

INSTRUCTOR'S SOLUTIONS MANUAL
TO ACCOMPANY

POWER SYSTEM
ANALYSIS AND DESIGN

FIFTH EDITION

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Contents

Chapter 2	1
Chapter 3	27
Chapter 4	71
Chapter 5	95
Chapter 6	137
Chapter 7	175
Chapter 8	195
Chapter 9	231
Chapter 10	303
Chapter 11	323
Chapter 12	339
Chapter 13	353
Chapter 14	379

Chapter 2

Fundamentals

ANSWERS TO MULTIPLE-CHOICE TYPE QUESTIONS

2.1	b	2.19	a
2.2	a	2.20	A. c
2.3	c		B. a
2.4	a		C. b
2.5	b	2.21	a
2.6	c	2.22	a
2.7	a	2.23	b
2.8	c	2.24	a
2.9	a	2.25	a
2.10	c	2.26	b
2.11	a	2.27	a
2.12	b	2.28	b
2.13	b	2.29	a
2.14	c	2.30	(i) c
2.15	a		(ii) b
2.16	b		(iii) a
2.17	A. a		(iv) d
	B. b	2.31	a
	C. a	2.32	a
2.18	c		

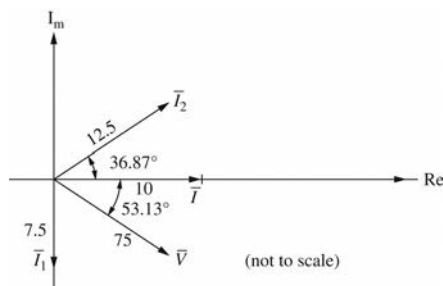
- 2.1 (a) $\bar{A}_1 = 5\angle 30^\circ = 5[\cos 30^\circ + j\sin 30^\circ] = 4.33 + j2.5$
 (b) $\bar{A}_2 = -3 + j4 = \sqrt{9+16} \angle \tan^{-1} \frac{4}{-3} = 5\angle 126.87^\circ = 5e^{j126.87^\circ}$
 (c) $\bar{A}_3 = (4.33 + j2.5) + (-3 + j4) = 1.33 + j6.5 = 6.635\angle 78.44^\circ$
 (d) $\bar{A}_4 = (5\angle 30^\circ)(5\angle 126.87^\circ) = 25\angle 156.87^\circ = -22.99 + j9.821$
 (e) $\bar{A}_5 = (5\angle 30^\circ)/(5\angle -126.87^\circ) = 1\angle 156.87^\circ = 1e^{j156.87^\circ}$

- 2.2 (a) $\bar{I} = 400\angle -30^\circ = 346.4 - j200$
 (b) $i(t) = 5\sin(\omega t + 15^\circ) = 5\cos(\omega t + 15^\circ - 90^\circ) = 5\cos(\omega t - 75^\circ)$
 $\bar{I} = (5/\sqrt{2})\angle -75^\circ = 3.536\angle -75^\circ = 0.9151 - j3.415$
 (c) $\bar{I} = (4/\sqrt{2})\angle -30^\circ + 5\angle -75^\circ = (2.449 - j1.414) + (1.294 - j4.83)$
 $= 3.743 - j6.244 = 7.28\angle -59.06^\circ$

- 2.3 (a) $V_{\max} = 359.3\text{ V}; I_{\max} = 100\text{ A}$
 (b) $V = 359.3/\sqrt{2} = 254.1\text{ V}; I = 100/\sqrt{2} = 70.71\text{ A}$
 (c) $\bar{V} = 254.1\angle 15^\circ\text{ V}; \bar{I} = 70.71\angle -85^\circ\text{ A}$

- 2.4 (a) $\bar{I}_1 = 10\angle 0^\circ \frac{-j6}{8 + j6 - j6} = 10 \frac{6\angle -90^\circ}{8} = 7.5\angle -90^\circ\text{ A}$
 $\bar{I}_2 = \bar{I} - \bar{I}_1 = 10\angle 0^\circ - 7.3\angle -90^\circ = 10 + j7.5 = 12.5\angle 36.87^\circ\text{ A}$
 $\bar{V} = \bar{I}_2(-j6) = (12.5\angle 36.87^\circ)(6\angle -90^\circ) = 75\angle -53.13^\circ\text{ V}$

(b)



- 2.5 (a) $v(t) = 277\sqrt{2}\cos(\omega t + 30^\circ) = 391.7\cos(\omega t + 30^\circ)\text{ V}$
 (b) $\bar{I} = \bar{V}/20 = 13.85\angle 30^\circ\text{ A}$
 $i(t) = 19.58\cos(\omega t + 30^\circ)\text{ A}$

$$(c) \quad \bar{Z} = j\omega L = j(2\pi 60)(10 \times 10^{-3}) = 3.771 \angle 90^\circ \Omega$$

$$\bar{I} = \bar{V} / \bar{Z} = (277 \angle 30^\circ) / (3.771 \angle 90^\circ) = 73.46 \angle -60^\circ \text{ A}$$

$$i(t) = 73.46 \sqrt{2} \cos(\omega t - 60^\circ) = 103.9 \cos(\omega t - 60^\circ) \text{ A}$$

$$(d) \quad \bar{Z} = -j25 \Omega$$

$$\bar{I} = \bar{V} / \bar{Z} = (277 \angle 30^\circ) / (25 \angle -90^\circ) = 11.08 \angle 120^\circ \text{ A}$$

$$i(t) = 11.08 \sqrt{2} \cos(\omega t + 120^\circ) = 15.67 \cos(\omega t + 120^\circ) \text{ A}$$

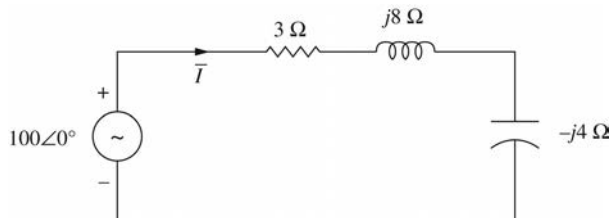
2.6 (a) $\bar{V} = (100 / \sqrt{2}) \angle -30^\circ = 70.7 \angle -30^\circ$; ω does not appear in the answer.

(b) $v(t) = 100 \sqrt{2} \cos(\omega t + 20^\circ)$; with $\omega = 377$,
 $v(t) = 141.4 \cos(377t + 20^\circ)$

(c) $\bar{A} = A \angle \alpha$; $\bar{B} = B \angle \beta$; $\bar{C} = \bar{A} + \bar{B}$
 $c(t) = a(t) + b(t) = \sqrt{2} \operatorname{Re}[\bar{C} e^{j\omega t}]$

The resultant has the same frequency ω .

2.7 (a) The circuit diagram is shown below:



(b) $\bar{Z} = 3 + j8 - j4 = 3 + j4 = 5 \angle 53.1^\circ \Omega$

(c) $\bar{I} = (100 \angle 0^\circ) / (5 \angle 53.1^\circ) = 20 \angle -53.1^\circ \text{ A}$

The current lags the source voltage by 53.1°

Power Factor = $\cos 53.1^\circ = 0.6$ Lagging

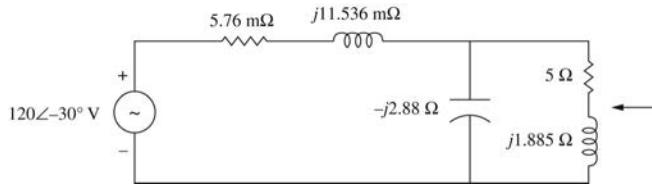
2.8 $\bar{Z}_{LT} = j(377)(30.6 \times 10^{-6}) = j11.536 \text{ m}\Omega$

$$\bar{Z}_{LL} = j(377)(5 \times 10^{-3}) = j1.885 \Omega$$

$$\bar{Z}_C = -j \frac{1}{(377)(921 \times 10^{-6})} = -j2.88 \Omega$$

$$\bar{V} = \frac{120 \sqrt{2}}{\sqrt{2}} \angle -30^\circ \text{ V}$$

The circuit transformed to phasor domain is shown below:



$$\begin{aligned}
 \mathbf{2.9} \quad \text{KVL: } 120\angle 0^\circ &= (60\angle 0^\circ)(0.1 + j0.5) + \bar{V}_{LOAD} \\
 \therefore \bar{V}_{LOAD} &= 120\angle 0^\circ - (60\angle 0^\circ)(0.1 + j0.5) \\
 &= 114.1 - j30.0 = 117.9\angle -14.7^\circ \text{ V} \leftarrow
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{2.10} \quad (\text{a}) \quad p(t) &= v(t)i(t) = [359.3 \cos(\omega t + 15^\circ)][100 \cos(\omega t - 85^\circ)] \\
 &= \frac{1}{2}(359.3)(100)[\cos 100^\circ + \cos(2\omega t - 70^\circ)] \\
 &= -3120 + 1.797 \times 10^4 \cos(2\omega t - 70^\circ) \text{ W}
 \end{aligned}$$

$$\begin{aligned}
 (\text{b}) \quad P &= VI \cos(\delta - \beta) = (254.1)(70.71) \cos(15^\circ + 85^\circ) \\
 &= -3120 \text{ W} \quad \text{Absorbed} \\
 &= +3120 \text{ W} \quad \text{Delivered}
 \end{aligned}$$

$$\begin{aligned}
 (\text{c}) \quad Q &= VI \sin(\delta - \beta) = (254.1)(70.71) \sin 100^\circ \\
 &= 17.69 \text{ kVAR Absorbed}
 \end{aligned}$$

(d) The phasor current $(-\bar{I}) = 70.71\angle -85^\circ + 180^\circ = 70.71\angle 95^\circ$ A leaves the positive terminal of the generator.

The generator power factor is then $\cos(15^\circ - 95^\circ) = 0.1736$ leading

$$\begin{aligned}
 \mathbf{2.11} \quad (\text{a}) \quad p(t) &= v(t)i(t) = 391.7 \times 19.58 \cos^2(\omega t + 30^\circ) \\
 &= 0.7669 \times 10^4 \left(\frac{1}{2}\right) [1 + \cos(2\omega t + 60^\circ)] \\
 &= 3.834 \times 10^3 + 3.834 \times 10^3 \cos(2\omega t + 60^\circ) \text{ W} \\
 P &= VI \cos(\delta - \beta) = 277 \times 13.85 \cos 0^\circ = 3.836 \text{ kW} \\
 Q &= VI \sin(\delta - \beta) = 0 \text{ VAR}
 \end{aligned}$$

$$\text{Source Power Factor} = \cos(\delta - \beta) = \cos(30^\circ - 30^\circ) = 1.0$$

$$\begin{aligned}
 (\text{b}) \quad p(t) &= v(t)i(t) = 391.7 \times 103.9 \cos(\omega t + 30^\circ) \cos(\omega t - 60^\circ) \\
 &= 4.07 \times 10^4 \left(\frac{1}{2}\right) [\cos 90^\circ + \cos(2\omega t - 30^\circ)] \\
 &= 2.035 \times 10^4 \cos(2\omega t - 30^\circ) \text{ W} \\
 P &= VI \cos(\delta - \beta) = 277 \times 73.46 \cos(30^\circ + 60^\circ) = 0 \text{ W}
 \end{aligned}$$

$$Q = VI \sin(\delta - \beta) = 277 \times 73.46 \sin 90^\circ = 20.35 \text{ kVAR}$$

$$pf = \cos(\delta - \beta) = 0 \text{ Lagging}$$

$$\begin{aligned} \text{(c) } p(t) &= v(t)i(t) = 391.7 \times 15.67 \cos(\omega t + 30^\circ) \cos(\omega t + 120^\circ) \\ &= 6.138 \times 10^3 \left(\frac{1}{2}\right) [\cos(-90^\circ) + \cos(2\omega t + 150^\circ)] = 3.069 \times 10^3 \cos(2\omega t + 150^\circ) \text{ W} \end{aligned}$$

$$P = VI \cos(\delta - \beta) = 277 \times 11.08 \cos(30^\circ - 120^\circ) = 0 \text{ W}$$

$$Q = VI \sin(\delta - \beta) = 277 \times 11.08 \sin(-90^\circ)$$

$$= -3.069 \text{ kVAR Absorbed} = +3.069 \text{ kVAR Delivered}$$

$$pf = \cos(\delta - \beta) = \cos(-90^\circ) = 0 \text{ Leading}$$

$$\mathbf{2.12} \quad \text{(a) } p_R(t) = (359.3 \cos \omega t)(35.93 \cos \omega t)$$

$$= 6455 + 6455 \cos 2\omega t \text{ W}$$

$$\text{(b) } p_x(t) = (359.3 \cos \omega t)[14.37 \cos(\omega t + 90^\circ)]$$

$$= 2582 \cos(2\omega t + 90^\circ)$$

$$= -2582 \sin 2\omega t \text{ W}$$

$$\text{(c) } P = V^2/R = (359.3/\sqrt{2})^2/10 = 6455 \text{ W Absorbed}$$

$$\text{(d) } Q = V^2/X = (359.3/\sqrt{2})^2/25 = 2582 \text{ VARs Delivered}$$

$$\text{(e) } (\beta - \delta) = \tan^{-1}(Q/P) = \tan^{-1}(2582/6455) = 21.8^\circ$$

$$\text{Power factor} = \cos(\delta - \beta) = \cos(21.8^\circ) = 0.9285 \text{ Leading}$$

$$\mathbf{2.13} \quad \bar{Z} = R - jx_c = 10 - j25 = 26.93 \angle -68.2^\circ \Omega$$

$$i(t) = (359.3/26.93) \cos(\omega t + 68.2^\circ)$$

$$= 13.34 \cos(\omega t + 68.2^\circ) \text{ A}$$

$$\text{(a) } p_R(t) = [13.34 \cos(\omega t + 68.2^\circ)][133.4 \cos(\omega t + 68.2^\circ)]$$

$$= 889.8 + 889.8 \cos[2(\omega t + 68.2^\circ)] \text{ W}$$

$$\text{(b) } p_x(t) = [13.34 \cos(\omega t + 68.2^\circ)][333.5 \cos(\omega t + 68.2^\circ - 90^\circ)]$$

$$= 2224 \sin[2(\omega t + 68.2^\circ)] \text{ W}$$

$$\text{(c) } P = I^2 R = (13.34/\sqrt{2})^2 10 = 889.8 \text{ W}$$

$$\text{(d) } Q = I^2 X = (13.34/\sqrt{2})^2 25 = 2224 \text{ VARs}$$

$$\text{(e) } pf = \cos[\tan^{-1}(Q/P)] = \cos[\tan^{-1}(2224/889.8)]$$

$$= 0.3714 \text{ Leading}$$

2.14 (a) $\bar{I} = 4\angle 0^\circ \text{ kA}$

$$\bar{V} = \bar{Z}\bar{I} = (2\angle -45^\circ)(4\angle 0^\circ) = 8\angle -45^\circ \text{ kV}$$

$$v(t) = 8\sqrt{2} \cos(\omega t - 45^\circ) \text{ kV}$$

$$\begin{aligned} p(t) &= v(t)i(t) = [8\sqrt{2} \cos(\omega t - 45^\circ)][4\sqrt{2} \cos \omega t] \\ &= 64 \left(\frac{1}{2}\right) [\cos(-45^\circ) + \cos(2\omega t - 45^\circ)] \\ &= 22.63 + 32 \cos(2\omega t - 45^\circ) \text{ MW} \end{aligned}$$

(b) $P = VI \cos(\delta - \beta) = 8 \times 4 \cos(-45^\circ - 0^\circ) = 22.63 \text{ MW Delivered}$

(c) $Q = VI \sin(\delta - \beta) = 8 \times 4 \sin(-45^\circ - 0^\circ)$
 $= -22.63 \text{ MVAR Delivered} = +22.63 \text{ MVAR Absorbed}$

(d) $pf = \cos(\delta - \beta) = \cos(-45^\circ - 0^\circ) = 0.707 \text{ Leading}$

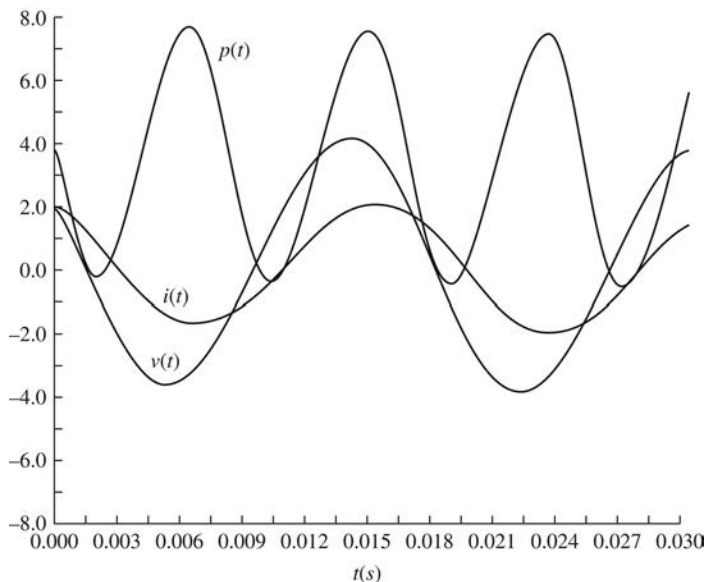
2.15 (a) $\bar{I} = [(4/\sqrt{2})\angle 60^\circ] / (2\angle 30^\circ) = \sqrt{2} \angle 30^\circ \text{ A}$

$$i(t) = 2 \cos(\omega t + 30^\circ) \text{ A with } \omega = 377 \text{ rad/s}$$

$$\begin{aligned} p(t) &= v(t)i(t) = 4 [\cos 30^\circ + \cos(2\omega t + 90^\circ)] \\ &= 3.46 + 4 \cos(2\omega t + 90^\circ) \text{ W} \end{aligned}$$

(b) $v(t)$, $i(t)$, and $p(t)$ are plotted below: (See next page)

(c) The instantaneous power has an average value of 3.46 W, and the frequency is twice that of the voltage or current.



2.16 (a) $\bar{Z} = 10 + j120\pi \times 0.04 = 10 + j15.1 = 18.1 \angle 56.4^\circ \Omega$

$pf = \cos 56.4^\circ = 0.553$ Lagging

(b) $\bar{V} = 120 \angle 0^\circ \text{ V}$

The current supplied by the source is

$$\bar{I} = (120 \angle 0^\circ) / (18.1 \angle 56.4^\circ) = 6.63 \angle -56.4^\circ \text{ A}$$

The real power absorbed by the load is given by

$$P = 120 \times 6.63 \times \cos 56.4^\circ = 440 \text{ W}$$

which can be checked by $I^2 R = (6.63)^2 10 = 440 \text{ W}$

The reactive power absorbed by the load is

$$Q = 120 \times 6.63 \times \sin 36.4^\circ = 663 \text{ VAR}$$

(c) Peak Magnetic Energy = $W = LI^2 = 0.04 (6.63)^2 = 1.76 \text{ J}$

$$Q = \omega W = 377 \times 1.76 = 663 \text{ VAR is satisfied.}$$

2.17 (a) $\bar{S} = \bar{V} \bar{I}^* = \bar{Z} \bar{I} \bar{I}^* = \bar{Z} |\bar{I}|^2 = j\omega LI^2$

$$Q = \text{Im}[\bar{S}] = \omega LI^2 \leftarrow$$

(b) $v(t) = L \frac{di}{dt} = -\sqrt{2} \omega L I \sin(\omega t + \theta)$

$$p(t) = v(t) \cdot i(t) = -2\omega L I^2 \sin(\omega t + \theta) \cos(\omega t + \theta)$$

$$= -\omega L I^2 \sin 2(\omega t + \theta) \leftarrow$$

$$= -Q \sin 2(\omega t + \theta) \leftarrow$$

Average real power P supplied to the inductor = 0 \leftarrow

Instantaneous power supplied (to sustain the changing energy in the magnetic field) has a maximum value of Q . \leftarrow

2.18 (a) $\bar{S} = \bar{V} \bar{I}^* = \bar{Z} \bar{I} \bar{I}^* = \text{Re}[\bar{Z} I^2] + j \text{Im}[\bar{Z} I^2]$

$$= P + jQ$$

$$\therefore P = Z I^2 \cos \angle Z; Q = Z I^2 \sin \angle Z \leftarrow$$

(b) Choosing $i(t) = \sqrt{2} I \cos \omega t$,

Then $v(t) = \sqrt{2} Z I \cos(\omega t + \angle Z)$

$$\therefore p(t) = v(t) \cdot i(t) = Z I^2 \cos(\omega t + \angle Z) \cdot \cos \omega t$$

$$= Z I^2 [\cos \angle Z + \cos(2\omega t + \angle Z)]$$

$$= Z I^2 [\cos \angle Z + \cos 2\omega t \cos \angle Z - \sin 2\omega t \sin \angle Z]$$

$$= P(1 + \cos 2\omega t) - Q \sin 2\omega t \leftarrow$$

$$(c) \bar{Z} = R + j\omega L + \frac{1}{j\omega C}$$

From part (a), $P = RI^2$ and $Q = Q_L + Q_C$

$$\text{where } Q_L = \omega LI^2 \text{ and } Q_C = -\frac{1}{\omega C} I^2$$

which are the reactive powers into L and C , respectively.

$$\text{Thus } p(t) = P(1 + \cos 2\omega t) - Q_L \sin 2\omega t - Q_C \sin 2\omega t \leftarrow$$

$$\left. \begin{array}{l} \text{If } \omega^2 LC = 1, \quad Q_L + Q_C = Q = 0 \\ \text{Then } \quad p(t) = P(1 + \cos 2\omega t) \end{array} \right\} \leftarrow$$

$$\begin{aligned} 2.19 \quad (a) \quad \bar{S} &= \bar{V} \bar{I}^* = \left(\frac{150}{\sqrt{2}} \angle 10^\circ \right) \left(\frac{5}{\sqrt{2}} \angle -50^\circ \right)^* = 375 \angle 60^\circ \\ &= 187.5 + j324.8 \end{aligned}$$

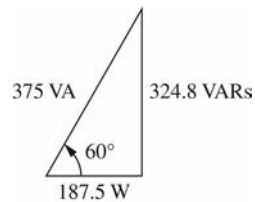
$$P = \text{Re} \bar{S} = 187.5 \text{ W Absorbed}$$

$$Q = \text{Im} \bar{S} = 324.8 \text{ VARs Absorbed}$$

$$(b) \quad pf = \cos(60^\circ) = 0.5 \text{ Lagging}$$

$$(c) \quad Q_S = P \tan \theta = 187.5 \tan[\cos^{-1} 0.5] = 187.5 \tan 60^\circ = 324.8 \text{ VARs}$$

$$Q_C = Q_L - Q_S = 324.8 - 324.8 = 0 \text{ VARs}$$



$$2.20 \quad \bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{20 \angle 30^\circ} = 0.05 \angle -30^\circ = (0.0433 - j0.025) \text{ S} = G_1 - jB_1$$

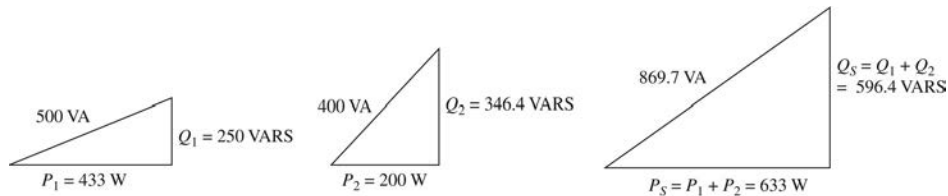
$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{25 \angle 60^\circ} = 0.04 \angle -60^\circ = (0.02 - j0.03464) \text{ S} = G_2 + jB_2$$

$$P_1 = V^2 G_1 = (100)^2 0.0433 = 433 \text{ W Absorbed}$$

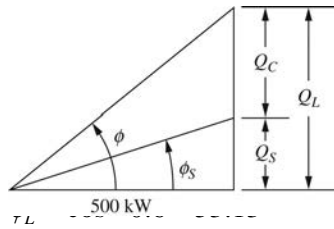
$$Q_1 = V^2 B_1 = (100)^2 0.025 = 250 \text{ VARs Absorbed}$$

$$P_2 = V^2 G_2 = (100)^2 0.02 = 200 \text{ W Absorbed}$$

$$Q_2 = V^2 B_2 = (100)^2 0.03464 = 346.4 \text{ VARs Absorbed}$$



2.21 (a)



$$Q_L = P \tan \phi_L = 500 \tan 53.13^\circ = 666.7 \text{ kVAR}$$

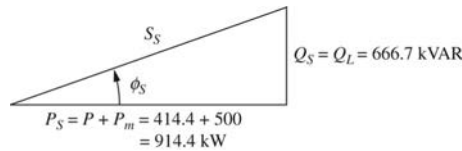
$$\phi_S = \cos^{-1} 0.9 = 25.84^\circ$$

$$Q_S = P \tan \phi_S = 500 \tan 25.84^\circ = 242.2 \text{ kVAR}$$

$$Q_C = Q_L - Q_S = 666.7 - 242.2 = 424.5 \text{ kVAR}$$

$$S_C = Q_C = 424.5 \text{ kVA}$$

(b) The Synchronous motor absorbs $P_m = \frac{(500)0.746}{0.9} = 414.4 \text{ kW}$ and $Q_m = 0 \text{ kVAR}$



$$\text{Source PF} = \cos \left[\tan^{-1} (666.7/914.4) \right] = 0.808 \text{ Lagging}$$

$$2.22 \text{ (a)} \quad \bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{(3 + j4)} = \frac{1}{5 \angle 53.13^\circ} = 0.2 \angle -53.13^\circ \\ = (0.12 - j0.16) \text{ S}$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{10} = 0.1 \text{ S}$$

$$P = V^2 (G_1 + G_2) \Rightarrow V = \sqrt{\frac{P}{G_1 + G_2}} = \sqrt{\frac{1100}{(0.12 + 0.1)}} = 70.71 \text{ V}$$

$$P_1 = V^2 G_1 = (70.71)^2 0.12 = 600 \text{ W}$$

$$P_2 = V^2 G_2 = (70.71)^2 0.1 = 500 \text{ W}$$

$$(b) \quad \bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 = (0.12 - j0.16) + 0.1 = 0.22 - j0.16 \\ = 0.272 \angle -36.03^\circ \text{ S}$$

$$I_S = V Y_{eq} = 70.71(0.272) = 19.23 \text{ A}$$

$$2.23 \quad \bar{S} = \bar{V}\bar{I}^* = (120\angle 0^\circ)(10\angle -30^\circ) = 1200\angle -30^\circ \\ = 1039.2 - j600$$

$$P = \text{Re}\bar{S} = 1039.2 \text{ W Delivered}$$

$$Q = \text{Im}\bar{S} = -600 \text{ VAR S Delivered} = +600 \text{ VAR S Absorbed}$$

$$2.24 \quad \bar{S}_1 = P_1 + jQ_1 = 10 + j0; \bar{S}_2 = 10\angle \cos^{-1} 0.9 = 9 + j4.359$$

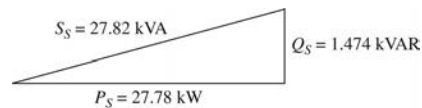
$$\bar{S}_3 = \frac{10 \times 0.746}{0.85 \times 0.95} \angle -\cos^{-1} 0.95 = 9.238\angle -18.19^\circ = 8.776 - j2.885$$

$$\bar{S}_S = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 27.78 + j1.474 = 27.82\angle 3.04^\circ$$

$$P_S = \text{Re}(\bar{S}_S) = 27.78 \text{ kW}$$

$$Q_S = \text{Im}(\bar{S}_S) = 1.474 \text{ kVAR}$$

$$S_S = |\bar{S}_S| = 27.82 \text{ kVA}$$



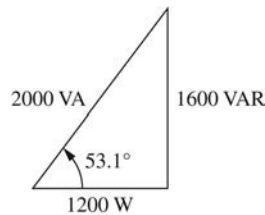
$$2.25 \quad \bar{S}_R = \bar{V}_R\bar{I}^* = R\bar{I}\bar{I}^* = I^2R = (20)^2 3 = 1200 + j0$$

$$\bar{S}_L = \bar{V}_L\bar{I}^* = (jX_L\bar{I})\bar{I}^* = jX_L I^2 = j8(20)^2 = 0 + j3200$$

$$\bar{S}_C = \bar{V}_C\bar{I}^* = (-jX_C\bar{I})\bar{I}^* = -jX_C I^2 = -j4(20)^2 = 0 - j1600$$

$$\text{Complex power absorbed by the total load } \bar{S}_{LOAD} = \bar{S}_R + \bar{S}_L + \bar{S}_C = 2000\angle 53.1^\circ$$

Power Triangle:

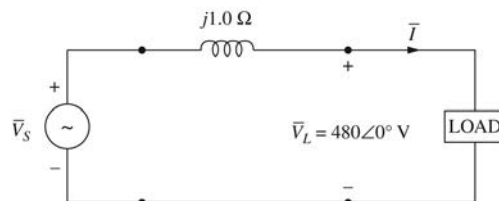


Complex power delivered by the source is

$$\bar{S}_{SOURCE} = \bar{V}\bar{I}^* = (100\angle 0^\circ)(20\angle -53.1^\circ)^* = 2000\angle 53.1^\circ$$

The complex power delivered by the source is equal to the total complex power absorbed by the load.

2.26 (a) The problem is modeled as shown in figure below:



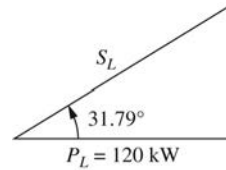
$$P_L = 120 \text{ kW}$$

$$pf_L = 0.85 \text{ Lagging}$$

$$\theta_L = \cos^{-1} 0.85 = 31.79^\circ$$

Power triangle for the load:

$$\begin{aligned}\bar{S}_L &= P_L + jQ_L = 141.18 \angle 31.79^\circ \text{ kVA} \\ I &= S_L / V = 141,180 / 480 = 294.13 \text{ A}\end{aligned}$$



$$\begin{aligned}Q_L &= P_L \tan(31.79^\circ) \\ &= 74.364 \text{ kVAR}\end{aligned}$$

Real power loss in the line is zero.

$$\begin{aligned}\text{Reactive power loss in the line is } Q_{LINE} &= I^2 X_{LINE} = (294.13)^2 1 \\ &= 86.512 \text{ kVAR}\end{aligned}$$

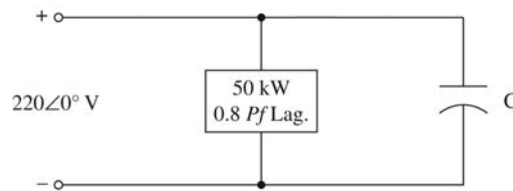
$$\therefore \bar{S}_S = P_S + jQ_S = 120 + j(74.364 + 86.512) = 200.7 \angle 53.28^\circ \text{ kVA}$$

The input voltage is given by $V_S = S_S / I = 682.4 \text{ V (rms)}$

The power factor at the input is $\cos 53.28^\circ = 0.6$ Lagging

$$\begin{aligned}\text{(b) Applying KVL, } \bar{V}_S &= 480 \angle 0^\circ + j1.0(294.13 \angle -31.79^\circ) \\ &= 635 + j250 = 682.4 \angle 21.5^\circ \text{ V (rms)} \\ (pf)_S &= \cos(21.5^\circ + 31.79^\circ) = 0.6 \text{ Lagging}\end{aligned}$$

2.27 The circuit diagram is shown below:



$$\begin{aligned}P_{old} &= 50 \text{ kW}; \cos^{-1} 0.8 = 36.87^\circ; \theta_{OLD} = 36.87^\circ; Q_{old} = P_{old} \tan(\theta_{old}) \\ &= 37.5 \text{ kVAR}\end{aligned}$$

$$\therefore \bar{S}_{old} = 50,000 + j37,500$$

$$\begin{aligned}\theta_{new} &= \cos^{-1} 0.95 = 18.19^\circ; \bar{S}_{new} = 50,000 + j50,000 \tan(18.19^\circ) \\ &= 50,000 + j16,430\end{aligned}$$

$$\text{Hence } \bar{S}_{cap} = \bar{S}_{new} - \bar{S}_{old} = -j21,070 \text{ VA}$$

$$\therefore C = \frac{21,070}{(377)(220)^2} = 1155 \mu\text{F} \leftarrow$$

$$2.28 \quad \bar{S}_1 = 12 + j6.667$$

$$\bar{S}_2 = 4(0.96) - j4[\sin(\cos^{-1} 0.96)] = 3.84 - j1.12$$

$$\bar{S}_3 = 15 + j0$$

$$\bar{S}_{TOTAL} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (30.84 + j5.547) \text{ kVA}$$

(i) Let \bar{Z} be the impedance of a series combination of R and X

$$\text{Since } \bar{S} = \bar{V}\bar{I}^* = \bar{V}\left(\frac{\bar{V}}{\bar{Z}}\right)^* = \frac{V^2}{\bar{Z}^*}, \text{ it follows that}$$

$$\bar{Z}^* = \frac{V^2}{\bar{S}} = \frac{(240)^2}{(30.84 + j5.547)10^3} = (1.809 - j0.3254) \Omega$$

$$\therefore \bar{Z} = (1.809 + j0.3254) \Omega \leftarrow$$

(ii) Let \bar{Z} be the impedance of a parallel combination of R and X

$$\text{Then } R = \frac{(240)^2}{(30.84)10^3} = 1.8677 \Omega$$

$$X = \frac{(240)^2}{(5.547)10^3} = 10.3838 \Omega$$

$$\therefore \bar{Z} = (1.8677 \parallel j10.3838) \Omega \leftarrow$$

2.29 Since complex powers satisfy KCL at each bus, it follows that

$$\bar{S}_{13} = (1 + j1) - (1 - j1) - (0.4 + j0.2) = -0.4 + j1.8 \leftarrow$$

$$\bar{S}_{31} = -\bar{S}_{13}^* = 0.4 + j1.8 \leftarrow$$

$$\text{Similarly, } \bar{S}_{23} = (0.5 + j0.5) - (1 + j1) - (-0.4 + j0.2) = -0.1 - j0.7 \leftarrow$$

$$\bar{S}_{32} = -\bar{S}_{23}^* = 0.1 - j0.7 \leftarrow$$

$$\text{At Bus 3, } \bar{S}_{G3} = \bar{S}_{31} + \bar{S}_{32} = (0.4 + j1.8) + (0.1 - j0.7) = 0.5 + j1.1 \leftarrow$$

2.30 (a) For load 1: $\theta_1 = \cos^{-1}(0.28) = 73.74^\circ$ Lagging

$$\bar{S}_1 = 125 \angle 73.74^\circ = 35 + j120$$

$$\bar{S}_2 = 10 - j40$$

$$\bar{S}_3 = 15 + j0$$

$$\bar{S}_{TOTAL} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 60 + j80 = 100 \angle 53.13^\circ \text{ kVA} = P + jQ$$

$$\therefore P_{TOTAL} = 60 \text{ kW}; Q_{TOTAL} = 80 \text{ kVAR}; \text{ kVA}_{TOTAL} = S_{TOTAL} = 100 \text{ kVA.} \leftarrow$$

$$\text{Supply } pf = \cos(53.13^\circ) = 0.6 \text{ Lagging} \leftarrow$$

$$(b) \bar{I}_{TOTAL} = \frac{\bar{S}^*}{\bar{V}^*} = \frac{100 \times 10^3 \angle -53.13^\circ}{1000 \angle 0^\circ} = 100 \angle -53.13^\circ \text{ A}$$

At the new *pf* of 0.8 lagging, P_{TOTAL} of 60kW results in the new reactive power Q' , such that

$$\theta' = \cos^{-1}(0.8) = 36.87^\circ$$

and $Q' = 60 \tan(36.87^\circ) = 45 \text{ kVAR}$

\therefore The required capacitor's kVAR is $Q_c = 80 - 45 = 35 \text{ kVAR} \leftarrow$

It follows then $X_c = \frac{V^2}{S_c^*} = \frac{(1000)^2}{j35000} = -j28.57 \Omega$

and $C = \frac{10^6}{2\pi(60)(28.57)} = 92.85 \mu\text{F} \leftarrow$

The new current is $I' = \frac{\bar{S}'^*}{\bar{V}^*} = \frac{60,000 - j45,000}{1000 \angle 0^\circ} = 60 - j45 = 75 \angle -36.87^\circ \text{ A}$

The supply current, in magnitude, is reduced from 100A to 75A \leftarrow

2.31 (a) $\bar{I}_{12} = \frac{V_1 \angle \delta_1 - V_2 \angle \delta_2}{X \angle 90^\circ} = \left(\frac{V_1}{X} \angle \delta_1 - 90^\circ \right) - \frac{V_2}{X} \angle \delta_2 - 90^\circ$

Complex power $\bar{S}_{12} = \bar{V}_1 \bar{I}_{12}^* = V_1 \angle \delta_1 \left[\frac{V_1}{X} \angle 90^\circ - \delta_1 - \frac{V_2}{X} \angle 90^\circ - \delta_2 \right]$

$$= \frac{V_1^2}{X} \angle 90^\circ - \frac{V_1 V_2}{X} \angle 90^\circ + \delta_1 - \delta_2$$

\therefore The real and reactive power at the sending end are

$$P_{12} = \frac{V_1^2}{X} \cos 90^\circ - \frac{V_1 V_2}{X} \cos(90^\circ + \delta_1 - \delta_2)$$

$$= \frac{V_1 V_2}{X} \sin(\delta_1 - \delta_2) \leftarrow$$

$$Q_{12} = \frac{V_1^2}{X} \sin 90^\circ - \frac{V_1 V_2}{X} \sin(90^\circ + \delta_1 - \delta_2)$$

$$= \frac{V_1}{X} [V_1 - V_2 \cos(\delta_1 - \delta_2)] \leftarrow$$

Note: If \bar{V}_1 leads \bar{V}_2 , $\delta = \delta_1 - \delta_2$ is positive and the real power flows from node 1 to node 2. If \bar{V}_1 Lags \bar{V}_2 , δ is negative and power flows from node 2 to node 1.

(b) Maximum power transfer occurs when $\delta = 90^\circ = \delta_1 - \delta_2 \leftarrow$

$$P_{MAX} = \frac{V_1 V_2}{X} \leftarrow$$

2.32 4 Mvar minimizes the real power line losses, while 4.5 Mvar minimizes the MVA power flow into the feeder.