2 s is reasonable.

2.20. Model: We will use the particle model and the constant-acceleration kinematic equations.

Visualize:

Pictorial representation



Solve: **(a)** Substituting the known values into $y_1 = y_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$, we get

$$-10\text{ m} = 0\text{ m} + 20\text{ (m/s)}t_1 + \frac{1}{2}(-9.8\text{ m/s}^2)t_1^2$$

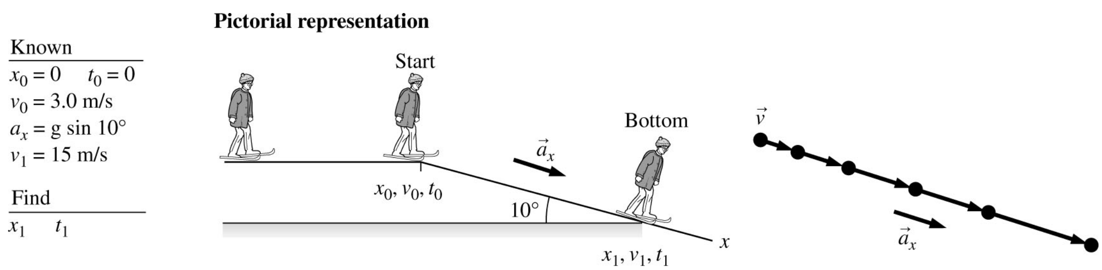
One of the roots of this equation is negative and is not relevant physically. The other root is $t_1 = 4.53$ s, which is the answer to part (b). Using $v_1 = v_0 + a\Delta t$, we obtain

$$v_1 = 20\text{(m/s)} + (-9.8\text{ m/s}^2)(4.53\text{ s}) = -24\text{ m/s}$$

(b) The time is 4.5 s.
Assess: A time of 4.5 s is a reasonable value. The rock's velocity as it hits the bottom of the hole has a negative sign because of its downward direction. The magnitude of 24 m/s compared to 20 m/s, when the rock was tossed up, is consistent with the fact that the rock travels an additional distance of 10 m into the hole.

Section 2.6 Motion on an Inclined Plane

2.21. Model: We will model the skier as a particle.
Visualize:

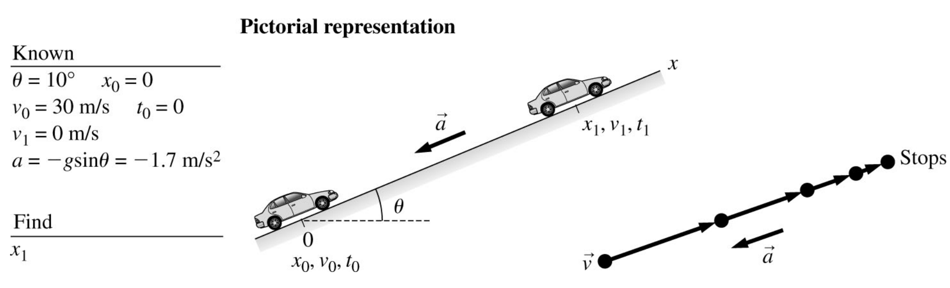


Note that the skier's motion on the horizontal, frictionless snow is not of any interest to us. Also note that the acceleration parallel to the incline is equal to $g\sin 10°$.
Solve: Using the following constant-acceleration kinematic equations,

$$v_{fx}^2 = v_{ix}^2 + 2a_x(x_f - x_i)$$
$$\Rightarrow (15\text{ m/s})^2 = (3.0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)\sin 10°(x_1 - 0\text{ m}) \Rightarrow x_1 = 64\text{ m}$$
$$v_{fx} = v_{ix} + a_x(t_f - t_i)$$
$$\Rightarrow (15\text{ m/s}) = (3.0\text{ m/s}) + (9.8\text{ m/s}^2)(\sin 10°)t \Rightarrow t = 7.1\text{ s}$$

Assess: A time of 7.1 s to cover 64 m is a reasonable value.

2.22. Model: Represent the car as a particle.
Visualize:



Solve: Note that the problem "ends" at a turning point, where the car has an instantaneous speed of 0 m/s before rolling back down. The rolling back motion is *not* part of this problem. If we assume the car rolls without friction, then we have motion on a frictionless inclined plane with an accleration $a = -g\sin\theta = -g\sin 10° = -1.7\ \text{m/s}^2$. Constant acceleration kinematics gives

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \Rightarrow 0\ \text{m}^2/\text{s}^2 = v_0^2 + 2ax_1 \Rightarrow x_1 = -\frac{v_0^2}{2a} = -\frac{(30\ \text{m/s})^2}{2(-1.7\ \text{m/s}^2)} = 265\ \text{m}$$

Notice how the two negatives canceled to give a positive value for x_1.

Assess: We must include the minus sign because the $\vec{a}$ vector points *down* the slope, which is in the negative *x*-direction.

Section 2.7 Instantaneous Acceleration

2.23. Solve: $x = (2t^2 - t + 1)$ m

(a) The position $t = 2$ s is $x_{2\text{s}} = [2(2)^2 - 2 + 1]$ m $= 7$ m.

(b) The velocity is the derivative $v = dx/dt$ and the velocity at $t = 2$ s is calculated as follows:

$$v = (4t - 1)\ \text{m/s} \Rightarrow v_{2\text{s}} = [4(2) - 1]\ \text{m/s} = 7\ \text{m/s}$$

(c) The acceleration is the derivative $a = dv/dt$ and the acceleration at $t = 2$ s is calculated as follows:

$$a = (4)\ \text{m/s}^2 \Rightarrow a_{2\text{s}} = 4\ \text{m/s}^2$$

2.24. Solve: The formula for the particle's position along the *x*-axis is given by

$$x_\text{f} = x_\text{i} + \int_{t_\text{i}}^{t_\text{f}} v_x\, dt$$

Using the expression for v_x we get

$$x_\text{f} = x_\text{i} + \tfrac{2}{3}[t_\text{f}^3 - t_\text{i}^3] \qquad a_x = \frac{dv_x}{dt} = \frac{d}{dt}(2t^2\ \text{m/s}) = 4t\ \text{m/s}^2$$

(a) The particle's position at $t = 1$ s is $x_{1\text{ s}} = 1\ \text{m} + \tfrac{2}{3}\ \text{m} = \tfrac{5}{3}$ m.

(b) The particle's speed at $t = 1$ s is $v_{1\text{ s}} = 2$ m/s.

(c) The particle's acceleration at $t = 1$ s is $a_{1\text{ s}} = 4\ \text{m/s}^2$.

2.25. Solve: The formula for the particle's velocity is given by

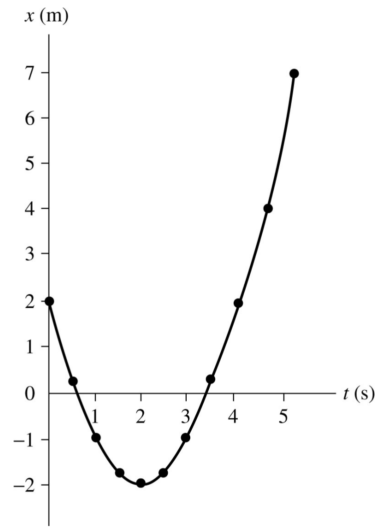
$$v_\text{f} = v_\text{i} + \text{area under the acceleration curve between } t_\text{i} \text{ and } t_\text{f}$$

For $t = 4$ s, we get

$$v_{4\text{ s}} = 8\ \text{m/s} + \frac{1}{2}(4\ \text{m/s}^2)4\ \text{s} = 16\ \text{m/s}$$

Assess: The acceleration is positive but decreases as a function of time. The initial velocity of 8.0 m/s will therefore increase. A value of 16 m/s is reasonable.

2.26. Solve: (a)



(b) To be completed by student.

(c) $\frac{dx}{dt} = v_x = 2t - 4 \Rightarrow v_x(\text{at } t = 1\text{ s}) = [2\text{ m/s}^2(1\text{ s}) - 4\text{ m/s}] = -2\text{ m/s}$

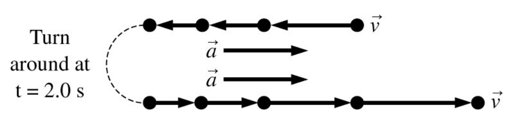
(d) There is a turning point at $t = 2$ s. At that time $x = -2$ m.

(e) Using the equation in part (c),

$$v_x = 4\text{ m/s} = (2t - 4)\text{ m/s} \Rightarrow t = 4$$

Since $x = (t^2 - 4t + 2)$ m, $x = 2$ m.

(f)

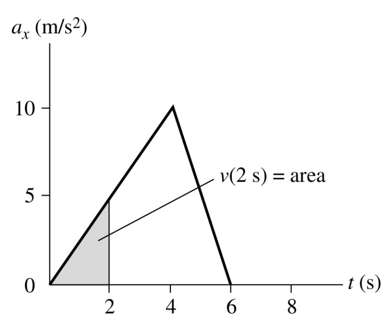


2.27. Solve: The graph for particle A is a straight line from $t = 2$ s to $t = 8$ s. The slope of this line is -10 m/s, which is the velocity at $t = 7.0$ s. The negative sign indicates motion toward lower values on the x-axis. The velocity of particle B at $t = 7.0$ s can be read directly from its graph. It is -20 m/s. The velocity of particle C can be obtained from the equation

$$v_f = v_i + \text{area under the acceleration curve between } t_i \text{ and } t_f$$

This area can be calculated by adding up three sections. The area between $t = 0$ s and $t = 2$ s is 40 m/s, the area between $t = 2$ s and $t = 5$ s is 45 m/s, and the area between $t = 5$ s and $t = 7$ s is -20 m/s. We get $(10\text{ m/s}) + (40\text{ m/s}) + (45\text{ m/s}) - (20\text{ m/s}) = 75$ m/s.

2.28. Visualize:



Solve: We will determine the object's velocity using graphical methods first and then using calculus. Graphically, $v(t) = v_0 +$ area under the acceleration curve from 0 to t. In this case, $v_0 = 0$ m/s. The area at each time t requested is a triangle.

$$t = 0 \text{ s} \qquad v(t = 0 \text{ s}) = v_0 = 0 \text{ m/s}$$

$$t = 2 \text{ s} \qquad v(t = 2 \text{ s}) = \frac{1}{2}(2 \text{ s})(5 \text{ m/s}) = 5 \text{ m/s}$$

$$t = 4 \text{ s} \qquad v(t = 4 \text{ s}) = \frac{1}{2}(4 \text{ s})(10 \text{ m/s}) = 20 \text{ m/s}$$

$$t = 6 \text{ s} \qquad v(t = 6 \text{ s}) = \frac{1}{2}(6 \text{ s})(10 \text{ m/s}) = 30 \text{ m/s}$$

$$t = 8 \text{ s} \qquad v(t = 8 \text{ s}) = v(t = 6 \text{ s}) = 30 \text{ m/s}$$

The last result arises because there is no additional area after $t = 6$ s. Let us now use calculus. The acceleration function $a(t)$ consists of three pieces and can be written:

$$a(t) = \begin{cases} 2.5t & 0 \text{ s} \le t \le 4 \text{ s} \\ -5t + 30 & 4 \text{ s} \le t \le 6 \text{ s} \\ 0 & 6 \text{ s} \le t \le 8 \text{ s} \end{cases}$$

These were determined by the slope and the y-intercept of each of the segments of the graph. The velocity function is found by integration as follows: For $0 \le t \le 4$ s,

$$v(t) = v(t = 0 \text{ s}) + \int_0^t a(t)dt = 0 + 2.5\frac{t^2}{2}\bigg|_0^t = 1.25t^2$$

This gives

$$t = 0 \text{ s} \quad v(t = 0 \text{ s}) = 0 \text{ m/s}$$

$$t = 2 \text{ s} \quad v(t = 2 \text{ s}) = 5 \text{ m/s}$$

$$t = 4 \text{ s} \quad v(t = 4 \text{ s}) = 20 \text{ m/s}$$

For $4 \text{ s} \le t \le 6 \text{ s}$,

$$v(t) = v(t = 4 \text{ s}) + \int_4^t a(t)dt = 20 \text{ m/s } + \left[\frac{-5t^2}{2} + 30t\right]_4^t = -2.5t^2 + 30t - 60$$

This gives:

$$t = 6 \text{ s} \quad v(t = 6 \text{ s}) = 30 \text{ m/s}$$

For $6 \text{ s} \le t \le 8 \text{ s}$,

$$v(t) = v(t = 6 \text{ s}) + \int_6^t a(t)dt = 30 \text{ m/s} + 0 \text{ m/s} = 30 \text{ m/s}$$

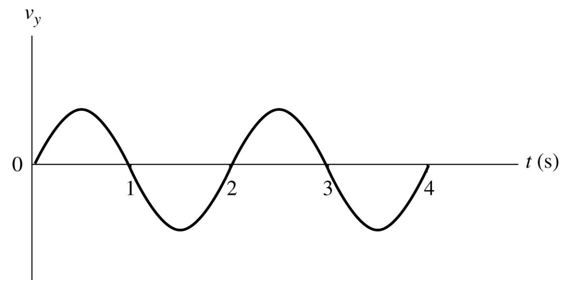
This gives:

$$t = 8 \text{ s} \quad v(t = 8 \text{ s}) = 30 \text{ m/s}$$

Assess: The same velocities are found using calculus and graphs, but the graphical method is easier for simple graphs.

2.29. Solve: (a) The velocity-versus-time graph is the derivative with respect to time of the distance-versus-time graph. The velocity is zero when the slope of the position-versus-time graph is zero, the velocity is most positive when the slope is most positive, and the velocity is most negative when the slope is most negative. The slope is zero at $t = 0$, 1 s, 2 s, 3 s, . . . ; the slope is most positive at t = 0.5 s, 2.5 s, . . ; and the slope is most negative at t = 1.5 s, 3.5 s, . . .

(b)



2.30. Solve: The given function for the velocity is $v_x = t^2 - 7t + 10$.

(a) The turning points are when the velocity changes sign. Set $v_x = 0$ and check that it actually changes sign at those times. The function factors into the product of two binomials:

$$v_x = (t-2)(t-5) \Rightarrow v_x = 0 \text{ when } t = 2 \text{ s and } t = 5 \text{ s}$$

Indeed, the function changes sign at those two times.

(b) The acceleration is given by the derivative of the velocity.

$$a_x = \frac{dv_x}{dt} = 2t - 7$$

Plug in the times from part (a): $a_x(2\text{ s}) = 2(2) - 7 = -3 \text{ m/s}^2$ and $a_x(5\text{ s}) = 2(5) - 7 = 3 \text{ m/s}^2$

Assess: This problem does not have constant acceleration so the kinematic equations do not apply, but $a = dv/dt$ always applies.

2.31. Solve: (a) The velocity-versus-time graph is given by the derivative with respect to time of the position function:

$$v_x = \frac{dx}{dt} = (6t^2 - 18t) \text{ m/s}$$

For $v_x = 0$ m/s, there are two solutions to the quadratic equation: $t = 0$ s and $t = 3$ s.

(b) At the first of these solutions,

$$x(\text{at } t = 0 \text{ s}) = 2(0 \text{ s})^3 - 9(0 \text{ s})^2 + 12 = 12 \text{ m}$$

The acceleration is the derivative of the velocity function:

$$a_x = \frac{dv_x}{dt} = (12t - 18) \text{ m/s}^2 \Rightarrow a\ (\text{at } t = 0 \text{ s}) = -18 \text{ m/s}^2$$

At the second solution,

$$x(\text{at } t = 3 \text{ s}) = 2(3 \text{ s})^3 - 9(3 \text{ s})^2 + 12 = -15 \text{ m} \qquad a_x(\text{at } t = 3 \text{ s}) = 12(3 \text{ s}) - 18 = +18 \text{ m/s}^2$$

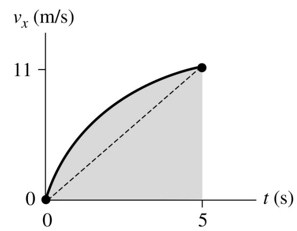
2.32. Model: Represent the object as a particle.

Solve: (a) Known information: $x_0 = 0$ m, $v_0 = 0$ m/s, $x_1 = 40$ m, $v_1 = 11$ m/s, $t_1 = 5$ s. *If* the acceleration is uniform (constant a), then the motion must satisfy the three equations

$$x_1 = \frac{1}{2}at_1^2 \Rightarrow a = 3.20 \text{ m/s}^2 \quad v_1 = at_1 \Rightarrow a = 2.20 \text{ m/s}^2 \quad v_1^2 = 2ax_1 \Rightarrow a = 1.51 \text{ m/s}^2$$

But each equation gives a different value of a. Thus the motion is *not* uniform acceleration.

(b) We know two points on the velocity-versus-time graph, namely at $t_0 = 0$ and $t_1 = 5$ s. What shape does the function have between these two points? *If* the acceleration was uniform, which it's not, then the graph would be a straight line. The area under the graph is the displacement Δx. From the figure you can see that $\Delta x = 27.5$ m for a straight-line graph. But we know that, in reality, $\Delta x = 40$ m. To get a larger Δx, the graph must bulge *upward* above the straight line. Thus the graph is curved, and it is concave downward.



2.33. Solve: The position is the integral of the velocity.

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = x_0 + \int_0^{t_1} kt^2 dt = x_0 + \tfrac{1}{3}kt^3\Big|_0^{t_1} = x_0 + \tfrac{1}{3}kt_1^3$$

We're given that $x_0 = -9.0$ m and that the particle is at $x_1 = 9.0$ m at $t_1 = 3.0$ s. Thus

$$9.0 \text{ m} = (-9.0 \text{ m}) + \tfrac{1}{3}k(3.0 \text{ s})^3 = (-9.0 \text{ m}) + k(9.0 \text{ s}^3)$$

Solving for k gives $k = 2.0 \text{ m/s}^3$.

2.34. Solve: (a) The velocity is the integral of the acceleration.

$$v_{1x} = v_{0x} + \int_{t_0}^{t_1} a_x dt = 0 \text{ m/s} + \int_0^{t_1} (10 - t)dt = \left(10t - \tfrac{1}{2}t^2\right)\Big|_0^{t_1} = 10t_1 - \tfrac{1}{2}t_1^2$$

The velocity is zero when

$$v_{1x} = 0 \text{ m/s} = \left(10t_1 - \tfrac{1}{2}t_1^2\right) = \left(10 - \tfrac{1}{2}t_1\right) \times t_1$$

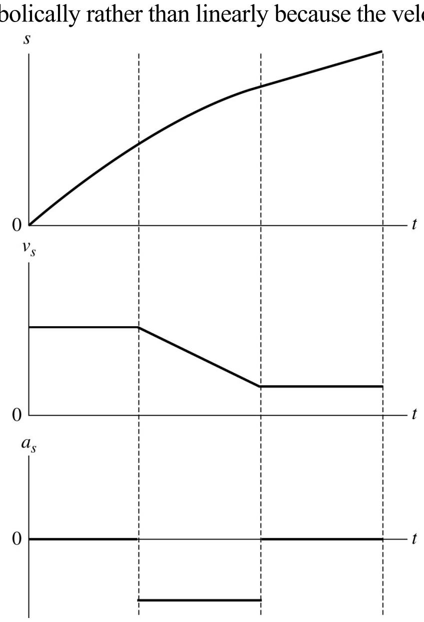
$$\Rightarrow t_1 = 0 \text{ s} \quad \text{or} \quad t_1 = 20 \text{ s}$$

The first solution is the initial condition. Thus the particle's velocity is again 0 m/s at $t_1 = 20$ s.

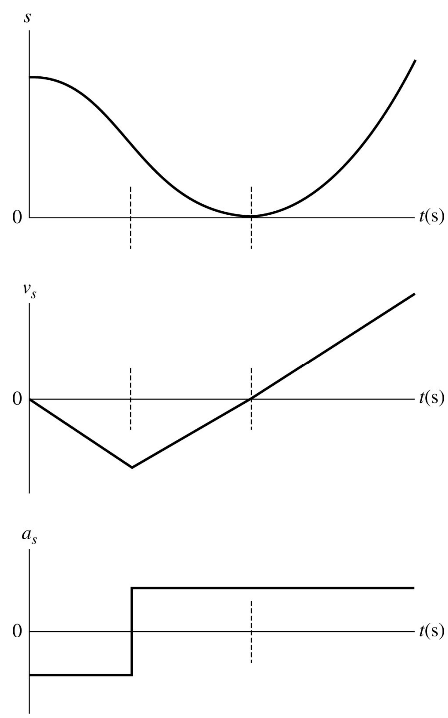
(b) Position is the integral of the velocity. At $t_1 = 20$ s, and using $x_0 = 0$ m at $t_0 = 0$ s, the position is

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = 0 \text{ m} + \int_0^{20} \left(10t - \tfrac{1}{2}t^2\right) dt = 5t^2\Big|_0^{20} - \tfrac{1}{6}t^3\Big|_0^{20} = 667 \text{ m}$$

2.35. Model: Represent the ball as a particle.
Visualize: Please refer to Figure P2.35.
Solve: In the first and third segments the acceleration a_s is zero. In the second segment the acceleration is negative and constant. This means the velocity v_s will be constant in the first two segments and will decrease linearly in the third segment. Because the velocity is constant in the first and third segments, the position s will increase linearly. In the second segment, the position will increase parabolically rather than linearly because the velocity decreases linearly with time.

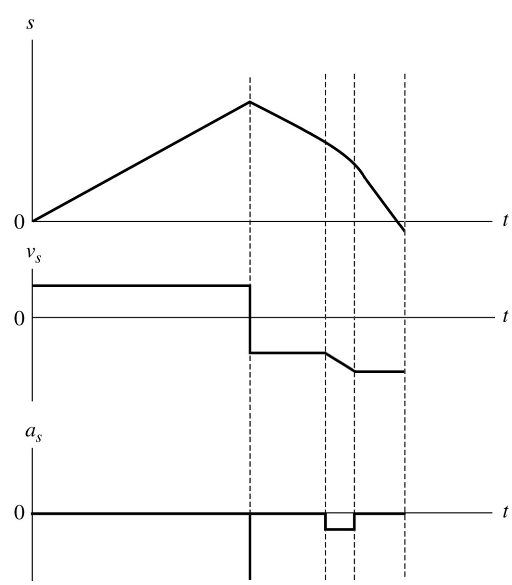


2.36. Model: Represent the ball as a particle.
Visualize: Please refer to Figure P2.36. The ball rolls down the first short track, then up the second short track, and then down the long track. s is the distance along the track measured from the left end (where $s = 0$). Label $t = 0$ at the beginning, that is, when the ball starts to roll down the first short track.
Solve: Because the incline angle is the same, the magnitude of the acceleration is the same on all of the tracks.



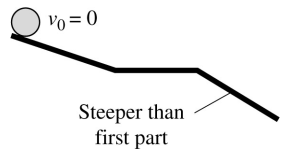
Assess: Note that the derivative of the s versus t graph yields the v_s versus t graph. And the derivative of the v_s versus t graph gives rise to the a_s versus t graph.

2.37. Model: Represent the ball as a particle.
Visualize: The ball moves to the right along the first track until it strikes the wall, which causes it to move to the left on a second track. The ball then descends on a third track until it reaches the fourth track, which is horizontal.
Solve:

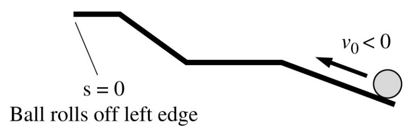


Assess: Note that the time derivative of the position graph yields the velocity graph, and the derivative of the velocity graph gives the acceleration graph.

2.38. Visualize: Please refer to Figure P2.38.
Solve:



2.39. Visualize: Please refer to Figure P2.39.
Solve:



2.40. Model: The plane is a particle and the constant-acceleration kinematic equations hold.
Solve: (a) To convert 80 m/s to mph, we calculate 80 m/s × 1 mi/1609 m × 3600 s/h = 179 mph.
(b) Using $a_s = \Delta v/\Delta t$, we have,

$$a_s(t = 0 \text{ to } t = 10 \text{ s}) = \frac{23 \text{ m/s} - 0 \text{ m/s}}{10 \text{ s} - 0 \text{ s}} = 2.3 \text{ m/s}^2 \qquad a_s(t = 20 \text{ s to } t = 30 \text{ s}) = \frac{69 \text{ m/s} - 46 \text{ m/s}}{30 \text{ s} - 20 \text{ s}} = 2.3 \text{ m/s}^2$$

For all time intervals a is 2.3 m/s^2.
(c) Using kinematics as follows:

$$v_{\text{fs}} = v_{\text{is}} + a(t_{\text{f}} - t_{\text{i}}) \Rightarrow 80 \text{ m/s} = 0 \text{ m/s} + (2.3 \text{ m/s}^2)(t_{\text{f}} - 0 \text{ s}) \Rightarrow t_{\text{f}} = 35 \text{ s}$$

(d) Using the above values, we calculate the takeoff distance as follows:

$$s_{\text{f}} = s_{\text{i}} + v_{\text{is}}(t_{\text{f}} - t_{\text{i}}) + \frac{1}{2}a_{\text{s}}(t_{\text{f}} - t_{\text{i}})^2 = 0 \text{ m} + (0 \text{ m/s})(35 \text{ s}) + \frac{1}{2}(2.3 \text{ m/s}^2)(35 \text{ s})^2 = 1410 \text{ m}$$

For safety, the runway should be $3 \times 1410 \text{ m} = 4230 \text{ m}$ or 2.6 mi. This is longer than the 2.4 mi long runway, so the takeoff is not safe.

2.41. Model: Represent the car as a particle.
Solve: (a) First, we will convert units:

$$60\frac{\text{miles}}{\text{hour}} \times \frac{1 \text{ hour}}{3600 \text{ s}} \times \frac{1610 \text{ m}}{1 \text{ mile}} = 27 \text{ m/s}$$

The motion is constant acceleration, so

$$v_1 = v_0 + a\Delta t \Rightarrow a = \frac{v_1 - v_0}{\Delta t} = \frac{(27 \text{ m/s} - 0 \text{ m/s})}{10 \text{ s}} = 2.7 \text{ m/s}^2$$

(b) The fraction is $a/g = 2.7/9.8 = 0.28$. So a is 28% of g.
(c) The distance is calculated as follows:

$$x_1 = x_0 + v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}a(\Delta t)^2 = 1.3 \times 10^2 \text{ m} = 4.3 \times 10^2 \text{ feet}$$

2.42. Model: Represent the spaceship as a particle.
Solve: (a) The known information is: $x_0 = 0$ m, $v_0 = 0$ m/s, $t_0 = 0$ s, $a = g = 9.8 \text{ m/s}^2$, and $v_1 = 3.0 \times 10^8$ m/s. Constant acceleration kinematics gives

$$v_1 = v_0 + a\Delta t \Rightarrow \Delta t = t_1 = \frac{v_1 - v_0}{a} = 3.06 \times 10^7 \text{ s}$$

The problem asks for the answer in days, so we need a conversion:

$$t_1 = (3.06\times10^7\text{ s})\times\frac{1\text{ hour}}{3600\text{ s}}\times\frac{1\text{ day}}{24\text{ hour}} = 3.54\times10^2\text{ days} \approx 3.6\times10^2\text{ days}$$

(b) The distance traveled is

$$x_1 - x_0 = v_0\Delta t + \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}at_1^2 = 4.6\times10^{15}\text{ m}$$

(c) The number of seconds in a year is

$$1\text{ year} = 365\text{ days}\times\frac{24\text{ hours}}{1\text{ day}}\times\frac{3600\text{ s}}{1\text{ hour}} = 3.15\times10^7\text{ s}$$

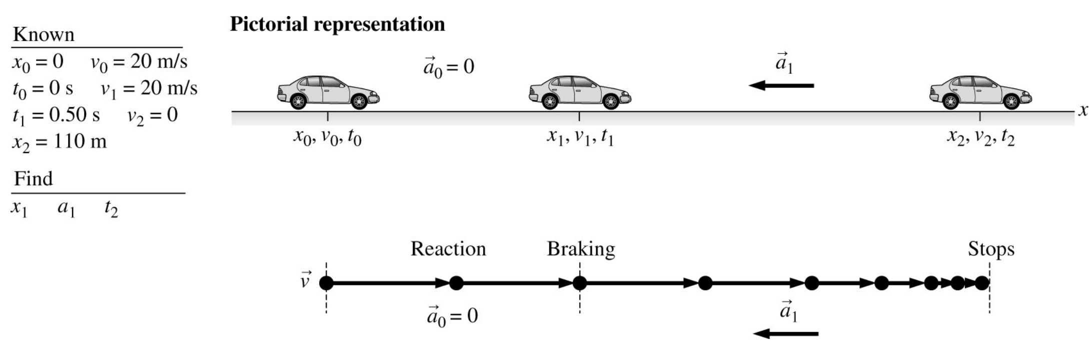
In one year light travels a distance

$$1\text{ light year} = (3.0\times10^8\text{ m/s})(3.15\times10^7\text{ s}) = 9.47\times10^{15}\text{ m}$$

The distance traveled by the spaceship is $4.6\times10^{15}\text{ m}/9.46\times10^{15}\text{ m} = 0.49$ of a light year.

Assess: Note that x_1 gives "Where is it?" rather than "How far has it traveled?" "How far" is represented by $x_1 - x_0$. They happen to be the same number in this problem, but that isn't always the case.

2.43. Model: The car is a particle and constant-acceleration kinematic equations hold.
Visualize:



Solve: (a) This is a two-part problem. During the reaction time,

$$x_1 = x_0 + v_0(t_1 - t_0) + 1/2a_0(t_1 - t_0)^2$$
$$= 0\text{ m} + (20\text{ m/s})(0.50\text{ s} - 0\text{ s}) + 0\text{ m} = 10\text{ m}$$

After reacting, $x_2 - x_1 = 110\text{ m} - 10\text{ m} = 100\text{ m}$, that is, you are 100 m away from the intersection.

(b) To stop successfully,

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow (0\text{ m/s})^2 = (20\text{ m/s})^2 + 2a_1(100\text{ m}) \Rightarrow a_1 = -2\text{ m/s}^2$$

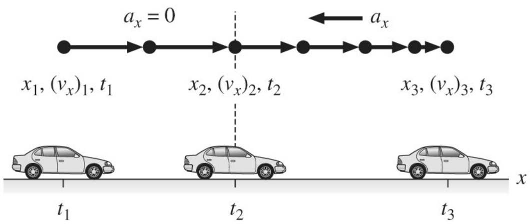
(c) The time it takes to stop once the brakes are applied can be obtained as follows:

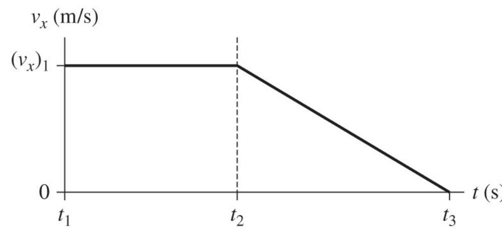
$$v_2 = v_1 + a_1(t_2 - t_1) \Rightarrow 0\text{ m/s} = 20\text{ m/s} + (-2\text{ m/s}^2)(t_2 - 0.50\text{ s}) \Rightarrow t_2 = 11\text{ s}$$

The total time to stop since the light turned red is 11.5 s.

2.44. Model: Model the car as a particle. Use the kinematic equations.

Visualize:





Solve:
(a) In the pictorial representation $d = x_2 - x_0$ and $t_1 = t_R$. At the end of the reaction time the position is $x_1 = x_0 + v_0 t_R$ and $v_1 = v_0$. We need to end up at rest with $v_2 = 0$. During the braking time we have $v_2^2 = v_1^2 + 2a(x_2 - x_1)$; solve that for *a* and set $v_2 = 0$.

$$a = \frac{v_2^2 - v_1^2}{2(x_2 - x_1)} = \frac{-v_1^2}{2(x_2 - x_1)} = \frac{-v_0^2}{2(x_2 - (x_0 + v_0 t_R))} = \frac{-v_0^2}{2(d - v_0 t_R)}$$

The required deceleration is the absolute value of *a*; (the denominator is positive because we were told $d > v_0 t_R$).

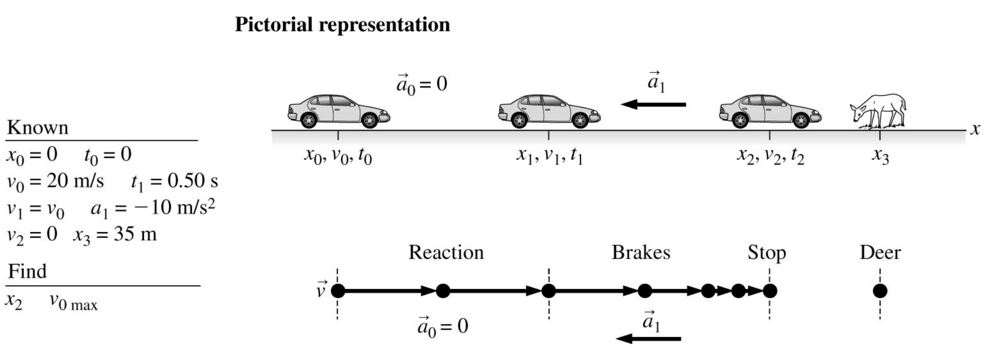
$$|a| = \frac{v_0^2}{2(d - v_0 t_R)}$$

(b) Given $v_0 = 21$ m/s, $d = 50$ m, and $t_R = 0.50$ s find the required deceleration using the answer from part (a).

$$|a| = \frac{v_0^2}{2(d - v_0 t_R)} = \frac{(21 \text{ m/s})^2}{2(50 \text{ m} - (21 \text{ m/s})(0.50 \text{ s}))} = 5.6 \text{ m/s}^2$$

Your car's maximum deceleration is greater than this, so yes, you can stop in time.
Assess: The units check out. The data given seem reasonable, and our answer is just less than the maximum deceleration.

2.45. Model: We will use the particle model and the constant-acceleration kinematic equations.
Visualize:



Solve: (a) To find x_2, we first need to determine x_1. Using $x_1 = x_0 + v_0(t_1 - t_0)$, we get $x_1 = 0 \text{ m} + (20 \text{ m/s})(0.50 \text{ s} - 0 \text{ s}) = 10$ m. Now,

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow 0 \text{ m}^2/\text{s}^2 = (20 \text{ m/s})^2 + 2(-10 \text{ m/s}^2)(x_2 - 10 \text{ m}) \Rightarrow x_2 = 30 \text{ m}$$

The distance between you and the deer is $(x_3 - x_2)$ or $(35\text{ m} - 30\text{ m}) = 5\text{ m}$.

(b) Let us find $v_{0\text{ max}}$ such that $v_2 = 0\text{ m/s}$ at $x_2 = x_3 = 35\text{ m}$. Using the following equation,

$$v_2^2 - v_{0\text{ max}}^2 = 2a_1(x_2 - x_1) \Rightarrow 0\text{ m}^2/\text{s}^2 - v_{0\text{ max}}^2 = 2(-10\text{ m/s}^2)(35\text{ m } - x_1)$$

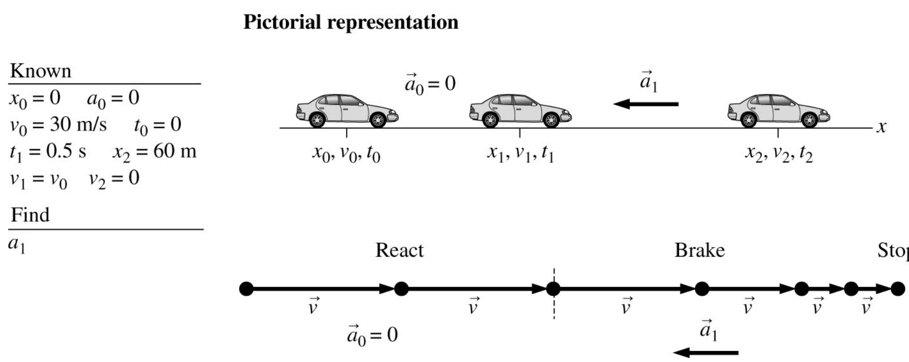
Also, $x_1 = x_0 + v_{0\text{ max}}(t_1 - t_0) = v_{0\text{ max}}(0.50\text{ s} - 0\text{ s}) = (0.50\text{ s})v_{0\text{ max}}$. Substituting this expression for x_1 in the above equation yields

$$-v_{0\text{ max}}^2 = (-20\text{ m/s}^2)[35\text{ m} - (0.50\text{ s})\, v_{0\text{ max}}] \Rightarrow v_{0\text{ max}}^2 + (10\text{ m/s})v_{0\text{ max}} - 700\text{ m}^2/\text{s}^2 = 0$$

The solution of this quadratic equation yields $v_{0\text{ max}} = 22\text{ m/s}$. (The other root is negative and unphysical for the present situation.)

Assess: An increase of speed from 20 m/s to 22 m/s is very reasonable for the car to cover an additional distance of 5 m with a reaction time of 0.50 s and a deceleration of 10 m/s^2.

2.46. Model: The car is represented as a particle.
Visualize:



Solve: (a) This is a two-part problem. First, we need to use the information given to determine the acceleration during braking. Second, we need to use that acceleration to find the stopping distance for a different initial velocity. First, the car coasts at constant speed before braking:

$$x_1 = x_0 + v_0(t_1 - t_0) = v_0 t_1 = (30\text{ m/s})(0.5\text{ s}) = 15\text{ m}$$

Then, the car brakes to a halt. Because we don't know the time interval, use

$$v_2^2 = 0 = v_1^2 + 2a_1(x_2 - x_1)$$

$$\Rightarrow a_1 = -\frac{v_1^2}{2(x_2 - x_1)} = -\frac{(30\text{ m/s})^2}{2(60\text{ m} - 15\text{ m})} = -10\text{ m/s}^2$$

We used $v_1 = v_0 = 30\text{ m/s}$. Note the minus sign, because $\vec{a}_1$ points to the left.

We can repeat these steps now with $v_0 = 40\text{ m/s}$. The coasting distance before braking is

$$x_1 = v_0 t_1 = (40\text{ m/s})(0.5\text{ s}) = 20\text{ m}$$

The position x_2 after braking is found using

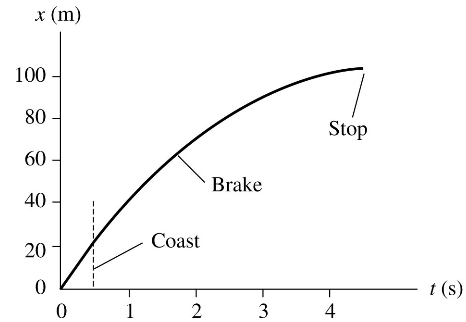
$$v_2^2 = 0 = v_1^2 + 2a_1(x_2 - x_1)$$

$$\Rightarrow x_2 = x_1 - \frac{v_1^2}{2a_1} = 20\text{ m} - \frac{(40\text{ m/s})^2}{2(-10\text{ m/s}^2)} = 100\text{ m}$$

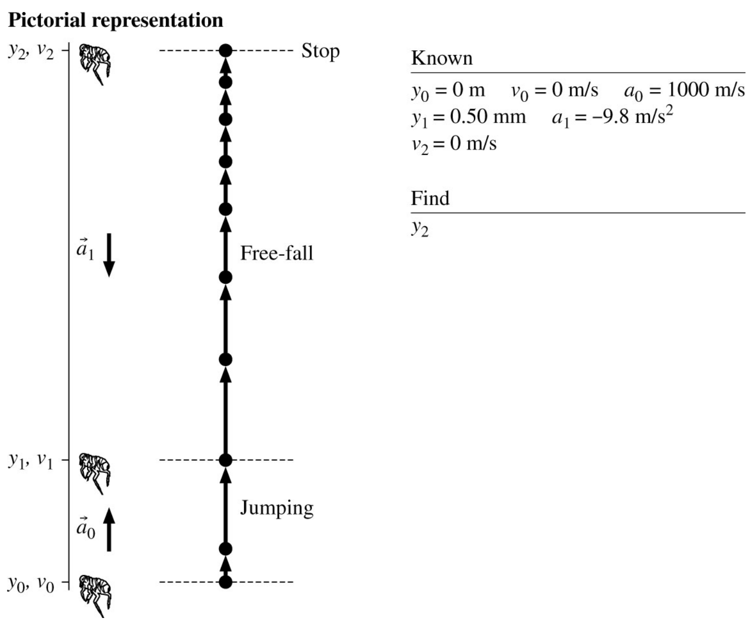
(b) The car coasts at a constant speed for 0.5 s, traveling 20 m. The graph will be a straight line with a slope of 40 m/s. For $t \geq 0.5$ the graph will be a parabola until the car stops at t_2. We can find t_2 from

$$v_2 = 0 = v_1 + a_1(t_2 - t_1) \Rightarrow t_2 = t_1 - \frac{v_1}{a_1} = 4.5\text{ s}$$

The parabola will reach zero slope $(v = 0 \text{ m/s})$ at $t = 4.5$ s. This is enough information to draw the graph shown in the figure.



2.47. Model: Model the flea as a particle. Both the initial acceleration phase and the free-fall phase have constant acceleration, so use the kinematic equations.
Visualize:



Solve: We can apply the kinematic equation $v_f^2 - v_i^2 = 2a\Delta y$ twice, once to find the take-off speed and then again to find the final height. In the first phase the acceleration is up (positive) and $v_0 = 0$.

$$v_1^2 = 2a_0(y_1 - y_0) = 2(1000 \text{ m/s}^2)(0.50\times10^{-3} \text{ m}) v_1 = 1.0 \text{ m/s}$$

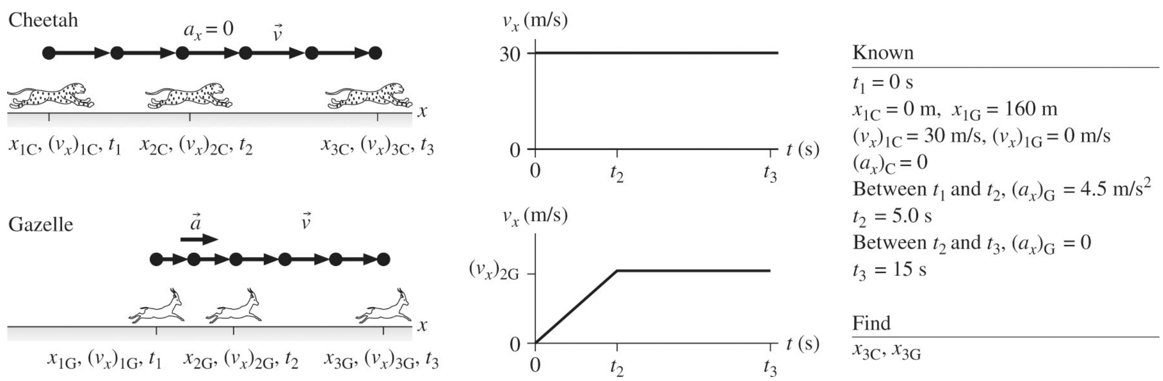
In the free fall phase the acceleration is $a_1 = -g$ and $v_1 = 1.0$ m/s and $v_2 = 0$ m/s.

$$y_2 - y_1 = \frac{v_2^2 - v_1^2}{2a_1} = \frac{-v_1^2}{2(-g)} = \frac{-(1.0 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)} = 5.1 \text{ cm}$$

So the final height is $y_2 = 5.1 \text{ cm} + y_1 = 5.1 \text{ cm} + 0.50 \text{ mm} = 5.2 \text{ cm}$.
Assess: This is pretty amazing–about 10–20 times the size of a typical flea.

2.48. Model: Model each of the animals as a particle and use kinematic equations. Assume that the time it takes the cheetah to accelerate to 30 m/s is negligible.

Visualize:



Solve: The cheetah is in uniform motion for the entire duration of the problem, so we can easily solve for its position at $t_3 = 25$ s:

$$x_{3C} = x_{1C} + (v_x)_{1C}\,\Delta t = 0 \text{ m} + (30 \text{ m/s})(15 \text{ s}) = 450 \text{ m}$$

The gazelle's motion has two phases: one of constant acceleration and then one of constant velocity. We can solve for the position and the velocity at t_2, the end of the first phase.

$$(v_x)_{2G} = (v_x)_{1G} + (a_x)_G\,\Delta t = 0 \text{ m/s} + (4.6 \text{ m/s}^2)(5.0 \text{ s}) = 23 \text{ m/s}$$

$$x_{2G} = x_{1G} + (v_x)_{1G}\,\Delta t + \frac{1}{2}(a_x)_G(\Delta t)^2 = 170 \text{ m} + 0 \text{ m} + \frac{1}{2}(4.6 \text{ m/s}^2)(5.0 \text{ s})^2 = 227.5 \text{ m}$$

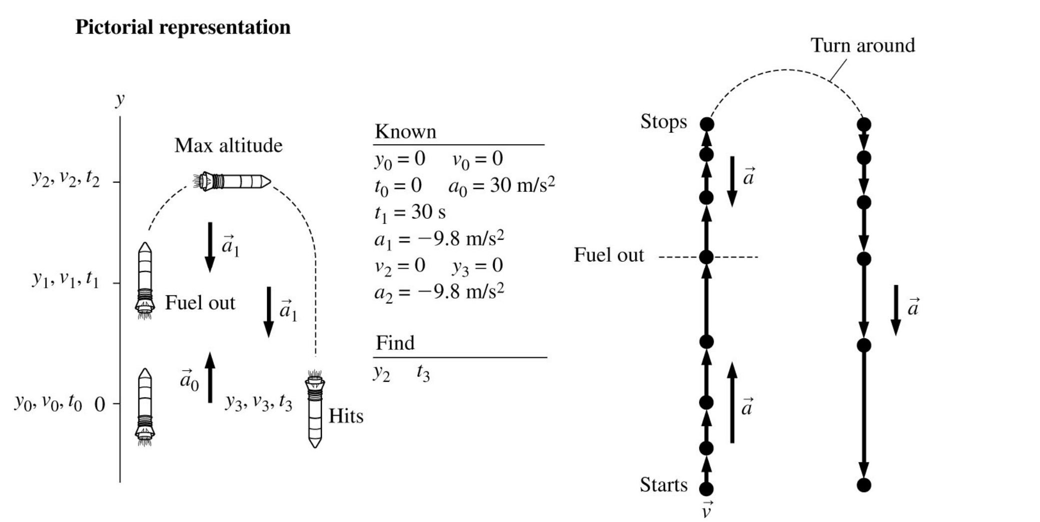
From t_2 to t_3 the gazelle moves at a constant speed, so we can use the equation for uniform motion to find its final position.

$$x_{3G} = x_{2G} + (v_x)_{2G}\,\Delta t = 227.5 \text{ m} + (23 \text{ m/s})(10.0 \text{ s}) = 457.5 \text{ m} \approx 460 \text{ m}$$

x_{3C} is 450m; x_{3G} is 460m. The gazelle is just a few meters ahead of the cheetah when the cheetah has to break off the chase, so the gazelle escapes.

Assess: The numbers in the problem statement are realistic, so we expect our results to mirror real life. The speed for the gazelle is close to that of the cheetah, which seems reasonable for two animals known for their speed. And the result is the most common occurrence–the chase is very close, but the gazelle gets away.

2.49. Model: The rocket is represented as a particle.
Visualize:



Solve: (a) There are three parts to the motion. Both the second and third parts of the motion are free fall, with $a = -g$. The maximum altitude is y_2. In the acceleration phase:

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a(t_1 - t_0)^2 = \frac{1}{2}at_1^2 = \frac{1}{2}(30\text{ m/s}^2)(30\text{ s})^2 = 13{,}500\text{ m}$$

$$v_1 = v_0 + a(t_1 - t_0) = at_1 = (30\text{ m/s}^2)(30\text{ s}) = 900\text{ m/s}$$

In the coasting phase,

$$v_2^2 = 0 = v_1^2 - 2g(y_2 - y_1) \Rightarrow y_2 = y_1 + \frac{v_1^2}{2g} = 13{,}500\text{ m} + \frac{(900\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 54{,}800\text{ m} = 54.8\text{ km}$$

The maximum altitude is 54.8 km ($\approx$ 33 miles).

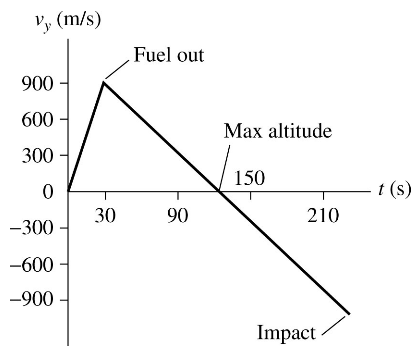
(b) The rocket is in the air until time t_3. We already know $t_1 = 30$ s. We can find t_2 as follows:

$$v_2 = 0\text{ m/s} = v_1 - g(t_2 - t_1) \Rightarrow t_2 = t_1 + \frac{v_1}{g} = 122\text{ s}$$

Then t_3 is found by considering the time needed to fall 54,800 m:

$$y_3 = 0\text{ m} = y_2 + v_2(t_3 - t_2) - \frac{1}{2}g(t_3 - t_2)^2 = y_2 - \frac{1}{2}g(t_3 - t_2)^2 \Rightarrow t_3 = t_2 + \sqrt{\frac{2y_2}{g}} = 228\text{ s}$$

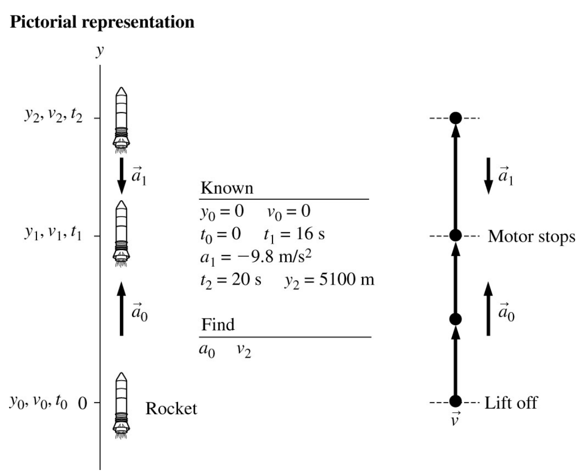
(c) The velocity increases linearly, with a slope of 30 (m/s)/s, for 30 s to a maximum speed of 900 m/s. It then begins to decrease linearly with a slope of -9.8(m/s)/s. The velocity passes through zero (the turning point at y_2) at $t_2 = 122$ s. The impact velocity at $t_3 = 228$ s is calculated to be $v_3 = v_2 - g(t_3 - t_2) = -1040$ m/s.



Assess: In reality, friction due to air resistance would prevent the rocket from reaching such high speeds as it falls, and the acceleration upward would not be constant because the mass changes as the fuel is burned, but that is a more complicated problem.

2.50. Model: We will model the rocket as a particle. Air resistance will be neglected.

Visualize:



Solve: **(a)** Using the constant-acceleration kinematic equations,

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + a_0(16 \text{ s} - 0 \text{ s}) = a_o(16 \text{ s})$$

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = \frac{1}{2}a_0(16 \text{ s} - 0 \text{ s})^2 = a_o(128 \text{ s}^2)$$

$$y_2 = y_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

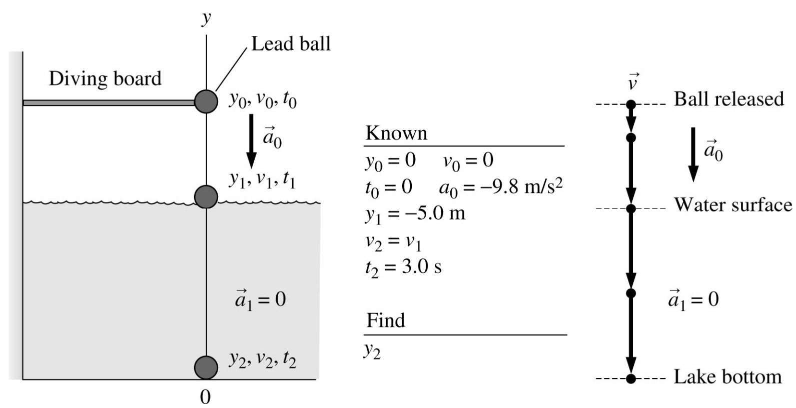
$$\Rightarrow 5100 \text{ m} = 128 \text{ s}^2 \; a_0 + 16 \text{ s} \; a_0(20 \text{ s} - 16 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(20 \text{ s} - 16 \text{ s})^2 \Rightarrow a_0 = 27 \text{ m/s}^2$$

(b) The rocket's speed as it passes through a cloud 5100 m above the ground can be determined using the kinematic equation:

$$v_2 = v_1 + a_1(t_2 - t_1) = (16 \text{ s})a_0 + (-9.8 \text{ m/s}^2)(4 \text{ s}) = 390 \text{ m/s}$$

Assess: 400 m/s ≈ 900 mph, which would be the final speed of a rocket that has been accelerating for 20 s at a rate of approximately 20 m/s^2 or 66 ft/s^2.

2.51. Model: We will model the lead ball as a particle and use the constant-acceleration kinematic equations.
Visualize:



Note that the particle undergoes free fall until it hits the water surface.
Solve: The kinematics equation $y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2$ becomes

$$-5.0 \text{ m} = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0)^2 \Rightarrow t_1 = 1.01 \text{ s}$$

Now, once again,

$$y_2 = y_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

$$\Rightarrow y_2 - y_1 = v_1(3.0 \text{ s} - 1.01 \text{ s}) + 0 \text{ m/s} = 1.99 \, v_1$$

v_1 is easy to determine since the time t_1 has been found. Using $v_1 = v_0 + a_0(t_1 - t_0)$, we get

$$v_1 = 0 \text{ m/s} - (9.8 \text{ m/s}^2)(1.01 \text{ s} - 0 \text{ s}) = -9.898 \text{ m/s}$$

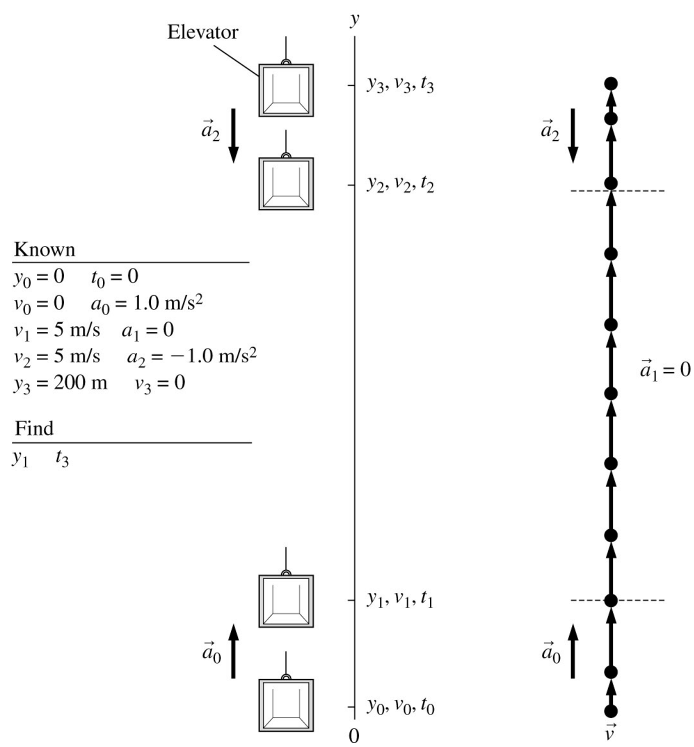
With this value for v_1, we go back to:

$$y_2 - y_1 = 1.99 v_1 = (1.99)(-9.898 \text{ m/s}) = -19.7 \text{ m}$$

$y_2 - y_1$ is the displacement of the lead ball in the lake and thus corresponds to the depth of the lake. The negative sign shows the direction of the displacement vector.
Assess: A depth of about 60 ft for a lake is not unusual.

2.52. Model: The elevator is a particle moving under constant-acceleration kinematic equations.
Visualize:

Pictorial representation



Solve: (a) To calculate the distance to accelerate up:

$$(v_1)^2 = v_0^2 + 2a_0(y_0 - y_0) \Rightarrow (5\text{ m/s})^2 = (0\text{ m/s})^2 + 2(1\text{ m/s}^2)(y_1 - 0\text{ m}) \Rightarrow y_1 = 12.5\text{ m}$$

(b) To calculate the time to accelerate up:

$$v_1 = v_0 + a_0(t_1 - t_0) \Rightarrow 5\text{ m/s} = 0\text{ m/s} + (1\text{ m/s}^2)(t_1 - 0\text{ s}) \Rightarrow t_1 = 5\text{ s}$$

To calculate the distance to decelerate at the top:

$$v_3^2 = v_2^2 + 2a_2(y_3 - y_2) \Rightarrow (0\text{ m/s})^2 = (5\text{ m/s})^2 + 2(-1\text{ m/s}^2)(y_3 - y_2) \Rightarrow y_3 - y_2 = 12.5\text{ m}$$

To calculate the time to decelerate at the top:

$$v_3 = v_2 + a_2(t_3 - t_2) \Rightarrow 0\text{ m/s} = 5\text{ m/s} + (-1\text{ m/s}^2)(t_3 - t_2) \Rightarrow t_3 - t_2 = 5\text{ s}$$

The distance moved up at 5 m/s is

$$y_2 - y_1 = (y_3 - y_0) - (y_3 - y_2) - (y_1 - y_0) = 200\text{ m} - 12.5\text{ m} - 12.5\text{ m} = 175\text{ m}$$

The time to move up 175 m is given by

$$y_2 - y_1 = v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 \Rightarrow 175\text{ m} = (5\text{ m/s})(t_2 - t_1) \Rightarrow (t_2 - t_1) = 35\text{ s}$$

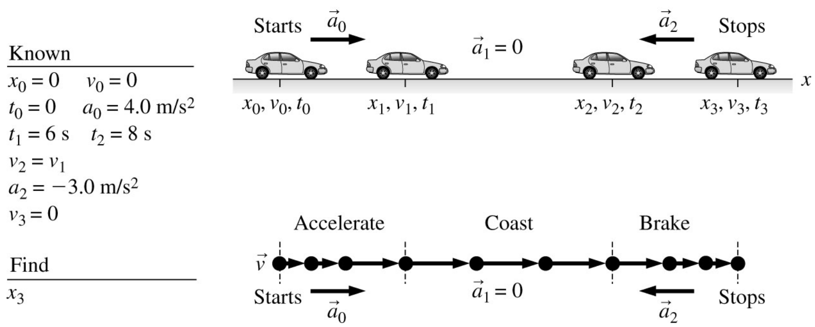
To total time to move to the top is

$$(t_1 - t_0) + (t_2 - t_1) + (t_3 - t_2) = 5\text{ s} + 35\text{ s} + 5\text{ s} = 45\text{ s}$$

Assess: To cover a distance of 200 m at 5 m/s (ignoring acceleration and deceleration times) will require a time of 40 s. This is comparable to the time of 45 s for the entire trip as obtained above.

2.53. Model: The car is a particle moving under constant-acceleration kinematic equations.
Visualize:

Pictorial representation



Solve: This is a three-part problem. First the car accelerates, then it moves with a constant speed, and then it decelerates.
First, the car accelerates:

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + (4.0 \text{ m/s}^2)(6 \text{ s} - 0 \text{ s}) = 24 \text{ m/s}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0 \text{ m} + \frac{1}{2}(4.0 \text{ m/s}^2)(6 \text{ s} - 0 \text{ s})^2 = 72 \text{ m}$$

Second, the car moves at v_1:

$$x_2 - x_1 = v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 = (24 \text{ m/s})(8 \text{ s} - 6 \text{ s}) + 0 \text{ m} = 48 \text{ m}$$

Third, the car decelerates:

$$v_3 = v_2 + a_2(t_3 - t_2) \Rightarrow 0 \text{ m/s} = 24 \text{ m/s} + (-3.0 \text{ m/s}^2)(t_3 - t_2) \Rightarrow (t_3 - t_2) = 8 \text{ s}$$

$$x_3 = x_2 + v_2(t_3 - t_2) + \frac{1}{2}a_2(t_3 - t_2)^2 \Rightarrow x_3 - x_2 = (24 \text{ m/s})(8 \text{ s}) + \frac{1}{2}(-3.0 \text{ m/s}^2)(8 \text{ s})^2 = 96 \text{ m}$$

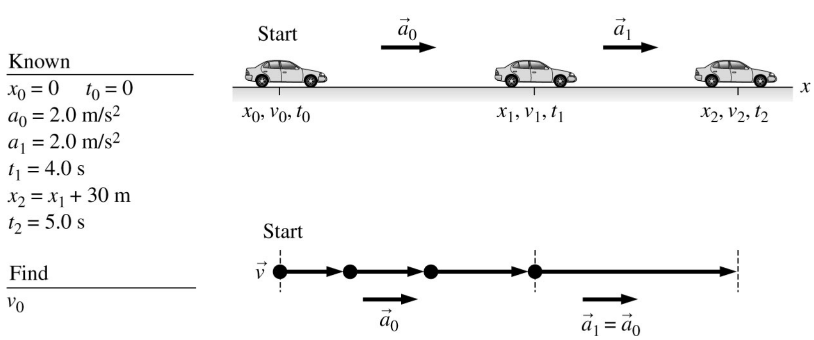
Thus, the total distance between stop signs is:

$$x_3 - x_0 = (x_3 - x_2) + (x_2 - x_1) + (x_1 - x_0) = 96 \text{ m} + 48\text{m} + 72 \text{ m} = 216 \text{ m}$$

Assess: A distance of approximately 600 ft in a time of around 10 s with an acceleration/deceleration of the order of 7 mph/s is reasonable.

2.54. Model: The car is a particle moving under constant linear acceleration.
Visualize:

Pictorial representation



Solve: Using the kinematic equation for position:

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 \Rightarrow x_1 + 30\text{ m} = x_1 + v_1(5.0\text{ s} - 4.0\text{ s}) + \frac{1}{2}(2\text{ m/s}^2)(5.0\text{ s} - 4.0\text{ s})^2$$

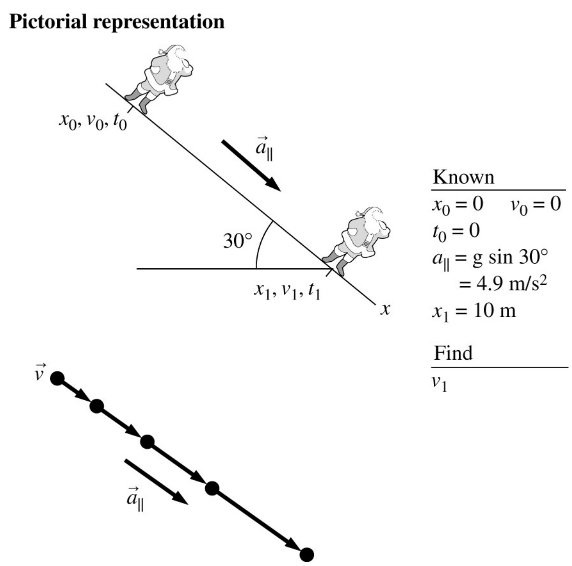
$$\Rightarrow 30\text{ m} = v_1(1.0\text{ s}) + \frac{1}{2}(2\text{ m/s}^2)(1.0\text{ s})^2 \Rightarrow v_1 = 29\text{ m/s}$$

And 4.0 seconds before:

$$v_1 = v_0 + a_0(t_1 - t_0) \Rightarrow 29\text{ m/s} = v_0 + (2\text{ m/s}^2)(4.0\text{ s} - 0\text{ s}) \Rightarrow v_0 = 21\text{ m/s}$$

Assess: 21 m/s $\approx$ 47 mph and is a reasonable value.

2.55. Model: Santa is a particle moving under constant-acceleration kinematic equations.
Visualize: Note that our x-axis is positioned along the incline.

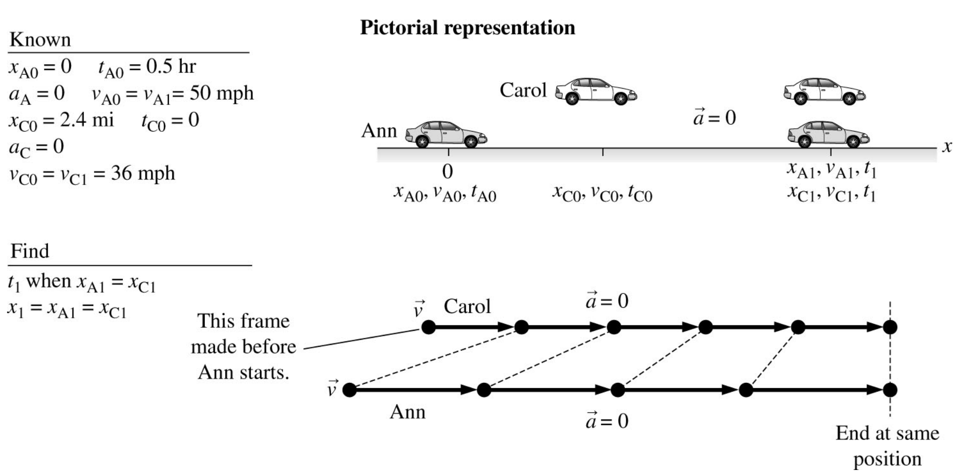


Solve: Using the following kinematic equation,

$$v_1^2 = v_0^2 + 2a_{\|}(x_1 - x_2) = (0\text{ m/s})^2 + 2(4.9\text{ m/s}^2)(10\text{ m} - 0\text{ m}) \Rightarrow v_1 = 9.9\text{ m/s}$$

Assess: Santa's speed of 20 mph as he reaches the edge is reasonable.

2.56. Model: The cars are represented as particles.
Visualize:



Solve: (a) Ann and Carol start from different locations at different times and drive at different speeds. But at time t_1 they have the *same* position. It is important in a problem such as this to express information in terms of *positions* (that is, coordinates) rather than distances. Each drives at a constant velocity, so using constant velocity kinematics gives

$$x_{A1} = x_{A0} + v_A(t_1 - t_{A0}) = v_A(t_1 - t_{A0}) \qquad x_{C1} = x_{C0} + v_C(t_1 - t_{C0}) = x_{C0} + v_C t_1$$

The critical piece of information is that Ann and Carol have the same position at t_1, so $x_{A1} = x_{C1}$. Equating these two expressions, we can solve for the time t_1 when Ann passes Carol:

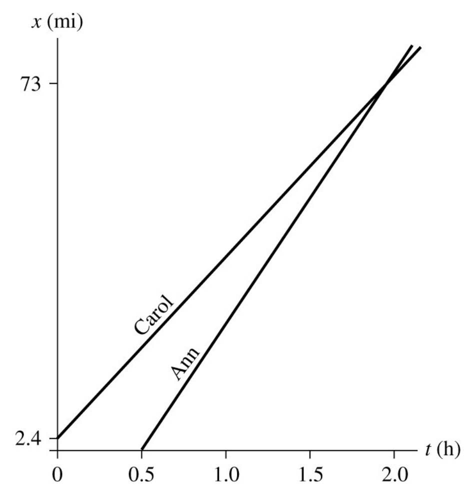
$$v_A(t_1 - t_{A0}) = x_{C0} + v_C t_1$$

$$\Rightarrow (v_A - v_C)t_1 = x_{C0} + v_A t_{A0}$$

$$\Rightarrow t_1 = \frac{x_{C0} + v_A t_{A0}}{v_A - v_C} = \frac{2.4\text{ mi} + (50\text{ mph})(0.5\text{ h})}{50\text{ mph} - 36\text{ mph}} = 1.96\text{ h} \approx 2.0\text{ h}$$

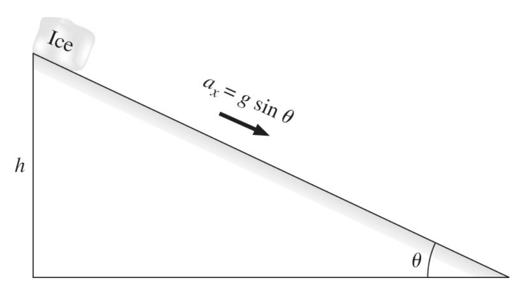
(b) Their position is $x_1 = x_{A1} = x_{C1} = x_{C0} + v_C t_1 = 72.86\text{ miles} \approx 73\text{ miles}$

(c) Note that Ann's graph doesn't start until $t = 0.5$ hours, but her graph has a steeper slope so it intersects Carol's graph at t $\approx$ 2.0 hours.



2.57. Model: Model the ice as a particle and use the kinematic equations for constant acceleration. Model the "very slippery block" and "smooth ramp"' as frictionless. Set the *x*-axis parallel to the ramp.

Visualize:



Note that the distance down the ramp is $\Delta x = h/\sin\theta$. Also $a_x = g\sin\theta$ down a frictionless ramp.

Solve:

(a) Use $v_f^2 = v_i^2 + 2a_x\Delta x$, where $v_i = 0$.

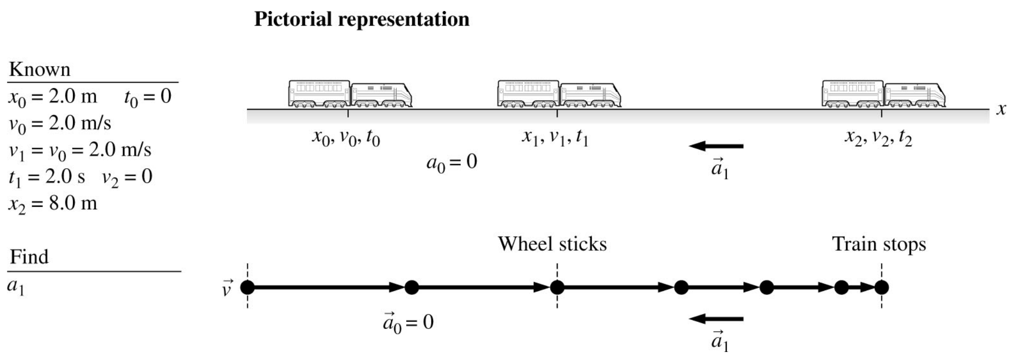
$$v_f^2 = 2a\Delta x \Rightarrow v_f = \sqrt{2(g\sin\theta)\frac{h}{\sin\theta}} = \sqrt{2gh}$$

(b) For $h = 0.30$ m,

$$v_f = \sqrt{2(9.8\text{ m/s}^2)(0.30\text{ m})} = 2.4\text{ m/s}$$

This is true for both angles as the answer is independent of the angle.
Assess: We will later learn how to solve this problem in an easier way with energy.

2.58. Model: We will model the toy train as a particle.
Visualize:



Solve: Using kinematics,

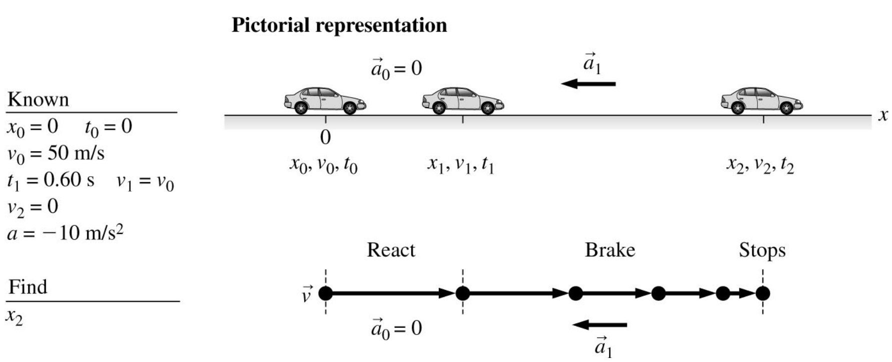
$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 2\text{ m} + (2.0\text{ m/s})(2.0\text{ s} - 0\text{ s}) + 0\text{ m} = 6.0\text{ m}$$

The acceleration can now be obtained as follows:

$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) \Rightarrow 0\text{ m}^2/\text{s}^2 = (2.0\text{ m/s})^2 + 2a_1(8.0\text{ m} - 6.0\text{ m}) \Rightarrow a_1 = -1.0\text{ m/s}^2$$

The magnitude is 1.0 m/s^2.
Assess: A deceleration of 1 m/s^2 in bringing the toy train to a halt over a distance of 2.0 m is reasonable.

2.59. Model: We will use the particle model and the kinematic equations at constant-acceleration.
Visualize:



Solve: To find x_2, let us use the kinematic equation

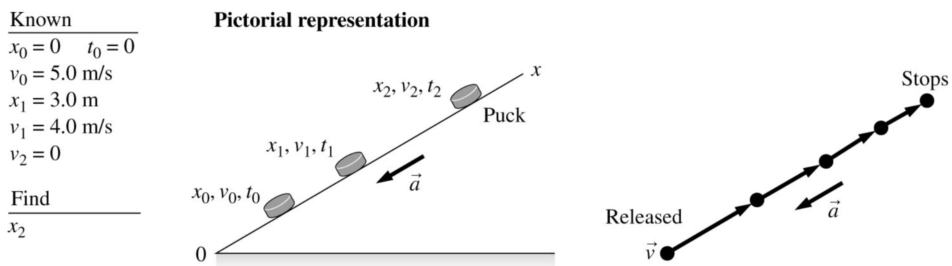
$$v_2^2 = v_1^2 + 2a_1(x_2 - x_1) = (0\text{ m/s})^2 = (50\text{ m/s})^2 + 2(-10\text{ m/s}^2)(x_2 - x_1) \Rightarrow x_2 = x_1 + 125\text{ m}$$

Since the nail strip is at a distance of 150 m from the origin, we need to determine x_1:

$$x_1 = x_0 + v_0(t_1 - t_0) = 0\text{ m} + (50\text{ m/s})(0.60\text{ s} - 0.0\text{ s}) = 30\text{ m}$$

Therefore, we can see that $x_2 = (30 + 125)\text{ m} = 155\text{ m}$. That is, he can't stop within a distance of 150 m. He is in jail.
Assess: Bob is driving at approximately 100 mph and the stopping distance is of the correct order of magnitude.

2.60. Model: We will use the particle model with constant-acceleration kinematic equations.
Visualize:



Solve: The acceleration, being the same along the incline, can be found as

$$v_1^2 = v_0^2 + 2a(x_1 - x_0) \Rightarrow (4.0 \text{ m/s})^2 = (5.0 \text{ m/s})^2 + 2a(3.0 \text{ m} - 0 \text{ m}) \Rightarrow a = -1.5 \text{ m/s}^2$$

We can also find the total time the puck takes to come to a halt as

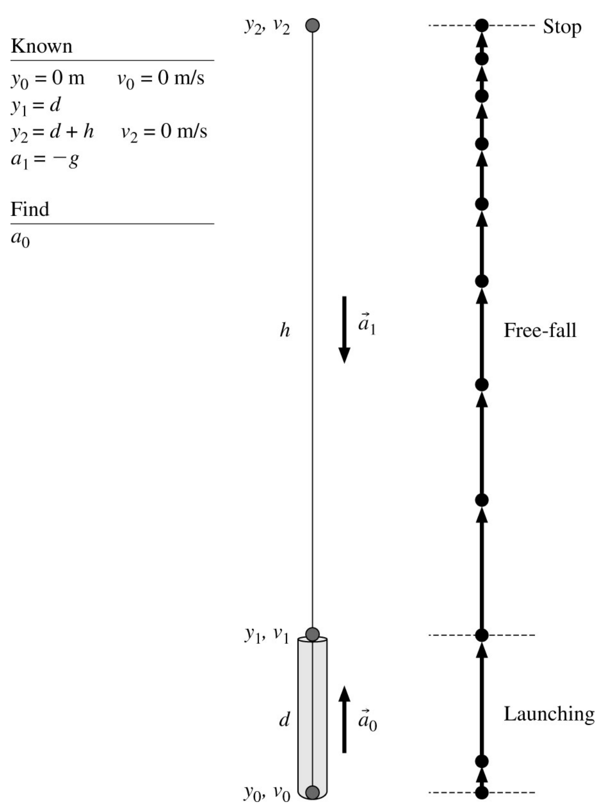
$$v_2 = v_0 + a(t_2 - t_0) \Rightarrow 0 \text{ m/s} = (5.0 \text{ m/s}) + (-1.5 \text{ m/s}^2)t_2 \Rightarrow t_2 = 3.3 \text{ s}$$

Using the above obtained values of a and t_2, we can find x_2 as follows:

$$x_2 = x_0 + v_0(t_2 - t_0) + \frac{1}{2}a(t_2 - t_0)^2 = 0 \text{ m} + (5.0 \text{ m/s})(3.3 \text{ s}) + \frac{1}{2}(-1.5 \text{ m/s}^2)(3.3 \text{ s})^2 = 8.3 \text{ m}$$

That is, the puck goes through a displacement of 8.3 m. Since the end of the ramp is 8.5 m from the starting position x_0 and the puck stops 0.2 m or 20 cm before the ramp ends, you are not a winner.

2.61. Model: Model the ball as a particle. Ignore air resistance.
Visualize:



Solve:

(a) We can apply the kinematic equation $v_f^2 - v_i^2 = 2a\Delta y$ twice, once for the launching phase and again for the free fall phase. In the launching phase the acceleration is up (positive), $v_0 = 0$ and $\Delta y = y_1 - y_0 = d$.

$$v_1^2 = 2a_0 d$$

In the free fall phase the acceleration is $a_1 = -g$, $v_2 = 0$, and $\Delta y = y_2 - y_1 = h$.

$$-v_1^2 = 2a_1 h = 2(-g)h$$

Cancel the negative signs and set the two expressions for v_1^2 equal to each other.

$$2a_0 d = 2gh$$

Solve for a_0.

$$a_0 = \frac{h}{d}g$$

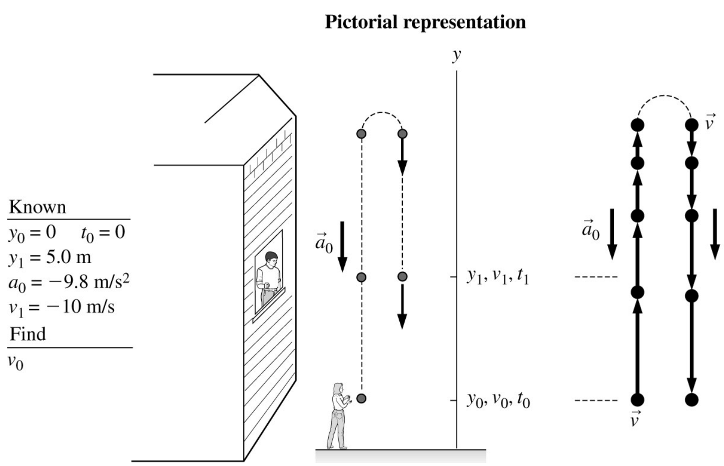
(b) For $h = 3.2\text{m}$, and $d = 0.45$ m we get

$$a_0 = \frac{h}{d}g = \frac{3.2\text{ m}}{0.45\text{ m}}(9.8\text{ m/s}^2) = 69.7\text{ m/s}^2 \approx 70\text{ m/s}^2$$

Assess: The answer is independent of the mass of the ball. The units check out.

2.62. Model: The ball is a particle that exhibits freely falling motion according to the constant-acceleration kinematic equations.

Visualize:

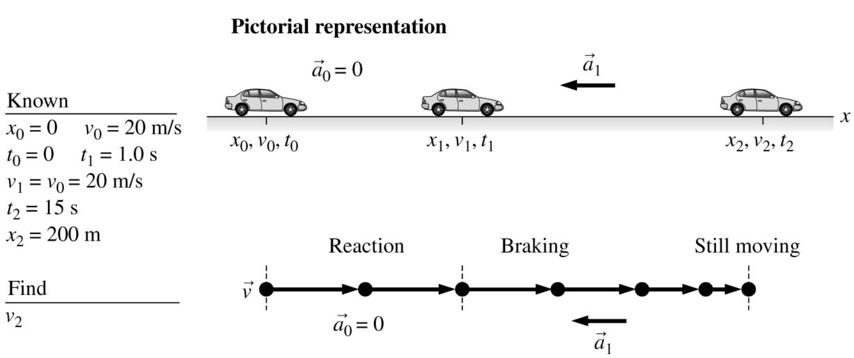


Solve: Using the known values, we have

$$v_1^2 = v_0^2 + 2a_0(y_1 - y_0) \Rightarrow (-10\text{ m/s})^2 = v_0^2 + 2(-9.8\text{ m/s}^2)(5.0\text{ m} - 0\text{ m}) \Rightarrow v_0 = 14\text{ m/s}$$

2.63. Model: The car is a particle that moves with constant linear acceleration.

Visualize:



Solve: The reaction time is 1.0 s, and the motion during this time is

$$x_1 = x_0 + v_0(t_1 - t_0) = 0\text{ m} + (20\text{ m/s})(1.0\text{ s}) = 20\text{ m}$$

During slowing down,

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 = 200\text{ m}$$

$$= 20\text{ m} + (20\text{ m/s})(15\text{ s} - 1.0\text{ s}) + \frac{1}{2}a_1(15\text{ s} - 1.0\text{ s})^2 \Rightarrow a_1 = -1.02\text{ m/s}^2$$

The final speed v_2 can now be obtained as

$$v_2 = v_1 + a_1(t_2 - t_1) = (20\text{ m/s}) + (-1.02\text{ m/s}^2)(15\text{ s} - 1\text{ s}) = 5.7\text{ m/s}$$

2.64. Solve: (a) The quantity $\frac{2P}{m} = \frac{2(3.6\times10^4\text{ W})}{1200\text{ kg}} = 60\text{ m}^2/\text{s}^3$. Thus

$$v_x = \sqrt{(60\text{ m}^2/\text{s}^3)t}$$

At $t = 10$ s, $v_x = \sqrt{(60\text{ m}^2/\text{s}^3)(10\text{ s})} = 24$ m/s (≈ 50 mph), and at $t = 20$ s, $v_x = 35$ m/s (≈ 75 mph).

(b) With $v_x = \sqrt{\frac{2P}{m}}t^{1/2}$, we have

$$a_x = \frac{dv_x}{dt} = \sqrt{\frac{2P}{m}} \times \frac{1}{2}t^{-1/2} = \sqrt{\frac{P}{2mt}}$$

(c) At $t = 1$ s, $a_x = \sqrt{\frac{P}{2mt}} = \sqrt{\frac{(3.6\times10^4\text{ W})}{2(1200\text{ kg})(1\text{ s})}} = 3.9\text{ m/s}^2$. Similarly, at $t = 10$ s, $a_x = 1.2\text{ m/s}^2$.

(d) Consider the limiting case of very short times. Note that $a_x \to \infty$ as $t \to 0$. This is physically impossible for the Alfa Romeo.

(e) We can use the relationship that $v_x = \frac{dx}{dt}$ and integrate to find $x(t)$. We have $v_x = \sqrt{\frac{2P}{m}}t^{1/2}$ and the initial condition $x_i = 0$ at $t_i = 0$. Thus

$$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$\text{and } x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{3/2} = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2}$$

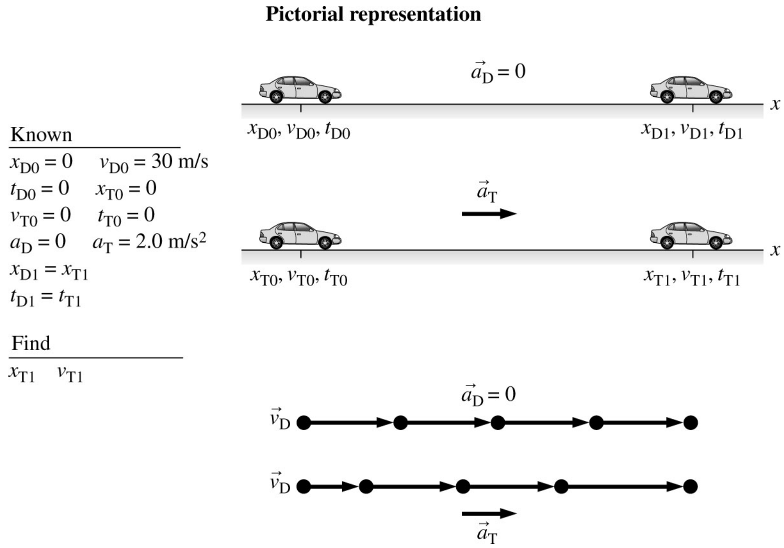
(f) Time to travel a distance x is found by solving the above equation for t.

$$t = \left[\frac{3}{2}\sqrt{\frac{m}{2P}}x\right]^{2/3}$$

For $x = 402$ m, $t = 18.2$ s.

2.65. Model: Both cars are particles that move according to the constant-acceleration kinematic equations.

Visualize:



Solve: (a) David's and Tina's motions are given by the following equations:

$$x_{D1} = x_{D0} + v_{D0}(t_{D1} - t_{D0}) + \frac{1}{2}a_D(t_{D1} - t_{D0})^2 = v_{D0}t_{D1}$$

$$x_{T1} = x_{T0} + v_{T0}(t_{T1} - t_{T0}) + \frac{1}{2}a_T(t_{T1} - t_{T0})^2 = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}a_T t_{T1}^2$$

When Tina passes David the distances are equal and $t_{D1} = t_{T1}$, so we get

$$x_{D1} = x_{T1} \Rightarrow v_{D0}t_{D1} = \frac{1}{2}a_T t_{T1}^2 \Rightarrow v_{D0} = \frac{1}{2}a_T t_{T1} \Rightarrow t_{T1} = \frac{2v_{D0}}{a_T} = \frac{2(30 \text{ m/s})}{2.0 \text{ m/s}^2} = 30 \text{ s}$$

Using Tina's position equation,

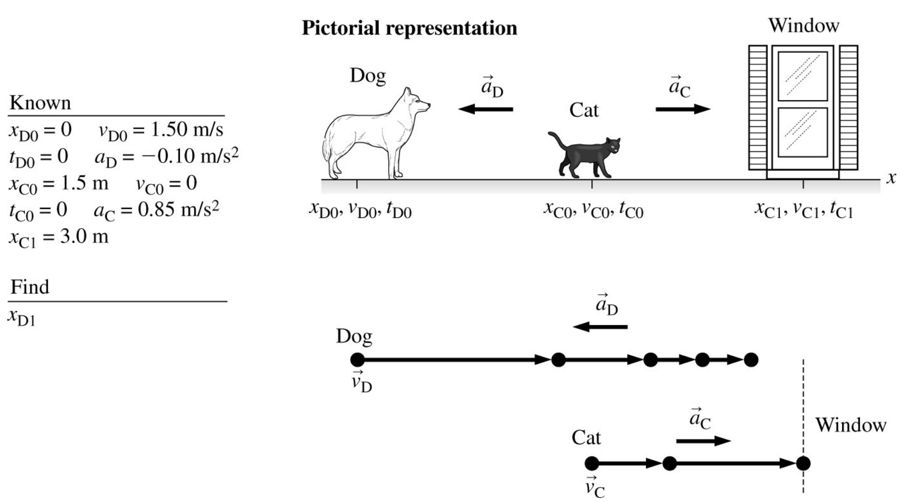
$$x_{T1} = \frac{1}{2}a_T t_{T1}^2 = \frac{1}{2}(2.0 \text{ m/s}^2)(30 \text{ s})^2 = 900 \text{ m}$$

(b) Tina's speed v_{T1} can be obtained from

$$v_{T1} = v_{T0} + a_T(t_{T1} - t_{T0}) = (0 \text{ m/s}) + (2.0 \text{ m/s}^2)(30 \text{ s} - 0 \text{ s}) = 60 \text{ m/s}$$

Assess: This is a high speed for Tina (~134 mph) and so is David's velocity (~67 mph). Thus the large distance for Tina to catch up with David (~0.6 miles) is reasonable.

2.66. Model: We will represent the dog and the cat in the particle model.
Visualize:



Solve: We will first calculate the time t_{C1} the cat takes to reach the window. The dog has exactly the same time to reach the cat (or the window). Let us therefore first calculate t_{C1} as follows:

$$x_{C1} = x_{C0} + v_{C0}(t_{C1} - t_{C0}) + \frac{1}{2}a_C(t_{C1} - t_{C0})^2$$

$$\Rightarrow 3.0 \text{ m} = 1.5 \text{ m} + 0 \text{ m} + \frac{1}{2}(0.85 \text{ m/s}^2)t_{C1}^2 \Rightarrow t_{C1} = 1.879 \text{ s}$$

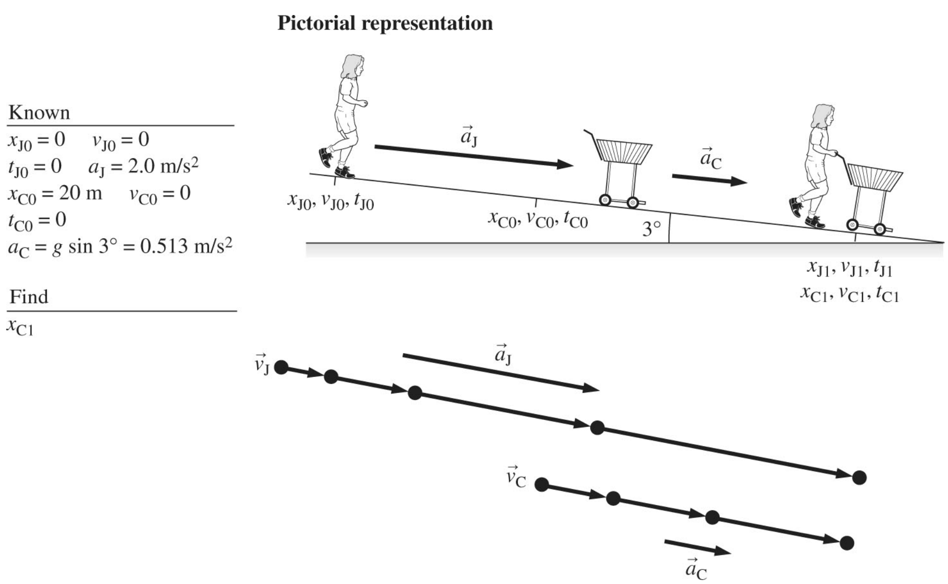
In the time $t_{D1} = 1.879$ s, the dog's position can be found as follows:

$$x_{D1} = x_{D0} + v_{D0}(t_{D1} - t_{D0}) + \frac{1}{2}a_D(t_{D1} - t_{D0})^2$$

$$= 0 \text{ m} + (1.50 \text{ m/s})(1.879 \text{ s}) + \frac{1}{2}(-0.10 \text{ m/s}^2)(1.879 \text{ s})^2 = 2.6 \text{ m}$$

That is, the dog is shy of reaching the cat by 0.4 m. The cat is safe.

2.67. Model: Jill and the grocery cart will be treated as particles that move according to the constant-acceleration kinematic equations.
Visualize:



Solve: The final position of Jill when the cart is caught is given by

$$x_{J1} = x_{J0} + v_{J0}(t_{J1} - t_{J0}) + \frac{1}{2}a_{J0}(t_{J1} - t_{J0})^2 = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}a_{J0}(t_{J1} - 0 \text{ s})^2 = \frac{1}{2}(2.0 \text{ m/s}^2)t_{J1}^2$$

The cart's position when it is caught is

$$x_{C1} = x_{C0} + v_{C0}(t_{C1} - t_{C0}) + \frac{1}{2}a_{C0}(t_{C1} - t_{C0})^2 = 20 \text{ m} + 0 \text{ m} + \frac{1}{2}(0.5 \text{ m/s}^2)(t_{C1} - 0 \text{ s})^2$$

$$= 20 \text{ m} + (0.25 \text{ m/s}^2)t_{C1}^2$$

Since $x_{J1} = x_{C1}$ and $t_{J1} = t_{C1}$, we get

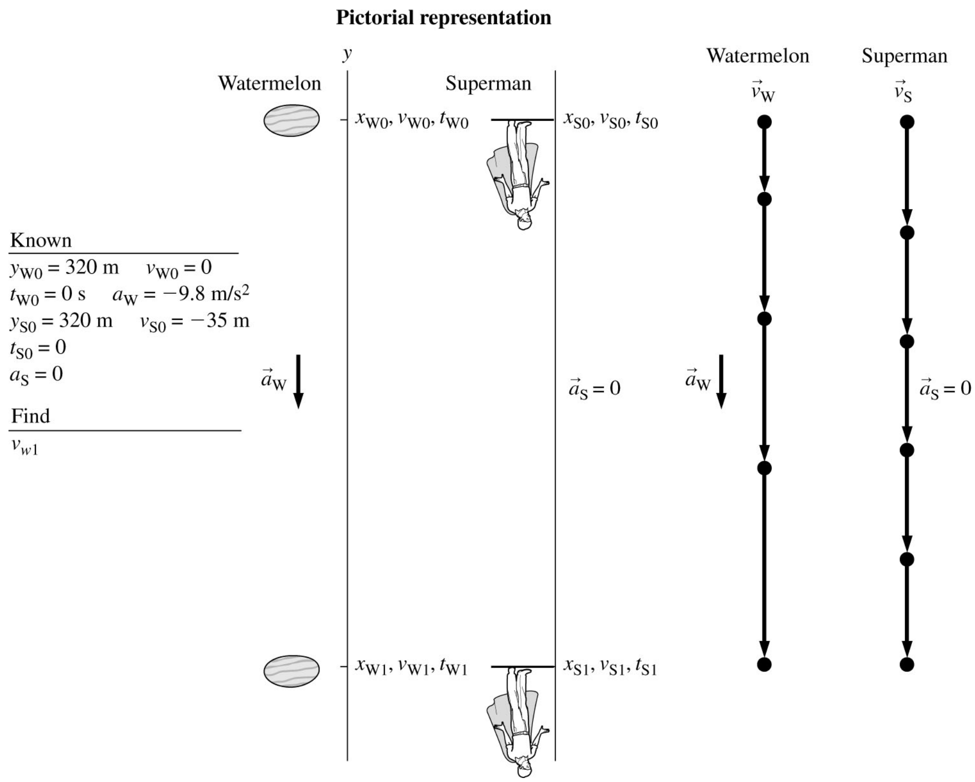
$$\frac{1}{2}(2.0)t_{J1}^2 = 20 \text{ s}^2 + 0.25t_{C1}^2 \Rightarrow 0.75t_{C1}^2 = 20 \text{ s}^2 \Rightarrow t_{C1} = 5.16 \text{ s}$$

$$\Rightarrow x_{C1} = 20 \text{ m} + (0.25 \text{ m/s}^2)t_{C1}^2 = 20 \text{ m} + (0.25 \text{ m/s}^2)(5.16 \text{ s})^2 = 26.7 \text{ m}$$

So, the cart has moved 6.7 m.

2.68. Model: The watermelon and Superman will be treated as particles that move according to constant-acceleration kinematic equations.

Visualize:



Solve: The watermelon's and Superman's position as they meet each other are

$$y_{W1} = y_{W0} + v_{W0}(t_{W1} - t_{W0}) + \frac{1}{2}a_{W0}(t_{W1} - t_{W0})^2$$

$$y_{S1} = y_{S0} + v_{S0}(t_{S1} - t_{S0}) + \frac{1}{2}a_{S0}(t_{S1} - t_{S0})^2$$

$$\Rightarrow y_{W1} = 320 \text{ m} + 0 \text{ m} + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_{W1} - 0 \text{ s})^2$$

$$\Rightarrow y_{S1} = 320 \text{ m} + (-35 \text{ m/s})(t_{S1} - 0 \text{ s}) + 0 \text{ m}$$

Because $t_{S1} = t_{W1}$,

$$y_{W1} = 320 \text{ m} - (4.9 \text{ m/s}^2)\, t_{W1}^2 \qquad y_{S1} = 320 \text{ m} - (35 \text{ m/s})\, t_{W1}$$

Since $y_{W1} = y_{S1}$,

$$320 \text{ m} - (4.9 \text{ m/s}^2)t_{W1}^2 = 320 \text{ m} - (35 \text{ m/s})t_{W1} \Rightarrow t_{W1} = 0 \text{ s and } 7.1 \text{ s}$$

Indeed, $t_{W1} = 0$ s corresponds to the situation when Superman arrives just as the watermelon is dropped off the Empire State Building. The other value, $t_{W1} = 7.1$ s, is the time when the watermelon will catch up with Superman. The speed of the watermelon as it passes Superman is

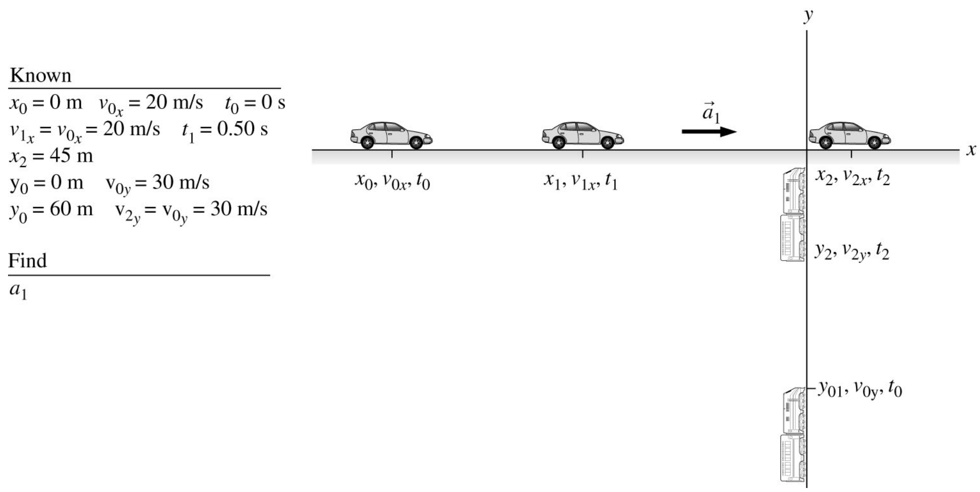
$$v_{W1} = v_{W0} + a_{W0}(t_{W1} - t_{W0}) = 0 \text{ m/s} + (-9.8 \text{ m/s}^2)(7.1 \text{ s} - 0 \text{ s}) = -70 \text{ m/s}$$

Note that the negative sign implies a downward velocity.

Assess: A speed of 140 mph for the watermelon is understandable in view of the significant distance (250 m) involved in the free fall.

2.69. Model: Treat the car and train in the particle model and use the constant acceleration kinematics equations.
Visualize:

Pictorial representation



Solve: In the particle model the car and train have no physical size, so the car has to reach the crossing at an infinitesimally sooner time than the train. Crossing at the same time corresponds to the minimum a_1 necessary to avoid a collision. So the problem is to find a_1 such that $x_2 = 45$ m when $y_2 = 60$ m.
The time it takes the train to reach the intersection can be found by considering its known constant velocity.

$$v_{0y} = v_{2y} = 30 \text{ m/s} = \frac{y_2 - y_0}{t_2 - t_0} = \frac{60 \text{ m}}{t_2} \Rightarrow t_2 = 2.0 \text{ s}$$

Now find the distance traveled by the car during the reaction time of the driver.

$$x_1 = x_0 + v_{0x}(t_1 - t_0) = 0 + (20 \text{ m/s})(0.50 \text{ s}) = 10 \text{ m}$$

The kinematic equation for the final position at the intersection can be solved for the minimum acceleration a_1.

$$x_2 = 45 \text{ m} = x_1 + v_{1x}(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$
$$= 10 \text{ m} + (20 \text{ m/s})(1.5 \text{ s}) + \frac{1}{2}a_1(1.5 \text{ s})^2$$
$$\Rightarrow a_1 = 4.4 \text{ m/s}^2$$

Assess: The acceleration of 4.4 m/s^2 = 2.0 miles/h/s is reasonable for an automobile to achieve. However, you should not try this yourself! Always pay attention when you drive! Train crossings are dangerous locations, and many people lose their lives at one each year.

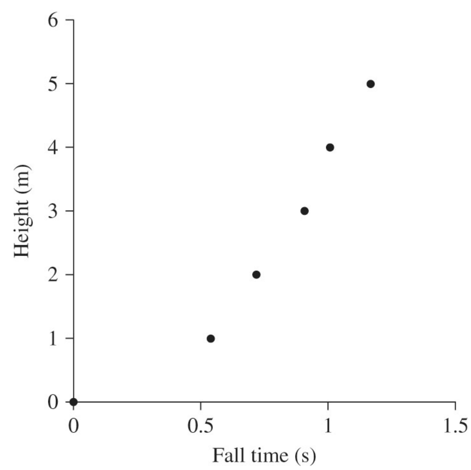
2.70. Model: Model the ball as a particle. Since the ball is heavy we ignore air resistance.

Visualize: We use the kinematic equation $\Delta y = v_0 \Delta t + \frac{1}{2}a(\Delta t)^2$, but we set the origin at the ground so $y_0 = h$ and $y_1 = 0$; this means $\Delta y = y_1 - y_0 = -h$. We release the ball from rest so at $t_0 = 0$ we have $v_0 = 0$ and $\Delta t = t$. We also note that $a = -g$ where g is the free-fall acceleration on Planet X. Making all these substitutions leaves

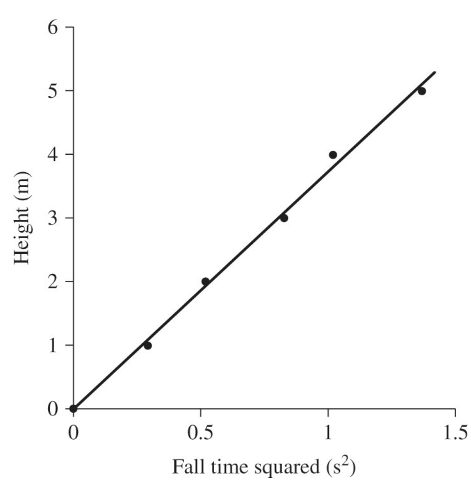
$$h = \left(\frac{1}{2}g\right)t^2$$

So we expect a graph of h vs. t^2 to produce a straight line whose slope is $g/2$ and whose intercept is zero. Compare to $y = mx + b$ where $y = h$, $m = g/2$, $x = t^2$, and $b = 0$.

Solve: First look at a graph of height vs. fall time and notice that it is not linear. It would be difficult to analyze. Even though the point $(0,0)$ is not a measured data point, it is valid to add to the data table and graph because it would take zero time to fall zero distance.



However, the theory has guided us to expect that a graph of height vs. fall time **squared** would be linear and the slope would be $g/2$. First we use a spreadsheet to square the fall times and then graph the height vs. fall time squared to see if it looks linear and that the intercept is close to zero.



It looks linear and $R^2 = 0.996$ tells us the linear fit is very good. We also see that the intercept is a very small negative number which is close to zero, so we have confidence in our model. The fit is not perfect and the intercept is not exactly zero probably because of uncertainties in timing the fall.

We now conclude that the slope of the best fit line $m = 3.7326$ is $g/2$ in the proper units, so $g = 2 \times 3.7326 \text{m/s}^2 = 7.5 \text{m/s}^2$ on Planet X.

Assess: The free-fall acceleration on Planet X is a little bit smaller than on earth, but is reasonable. It is customary to put the independent variable on the horizontal axis and the dependent variable along the vertical axis. Had we done so here we would have graphed t^2 vs. h and the slope would have been $2/g$. Our answer to the question would be the same.

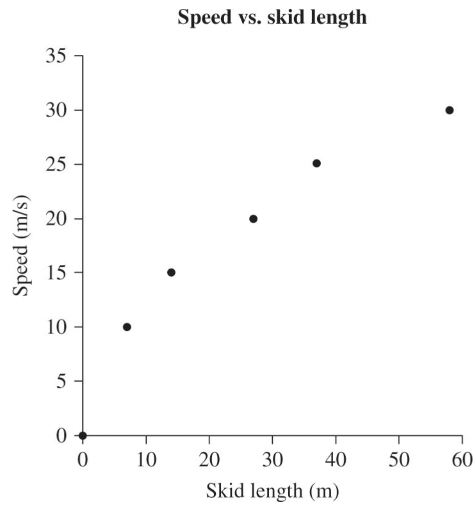
2.71. Model: Model the car as a particle. Ignore air resistance. Hard braking means the wheels are locked (not turning) and the car is in full skid. For convenience, assume the car is skidding to the right.

Visualize: We use the kinematic equation $v_f^2 - v_i^2 = 2a\Delta x.$ In this case $v_f = 0$ and $a < 0,$ but since we want the deceleration (which is the absolute value of the acceleration) we drop the negative signs. Relabel v_i as v. We'll call the beginning of the skid mark the origin so that $x_i = 0$ and the skid length is $\Delta x = x.$ Making these substitutions leaves

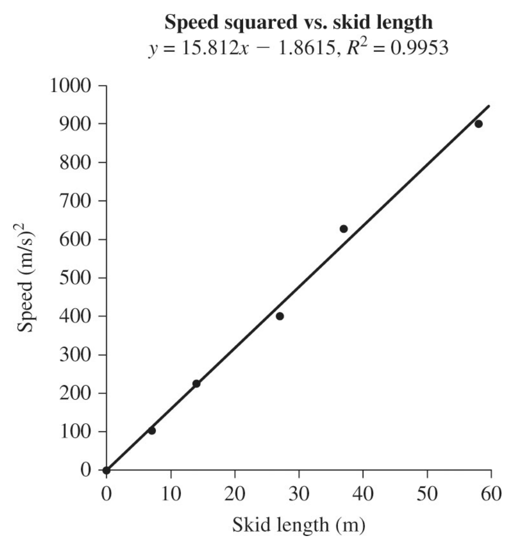
$$v^2 = (2a)x$$

So we expect a graph of v^2 vs. x to produce a straight line whose slope is $2a$ and whose intercept is zero. Compare to $y = mx + b$ where $y = v^2,$ $m = 2a,$ $x = x,$ and $b = 0.$

Solve: First look at a graph of the data of speed vs. skid length and notice that it is not linear. It would be difficult to analyze. We added the point $(0,0)$ to the data table and graph because we are sure that if the speed were zero the skid length would also be zero.



However, the theory has guided us to expect that a graph of speed **squared** vs. skid length would be linear and the slope would be $2a$. First we use a spreadsheet to square the speed and then graph the speed squared vs. skid length to see if it looks linear and that the intercept is close to zero. Only if it is linear is the deceleration constant, independent of speed.



(a) It looks linear and $R^2 = 0.995$ tells us the linear fit is very good. This means the deceleration (involved in the slope) is constant, independent of speed. We also see that the intercept is a very small negative number which is close to zero, so we have confidence in our model. The fit is not perfect and the intercept is not exactly zero probably because of uncertainties in measuring the speed.

(b) We now conclude that the slope of the best fit line $m = 15.812$ is $2a$ in the proper units, so the deceleration is $a = \frac{1}{2} \times 15.812\ \text{m/s}^2 = 7.9\ \text{m/s}^2$.

Assess: The value of $7.9\ \text{m/s}^2$ seems reasonable for hard braking. It is customary to put the independent variable on the horizontal axis and the dependent variable along the vertical axis. Had we done so here we would have graphed x vs. v^2 and the slope would have been $1/2a$. Our answer to the question would be the same.

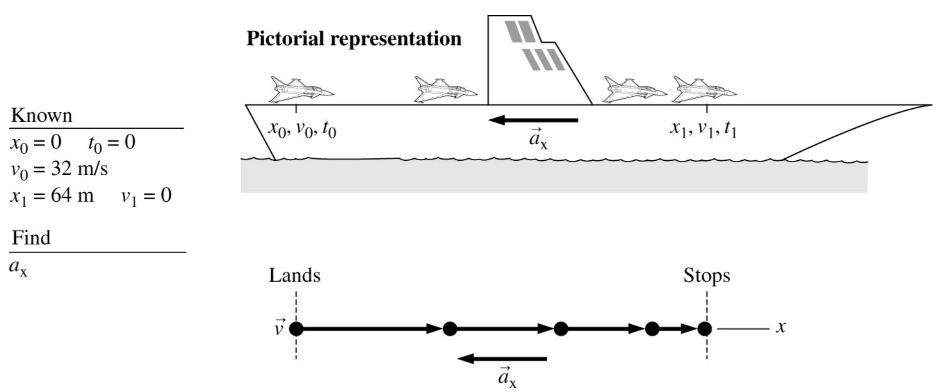
2.72. Solve: A comparison of the given equation with the constant-acceleration kinematics equation

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2$$

yields the following information: $x_0 = 0$ m, $x_1 = 64$ m, $t_0 = 0, t_1 = 4$ s, and $v_0 = 32$ m/s.

(a) After landing on the deck of a ship at sea with a velocity of 32 m/s, a fighter plane is observed to come to a complete stop in 4.0 seconds over a distance of 64 m. Find the plane's deceleration.

(b)



(c) $64\ \text{m} = 0\ \text{m} + (32\ \text{m/s})(4\ \text{s} - 0\ \text{s}) + \frac{1}{2}a_x(4\ \text{s} - 0\ \text{s})^2 \qquad 64\ \text{m} = 128\ \text{m} + (8\ \text{s}^2)a_x \Rightarrow a_x = -8\ \text{m/s}^2$

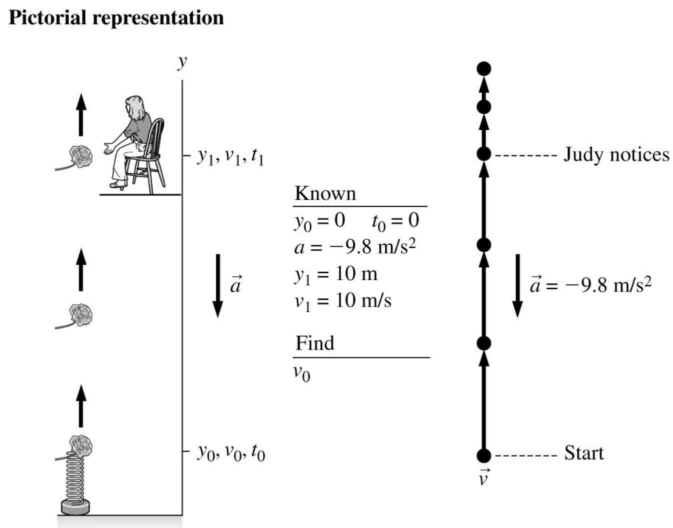
The deceleration is the absolute value of the acceleration, or $8\ \text{m/s}^2$.

2.73. Solve: (a) A comparison of this equation with the constant-acceleration kinematic equation

$$(v_{1y})^2 = v_{0y}^2 + 2(a_y)(y_1 - y_0)$$

yields the following information: $y_0 = 0$ m, $y_1 = 10$ m, $a_y = -9.8\ \text{m/s}^2$, and $v_{1y} = 10$ m/s. It is clearly a problem of free fall. On a romantic Valentine's Day, John decided to surprise his girlfriend, Judy, in a special way. As he reached her apartment building, he found her sitting in the balcony of her second floor apartment 10 m above the first floor. John quietly armed his spring-loaded gun with a rose, and launched it straight up to catch her attention. Judy noticed that the flower flew past her at a speed of 10 m/s. Judy is refusing to kiss John until he tells her the initial speed of the rose as it was released by the spring-loaded gun. Can you help John on this Valentine's Day?

(b)



(c) $(10 \text{ m/s})^2 = v_{0y}^2 - 2(9.8 \text{ m/s}^2)(10 \text{ m} - 0 \text{ m}) \Rightarrow v_{0y} = 17.2 \text{ m/s}$

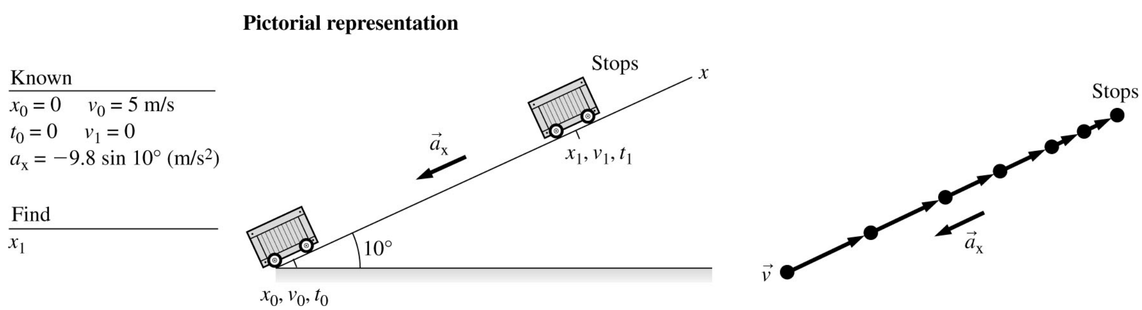
Assess: The initial velocity of 17.2 m/s, compared to a velocity of 10 m/s at a height of 10 m, is very reasonable.

2.74. Solve: A comparison with the constant-acceleration kinematics equation

$$(v_{1x})^2 = (v_{0x})^2 + 2a_x(x_1 - x_0)$$

yields the following quantities: $x_0 = 0$ m, $v_{0x} = 5$ m/s, $v_{1x} = 0$ m/s, and $a_x = -(9.8 \text{ m/s}^2)\sin 10°$.

(a) A wagon at the bottom of a frictionless 10° incline is moving up at 5 m/s. How far up the incline does it move before reversing direction and then rolling back down?
(b)

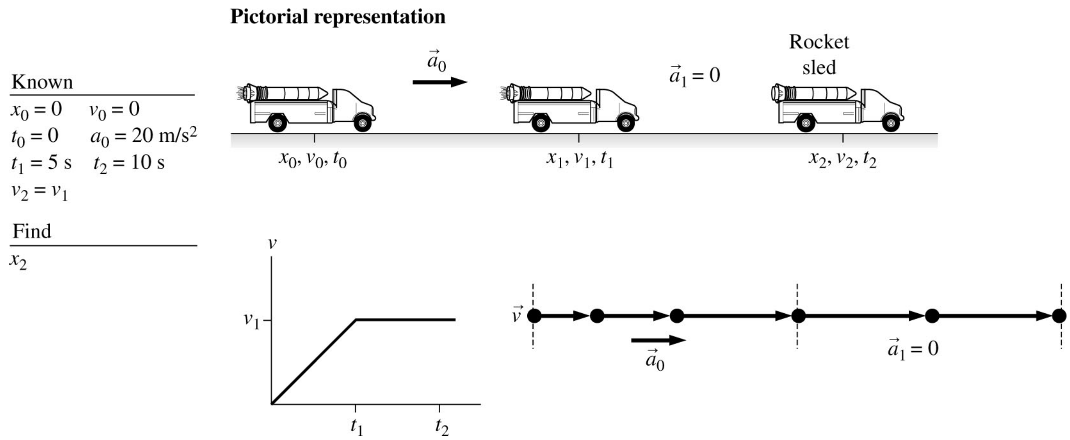


(c) $(0 \text{ m/s})^2 = (5 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)\sin 10°(x_1 - 0 \text{ m})$

$\Rightarrow 25(\text{m/s})^2 = 2(9.8 \text{ m/s}^2)(0.174)x_1 \Rightarrow x_1 = 7.3 \text{ m}$

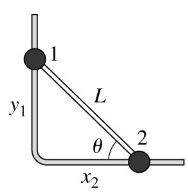
2.75. Solve: (a) From the first equation, the particle starts from rest and accelerates for 5 s. The second equation gives a position consistent with the first equation. The third equation gives a subsequent position following the second equation with zero acceleration. A rocket sled accelerates from rest at 20 m/s^2 for 5 s and then coasts at constant speed for an additional 5 s. Draw a graph showing the velocity of the sled as a function of time up to t = 10 s. Also, how far does the sled move in 10 s?

(b)



(c) $x_1 = \frac{1}{2}(20 \text{ m/s}^2)(5 \text{ s})^2 = 250 \text{ m}$ $\quad v_1 = 20 \text{ m/s}^2(5 \text{ s}) = 100 \text{ m/s}$ $\quad x_2 = 250 \text{ m} + (100 \text{ m/s})(5 \text{ s}) = 750 \text{ m}$

2.76. Model: The masses are particles.
Visualize:



Solve: The rigid rod forms the hypotenuse of a right triangle, which defines a relationship between x_2 and y_1:

$$x_2^2 + y_1^2 = L^2 .$$

Taking the time derivative of both sides yields

$$2x_2 \frac{dx_2}{dt} + 2y_1 \frac{dy_1}{dt} = 0$$

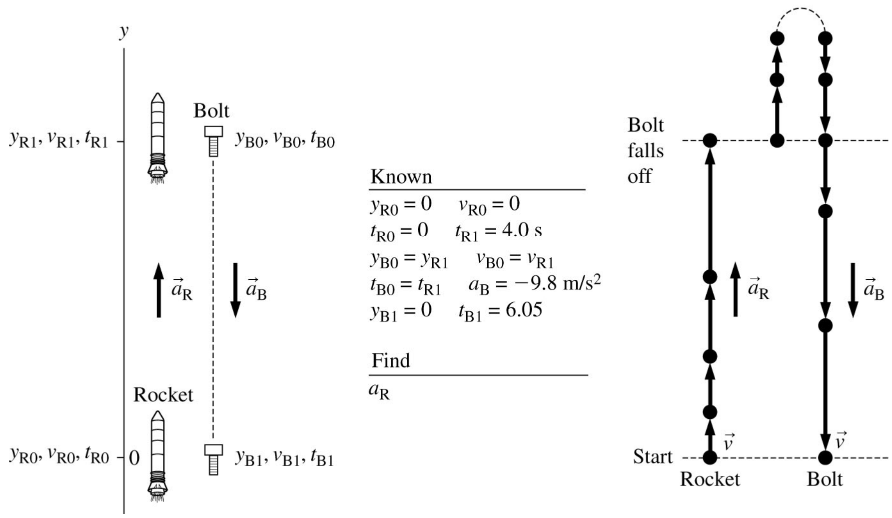
We can now use $v_{2x} = \frac{dx_2}{dt}$ and $v_{1y} = \frac{dy_1}{dt}$ to write $x_2 v_{2x} + y_1 v_{1y} = 0.$

Thus $v_{2x} = -\left(\frac{y_1}{x_2}\right) v_{1y}$. But from the figure, $\frac{y_1}{x_2} = \tan\theta \Rightarrow v_{2x} = -v_{1y} \tan\theta.$

Assess: As x_2 decreases $(v_{2x} < 0)$, y_1 increases $(v_{1y} > 0)$, and vice versa.

2.77. Model: The rocket and the bolt will be represented as particles to investigate their motion.
Visualize:

Pictorial representation



The initial velocity of the bolt as it falls off the side of the rocket is the same as that of the rocket, that is, $v_{B0} = v_{R1}$ and it is positive since the rocket is moving upward. The bolt continues to move upward with a deceleration equal to $g = 9.8\ \text{m/s}^2$ before it comes to rest and begins its downward journey.

Solve: To find a_R we look first at the motion of the rocket:

$$y_{R1} = y_{R0} + v_{R0}(t_{R1} - t_{R0}) + \frac{1}{2}a_R(t_{R1} - t_{R0})^2$$
$$= 0\ \text{m} + 0\ \text{m/s} + \frac{1}{2}a_R(4.0\ \text{s} - 0\ \text{s})^2 = 8a_R$$

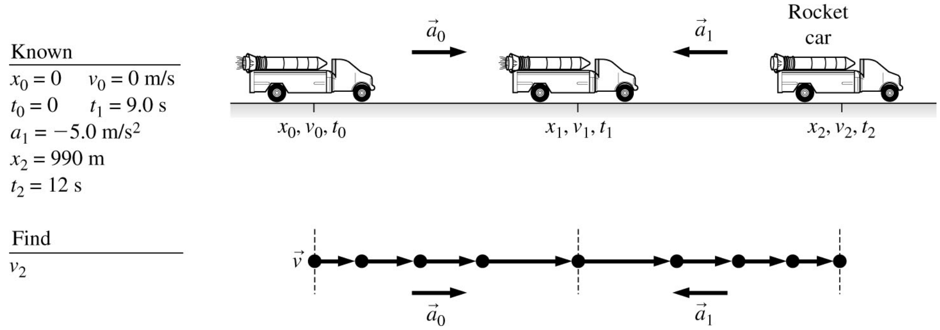
To find a_R we must determine the magnitude of y_{R1} or y_{B0}. Let us now look at the bolt's motion:

$$y_{B1} = y_{B0} + v_{B0}(t_{B1} - t_{B0}) + \frac{1}{2}a_B(t_{B1} - t_{B0})^2$$
$$0 = y_{R1} + v_{R1}(6.0\ \text{s} - 0\ \text{s}) + \frac{1}{2}(-9.8\ \text{m/s}^2)(6.0\ \text{s} - 0\ \text{s})^2$$
$$\Rightarrow y_{R1} = 176.4\ \text{m} - (6.0\ \text{s})\ v_{R1}$$

Since $v_{R1} = v_{R0} + a_R(t_{R1} - t_{R0}) = 0\ \text{m/s} + 4\ a_R = 4\ a_R$ the above equation for y_{R1} yields $y_{R1} = 176.4 - 6.0(4a_R)$. We know from the first part of the solution that $y_{R1} = 8a_R$. Therefore, $8a_R = 176.4 - 24.0a_R$ and hence $a_R = 5.5\ \text{m/s}^2$.

2.78. Model: The rocket car is a particle that moves according to the constant-acceleration equations of motion.
Visualize:

Pictorial representation



Solve: This is a two-part problem. For the first part,

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \frac{1}{2}a_0(9.0\text{ s} - 0\text{ s})^2 = \frac{1}{2}(81\text{ s}^2)a_0$$

$$v_1 = v_0 + a_0(t_1 - t_0) = 0\text{ m/s} + a_0(9.0\text{ s} - 0\text{ s}) = (9.0\text{ s})a_0$$

During the second part of the problem,

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

$$\Rightarrow 990\text{ m} = \frac{1}{2}(81\text{ s}^2)a_0 + (9.0\text{ s})a_0(12\text{ s} - 9.0\text{ s}) + \frac{1}{2}(-5.0\text{ m/s}^2)(12\text{ s} - 9.0\text{ s})^2$$

$$\Rightarrow a_0 = 15\text{ m/s}^2$$

This leads to:

$$v_1 = (9.0\text{ s})a_0 = (9.0\text{ s})(15\text{ m/s}^2) = 135\text{ m/s}$$

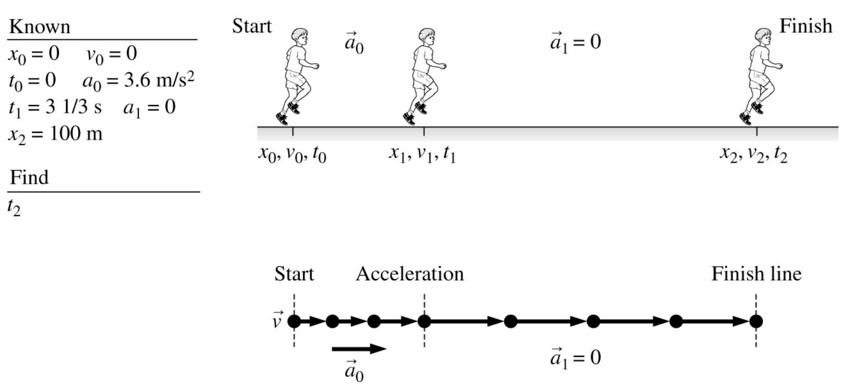
Using this value of v_1, we can now calculate v_2 as follows:

$$v_2 = v_1 + a_1(t_2 - t_1) = (135\text{ m/s}) + (-5.0\text{ m/s}^2)(12\text{ s} - 9.0\text{ s}) = 120\text{ m/s}$$

That is, the car's speed as it passes the judges is 120 m/s.
Assess: This is a very fast motion (~250 mph), but the acceleration is large and the long burn time of 9 s yields a high velocity.

2.79. Model: Use the particle model.
Visualize:

Pictorial representation



Solve: (a) Substituting into the constant-acceleration kinematic equation

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 \Rightarrow 100\text{ m} = x_1 + v_1\left(t_2 - \frac{10}{3}\right) + 0\text{ m}$$

$$t_2 = \frac{100 - x_1}{v_1} + \frac{10}{3}$$

Let us now find v_1 and x_1 as follows:

$$v_1 = v_0 + a_0(t_1 - t_0) = 0\text{ m/s} + (3.6\text{ m/s}^2)\left(\frac{10}{3}\text{ s} - 0\text{ s}\right) = 12\text{ m/s}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \frac{1}{2}(3.6\text{ m/s}^2)\left(\frac{10}{3}\text{ s} - 0\text{ s}\right)^2 = 20\text{ m}$$

The expression for t_2 can now be solved as

$$t_2 = \frac{100\text{ m} - 20\text{ m}}{12\text{ m/s}} + \frac{10\text{ s}}{3} = 10\text{ s}$$

(b) The top speed = 12 m/s which means $v_1 = 12$ m/s. To find the acceleration so that the sprinter can run the 100-meter dash in 9.9 s, we use

$$v_1 = v_0 + a_0(t_1 - t_0) \Rightarrow 12 \text{ m/s} = 0 \text{ m/s} + a_0 t_1 \Rightarrow t_1 = \frac{12 \text{ m/s}}{a_0}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}a_0 t_1^2 = \frac{1}{2}a_0 t_1^2$$

Since $x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$, we get

$$100 \text{ m} = \frac{1}{2}a_0 t_1^2 + (12 \text{ m/s})(9.9 \text{ s} - t_1) + 0 \text{ m}$$

Substituting the above equation for t_1 in this equation,

$$100 \text{ m} = \left(\frac{1}{2}\right)a_0\left(\frac{12 \text{ m/s}}{a_0}\right)^2 + (12 \text{ m/s})\left(9.9 \text{ s} - \frac{12 \text{ m/s}}{a_0}\right) \Rightarrow a_0 = 3.8 \text{ m/s}^2$$

(c) We see from parts (a) and (b) that the acceleration has to be increased from 3.6 m/s^2 to 3.8 m/s^2 for the sprint time to be reduced from 10 s to 9.9 s, that is, by 1%. This decrease of time by 1% corresponds to an increase of acceleration by

$$\frac{3.8 - 3.6}{3.6} \times 100\% = 5.6\%$$

2.80. Solve: (a) The acceleration is the time derivative of the velocity.

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[a(1 - e^{-bt})] = abe^{-bt}$$

With $a = 11.81$ m/s and $b = 0.6887 \text{ s}^{-1}$, $a_x = 8.134e^{-0.6887t}$ m/s^2. At the times $t = 0$ s, 2 s, and 4 s, this has the values 8.134 m/s^2, 2.052 m/s^2, and 0.5175 m/s^2.

(b) Since $v_x = \dfrac{dx}{dt}$, the position x is the integral of the velocity. With $v_x = \dfrac{dx}{dt} = a - ae^{-bt}$ and the initial condition that $x_i = 0$ m at $t_i = 0$ s,

$$\int_o^x dx = \int_o^t a\,dt - \int_o^t ae^{-bt}dt$$

Thus

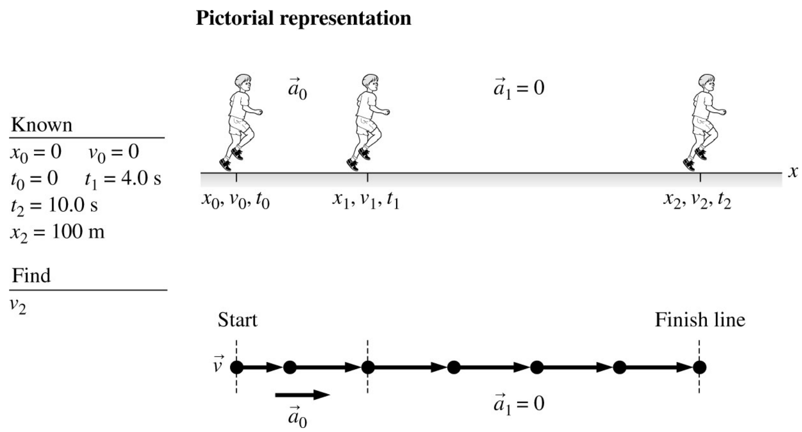
$$x = at\Big|_o^t + \frac{a}{b}e^{-bt}\Big|_o^t = at + \frac{a}{b}e^{-bt} - \frac{a}{b}$$

This can be written a little more neatly as

$$x = \frac{a}{b}(bt + e^{-bt} - 1)$$
$$= 17.15(0.6887t + e^{-0.6887t} - 1) \text{ m}$$

(c) By trial and error, t = 9.92 s yields x = 100.0 m.
Assess: Lewis's actual time was 9.93 s.

2.81. Model: We will use the particle-model to represent the sprinter and the equations of kinematics.
Visualize:



Solve: Substituting into the constant-acceleration kinematic equations,

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0 \text{ m} + 0 \text{ m} + \frac{1}{2}a_0(4 \text{ s} - 0 \text{ s})^2 = \frac{1}{2}a_0 t_1^2 = \frac{1}{2}a_0(4.0 \text{ s})^2$$

$$\Rightarrow x_1 = (8 \text{ s}^2)a_0$$

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + a_0(4.0 \text{ s} - 0 \text{ s}) \Rightarrow v_1 = (4.0 \text{ s})\ a_0$$

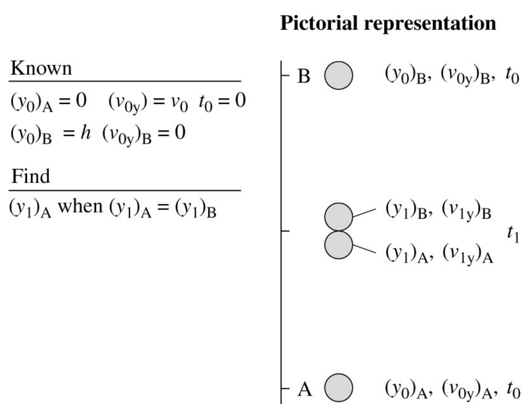
From these two results, we find that $x_1 = (2 \text{ s})v_1$. Now,

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

$$\Rightarrow 100 \text{ m} = (2 \text{ s})v_1 + v_1(10 \text{ s} - 4 \text{ s}) + 0 \text{ m} \Rightarrow v_1 = 12.5 \text{ m/s}$$

Assess: Using the conversion 2.24 mph = 1 m/s, $v_1 = 12.5 \text{ m/s} = 28$ mph. This speed as the sprinter reaches the finish line is physically reasonable.

2.82. Model: The balls are particles undergoing constant acceleration.
Visualize:



Solve: (a) The positions of each of the balls at t_1 is found from kinematics.

$$(y_1)_A = (y_0)_A + (v_{0y})_A t_1 - \frac{1}{2}g t_1^2 = v_0 t_1 - \frac{1}{2}g t_1^2$$

$$(y_1)_B = (y_0)_B + (v_{0y})_B t_1 - \frac{1}{2}g t_1^2 = h - \frac{1}{2}g t_1^2$$

In the particle model the balls have no physical extent, so they meet when $(y_1)_A = (y_1)_B$. This means

$$v_0 t_1 - \frac{1}{2}g t_1^2 = h - \frac{1}{2}g t_1^2 \Rightarrow t_1 = \frac{h}{v_0}$$

Thus the collision height is $y_{\text{coll}} = h - \frac{1}{2}g t_1^2 = h - \frac{gh^2}{2v_0^2}$.

(b) We need the collision to occur while $y_{\text{coll}} \geq 0$. Thus

$$h - \frac{gh^2}{2v_0^2} \geq 0 \Rightarrow 1 \geq \frac{gh}{2v_0^2} \Rightarrow h \leq \frac{2v_0^2}{g}$$

So $h_{\max} = \frac{2v_0^2}{g}$.

(c) Ball A is at its highest point when its velocity $(v_{1y})_A = 0$.

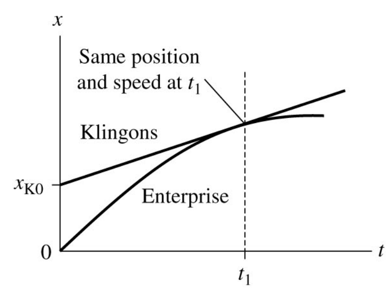
$$(v_{1y})_A = (v_{0y})_A - g t_1 \Rightarrow 0 = v_0 - g t_1 \Rightarrow t_1 = \frac{v_0}{g}$$

In (a) we found that the collision occurs at $t_1 = \frac{h}{v_0}$. Equating these, $\frac{h}{v_0} = \frac{v_0}{g} \Rightarrow h = \frac{v_0^2}{g}$.

Assess: Interestingly, the height at which a collision occurs while Ball A is at its highest point is exactly half of $h_{\max}$.

2.83. Model: The space ships are represented as particles.

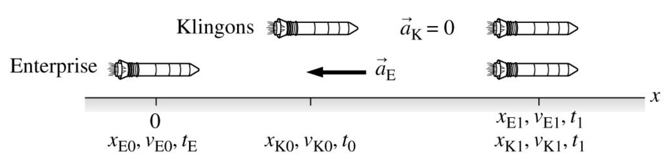
Visualize:

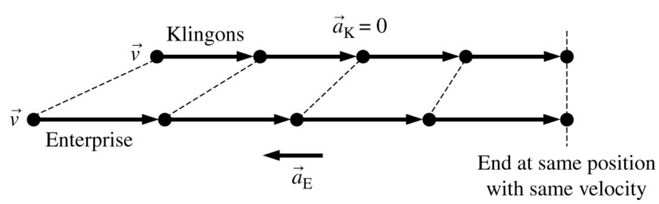


Pictorial representation

Known
$x_{E0} = 0$ $v_{E0} = 50$ km/s
$t_0 = 0$ $x_{K0} = 100$ km
$v_{K0} = 20$ km/s $a_K = 0$
$v_{K1} = v_{K0}$ $x_{E1} = x_{K1}$
$v_{E1} = v_{K1}$

Find
$a_E = a$





Solve: The difficulty with this problem is how to describe "barely avoid." The Klingon ship is moving with constant speed, so its position-versus-time graph is a straight line from $x_{K0} = 100$ km. The Enterprise will be decelerating, so its graph is a parabola with decreasing slope. The Enterprise doesn't have to stop; it merely has to slow quickly enough to match the Klingon ship speed at the point where it has caught up with the Klingon ship. (You do the same thing in your car when you are coming up on a slower car; you decelerate to match its speed just as you come up on its rear bumper.) Thus the parabola of the Enterprise will be tangent to the straight line of the Klingon ship, showing that the two ships have the same speed (same slopes) when they are at the same position. Mathematically, we can say that at time t_1 the two ships will have the same position $(x_{E1} = x_{K1})$ *and* the same velocity $(v_{E1} = v_{K1})$. Note that we are using the particle model, so the ships have zero length. At time t_1,

$$x_{K1} = x_{K0} + v_{K0}t_1 \qquad v_{K1} = v_{K0}$$

$$x_{E1} = v_{E0}t_1 + \frac{1}{2}at_1^2 \qquad v_{E1} = v_{E0} + at_1$$

Equating positions and velocities at t_1 :

$$x_{K0} + v_{K0}t_1 = v_{E0}t_1 + \frac{1}{2}at_1^2 \qquad v_{K0} = v_{E0} + at_1$$

We have two simultaneous equations in the two unknowns a and t_1. From the velocity equation,

$$t_1 = (v_{K0} - v_{E0})/a$$

Substituting into the position equation gives

$$x_{K0} = -(v_{K0} - v_{E0}) \cdot \frac{(v_{K0} - v_{E0})}{a} + \frac{1}{2}a \cdot \left(\frac{(v_{K0} - v_{E0})}{a}\right)^2 = -\frac{(v_{K0} - v_{E0})^2}{2a}$$

$$\Rightarrow a = -\frac{(v_{K0} - v_{E0})^2}{2x_{K0}} = -\frac{(20{,}000 \text{ m/s} - 50{,}000 \text{ m/s})^2}{2(100{,}000 \text{ m})} = -4500 \text{ m/s}^2$$

The magnitude of the acceleration is 4500 m/s^2.

Assess: The deceleration is 4500 m/s^2, which is a rather extreme $\approx 460g$. Fortunately, the Enterprise has other methods to keep the crew from being killed.