## CHAPTER THREE

## Forces

## MULTIPLE CHOICE QUESTIONS

## Multiple Choice 3.1

Correct Answer (e).

$$
M_{\mathrm{p}}=\frac{1}{2} M_{\mathrm{E}} \text { and } R_{\mathrm{p}}=2 R_{\mathrm{E}}
$$

If the acceleration on Earth and the planet are:

$$
g_{\mathrm{E}}=\frac{G M_{\mathrm{E}}}{R_{\mathrm{E}}^{2}} \text { and } g_{\mathrm{P}}=\frac{G M_{\mathrm{P}}}{R_{\mathrm{P}}^{2}},
$$

respectively, the ratio $\frac{g_{\mathrm{P}}}{g_{\mathrm{E}}}$ is:

$$
\begin{gathered}
\frac{g_{\mathrm{P}}}{g_{\mathrm{E}}}=\frac{\frac{G M_{\mathrm{P}}}{R_{\mathrm{P}}^{2}}}{\frac{G M_{\mathrm{E}}}{R_{\mathrm{E}}^{2}}}=\frac{M_{\mathrm{P}} R_{\mathrm{E}}^{2}}{M_{\mathrm{E}} R_{\mathrm{P}}^{2}}=\frac{\left(\frac{1}{2} M_{\mathrm{E}}^{2} R_{\mathrm{E}}^{2}\right)}{M_{\mathrm{E}}\left(4 R_{\mathrm{E}}\right)}=\frac{1}{8} \\
\therefore \frac{g_{\mathrm{P}}}{g_{\mathrm{E}}}=\frac{1}{8} \\
g_{\mathrm{p}}=\frac{1}{8} g_{\mathrm{E}} .
\end{gathered}
$$

## Multiple Choice 3.2

Correct Answer (d). The gravitational force between two objects of masses $m_{1}$ and $m_{2}$ at a distance $d$ is:

$$
F_{\mathrm{g}}=G \frac{m_{1} m_{2}}{d^{2}}
$$

and:

$$
\begin{gathered}
G=6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}} \\
F_{\mathrm{g}}=\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right) \frac{(12 \mathrm{~kg})(25 \mathrm{~kg})}{(1.2 \mathrm{~m})^{2}} \\
F_{\mathrm{g}}=1.39 \times 10^{-8}=1.4 \times 10^{-8} \mathrm{~N} .
\end{gathered}
$$

## Multiple Choice 3.3

Correct Answer (a). The gravitational force is:

$$
F_{\mathrm{g}}=G \frac{m_{1} m_{2}}{d^{2}}
$$

and:

$$
\begin{gathered}
G=6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}} \\
d_{\mathrm{R}}=R_{\mathrm{E}}, \quad d_{2 \mathrm{R}}=R_{\mathrm{E}}+R_{\mathrm{E}}=2 R_{\mathrm{E}} \\
F_{\mathrm{g}, 2 \mathrm{R}}=G \frac{m_{\mathrm{c}} M_{\mathrm{E}}}{\left(2 R_{\mathrm{E}}\right)^{2}}=\frac{1}{4}\left(G \frac{m_{\mathrm{c}} M_{\mathrm{E}}}{\left(R_{\mathrm{E}}\right)^{2}}\right)=\frac{1}{4} F \\
w_{2 \mathrm{R}}=F_{\mathrm{g}, 2 \mathrm{R}}=\frac{1}{4} F=\frac{9.80 \mathrm{~N}}{4}=2.45 \mathrm{~N} .
\end{gathered}
$$

## Multiple Choice 3.4

Correct Answer (b). The gravitational force is:

$$
F_{\mathrm{g}}=G \frac{m_{1} m_{2}}{d^{2}}
$$

and:

$$
\begin{gathered}
G=6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}} \\
R_{\text {Orbit }}= \\
\frac{R_{\mathrm{E}}+h=6378+559=6937 \mathrm{~km}}{\frac{w}{\text { Earth }}^{w_{\text {Orbit }}}=} \frac{F_{g, \text { Earth }}}{F_{g, \text { orbit }}}=\frac{G \frac{m_{\mathrm{HS}} M_{\mathrm{E}}}{\left(R_{\mathrm{E}}\right)^{2}}}{G \frac{m_{\mathrm{HS}} M_{\mathrm{E}}}{\left(R_{\text {Orbit }}\right)^{2}}=\left(\frac{R_{\text {Orbit }}}{R_{\text {Earth }}}\right)^{2}} \\
\frac{w_{\text {Earth }}}{w_{\text {Orbit }}}=\left(\frac{6937}{6378}\right)^{2}=1.18
\end{gathered}
$$

## Multiple Choice 3.5

Correct Answer (a). The acceleration on the planets are:

$$
g_{1}=\frac{G M_{1}}{r_{1}^{2}} \text { and } g_{2}=\frac{G M_{2}}{r_{2}^{2}}
$$

respectively, so the ratio $\frac{g_{2}}{g_{1}}$ is:

$$
\frac{g_{2}}{g_{1}}=\frac{\frac{G M_{2}}{r_{2}^{2}}}{\frac{G M_{1}}{r_{1}^{2}}}=\frac{M_{2} r_{1}^{2}}{M_{1} r_{2}^{2}}
$$

The masses of the two planets can be written as:

$$
\begin{gathered}
M_{1}=\rho V_{1}=\rho\left(\frac{4}{3} \pi r_{1}^{2}\right) \\
M_{2}=\rho V_{2}=\rho\left(\frac{4}{3} \pi r_{2}^{2}\right) \\
\text { So: } \frac{g_{2}}{g_{1}}=\frac{M_{2} r_{1}^{2}}{M_{1} r_{2}^{2}}=\frac{\rho\left(\frac{4}{5} \pi r_{2}^{3}\right) r_{1}^{2}}{\rho\left(\frac{4}{5} \pi r_{1}^{3}\right) r_{2}^{2}}=\frac{r_{2}}{r_{1}} \\
\frac{g_{2}}{g_{1}}=\frac{r_{2}}{r_{1}}
\end{gathered}
$$

## Multiple Choice 3.6

Correct Answer (d). From MC 3.5, the ratio of $\frac{g_{2}}{g_{1}}$ is:

$$
\frac{g_{2}}{g_{1}}=\frac{M_{2} r_{1}^{2}}{M_{1} r_{2}^{2}} \Rightarrow \frac{g_{\mathrm{P}}}{g_{\mathrm{E}}}=\frac{M_{\mathrm{P}} r_{\mathrm{E}}^{2}}{M_{\mathrm{E}} r_{\mathrm{P}}^{2}}
$$

So $\frac{M_{\mathrm{P}}}{g_{\mathrm{E}}}=\frac{g_{\mathrm{P}} r_{\mathrm{P}}^{2}}{g_{\mathrm{E}} r_{\mathrm{E}}^{2}}$

$$
\frac{M_{\mathrm{P}}}{M_{\mathrm{E}}}=\frac{\frac{1}{2} g_{\mathrm{E}}\left(\frac{1}{2} r_{\mathrm{E}}\right)^{2}}{g_{\mathrm{E}} r_{\mathrm{E}}^{2}}=\frac{1}{8}=0.12
$$

## Multiple Choice 3.7

Correct Answer (c).

## Multiple Choice 3.8

Correct Answer (d).

## Multiple Choice 3.9

Correct Answer (d). The force between two charges $q_{1}$ and $q_{2}$ is:

$$
F=\frac{k q_{1} q_{2}}{r^{2}}
$$

The force is proportional to $1 / r^{2}$. That means doubling the distance quarters the force. In this problem, we decrease the distance by 5 times, so the force increases by $5^{2}$ times. So:

$$
\begin{gathered}
\frac{F_{2}}{F_{1}}=\frac{k \frac{q_{1} q_{2}}{r_{2}^{2}}}{k \frac{q_{1} q_{2}}{r_{1}^{2}}}=\frac{r_{1}^{2}}{r_{2}^{2}}=\left(\frac{r_{1}}{r_{2}}\right)^{2} \\
F_{2}=F_{1}\left(\frac{r_{1}}{r_{2}}\right)^{2}=3 \times 10^{-7} \mathrm{~N}\left(\frac{10 \mathrm{~cm}}{2 \mathrm{~cm}}\right)^{2} \\
F_{2}=7.5 \times 10^{-6} \mathrm{~N} .
\end{gathered}
$$

## Multiple Choice 3.10

Correct Answer (c). The electric force is proportional to $1 / r^{2}$, so if we increase the distance by a factor of three, the force is reduced by a factor of $3^{2}$, or 9 .

## Multiple Choice 3.11

Correct Answer (d). The electric force is proportional to $q$, so if we double the magnitude of one the charges, the magnitude of the force between them increases to twice its former magnitude.

Multiple Choice 3.12 Correct Answer (c). Easy way: Doubling the distance reduces the force by a factor of 4 . Since $F$ is proportional to $q$, doubling one of the charges doubles the force.

Combining these factors effects we get a factor of:

$$
\left(\frac{1}{4}\right)(2)=\frac{1}{2},
$$

So, the force is reduced by a factor of 2 .
Longer way: Let

$$
F_{1}=\frac{k q_{1} q_{2}}{r_{1}^{2}}
$$

then

$$
F_{2}=\frac{k\left(2 q_{1}\right) q_{2}}{\left(2 r_{1}\right)^{2}}=\frac{k q_{1} q_{2}}{2 r_{1}^{2}}=\frac{1}{2} F_{1} .
$$

## Multiple Choice 3.13

Correct Answer (e). The electric force between two charges is:

$$
F_{12}=F_{21}=k \frac{q_{1} q_{2}}{r_{12}^{2}}
$$

First, we calculate the forces $q_{2}$ and $q_{3}$ exert on $q_{1}$. Then, find the $x$ and $y$ components of the forces, which are the components of the
net force on $q_{1}$. The forces $q_{2}$ and $q_{3}$ each exerts on $q_{1}$ are:

$$
F_{12}=\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(15 \times 10^{-6} \mathrm{C}\right)\left(13 \times 10^{-6} \mathrm{C}\right)}{\left(7.5 \times 10^{-2} \mathrm{~m}\right)^{2}}
$$

## Figure 1

$$
\begin{gathered}
F_{13}=\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(15 \times 10^{-6} \mathrm{C}\right)\left(13 \times 10^{-6} \mathrm{C}\right)}{\left(12.5 \times 10^{-2} \mathrm{~m}\right)^{2}} \\
F_{13}=112 \mathrm{~N} .
\end{gathered}
$$

Figure 1 shows $F_{12}, F_{13}$, and its $x$ - and $y$-components, which are:

$$
\begin{aligned}
& F_{13, x}=F_{13} \cos 37^{\circ}=89 \mathrm{~N} \\
& F_{13, y}=F_{13} \sin 37^{\circ}=67 \mathrm{~N}
\end{aligned}
$$

So:

$$
F_{\mathrm{Net}, x}=F_{13, x} \approx 89 \mathrm{~N}
$$

$$
F_{\mathrm{Net}, y}=F_{12}+F_{13, y}=312+67 \approx 380 \mathrm{~N} .
$$

## Multiple Choice 3.14

Correct Answer (a). The electric force between two charges is shown in Multiple Choice 3.13:

$$
F_{12}=k \frac{q_{1} q_{2}}{r_{12}^{2}}
$$

Since $q_{1}=q_{2}=q_{3}$, and they are located on the corners of a square (see Figure 2):


Figure 2
So:

$$
\begin{gathered}
r_{12}=r_{23}=a \\
r_{13}=\sqrt{\left(r_{12}^{2}+r_{23}^{2}\right)}=\sqrt{\left(a^{2}+a^{2}\right)}=\sqrt{2} a .
\end{gathered}
$$

Therefore:

$$
\frac{F_{13}}{F_{21}}=\frac{k \frac{q_{1} q_{3}}{(\sqrt{2} a)^{2}}}{k \frac{q_{1} q_{2}}{(a)^{2}}}=\frac{1}{2} .
$$

## Multiple Choice 3.15

Correct Answer (d). The strong nuclear force is the strongest of the four fundamental forces and holds the protons together in the nucleus.

## Multiple Choice 3.16

Correct Answer (c). The strong nuclear force is about 100 times larger than the electric force over the same distance and only acts over very short distances.

## Multiple Choice 3.17

Correct Answer (d). The strong nuclear force does not get weaker with distance. For two quarks, it reaches a constant value of about $10,000 \mathrm{~N}$. The weak force diminishes with distance.

## Multiple Choic 3.18

Correct Answer (b).

## Multiple Choice 3.19

Correct Answer (e). Since the kicker's foot is no longer in contact with the ball, it no longer exerts a force on the ball. The force exerted by the floor consists of two parts, a normal force $N$ and a frictional force that makes the ball rotate. The force of gravity also acts upon the ball.

## Multiple Choice 3.20

Correct Answer (c). The forces $T$ and $F$ are contact forces. If the muscles were suddenly cut, the tension would disappear. Similarly, if the dumbbell were released, the force $F$ would disappear.

## Multiple Choice 3.21

Correct Answer (b). Figure 3 shows the normal force and the horizontal component of the applied force $\vec{F}$.


Figure 3
According to Figure 3:

$$
\begin{gathered}
N-F \cos \theta=0 \\
N=F \cos \theta .
\end{gathered}
$$

## Multiple Choice 3.22

Correct Answer (b). When you press your book horizontally against the wall, you prevent it from falling. So, the friction force is upward.

## Multiple Choice 3.23

Correct Answer (c). The direction of the friction force exerted on the truck is to the west. So, the direction of the friction force exerted on the box by the truck is due east.

## Multiple Choice 3.24

Correct Answer (d). The maximum force of static friction is:

$$
f_{\mathrm{s}, \mathrm{Max}}=\mu_{\mathrm{s}} N .
$$

Since $N=F \cos \theta$ :

$$
f_{\mathrm{s}, \operatorname{Max}}=\mu_{\mathrm{s}} N=\mu_{\mathrm{s}} F \cos \theta .
$$

## Multiple Choice 3.25

Correct Answer (c). The component of weight acting down the incline is $m g \sin \theta$. Since the block is stationary, static friction must balance this force (See Figure 4).


Figure 4

## Multiple Choice 3.26

Correct Answer (c). The normal force exerted by the plane must balance the component of the weight perpendicular to the plane. This component is $M g \cos \theta$. See Figure 4.

## Multiple Choice 3.27

Correct Answer (a). The magnitude of the force exerted by a spring stretched a distance $x$ from its equilibrium position is $F=k x$. If $x$ is doubled, then the force must be doubled.

## Multiple Choice 3.28

Correct Answer (c).
Since $F=k x$, if applied force $F$ increases 16 times and $k$ is increased 4 times, $x$ will change to 4 times the first spring stretch.

## Multiple Choice 3.29

Correct Answer (e). Both marbles move with constant velocities, which imply that the net force on each marble is zero. This, in turn, implies that the viscous force on the first marble is equal to its weight and the viscous force on the second marble is equal to the weight. This, in other words, means that the ratio of the magnitudes of the viscous forces is equal to the ratio of the weights and, therefore, equal to the ratio of the masses. We conclude that the ratio of the amplitudes of the viscous forces is equal to the cubic power of the ratio of the diameters: 8. This answer can also be obtained using the expression for the viscous force $F_{\text {vis }}=$ $6 \pi \eta r v$. The ratio of the viscous forces is equal to:

$$
\frac{F_{\mathrm{vi}, 1}}{F_{\mathrm{vi}, 2}}=\frac{r_{1} v_{1}}{r_{2} v_{2}}=\frac{r_{1} v_{1}}{\left(2 r_{1}\right)\left(4 v_{1}\right)}=\frac{1}{8} .
$$

## Multiple Choice 3.30

Correct Answer (b).

## Multiple Choice 3.31

Correct Answer (c). The hanging mass $M$ is in equilibrium, the same as the cross point of three forces $T_{1}, T_{2}, T_{3}$. Draw free body diagram, Figure 5:


Figure 5
According to free body diagram of Figure 5, we have:

$$
\begin{gathered}
\left\{\begin{array}{l}
x: \Rightarrow T_{1, x}-T_{2, x}=0 \\
y: \Rightarrow T_{2, y}-T_{3}=0
\end{array}\right. \\
T_{3}=w=m g \\
\therefore T_{1, x}=T_{2, x} \\
T_{1}-T_{2} \cos 60^{\circ}=0 \\
T_{2}=T_{1} / \cos 60^{\circ}=21 / 0.5=42 \mathrm{~N} \\
w=T_{3}=T_{2, y}=T_{2} \sin 60^{\circ}=42 \sin 60^{\circ} \\
=36 \mathrm{~N} .
\end{gathered}
$$

## Multiple Choice 3.32

Correct Answer (d). The hanging mass $M$ is in equilibrium, same as the cross point of three forces $T_{1}, T_{2}, T_{3}$. Draw free body diagram, Figure 6:


Figure 6
According to free body diagram of Figure 6, we have:

$$
\left.\begin{array}{c}
\left\{\begin{array}{c}
x: \Rightarrow T_{2, x}-T_{1, x}=0 \\
y: \Rightarrow T_{2, y}+T_{1, y}-T_{3}=0
\end{array}\right. \\
T_{3}=w=m g \\
\therefore T_{1, x}=T_{2, x}
\end{array}\right\} \begin{gathered}
T_{1} \cos 60^{\circ}-T_{2} \cos 30^{\circ}=0 \\
T_{1}=T_{2} \cos 30^{\circ} / \cos 60^{\circ}=41(1.73)=71 \mathrm{~N} .
\end{gathered}
$$

## CONCEPTUAL QUESTIONS

## Conceptual Question 3.1

(a)
(A) Two forces: the weight and the tension
(B) The weight and the normal to the bowl (along the radius)
(C) The weight and the normal to the bowl (vertical, in this case)
(D) The weight and the normal force due to the table
(E) The weight, the normal due to the incline, and the tension (no friction)
(b) The free body diagrams need to be drawn.
(c) In case (B) of part (a), the object is not in static equilibrium.

## Conceptual Question 3.2

It is the influence of gravity. You have your head down, helping to push more blood to your head, thus more oxygen to your brain. When you stand rapidly, the blood is not supplied as before; it rushes out, which is the influence of gravity pull.

## Conceptual Question 3.3

Both forces are contact forces, that is, contact is required for a force to be exerted. This is in contrast to a force like gravity, which can act over a distance, with no contact. The main difference between the normal force and the spring force is that the normal force is a constant force, whereas the spring force varies with distance.

## Conceptual Question 3.4

Correct Answer (A).

## Conceptual Question 3.5

The weight of the body part can be neglected if it is perfectly balanced by another force.

## Conceptual Question 3.6

If you turn the adjustable inclined plane up, increasing the inclined angle, the mass on the plane starts to move down in a specific inclined angle. If you increase the inclined angle more, the mass moves down the inclined plane. So, at the angle that it starts to move, the total force parallel to the inclined plane acting on the object is zero. Thus, you may calculate the normal force on the object by the inclined plane. Its magnitude is equal to the magnitude of the perpendicular component of the weight. Then you can find the maximum force of friction, which is equal to the coefficient of static friction times the normal force. The maximum force of friction should be equal to the magnitude of the parallel component of the weight.

## Conceptual Question 3.7

The magnitude of the normal force on the book is equal to the force you exert on the book. Since you prevent the book falling down, the friction from the surface of the wall is upward, in the direction against gravity.

## Conceptual Question 3.8

It is easier for you if you pull the sled since, when you pull it at angle $\theta$, the vertical component of your force is upward and reduces the magnitude of the normal force. The magnitude of the normal force, in this case, will be less than the weight of your sister plus the sled. So, having less normal force gives less force of friction. This is in contrast to when you push it. Since the push will increase the normal force, so does it the friction.

## Conceptual Question 3.9

With the force of gravity, weight decreases with the distance from the surface of Earth. Therefore, you should buy the gold at a higher altitude. Since Calgary is at a higher altitude than Halifax, you should buy gold in Calgary and sell it in Halifax. Since the mass is the same everywhere, it does not make any difference where you buy your gold by mass, whether in Montreal or Toronto.

## Conceptual Question 3.10

Doubling the distance reduces the force by a factor of 4 . So, at 2R, the gravitational force on him would be one fourth (a quarter) of his weight (gravitational force) on Earth. So, the gravitational force on him at 4 R would be one fourth of the gravitational force at $2 R$ and one sixteenth of that on the surface of Earth.

## ANALYTICAL PROBLEMS

## Problem 3.1

The distance between the sphere centres is $r=0.5 m+0.5 m=1.0 m$. The gravitational force between them is:

$$
\begin{aligned}
F & =\frac{G m m}{r^{2}}=\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)(15 \mathrm{~kg})(15 \mathrm{~kg})}{(1.0 \mathrm{~m})^{2}} \\
& =1.50 \times 10^{-8} \mathrm{~N} .
\end{aligned}
$$

If the surface of spheres are separated by 2.0 m , then:

$$
r=0.5 m+2.0 m+0.5 m=3.0 m \text {. }
$$

The gravitational force between them is now:

$$
\begin{aligned}
F & =\frac{G m m}{r^{2}}=\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)(15 \mathrm{~kg})(15 \mathrm{~kg})}{(3.0 \mathrm{~m})^{2}} \\
& =1.68 \times 10^{-9} \mathrm{~N} .
\end{aligned}
$$

## Problem 3.2

Take the radius of the Earth:

$$
R_{\mathrm{E}}=6.37 \times 10^{6} \mathrm{~m} .
$$

The acceleration of gravity is:

$$
\begin{aligned}
g & =\frac{G M_{\mathrm{E}}}{R_{\mathrm{E}}^{2}}=\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(6.00 \times 10^{24}\right)}{\left(6.37 \times 10^{6} \mathrm{~m}\right)^{2}} \\
& =9.86 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} .
\end{aligned}
$$

At 9800 km above the Earth's surface, the gravitational acceleration is:

$$
\begin{aligned}
g & =\frac{G M_{\mathrm{E}}}{\left(R_{\mathrm{E}}+9.8 \times 10^{6}\right)^{2}} \\
& =9.86 \times\left(\frac{R_{\mathrm{E}}}{R_{\mathrm{E}}+9.8 \times 10^{6}}\right)^{2} \\
& =1.53 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} .
\end{aligned}
$$

## Problem 3.3

The gravitational force between the spheres is:

$$
F=\frac{G m m}{r^{2}}=0.50 \mathrm{~N} .
$$

Solving for the mass $m$, we get:

$$
\begin{aligned}
m^{2}= & \frac{F r^{2}}{G} \\
\text { so } m & =\sqrt{\frac{F r^{2}}{G}}=\sqrt{\frac{(0.5 \mathrm{~N})(2.00 \mathrm{~m})^{2}}{6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}}} \\
& =1.73 \times 10^{5} \mathrm{~kg} .
\end{aligned}
$$

## Problem 3.4

(a) Known:
$M_{\mathrm{E}}=6.00 \times 10^{24} \mathrm{~kg}, M_{\mathrm{m}}=7.40 \times 10^{22} \mathrm{~kg}$, and $R_{\mathrm{EM}}=3.84 \times 10^{8} \mathrm{~m}$.

$$
\begin{aligned}
& F_{\mathrm{EM}}=\frac{G M_{\mathrm{E}} M_{\mathrm{M}}}{R_{\mathrm{EM}}^{2}} \\
& =\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(6.00 \times 10^{24} \mathrm{~kg}\right)\left(7.40 \times 10^{22} \mathrm{~kg}\right)}{\left(3.84 \times 10^{8} \mathrm{~m}\right)^{2}} \\
& =2.00 \times 10^{20} \mathrm{~N} .
\end{aligned}
$$

(b) $F_{\text {Net }}=F_{\text {mbyE }}-F_{\text {mby M }}=0$

$$
\Rightarrow F_{\mathrm{mbyE}}=F_{\mathrm{mby} \mathrm{M}}
$$

$$
\begin{gathered}
G \frac{m M_{\mathrm{E}}}{R_{\mathrm{mE}}^{2}}=G \frac{m M_{\mathrm{M}}}{R_{\mathrm{mM}}^{2}} \\
\frac{M_{\mathrm{E}}}{M_{\mathrm{M}}}=\frac{R_{\mathrm{mE}}^{2}}{R_{\mathrm{mM}}^{2}} \\
\frac{R_{\mathrm{mE}}}{R_{\mathrm{mM}}}=\sqrt{\frac{M_{\mathrm{E}}}{M_{\mathrm{M}}}}=\sqrt{\frac{6.00 \times 10^{24} \mathrm{~kg}}{7.40 \times 10^{22} \mathrm{~kg}}}=9.00 \\
\Rightarrow R_{\mathrm{mE}}=9.00 R_{\mathrm{mM}}
\end{gathered}
$$

and:

$$
\begin{gathered}
R_{\mathrm{mE}}+R_{\mathrm{mM}}=R_{\mathrm{EM}}=3.84 \times 10^{8} \mathrm{~m} \\
9.00 R_{\mathrm{mM}}+R_{\mathrm{mM}}=3.84 \times 10^{8} \mathrm{~m} \\
R_{\mathrm{mM}}=3.84 \times 10^{7} \mathrm{~m} \\
R_{\mathrm{mE}}=9.00\left(3.84 \times 10^{7} \mathrm{~m}\right)=3.45 \times 10^{8} \mathrm{~m}
\end{gathered}
$$

## Problem 3.5

The ratio of the Sun and Earth's gravitational force on the Moon is:

$$
\begin{gathered}
\frac{F_{\mathrm{SM}}}{F_{\mathrm{EM}}}=\frac{\frac{G M_{\mathrm{S}} M_{\mathrm{M}}}{R_{\mathrm{SM}}^{2}}}{\frac{G M_{\mathrm{E}} M_{\mathrm{M}}}{R_{\mathrm{EM}}^{2}}}=\frac{M_{\mathrm{S}} R_{\mathrm{EM}}^{2}}{M_{\mathrm{E}} R_{\mathrm{SM}}^{2}} \\
\frac{F_{\mathrm{SM}}}{F_{\mathrm{EM}}}=\frac{\left(2.0 \times 10^{30} \mathrm{~kg}\right)\left(3.84 \times 10^{8} \mathrm{~m}\right)^{2}}{\left(6.0 \times 10^{24} \mathrm{~kg}\right)\left(1.5 \times 10^{11} \mathrm{~m}\right)^{2}} \\
\frac{F_{\mathrm{SM}}}{F_{\mathrm{EM}}}=2.2 .
\end{gathered}
$$

Or we can calculate the gravitational force of Sun and Earth on the Moon separately and then find the ratio of them. We calculated the force of Earth on the Moon in
the previous problem. The force of the Sun on the Moon is:

$$
\begin{aligned}
& F_{\mathrm{SM}}=G \frac{M_{\mathrm{S}} M_{\mathrm{M}}}{R_{\mathrm{SM}}^{2}} \\
& =\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(2.00 \times 10^{30} \mathrm{~kg}\right)\left(7.40 \times 10^{22} \mathrm{~kg}\right)}{\left(1.5 \times 10^{11} \mathrm{~m}\right)^{2}} \\
& =4.4 \times 10^{20} \mathrm{~N} .
\end{aligned}
$$

So, the ratio of these two forces are:

$$
\frac{F_{\mathrm{SM}}}{F_{\mathrm{EM}}}=\frac{4.4 \times 10^{20} \mathrm{~N}}{2.00 \times 10^{20} \mathrm{~N}}=2.2 .
$$

## Problem 3.6

The ratio of the Sun and the Moon's gravitational force on a person on Earth is:

$$
\frac{F_{\mathrm{Sm}}}{F_{\mathrm{Mm}}}=\frac{\frac{G M_{\mathrm{S}} m}{R_{\mathrm{SE}}^{2}}}{\frac{G M_{\mathrm{M}} m}{R_{\mathrm{ME}}^{2}}}=\frac{M_{\mathrm{S}} R_{\mathrm{ME}}^{2}}{M_{\mathrm{M}} R_{\mathrm{SE}}^{2}}
$$

$$
\begin{aligned}
& R_{\mathrm{SE}}= R_{\mathrm{SM}}+R_{\mathrm{ME}} \\
&= 3.84 \times 10^{8} \mathrm{~m}+1.5 \times 10^{11} \mathrm{~m} \\
&= 1.50384 \times 10^{11} \mathrm{~m} \\
& \frac{F_{\mathrm{Sm}}}{F_{\mathrm{Mm}}}=\frac{\left(2.0 \times 10^{30} \mathrm{~kg}\right)\left(3.84 \times 10^{8} \mathrm{~m}\right)^{2}}{\left(7.40 \times 10^{22} \mathrm{~kg}\right)\left(1.50384 \times 10^{11} \mathrm{~m}\right)^{2}} \\
& \frac{F_{\mathrm{Sm}}}{F_{\mathrm{Mm}}}=1.77 \times 10^{2} .
\end{aligned}
$$

So, the Sun's gravitational force is larger than the Moon's gravitational force on a person on Earth and on Earth itself.

## Problem 3.7

The acceleration of gravity is:

$$
\begin{gathered}
g_{\mathrm{Ma}}=\frac{G M_{\mathrm{Ma}}}{R_{\mathrm{Ma}}^{2}} \\
R=\sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(6.40 \times 10^{23} \mathrm{~kg}\right)}{3.62 \frac{\mathrm{~m}}{\mathrm{~g}_{\mathrm{Ma}}^{2}}}} \\
R_{\mathrm{Ma}}=3.43 \times 10^{6} \mathrm{~m} .
\end{gathered}
$$

## Problem 3.8

The maximum force of gravity happens when they are closest to each other, touching each other. So:

$$
\begin{aligned}
R & =R_{\mathrm{bi}}+R_{\mathrm{bo}} \\
& =2.8 \times 10^{-2} \mathrm{~m}+11 \times 10^{-2} \mathrm{~m} \\
& =13.8 \times 10^{-2} \mathrm{~m}=1.38 \times 10^{-1} \mathrm{~m} .
\end{aligned}
$$

The gravitational force between the balls is

$$
\begin{gathered}
F=\frac{G m_{\mathrm{bi}} m_{\mathrm{bo}}}{R^{2}} \\
F=\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)(0.380 \mathrm{~kg})(7.0 \mathrm{~kg})}{\left(1.38 \times 10^{-1} \mathrm{~m}\right)^{2}} \\
F=9.3 \times 10^{-9} \mathrm{~N} .
\end{gathered}
$$

## Problem 3.9

(a) The net gravitation force on $m_{3}$ is:

$$
F_{3, \mathrm{Net}}=F_{31}-F_{32}
$$

$$
F_{3, \text { Net }}=\frac{G m_{3} m_{1}}{r_{31}^{2}}-\frac{G m_{3} m_{2}}{r_{32}^{2}}
$$

$$
\begin{gathered}
r_{31}=r_{32}=d_{12} / 2=0.6 \mathrm{~m} / 2=0.3 \mathrm{~m} \\
F_{3, \text { Net }}=G \frac{m_{3}}{r_{31}^{2}}\left(m_{1}-m_{2}\right) \\
F_{3, \mathrm{Net}}=\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(\frac{75.0 \mathrm{~kg}}{(0.3 \mathrm{~m})^{2}}\right) \\
\times(750 \mathrm{~kg}-300 \mathrm{~kg})=2.50 \times 10^{-5} \mathrm{~N} \\
F_{3, \mathrm{Net}}=2.50 \times 10^{-5} \mathrm{~N} .
\end{gathered}
$$

(b)

$$
\begin{gathered}
F_{3, \text { Net }}=F_{31}-F_{32}=0 \\
F_{31}=F_{32} \\
\frac{G m_{3} m_{1}}{r_{31}^{2}}=\frac{G m_{3} m_{2}}{r_{32}^{2}} \\
\frac{m_{1}}{r_{31}^{2}}=\frac{m_{2}}{r_{32}^{2}}
\end{gathered}
$$

$$
\begin{gathered}
\frac{r_{32}}{r_{31}}=\sqrt{\frac{m_{2}}{m_{1}}} \\
\frac{r_{32}}{r_{31}}=\sqrt{\frac{300}{750}}=0.63 \\
r_{32}=0.63 r_{31}
\end{gathered}
$$

Also:

$$
\begin{gathered}
r_{31}+r_{32}=d_{12}=0.6 \mathrm{~m} \\
r_{31}+0.63 r_{31}=1.63 r_{31}=0.6 \mathrm{~m} \\
r_{31}=0.6 / 1.63 \mathrm{~m}=0.37 \mathrm{~m} \\
r_{32}=0.63(0.37 \mathrm{~m})=0.23 \mathrm{~m}
\end{gathered}
$$

## Problem 3.10

$$
\begin{gathered}
F_{\mathrm{Net}, \mathrm{M}}=F_{\mathrm{MbyS}}-F_{\mathrm{Mby} \mathrm{E}} \\
F_{\mathrm{Net}}=G \frac{M_{\mathrm{S}} M_{\mathrm{M}}}{R_{\mathrm{SM}}^{2}}-G \frac{M_{\mathrm{E}} M_{\mathrm{M}}}{R_{\mathrm{EM}}^{2}} \\
F_{\mathrm{Net}}=G M_{\mathrm{M}}\left(\frac{M_{\mathrm{S}}}{R_{\mathrm{SM}}^{2}}-\frac{M_{\mathrm{E}}}{R_{\mathrm{EM}}^{2}}\right) \\
F_{\mathrm{Net}}=\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(7.40 \times 10^{22} \mathrm{~kg}\right) \\
\times\left(\frac{2.0 \times 10^{30} \mathrm{~kg}}{\left(1.5 \times 10^{11} \mathrm{~m}\right)^{2}}-\frac{6.0 \times 10^{24} \mathrm{~kg}}{\left(3.84 \times 10^{8} \mathrm{~m}\right)^{2}}\right) \\
F_{\mathrm{Net}}=2.38 \times 10^{20} \mathrm{~N}
\end{gathered}
$$

## Problem 3.11

The distance halfway between Earth and the Moon is:

$$
r=d_{\mathrm{EM}} / 2=3.84 \times 10^{8} \mathrm{~m} / 2=1.92 \times 10^{8} \mathrm{~m}
$$

The net gravitational force on Apollo exerted by Earth and the Moon is:

$$
\begin{gathered}
F_{\mathrm{Net}, \mathrm{~A}}=F_{\mathrm{AbyE}}-F_{\mathrm{Aby} \mathrm{M}} \\
F_{\mathrm{Net}}=G \frac{M_{\mathrm{E}} M_{\mathrm{A}}}{R_{\mathrm{EA}}^{2}}-G \frac{M_{\mathrm{M}} M_{\mathrm{A}}}{R_{\mathrm{MA}}^{2}} \\
F_{\mathrm{Net}}=G \frac{M_{\mathrm{A}}}{R^{2}}\left(M_{\mathrm{E}}-M_{\mathrm{M}}\right) \\
F_{\mathrm{Net}}=\left(6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}\right)\left(\frac{3.01 \times 10^{4} \mathrm{~kg}}{\left(1.92 \times 10^{8} \mathrm{~m}\right)^{2}}\right) \\
\times\left(6.0 \times 10^{24} \mathrm{~kg}-7.40 \times 10^{22} \mathrm{~kg}\right) \\
=3.23 \times 10^{2} \mathrm{~N} .
\end{gathered}
$$

## Problem 3.12

According to Newton's third law, the force is equal in magnitude and opposite in direction on each charge. The magnitude of the force is given by:

$$
\begin{aligned}
F & =\frac{k q_{1} q_{2}}{r^{2}} \\
& =\frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(1.00 \times 10^{-6} \mathrm{C}\right)\left(1.00 \times 10^{-6} \mathrm{C}\right)}{(1.00 \mathrm{~m})^{2}} \\
& =9.00 \times 10^{-3} \mathrm{~N} .
\end{aligned}
$$

## Problem 3.13

The electron and proton have the same magnitude of charge, that is, $e=1.6 \times 10^{-19} \mathrm{C}$. The magnitude of the electric force between the two is:

$$
F=\frac{k e^{2}}{r^{2}}=1.00 \mathrm{~N}
$$

Rearranging and plugging in numbers:

$$
\begin{aligned}
& \frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2}}{1.00 \mathrm{~N}}=r^{2} \\
& r=1.52 \times 10^{-14} \mathrm{~m} .
\end{aligned}
$$

## Problem 3.14

The electric force between two charges is:

$$
F=\frac{k q_{1} q_{2}}{R^{2}}
$$

So, the distance between them is given by:

$$
\begin{gathered}
R=\sqrt{\frac{k q_{1} q_{2}}{F}} \\
R=\sqrt{\frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(6.5 \times 10^{-6} \mathrm{C}\right)\left(9.4 \times 10^{-6} \mathrm{C}\right)}{0.78 \mathrm{~N}}} \\
R=0.84 \mathrm{~m} .
\end{gathered}
$$

## Problem 3.15

The electric force between two identical charges is:

$$
F=\frac{k q q}{R^{2}}=\frac{k q^{2}}{R^{2}}
$$

So, the charge is given by:

$$
\begin{gathered}
q=\sqrt{\frac{F}{k}} R \\
q=\sqrt{\frac{12 \mathrm{~N}}{9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}}}(0.1 \mathrm{~m}) \\
=3.65 \times 10^{-6} \mathrm{C}=3.65 \mu \mathrm{C} .
\end{gathered}
$$

Problem 3.16
Both the electric and gravitational force have the same $1 / r^{2}$ dependence; so, the $r^{2}$ will cancel out when we take the ratio of the two forces. Also, the electric force between two protons or two electrons is the same because protons and electrons have the same amount of charge. The ratio of the electric force to the gravitational force for two masses with the same charge can be written as:

$$
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{\frac{k e^{2}}{r^{2}}}{\frac{G m_{1} m_{2}}{r^{2}}}=\frac{k e^{2}}{G m_{1} m_{2}} .
$$

(a) Two protons

$$
\begin{gathered}
m_{1}=m_{2}=m_{\mathrm{p}}=1.67 \times 10^{-27} \mathrm{~kg}, \mathrm{so}: \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{k e^{2}}{G m_{\mathrm{p}} m_{\mathrm{p}}}=\frac{k}{G}\left(\frac{e}{m_{\mathrm{p}}}\right)^{2} \\
F_{\mathrm{g}} \\
=\frac{9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}}{6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}}\left(\frac{1.6 \times 10^{-19} \mathrm{C}}{1.67 \times 10^{-27} \mathrm{~kg}}\right)^{2} \\
=1.24 \times 10^{36} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=1.24 \times 10^{36} .
\end{gathered}
$$

(b) Two electrons

$$
m_{1}=m_{2}=m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg} \text {, so: }
$$

$$
\begin{gathered}
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{\frac{k e^{2}}{r^{2}}}{\frac{G m_{1} m_{2}}{r^{2}}}=\frac{k e^{2}}{G m_{1} m_{2}} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{k e^{2}}{G m_{\mathrm{e}} m_{\mathrm{e}}}=\frac{k}{G}\left(\frac{e}{m_{\mathrm{e}}}\right)^{2} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}}{6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}}\left(\frac{1.6 \times 10^{-19} \mathrm{C}}{9.11 \times 10^{-31} \mathrm{~kg}}\right)^{2} \\
=4.17 \times 10^{42} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=4.17 \times 10^{42} .
\end{gathered}
$$

(c) A proton $m_{1}=m_{\mathrm{p}}=1.67 \times 10^{-27} \mathrm{~kg}$ and an electron $m_{1}=m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg}$, so:

$$
\begin{gathered}
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{k e^{2}}{G m_{\mathrm{p}} m_{\mathrm{e}}}=\frac{k}{G} \frac{e^{2}}{m_{\mathrm{p}} m_{\mathrm{e}}} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=\frac{9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}}{6.67 \times 10^{-11} \frac{\mathrm{Nm}^{2}}{\mathrm{~kg}^{2}}} \\
\times\left(\frac{\left(1.6 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(1.67 \times 10^{-27} \mathrm{~kg}\right)\left(9.11 \times 10^{-31} \mathrm{~kg}\right)}\right. \\
=2.27 \times 10^{39} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}}=2.27 \times 10^{39} .
\end{gathered}
$$

The ratio is largest for the force between two electrons. The electric force will be the same in all three cases, but the gravitational force will be smallest when the product of the masses is smallest; that is, in the force between two electrons.

## Problem 3.17

The electron and proton have the same magnitude of charge; that is, $e=1.6 \times$ $10^{-19} \mathrm{C}$. The electric force between two charges is:

$$
F=\frac{k q_{1} q_{2}}{R^{2}}
$$

So, the ratio between two electric forces is:

$$
\begin{gathered}
\frac{F_{\mathrm{P}}}{F_{\mathrm{E}}}=\frac{\frac{k q_{\mathrm{p}} q_{\mathrm{p}}}{R_{\mathrm{p}}^{2}}}{\frac{k q_{\mathrm{p}} q_{\mathrm{e}}}{R_{\mathrm{e}}^{2}}}=\frac{\frac{k e e}{R_{\mathrm{p}}^{2}}}{\frac{k e e}{R_{\mathrm{e}}^{2}}}=\frac{R_{\mathrm{e}}^{2}}{R_{\mathrm{p}}^{2}} \\
\frac{F_{\mathrm{p}}}{F_{\mathrm{E}}}=\frac{R_{\mathrm{e}}^{2}}{R_{\mathrm{p}}^{2}}=\left(\frac{10^{-10} \mathrm{~m}}{10^{-15} \mathrm{~m}}\right)^{2}=1.0 \times 10^{10} .
\end{gathered}
$$

## Problem 3.18



Figure 7
The forces $\vec{F}_{1}$ and $\overrightarrow{F_{2}}$ are of the same magnitude but have different directions. As can be seen in Figure 7, above, the $x$-components of the forces cancel and the $y$-components add. The resultant $F R$ is simply twice either component; that is: $F R=2 F_{1} \cos 30^{\circ}=2 F_{2} \cos 30^{\circ}$

$$
\begin{gathered}
F=\frac{k q_{1} q_{2}}{R^{2}} \\
F_{1}=\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right) \\
\times \frac{\left(2 \times 10^{-9} \mathrm{C}\right)\left(4 \times 10^{-9} \mathrm{C}\right)}{\left(4 \times 10^{-2} \mathrm{~m}\right)^{2}} \\
=4.5 \times 10^{-5} \mathrm{~N}
\end{gathered}
$$

## Problem 3.19

The magnitude of the force will be the same but the direction will be in the opposite direction. The force will be $7.79 \times 10^{-5} \mathrm{~N}$ in the negative $y$-direction.

## Problem 3.20

Singly charged ions each carry one unit of electronic charge ( $e=1.60 \times 10^{-19} \mathrm{C}$ ), so the force between these ions is:

$$
\begin{aligned}
F_{1} & =F_{2}=\frac{k q_{1} q_{2}}{r^{2}} \\
& =\frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)}{\left(2.82 \times 10^{-10} \mathrm{~m}\right)^{2}} \\
& =2.90 \times 10^{-9} \mathrm{~N} .
\end{aligned}
$$

## Problem 3.21

The forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are two forces exerted on $q_{3}$ by two other charges, $q_{1}$ and $q_{2}$, and are given by:
$F=k \frac{q q^{\prime}}{r^{2}}$
$F_{31}=\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(6.60 \times 10^{-9} \mathrm{C}\right)\left(6.75 \times 10^{-9} \mathrm{C}\right)}{(5.00 \mathrm{~m})^{2}}$
$=1.60 \times 10^{-8} \mathrm{~N}$

$$
\begin{aligned}
F_{32} & =\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right) \frac{\left(5.40 \times 10^{-9} \mathrm{C}\right)\left(6.75 \times 10^{-9} \mathrm{C}\right)}{(4.00 \mathrm{~m})^{2}} \\
& =2.05 \times 10^{-8} \mathrm{~N} .
\end{aligned}
$$



Figure 8

As Figure 8 shows, $\vec{F}_{31}$ makes an angle $\theta$ with horizontal:

$$
\theta=\tan ^{-1}\left(\frac{3.00 \mathrm{~m}}{4.00 \mathrm{~m}}\right)=36.9^{\circ}
$$

Now, calculate the components of $\vec{F}_{31}$ :

$$
\begin{gathered}
F_{31, x}=1.60 \times 10^{-8} \mathrm{~N} \times \cos 36.9^{\circ}=1.28 \times 10^{-8} \mathrm{~N} \\
F_{31, y}=1.60 \times 10^{-8} \mathrm{~N} \times \sin 36.9^{\circ}=9.61 \times 10^{-9} \mathrm{~N} \\
F R_{x}=F_{31, x}-F_{32} \\
F R_{x}=1.28 \times 10^{-8} \mathrm{~N}-2.05 \times 10^{-8} \mathrm{~N} \\
=-7.70 \times 10^{-9} \mathrm{~N}
\end{gathered}
$$

$$
F R_{y}=F_{31, y}=9.61 \times 10^{-9} \mathrm{~N}
$$

According to Figure 9, which shows the $x$ and $y$-components of $F R$, we get:


Figure 9

$$
\begin{aligned}
F R & =\sqrt{\left(F R_{x}^{2}+F R_{y}^{2}\right)} \\
& =\sqrt{\left(-7.70 \times 10^{-9} \mathrm{~N}\right)^{2}+\left(9.61 \times 10^{-9} \mathrm{~N}\right)^{2}} \\
& =1.23 \times 10^{-8} \mathrm{~N} \\
\varphi & =\tan ^{-1}\left(\frac{F R_{y}}{F R_{x}}\right)=\tan ^{-1}\left(\frac{9.61 \times 10^{-9} \mathrm{~N}}{-7.70 \times 10^{-9} \mathrm{~N}}\right) \\
& =\tan ^{-1}(-1.24)=-51^{\circ}=129^{\circ} .
\end{aligned}
$$

## Problem 3.22

The forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are two forces exerted on $q_{3}$ by two other charges, $q_{1}$ and $q_{2}$, and are given by:

$$
F=k \frac{q q^{\prime}}{r^{2}}
$$

To have a zero resultant force on negative charge $q_{3}$ requires that it is positioned between two positive charges, as Figure 10, below, shows.


Figure 10
Thus:

$$
r_{31}+r_{32}=4.0 \mathrm{~m}
$$

and:

$$
\begin{gathered}
F_{32}-F_{31}=0 \\
k \frac{\left|q_{1}\right|\left|q_{3}\right|}{r_{31}^{2}}=k \frac{\left|q_{2}\right|\left|q_{3}\right|}{r_{32}^{2}} \\
r_{32}=r_{31} \sqrt{\frac{\left|q_{2}\right|}{\left|q_{1}\right|}}=r_{31} \sqrt{\frac{25}{9}} \\
r_{32}=\frac{5}{3} r_{31} .
\end{gathered}
$$

Substituting it into the top equation gives:

$$
\begin{gathered}
r_{31}+\frac{5}{3} r_{31}=4.0 \mathrm{~m} \\
r_{31}=1.5 \mathrm{~m} \\
r_{32}=2.5 \mathrm{~m}
\end{gathered}
$$

## Problem 3.23

The force between two charges is:

$$
F=k \frac{q q^{\prime}}{r^{2}} .
$$

Therefore:

$$
\begin{aligned}
F_{12} & =k \frac{q_{1} q_{2}}{r_{12}^{2}} \\
& =\frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(10^{-4} \mathrm{C}\right)\left(45 \times 10^{-6} \mathrm{C}\right)}{(4 \mathrm{~m})^{2}} \\
& =2.53 \mathrm{~N} \\
F_{14} & =k \frac{q_{1} q_{4}}{r_{14}^{2}} \\
& =\frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(10^{-4} \mathrm{C}\right)\left(25 \times 10^{-6} \mathrm{C}\right)}{(5 \mathrm{~m})^{2}} \\
& =0.90 \mathrm{~N}
\end{aligned}
$$

$$
F_{13}=k \frac{q_{1} q_{3}}{r_{13}^{2}}
$$

$$
=\frac{\left(9 \times 10^{9} \frac{\mathrm{Nm}^{2}}{\mathrm{C}^{2}}\right)\left(10^{-4} \mathrm{C}\right)\left(125 \times 10^{-6} \mathrm{C}\right)}{\left(\sqrt{\left[(4 \mathrm{~m})^{2}+(5 \mathrm{~m})^{2}\right]}\right)^{2}}
$$

$$
=2.74 \mathrm{~N} .
$$

These are the magnitudes of the three forces acting on the charge $q_{1}$, as shown in Figure 11. We now evaluate the components of the resultant force along the vertical and horizontal axes.


Figure 11

$$
\begin{aligned}
& \begin{aligned}
F_{\text {net }, x}= & F_{14}-F_{13} \times \frac{5}{\sqrt{41}} \\
\quad= & 0.9-2.74 \times \frac{5}{\sqrt{41}}=-1.24 \mathrm{~N} \\
F_{\text {net }, y} & =-F_{12}+F_{13} \times \frac{4}{\sqrt{41}} \\
& =-2.53+2.74 \times \frac{4}{\sqrt{41}}=-0.82 \mathrm{~N}
\end{aligned}
\end{aligned}
$$

The magnitude of the net force and its direction are given by:

$$
\begin{gathered}
F_{\text {net }}=\sqrt{F_{\text {net }, x}^{2}+F_{\text {net }, y}^{2}}=\sqrt{2.21}=1.48 \mathrm{~N} \\
\\
\tan (\theta)=\frac{-0.82}{-1.24}=0.66 \\
\\
\theta=213.5^{\circ} .
\end{gathered}
$$

## Problem 3.24

Take the crate and the person on it as one object with a mass equal to the sum of the mass of crate and the mass of person. The normal force of the floor on the object is given as:

$$
\begin{aligned}
N_{\text {crate }} & =\left(m_{\text {crate }}+m_{\text {person }}\right) g \\
& =(45 \mathrm{~kg}+75 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)=1176 \mathrm{~N} .
\end{aligned}
$$

And the normal force on the person exerted by the crate is:

$$
\begin{aligned}
N_{\text {person }} & =\left(m_{\text {person }}\right) g=(75 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
& =735 \mathrm{~N} .
\end{aligned}
$$

## Problem 3.25



Figure 12

$$
\vec{w}\left\{\begin{array}{l}
w_{x}=m g \sin 35^{\circ} \\
w_{y}=m g \cos 35^{\circ}
\end{array}\right.
$$

In the $y$-direction:

$$
\begin{aligned}
& \Sigma F_{y}=0 \\
& F_{\mathrm{N}}-m g \cos 35^{\circ}=0 \\
& \begin{aligned}
F_{\mathrm{N}} & =m g \cos 35^{\circ} \\
& =(5.8 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)\left(\cos 35^{\circ}\right) \\
& =46.6 \mathrm{~N} .
\end{aligned}
\end{aligned}
$$

## Problem 3.26

(a) The weight of the man is $W=m g$; that is, $W=70(\mathrm{~kg}) \times 9.8\left(\mathrm{~m} / \mathrm{s}^{2}\right)=687 \mathrm{~N}$.
(b) The normal force acting on the man is equal and opposite to his weight.
(c) The man will read 687 N in principle. However, if the scale is not calibrated properly to zero, the weight might be off by the error in calibration. Moreover, the scale has a certain accuracy that may be greater than 1 N , which in turn means that there will be a round-off error.

## Problem 3.27

The force of static friction is equal to your applied force. So:

$$
f_{\mathrm{s}}=F_{\mathrm{App}}=100 \mathrm{~N}
$$

No, since we do not know the maximum force of static friction.

## Problem 3.28



Figure 13
Figure 13 shows the free body diagram of the refrigerator.

$$
N-w=0 \Rightarrow N=w=m g
$$

(a) $f_{\mathrm{s}, \mathrm{Max}}=\mu_{\mathrm{s}} N=\mu_{\mathrm{s}} m g$

$$
\begin{aligned}
& f_{\mathrm{s}, \operatorname{Max}}=(0.55)(67 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)=361 \mathrm{~N} \\
& \because F_{\mathrm{App}}=276 \mathrm{~N}<f_{\mathrm{s}, \operatorname{Max}}=361 \mathrm{~N}
\end{aligned}
$$

No, the person does not move the refrigerator.
(b) The largest push before it begins to move is:

$$
F_{\mathrm{App}}=f_{\mathrm{s}, \mathrm{Max}}=361 \mathrm{~N} .
$$

## Problem 3.29

Figure 14 shows the person's free body diagram of the climber. The climber is stationary, so:

$$
F_{\text {net }, x}=F_{\text {net }, y}=0 .
$$



Figure 14
(a) In the $y$-direction:

$$
\begin{aligned}
& F_{\text {net, },}=N-w_{y}=N-m g \cos 36^{\circ}=0 \\
& \begin{aligned}
N & =m g \cos 36^{\circ}=(64 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \cos 36^{\circ} \\
& =5.07 \times 10^{2} \mathrm{~N} .
\end{aligned}
\end{aligned}
$$

(b) In the $x$-direction:

$$
\begin{aligned}
& F_{\text {net }, x}=f_{\mathrm{s}}-w_{x}=f_{\mathrm{s}}-m g \sin 36^{\circ}=0 \\
& \begin{array}{l}
f_{\mathrm{s}}=m g \sin 36^{\circ}=(64 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \sin 36^{\circ} \\
\quad=3.7 \times 10^{2} \mathrm{~N}
\end{array}
\end{aligned}
$$

(c) The maximum static frictional force is:

$$
\begin{gathered}
f_{\mathrm{s}, \mathrm{Max}}=\mu_{\mathrm{s}} \mathrm{~N}=(0.86)\left(7.07 \times 10^{2} \mathrm{~N}\right) \\
f_{\mathrm{s}, \mathrm{Max}}=436 \mathrm{~N}
\end{gathered}
$$

The actual frictional force is much less than this.

## Problem 3.30

If you draw a free body diagram of the sea lion on inclined plane, it is similar to the free body diagram of the previous problem, Figure 14 in Problem 3.29, except the inclined angle is $40^{\circ}$. So:
(a) $F_{\text {net }, y}=N-w_{y}=N-m g \cos 40^{\circ}=0$

$$
\begin{aligned}
N & =m g \cos 40^{\circ} \\
& =(480 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \cos 40^{\circ} \\
& =3.6 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

(b) $F_{\mathrm{net}, x}=f_{\mathrm{s}}-w_{x}=f_{\mathrm{s}}-m g \sin 40^{\circ}=0$

$$
\begin{aligned}
f_{\mathrm{s}}=m g \sin 36^{\circ}=(480 \mathrm{~kg}) & \left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \sin 36^{\circ} \\
& =3.0 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

(c) $\begin{aligned} f_{\mathrm{s}, \text { Max }} & =\mu_{\mathrm{s}} N=(0.96)\left(3.6 \times 10^{2} \mathrm{~N}\right) \\ & =3.5 \times 10^{3} \mathrm{~N}\end{aligned}$

## Problem 3.31

Its free body diagram is similar to Figure 13 in Problem 3.28.

$$
\begin{gathered}
N-w=0 \Rightarrow N=w=m g \\
f_{\mathrm{s}, \operatorname{Max}}=\mu_{\mathrm{s}} N=\mu_{\mathrm{s}} m g \\
f_{\mathrm{s}, \text { Max }}=(0.34)(45.5 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \\
f_{\mathrm{s}, \operatorname{Max}}=152 \mathrm{~N} \\
F_{\mathrm{App}, \operatorname{Min}}=f_{\mathrm{s}, \operatorname{Max}}=152 \mathrm{~N}
\end{gathered}
$$

## Problem 3.32

Two forces are exerted on a mass on spring, weight and spring force, so:

$$
F_{\mathrm{s}}-w=0
$$

First, we calculate the spring coefficient:

$$
k x-m g=0
$$

$$
\begin{aligned}
k & =\frac{m g}{x} \\
& =\frac{(4.40 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{2.50 \times 10^{-2} \mathrm{~m}} \\
& =1.72 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}
\end{aligned}
$$

So, for new mass, the stretch is:

$$
\begin{aligned}
x & =\frac{m g}{x} \\
& =\frac{(6.80 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{1.72 \times 10^{3} \frac{\mathrm{~N}}{\mathrm{~m}}} \\
& =3.87 \times 10^{-2} \mathrm{~m}=3.87 \mathrm{~cm} .
\end{aligned}
$$

## Problem 3.33

When you stretched a spring by applying a force, the spring applies the same amount of force on you in the opposite direction; so:

$$
\begin{gathered}
F_{\mathrm{App}}=F_{\mathrm{s}}=k x \\
\therefore \frac{F_{\mathrm{App}, 2}}{F_{\mathrm{App}, 1}}=\frac{k x_{2}}{k x_{1}}=\frac{x_{2}}{x_{1}} \\
\frac{F_{\mathrm{App}, 2 \mathrm{~L}}}{27.0 \mathrm{~N}}=\frac{(15.0+18.0) \times 10^{-2} \mathrm{~m}}{15.0 \times 10^{-2} \mathrm{~m}} \\
F_{\mathrm{App}, 2}=(27.0 \mathrm{~N})(2.2)=59.4 \mathrm{~N} \\
\Delta F=F_{\mathrm{App}, 2}-F_{\mathrm{App}, 1}=59.4 \mathrm{~N}-27.0 \mathrm{~N} \\
\Delta F=F_{\mathrm{App}, 2}-F_{\mathrm{App}, 1}=32.4 \mathrm{~N} .
\end{gathered}
$$

## Problem 3.34

The cchandelier is in static equilibrium so $\sum \vec{F}=0$. There are no forces to consider in the $x$-direction. In the $y$-direction, there are two forces, the weight and the tension on the chain. Figure 15 shows the free body diagram:


Figure 15

$$
\begin{gathered}
\sum F_{y}=0 \\
T-w=T-m g=0 \\
T=(11 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)=109 \mathrm{~N}
\end{gathered}
$$

## Problem 3.35

The climber is in static equilibrium, so $\sum \vec{F}=0$. There are no forces to consider in the $x$-direction. In the $y$-direction, there are two forces: the weight of the climber and the tension on the rope. Its free body diagram is similar to Figure 15 in Problem 3.34.

$$
\begin{gathered}
\sum F_{\mathrm{y}}=0 \\
T-w=T-m g=0 \\
T=(85 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)=833 \mathrm{~N}
\end{gathered}
$$

## Problem 3.36

The weight $W$ of the climber is 76.0 kg . The free body diagram for the climber is shown in Figure 16. Since the climber is in static equilibrium:


Figure 16

$$
\begin{aligned}
& \sum \vec{F}=0 \\
& \text { so } \sum F_{\mathrm{x}}=0 \text { and } \sum F_{\mathrm{y}}=0
\end{aligned}
$$

The $x$-component gives:

$$
T_{\mathrm{R}} \cos 18.5^{\circ}-T_{\mathrm{L}} \cos 11^{\circ}=0
$$

The $y$-component gives:

$$
\begin{gathered}
T_{\mathrm{R}} \sin 18.5^{\circ}+T_{\mathrm{L}} \sin 11^{\circ}-w=0 \\
w=m g=(76.0 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)=745 \mathrm{~N}
\end{gathered}
$$

Solving these two equations in two unknowns gives:

$$
\begin{aligned}
& T_{\mathrm{R}}=1.48 \times 10^{3} \mathrm{~N} \\
& T_{\mathrm{L}}=1.43 \times 10^{3} \mathrm{~N}
\end{aligned}
$$

## Problem 3.37

Case (a):


Figure 17a
Case (b): In this case, there are two contact forces between block A and B: one parallel to the incline $f_{\mathrm{s}, \mathrm{AB}}=f_{\mathrm{s}, \mathrm{BA}}$ and one perpendicular to the incline $N_{\mathrm{AB}}=N_{\mathrm{BA}} \cdot N_{\mathrm{AB}}$ is a normal force exerted on A by B and an equal and opposite normal force $N_{\mathrm{BA}}$ exerted on B by A .


Figure 17b

Case (c): This case is similar to case (a).


Figure 17c

## Problem 3.38

Block A:


Figure 18a
Block B: In this case, $N_{\mathrm{BA}}$ and $N_{\mathrm{AB}}$ are two contact (normal) forces that blocks A and B exerts on each other. They are equal magnitude and in opposite directions.


Figure 18b
Block C:


Figure 18c

Problem 3.39


Figure 19

In this case, $N_{\mathrm{MB}}$ and $N_{\mathrm{BM}}$ are two contact (normal) forces acting on the magnet and the box. They are an interaction pair and are equal and in opposite directions. Note that, when the magnet exerts magnetic force $F_{\mathrm{M}}$ on the box, the box also exerts magnetic force on the magnet (interaction pair).

## Problem 3.40

Figure 3.47 shows a rock climber climbing up Devil's Tower in Wyoming.


Figure 3.47
The forces on the climber are her weight, her fingers pulling inward and up against the rock, and the force exerted by the climber's legs on the rock. The woman cannot be considered a simple point object in this case. Her hands pull inward, producing an outward normal force exerted by the rock. She supports her weight primarily by having her legs at a large angle, so that they can push outward on the rock. The outward component of force increases the normal force exerted by the rock wall. This, in turn, increases the frictional force, which is parallel to the wall and upward. This supports most of her weight. The frictional force $f_{\mathrm{s}, \mathrm{h}}$ produced by her hands can also support some of the weight. In the first force diagram below, note that the normal forces $N_{\mathrm{h}}$ and $N_{\mathrm{l}}$ exerted by the wall on the woman
are out on the hands and in on the legs. The four upward forces represent static frictional forces, which we label $f_{\mathrm{s}, \mathrm{h}}$ and $f_{\mathrm{s}, 1}$.The second diagram, Figure 20 b , shows a simplified free body diagram.


Figure 20a


Figure 20b

## Problem 3.41

Figure 21 shows the free body diagram for the picture.


Figure 21

Since the picture is in equilibrium, the net force on it is zero. Therefore:

$$
\begin{gathered}
F_{\mathrm{net}, x}=T_{1, x}-T_{2, x}=0 \\
F_{\mathrm{net}, y}=T_{1, y}+T_{2, y}-w=0 .
\end{gathered}
$$

From the first equation (the $x$-component of the net force), we get:

$$
\begin{gathered}
F_{\mathrm{net}, x}=T_{1} \cos 45^{\circ}-T_{2,} \cos 45^{\circ}=0 \\
T_{1}=T_{2,}
\end{gathered}
$$

Substituting in the second equation and solving it gives:

$$
\begin{gathered}
F_{\text {net }, y}=T_{1} \sin 45^{\circ}+T_{2} \sin 45^{\circ}-w=0 \\
F_{\text {net }, y}=T_{1} \sin 45^{\circ}+T_{1} \sin 45^{\circ}-w=0 \\
2 T_{1} \sin 45^{\circ}=w=m g \\
T_{1}=\frac{m g}{2 \sin 45^{\circ}} \\
=\frac{(4.0 \mathrm{~kg})\left(9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)}{\left(2 \sin 45^{\circ}\right)} \\
=27.7 \mathrm{~N}
\end{gathered}
$$

$$
T_{1}=T_{2,}=27.7 \mathrm{~N}
$$

## Problem 3.42

The free body diagram for each arm is similar to that of Figure 4.51 in Example 4.29 in the textbook.

The force balance for each arm can be written as:

$$
\mathrm{T}-\mathrm{F}-\mathrm{F}_{\mathrm{arm}}=0
$$

and the force balance for the bar is:

$$
2 \mathrm{~F}-\mathrm{W}_{\mathrm{bar}}=0
$$

Combining the two equations, we find the expression for the tension on the shoulder:

$$
\mathrm{T}=\mathrm{F}_{\mathrm{arm}}+\mathrm{W}_{\mathrm{bar}} / 2
$$

Using the Table 4.1, we can estimate the tension:

$$
\mathrm{T}=4.6 \times 9.8+100 \times 9.8 / 2=535 \mathrm{~N}
$$

## Problem 3.43

Each hand pulls down the bar with a force $F_{\text {hand }}$. The cable attached to the bar provides a tension T, such that:

$$
T=2 F_{\text {hand }} .
$$

On the other hand, the system is assumed to be in equilibrium and, therefore, the tension has to be balanced by the weight of the arms, trunk, and head:

$$
T=W_{\text {amsstrunk+head }}
$$

Combining the two equations, we find:

$$
F_{\text {hand }}=W_{\text {armsstrunk } k \text { head }} / 2 .
$$

Using Table 4.1, we can determine the weight of arms, trunk and head:

$$
\begin{aligned}
F_{\text {hand }} & =9.8 \times(2 \times 70 \times 0.065+70 \times 0.48+70 \times 0.07) / 2 \\
& =233 \mathrm{~N} .
\end{aligned}
$$

