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Preface

This manual contains solutions/answers to all exercises in the text *Precalculus: Functions and Graphs, Twelfth Edition*, by Earl W. Swokowski and Jeffery A. Cole. A *Student's Solutions Manual* is also available; it contains solutions for the odd-numbered exercises in each section and for the Discussion Exercises, as well as solutions for all the exercises in the Review Sections and for the Chapter Tests.

For most problems, a reasonably detailed solution is included. It is my hope that by merely browsing through the solutions, professors will save time in determining appropriate assignments for their particular class.

I appreciate feedback concerning errors, solution correctness or style, and manual style—comments from professors using previous editions have greatly strengthened the ancillary package as well as the text. Any comments may be sent directly to me at the address below, at jeff.cole@anokaramsey.edu, or in care of the publisher: Brooks/Cole | Cengage Learning, 20 Davis Drive, Belmont, CA 94002-3098.

I would like to thank: Marv Riedesel and Mary Johnson for accuracy checking of the new exercises; Andrew Bulman-Fleming, for manuscript preparation; Brian Morris and the late George Morris, of Scientific Illustrators, for creating the mathematically precise art package; and Cynthia Ashton, of Cengage Learning, for checking the manuscript. I dedicate this book to my children, Becky and Brad.

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To the Instructor

In the chapter review sections, the solutions are abbreviated since more detailed solutions were given in chapter sections. In easier groups of exercises, representative solutions are shown. When appropriate, only the answer is listed.

All figures have been plotted using computer software, offering a high degree of precision. The calculator graphs are from various TI screens. When possible, we tried to make each piece of art with the same scale to show a realistic and consistent graph.

This manual was done using $\mathbb{EXP}$: *The Scientific Word Processor*.

The following notations are used in the manual.

Note: Notes to the instructor/student pertaining to hints on instruction or conventions to follow.

{ }	{ comments to the reader are in braces }
LS	{ Left Side of an equation }
RS	{ Right Side of an equation }
$\Rightarrow$	{ implies, next equation, logically follows }
$\Leftrightarrow$	{ if and only if, is equivalent to }
•	{ bullet, used to separate problem statement from solution or explanation }
★	{ used to identify the answer to the problem }
§	{ *section* references }
$\forall$	{ For all, i.e., $\forall x$ means "for all x". }
$\mathbb{R} - \{a\}$	{ The set of all real numbers except a. }
$\therefore$	{ therefore }
QI–QIV	{ quadrants I, II, III, IV }

Chapter 1: Fundamental Concepts of Algebra

1.1 Exercises

1 (a) Since x and y have opposite signs, the product xy is negative.

(b) Since $x^2 > 0$ and $y > 0$, the product x^2y is positive.

(c) Since $x < 0$ $\{x$ is negative$\}$ and $y > 0$ $\{y$ is positive$\}$, the quotient $\dfrac{x}{y}$ is negative.

Thus, $\dfrac{x}{y} + x$ is the sum of two negatives, which is negative.

(d) Since $y > 0$ and $x < 0$, $y - x > 0$.

2 (a) Since x and y have opposite signs, the quotient $\dfrac{x}{y}$ is negative.

(b) Since $x < 0$ and $y^2 > 0$, the product xy^2 is negative.

(c) Since $x - y < 0$ and $xy < 0$, $\dfrac{x-y}{xy} > 0$.

(d) Since $y > 0$ and $y - x > 0$, $y(y - x) > 0$.

3 (a) Since -7 is to the left of -4 on a coordinate line, $-7 \boxed{<} -4$.

(b) Using a calculator, we see that $\frac{\pi}{2} \approx 1.57$. Hence, $\frac{\pi}{2} \boxed{>} 1.5$.

(c) $\sqrt{225} \boxed{=} 15$ ***Note:*** $\sqrt{225} \neq \pm 15$

4 (a) Since -3 is to the right of -6 on a coordinate line, $-3 \boxed{>} -6$.

(b) Using a calculator, we see that $\frac{\pi}{4} \approx 0.79$. Hence, $\frac{\pi}{4} \boxed{<} 0.8$.

(c) $\sqrt{289} \boxed{=} 17$ ***Note:*** $\sqrt{289} \neq \pm 17$

5 (a) Since $\frac{1}{11} = 0.\overline{09} = 0.0909\ldots$, $\frac{1}{11} \boxed{>} 0.09$.

(b) Since $\frac{2}{3} = 0.\overline{6} = 0.6666\ldots$, $\frac{2}{3} \boxed{>} 0.666$.

(c) Since $\frac{22}{7} = 3.\overline{142857}$ and $\pi \approx 3.141593$, $\frac{22}{7} \boxed{>} \pi$.

6 (a) Since $\frac{1}{7} = 0.\overline{142857}$, $\frac{1}{7} \boxed{<} 0.143$.

(b) Since $\frac{5}{6} = 0.8\overline{3} = 0.8333\ldots$, $\frac{5}{6} \boxed{>} 0.833$.

(c) Since $\sqrt{2} \approx 1.414$, $\sqrt{2} \boxed{>} 1.4$.

7 (a) "x is negative" is equivalent to $x < 0$. We symbolize this by writing "x is negative $\Leftrightarrow$ $x < 0$."

(b) y is nonnegative $\Leftrightarrow$ $y \geq 0$

(c) q is less than or equal to π $\Leftrightarrow$ $q \leq \pi$

(d) d is between 4 and 2 $\Leftrightarrow$ $2 < d < 4$

(e) t is not less than 5 $\Leftrightarrow$ $t \geq 5$

(f) The negative of z is not greater than 3 $\Leftrightarrow$ $-z \leq 3$

(g) The quotient of p and q is at most 7 $\Leftrightarrow$ $\dfrac{p}{q} \leq 7$

(h) The reciprocal of w is at least 9 $\Leftrightarrow$ $\dfrac{1}{w} \geq 9$

(i) The absolute value of x is greater than 7 $\Leftrightarrow$ $|x| > 7$

Note: An informal definition of absolute value that may be helpful is

$$|\textit{something}| = \begin{cases} \textit{itself} & \text{if } \textit{itself} \text{ is positive or zero} \\ -(\textit{itself}) & \text{if } \textit{itself} \text{ is negative} \end{cases}$$

8 (a) b is positive $\Leftrightarrow$ $b > 0$

(b) s is nonpositive $\Leftrightarrow$ $s \leq 0$

(c) w is greater than or equal to -4 $\Leftrightarrow$ $w \geq -4$

(d) c is between $\frac{1}{5}$ and $\frac{1}{3}$ $\Leftrightarrow$ $\frac{1}{5} < c < \frac{1}{3}$

(e) p is not greater than -2 $\Leftrightarrow$ $p \leq -2$

(f) The negative of m is not less than -2 $\Leftrightarrow$ $-m \geq -2$

(g) The quotient of r and s is at least $\frac{1}{5}$ $\Leftrightarrow$ $\frac{r}{s} \geq \frac{1}{5}$

(h) The reciprocal of f is at most 14 $\Leftrightarrow$ $\frac{1}{f} \leq 14$

(i) The absolute value of x is less than 4 $\Leftrightarrow$ $|x| < 4$

9 (a) $|-3-4| = |-7| = -(-7)$ {since $-7 < 0$} $= 7$

(b) $|-5| - |2| = -(-5) - 2 = 5 - 2 = 3$

(c) $|7| + |-4| = 7 + [-(-4)] = 7 + 4 = 11$

10 (a) $|-11+1| = |-10| = -(-10)$ { since $-10 < 0$ } $= 10$

(b) $|6| - |-3| = 6 - [-(-3)] = 6 - 3 = 3$

(c) $|8| + |-9| = 8 + [-(-9)] = 8 + 9 = 17$

11 (a) $(-5)|3-6| = (-5)|-3| = (-5)[-(-3)] = (-5)(3) = -15$

(b) $|-6|/(-2) = -(-6)/(-2) = 6/(-2) = -3$

(c) $|-7| + |4| = -(-7) + 4 = 7 + 4 = 11$

12 (a) $(4)|6-7| = (4)|-1| = (4)[-(-1)] = (4)(1) = 4$

(b) $5/|-2| = 5/[-(-2)] = 5/2$

(c) $|-1| + |-9| = -(-1) + [-(-9)] = 1 + 9 = 10$

13 (a) Since $(4-\pi)$ is positive, $|4-\pi| = 4-\pi$.

(b) Since $(\pi - 4)$ is negative, $|\pi - 4| = -(\pi - 4) = 4 - \pi$.

(c) Since $\left(\sqrt{2} - 1.5\right)$ is negative, $\left|\sqrt{2} - 1.5\right| = -\left(\sqrt{2} - 1.5\right) = 1.5 - \sqrt{2}$.

14 (a) Since $\left(\sqrt{3} - 1.7\right)$ is positive, $\left|\sqrt{3} - 1.7\right| = \sqrt{3} - 1.7$.

(b) Since $\left(1.7 - \sqrt{3}\right)$ is negative, $\left|1.7 - \sqrt{3}\right| = -\left(1.7 - \sqrt{3}\right) = \sqrt{3} - 1.7$.

(c) $\left|\frac{1}{5} - \frac{1}{3}\right| = \left|\frac{3}{15} - \frac{5}{15}\right| = \left|-\frac{2}{15}\right| = -\left(-\frac{2}{15}\right) = \frac{2}{15}$

15 (a) $d(A,B) = |7-3| = |4| = 4$

(b) $d(B,C) = |-5-7| = |-12| = 12$

(c) $d(C,B) = d(B,C) = 12$

(d) $d(A,C) = |-5-3| = |-8| = 8$

16 (a) $d(A,B) = |-2-(-6)| = |4| = 4$

(b) $d(B,C) = |4-(-2)| = |6| = 6$

(c) $d(C,B) = d(B,C) = 6$

(d) $d(A,C) = |4-(-6)| = |10| = 10$

17 (a) $d(A,B) = |1-(-9)| = |10| = 10$

(b) $d(B,C) = |10-1| = |9| = 9$

(c) $d(C,B) = d(B,C) = 9$

(d) $d(A,C) = |10-(-9)| = |19| = 19$

18 (a) $d(A,B) = |-4-8| = |-12| = 12$

(b) $d(B,C) = |-1-(-4)| = |3| = 3$

(c) $d(C,B) = d(B,C) = 3$

(d) $d(A,C) = |-1-8| = |-9| = 9$

Note: Because $|a| = |-a|$, the answers to Exercises 19–24 could have a different form. For example, $|-3-x| \geq 8$ is equivalent to $|x+3| \geq 8$.

19 $A = x$ and $B = 7$, so $d(A,B) = |7-x|$. Thus, "$d(A,B)$ is less than 2" can be written as $|7-x| < 2$.

20 $d(A,B) = \left|-\sqrt{2} - x\right| \Rightarrow \left|-\sqrt{2} - x\right| > 1$

21 $d(A,B) = |-3-x| \Rightarrow |-3-x| \geq 8$

22 $d(A,B) = |4-x| \Rightarrow |4-x| \leq 5$

23 $d(A, B) = |x - 4| \quad \Rightarrow \quad |x - 4| \le 3$

24 $d(A, B) = |x - (-2)| = |x + 2| \quad \Rightarrow \quad |x + 2| \ge 4$

Note: In Exercises 25–32, you may want to substitute a permissible value for the variable to first test if the expression inside the absolute value symbol is positive or negative.

25 Pick an arbitrary value for x that is less than -3, say -5.
Since $3 + (-5) = -2$ is negative, we conclude that if $x < -3$, then $3 + x$ is negative.
Hence, $|3 + x| = -(3 + x) = -x - 3$.

26 If $x > 5$, then $5 - x < 0$, and $|5 - x| = -(5 - x) = x - 5$.

27 If $x < 2$, then $2 - x > 0$, and $|2 - x| = 2 - x$.

28 If $x \ge -7$, then $7 + x \ge 0$, and $|7 + x| = 7 + x$.

29 If $a < b$, then $a - b < 0$, and $|a - b| = -(a - b) = b - a$.

30 If $a > b$, then $a - b > 0$, and $|a - b| = a - b$.

31 Since $x^2 + 4 > 0$ for every x, $|x^2 + 4| = x^2 + 4$.

32 Since $-x^2 - 1 < 0$ for every x, $|-x^2 - 1| = -(-x^2 - 1) = x^2 + 1$.

33 $\text{LS} = \dfrac{ab + ac}{a} = \dfrac{ab}{a} + \dfrac{ac}{a} = b + c \boxed{\neq} \text{RS}$ (which is $b + ac$).

34 $\text{LS} = \dfrac{ab + ac}{a} = \dfrac{ab}{a} + \dfrac{ac}{a} = b + c \boxed{=} \text{RS}$.

35 $\text{LS} = \dfrac{b + c}{a} = \dfrac{b}{a} + \dfrac{c}{a} \boxed{=} \text{RS}$.

36 $\text{LS} = \dfrac{a + c}{b + d} = \dfrac{a}{b + d} + \dfrac{c}{b + d} \boxed{\neq} \text{RS} \left(\text{which is } \dfrac{a}{b} + \dfrac{c}{d}\right)$.

37 $\text{LS} = (a \div b) \div c = \dfrac{a}{b} \cdot \dfrac{1}{c} = \dfrac{a}{bc}$. $\text{RS} = a \div (b \div c) = a \div \dfrac{b}{c} = a \cdot \dfrac{c}{b} = \dfrac{ac}{b}$. $\text{LS} \boxed{\neq} \text{RS}$

38 $\text{LS} = (a - b) - c = a - b - c$. $\text{RS} = a - (b - c) = a - b + c$. $\text{LS} \boxed{\neq} \text{RS}$

39 $\text{LS} = \dfrac{a - b}{b - a} = \dfrac{-(b - a)}{b - a} = -1 \boxed{=} \text{RS}$.

40 $\text{LS} = -(a + b) = -a - b \boxed{\neq} \text{RS}$ (which is $-a + b$).

41 (a) On the TI-83/4 Plus, the absolute value function is choice 1 under MATH, NUM.
Enter abs(3.2^2-√(4.27)). $\left|3.2^2 - \sqrt{4.27}\right| \approx 8.1736$

(b) $\sqrt{(15.6 - 1.5)^2 + (4.3 - 5.4)^2} \approx 14.1428$

42 (a) $\dfrac{3.42 - 1.29}{5.83 + 2.64} \approx 0.2515$

(b) $\pi^3 \approx 31.0063$

43 (a) $\dfrac{1.2 \times 10^3}{3.1 \times 10^2 + 1.52 \times 10^3} \approx 0.6557 = 6.557 \times 10^{-1}$

Note: For the TI-83/4 Plus, use 1.2E3/(3.1E2 + 1.52E3), where E is obtained by pressing [2nd] [EE].

(b) $\left(1.23 \times 10^{-4}\right) + \sqrt{4.5 \times 10^3} \approx 67.08 = 6.708 \times 10^1$

44 (a) $\sqrt{|3.45 - 1.2 \times 10^4| + 10^5} \approx 334.7 = 3.347 \times 10^2$

(b) $(1.79 \times 10^2) \times (9.84 \times 10^3) = 1{,}761{,}360 \approx 1.761 \times 10^6$

45 Construct a right triangle with sides of lengths $\sqrt{2}$ and 1. The hypotenuse will have length $\sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$. Next construct a right triangle with sides of lengths $\sqrt{3}$ and $\sqrt{2}$. The hypotenuse will have length $\sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{5}$.

46 Use $C = 2\pi r$ with $r = 1$, 2, and 10 to obtain 2π, 4π, and 20π units from the origin.

47 The large rectangle has area = width × length = $a(b+c)$. The sum of the areas of the two small rectangles is $ab + ac$. Since the areas are the same, we have $a(b+c) = ab + ac$.

48 $x_1 = \dfrac{3}{2}$ and $n = 2 \quad \Rightarrow \quad x_2 = \dfrac{1}{2}\left(x_1 + \dfrac{n}{x_1}\right) = \dfrac{1}{2}\left(\dfrac{3}{2} + \dfrac{2}{\frac{3}{2}}\right) = \dfrac{1}{2}\left(\dfrac{3}{2} + \dfrac{4}{3}\right) = \dfrac{1}{2}\left(\dfrac{17}{6}\right) = \dfrac{17}{12}$.

$$x_3 = \frac{1}{2}\left(x_2 + \frac{2}{x_2}\right) = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{\frac{17}{12}}\right) = \frac{1}{2}\left(\frac{17}{12} + \frac{24}{17}\right) = \frac{1}{2}\left(\frac{577}{204}\right) = \frac{577}{408}$$

49 (a) Since the decimal point is 5 places to the right of the first nonzero digit, $427{,}000 = 4.27 \times 10^5$.

(b) Since the decimal point is 8 places to the left of the first nonzero digit, $0.000\,000\,093 = 9.3 \times 10^{-8}$.

(c) Since the decimal point is 8 places to the right of the first nonzero digit, $810{,}000{,}000 = 8.1 \times 10^8$.

50 (a) $85{,}200 = 8.52 \times 10^4$ (b) $0.000\,005\,4 = 5.4 \times 10^{-6}$

(c) $24{,}900{,}000 = 2.49 \times 10^7$

51 (a) Moving the decimal point 5 places to the right, we have $8.3 \times 10^5 = 830{,}000$.

(b) Moving the decimal point 12 places to the left, we have $2.9 \times 10^{-12} = 0.000\,000\,000\,002\,9$.

(c) Moving the decimal point 8 places to the right, we have $5.64 \times 10^8 = 564{,}000{,}000$.

52 (a) $2.3 \times 10^7 = 23{,}000{,}000$ (b) $7.01 \times 10^{-9} = 0.000\,000\,007\,01$

(c) $1.25 \times 10^{10} = 12{,}500{,}000{,}000$

53 Since the decimal point is 24 places to the left of the first nonzero digit,

$$0.000\,000\,000\,000\,000\,000\,000\,001\,7 = 1.7 \times 10^{-24}.$$

54 $9.1 \times 10^{-3^1} = 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,91$

55 It is helpful to write the units of any fraction, and then "cancel" those units to determine the units of the final answer. $\dfrac{186{,}000 \text{ miles}}{\text{second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{24 \text{ hours}}{1 \text{ day}} \cdot \dfrac{365 \text{ days}}{1 \text{ year}} \cdot 1 \text{ year} \approx 5.87 \times 10^{12}$ mi

56 (a) 100 billion $= 100{,}000{,}000{,}000 = 1 \times 10^{11}$

(b) $d \approx (100{,}000 \text{ yr})\left(5.87 \times 10^{12} \dfrac{\text{mi}}{\text{yr}}\right) = 5.87 \times 10^{17}$ mi

57 $$\frac{\dfrac{1.01 \text{ grams}}{\text{mole}}}{\dfrac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}}} \cdot 1 \text{ atom} = \frac{1.01 \text{ grams}}{6.02 \times 10^{23}} \approx 0.1678 \times 10^{-23} \text{ g} = 1.678 \times 10^{-24} \text{ g}$$

58 $(2.5 \text{ million})(0.00035\%) = (2.5 \times 10^6)(3.5 \times 10^{-6}) = 8.75 \approx 9$ halibut

59 $\dfrac{24 \text{ frames}}{\text{second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot 48 \text{ hours} = 4.1472 \times 10^6$ frames

60 $\dfrac{2 \times 10^{11} \text{ calculations}}{\text{second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{24 \text{ hours}}{1 \text{ day}} \cdot 60 \text{ days} = 1.0368 \times 10^{18}$ calculations

61 (a) $1 \text{ ft}^2 = 144 \text{ in}^2$, so the force on one square foot of a wall is $144 \text{ in}^2 \times 1.4 \text{ lb/in}^2 = 201.6$ lb.

(b) The area of the wall is $40 \times 8 = 320 \text{ ft}^2$, or $320 \text{ ft}^2 \times 144 \text{ in}^2/\text{ft}^2 = 46{,}080 \text{ in}^2$.

The total force is $46{,}080 \text{ in}^2 \times 1.4 \text{ lb/in}^2 = 64{,}512$ lb.

Converting to tons, we have $64{,}512 \text{ lb}/(2000 \text{ lb/ton}) = 32.256$ tons.

62 (a) We start with 400 adults, 150 yearlings, and 200 calves {total = 750}

Number of Adults = surviving adults + surviving yearlings
$= (0.90)(400) + (0.80)(150) = \underline{480}$

Number of Yearlings = surviving calves
$= (0.75)(200) = \underline{150}$

Number of Calves = number of female adults
$= (0.50)(480) = \underline{240}$

(b) 75% of last spring's calves equal the number of this year's yearlings (150), so the number of calves is $\underline{200}$.

The number of calves is equal to the number of adult females and this is one-half of the number of adults, so the number of adults is $\underline{400}$.

90% of these (360) are part of the 400 adults this year. The other 40 adults represent 80% of last year's yearlings, so the number of yearlings is $\underline{50}$.

1.2 Exercises

1 $\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) = \frac{16}{81}$

Note: Do not confuse $(-x)^4$ and $-x^4$ since $(-x)^4 = x^4$ and $-x^4$ is the negative of x^4.

2 $(-3)^3 = -27 = \frac{-27}{1}$

3 $\dfrac{2^{-3}}{3^{-2}} = \dfrac{3^2}{2^3} = \dfrac{9}{8}$

Note: Remember that negative exponents don't necessarily give negative results—that is, $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$, not $-\dfrac{1}{8}$.

4 $\dfrac{2^0 + 0^2}{2 + 0} = \dfrac{1 + 0}{2} = \dfrac{1}{2}$

5 $-2^4 + 3^{-1} = -16 + \frac{1}{3} = -\frac{48}{3} + \frac{1}{3} = -\frac{47}{3}$

6 $\left(-\frac{3}{2}\right)^4 - 2^{-4} = \frac{81}{16} - \frac{1}{16} = \frac{80}{16} = \frac{5}{1}$

7 $9^{5/2} = \left(\sqrt{9}\right)^5 = 3^5 = \frac{243}{1}$

8 $16^{-3/4} = 1/16^{3/4} = 1/\left(\sqrt[4]{16}\right)^3 = 1/2^3 = \frac{1}{8}$

9 $(-0.008)^{2/3} = \left(\sqrt[3]{-0.008}\right)^2 = (-0.2)^2 = 0.04 = \frac{4}{100} = \frac{1}{25}$

10 $(0.008)^{-2/3} = 1/(0.008)^{2/3} = 1/\left(\sqrt[3]{0.008}\right)^2 = 1/(0.2)^2 = 1/(0.04) = \frac{25}{1}$

11 $\left(\frac{1}{2}x^4\right)(16x^5) = \left(\frac{1}{2} \cdot 16\right)x^{4+5} = 8x^9$

12 $(-3x^{-2})(4x^4) = (-3 \cdot 4)x^{-2+4} = -12x^2$

13 A common mistake is to write $x^3x^2 = x^6$, and another is to write $(x^2)^3 = x^5$.
The following solution illustrates the proper use of the exponent rules.

$$\frac{(2x^3)(3x^2)}{(x^2)^3} = \frac{(2\cdot 3)x^{3+2}}{x^{2\cdot 3}} = \frac{6x^5}{x^6} = 6x^{5-6} = 6x^{-1} = \frac{6}{x}$$

14 $\dfrac{(2x^2)^3y^2}{4x^4y^2} = \dfrac{8x^6}{4x^4} = 2x^2$

15 $\left(\frac{1}{6}a^5\right)(-3a^2)\left(4a^7\right) = \frac{1}{6}\cdot(-3)\cdot 4\cdot a^{5+2+7} = -2a^{14}$

16 $(-4b^3)\left(\frac{1}{6}b^2\right)\left(-9b^4\right) = (-4)\cdot\frac{1}{6}\cdot(-9)\cdot b^{3+2+4} = 6b^9$

17 $\dfrac{(6x^3)^2}{(2x^2)^3}\cdot\left(3x^2\right)^0 = \dfrac{6^2x^{3\cdot 2}}{2^3x^{2\cdot 3}}\cdot 1$ {an expression raised to the zero power is equal to 1} $= \dfrac{36x^6}{8x^6} = \dfrac{36}{8} = \dfrac{9}{2}$

18 $\dfrac{(3y^3)(2y^2)^2}{(y^4)^3}\cdot\left(5y^3\right)^0 = \dfrac{(3y^3)\left(4y^4\right)}{y^{12}}\cdot 1 = \dfrac{12y^7}{y^{12}} = \dfrac{12}{y^5}$

19 $\left(3u^7v^3\right)\left(4u^4v^{-5}\right) = 12u^{7+4}v^{3+(-5)} = 12u^{11}v^{-2} = \dfrac{12u^{11}}{v^2}$

20 $(x^2yz^3)(-2xz^2)(x^3y^{-2}) = -2x^{2+1+3}y^{1-2}z^{3+2} = -2x^6y^{-1}z^5 = \dfrac{-2x^6z^5}{y}$

21 $\left(8x^4y^{-3}\right)\left(\frac{1}{2}x^{-5}y^2\right) = 4x^{4-5}y^{-3+2} = 4x^{-1}y^{-1} = \dfrac{4}{xy}$

22 $\left(\dfrac{4a^2b}{a^3b^2}\right)\left(\dfrac{5a^2b}{2b^4}\right) = \dfrac{20a^{2+2}b^{1+1}}{2a^3b^{2+4}} = \dfrac{20a^4b^2}{2a^3b^6} = \dfrac{10a^{4-3}b^{2-6}}{1} = \dfrac{10a}{b^4}$

23 $\left(\dfrac{1}{3}x^4y^{-3}\right)^{-2} = \left(\dfrac{1}{3}\right)^{-2}\left(x^4\right)^{-2}\left(y^{-3}\right)^{-2} = \left(\dfrac{3}{1}\right)^2x^{-8}y^6 = 3^2x^{-8}y^6 = \dfrac{9y^6}{x^8}$

24 $(-2xy^2)^5\left(\dfrac{x^7}{8y^3}\right) = (-32x^5y^{10})\left(\dfrac{x^7}{8y^3}\right) = -4x^{12}y^7$

25 $(3y^3)^4(4y^2)^{-3} = 3^4y^{12}\cdot 4^{-3}y^{-6} = 81y^6\cdot\dfrac{1}{4^3} = \dfrac{81}{64}y^6$

26 $(-3a^2b^{-5})^3 = -27a^6b^{-15} = -\dfrac{27a^6}{b^{15}}$

27 $\left(-2r^4s^{-3}\right)^{-2} = (-2)^{-2}r^{-8}s^6 = \dfrac{s^6}{(-2)^2r^8} = \dfrac{s^6}{4r^8}$

28 $(2x^2y^{-5})(6x^{-3}y)\left(\frac{1}{3}x^{-1}y^3\right) = 4x^{-2}y^{-1} = \dfrac{4}{x^2y}$

29 $(5x^2y^{-3})\left(4x^{-5}y^4\right) = 20x^{2-5}y^{-3+4} = 20x^{-3}y^1 = \dfrac{20y}{x^3}$

30 $(-2r^2s)^5(3r^{-1}s^3)^2 = (-32r^{10}s^5)(9r^{-2}s^6) = -288r^8s^{11}$

31 $\left(\dfrac{3x^5y^4z}{x^0y^{-3}z}\right)^2$ {remember that $x^0 = 1$, cancel z} $= \dfrac{9x^{10}y^8}{y^{-6}} = 9x^{10}y^{8-(-6)} = 9x^{10}y^{14}$

32 $(4a^2b)^4\left(\dfrac{-a^3}{2b}\right)^2 = \left(256a^8b^4\right)\left(\dfrac{a^6}{4b^2}\right) = 64a^{14}b^2$

33 $\left(-5a^{3/2}\right)\left(2a^{1/2}\right) = -5\cdot 2a^{(3/2)+(1/2)} = -10a^{4/2} = 8a^2$

34 $\left(-6x^{7/5}\right)\left(2x^{8/5}\right) = -6 \cdot 2x^{(7/5)+(8/5)} = -12x^{15/5} = -12x^3$

35 $\left(3x^{5/6}\right)\left(8x^{2/3}\right) = 3 \cdot 8x^{(5/6)+(4/6)} = 24x^{9/6} = 24x^{3/2}$

36 $(8r)^{1/3}\left(2r^{1/2}\right) = \left(2r^{1/3}\right)\left(2r^{1/2}\right) = 4r^{(2/6)+(3/6)} = 4r^{5/6}$

37 $\left(27a^6\right)^{-2/3} = 27^{-2/3}a^{-12/3} = \dfrac{a^{-4}}{27^{2/3}} = \dfrac{1}{\left(\sqrt[3]{27}\right)^2 a^4} = \dfrac{1}{3^2\, a^4} = \dfrac{1}{9a^4}$

38 $\left(25z^4\right)^{-3/2} = 25^{-3/2}z^{-12/2} = \dfrac{z^{-6}}{25^{3/2}} = \dfrac{1}{\left(\sqrt{25}\right)^3 z^6} = \dfrac{1}{5^3\, z^6} = \dfrac{1}{125z^6}$

39 $\left(8x^{-2/3}\right)x^{1/6} = 8x^{(-4/6)+(1/6)} = 8x^{-3/6} = \dfrac{8}{x^{1/2}}$

40 $\left(3x^{1/2}\right)\left(-2x^{5/2}\right) = -6x^{(1/2)+(5/2)} = -6x^3$

41 $\left(\dfrac{-8x^3}{y^{-6}}\right)^{2/3} = \dfrac{(-8)^{2/3}(x^3)^{2/3}}{(y^{-6})^{2/3}} = \dfrac{\left(\sqrt[3]{-8}\right)^2 x^{(3)(2/3)}}{y^{(-6)(2/3)}} = \dfrac{(-2)^2x^2}{y^{-4}} = \dfrac{4x^2}{y^{-4}} = 4x^2y^4$

42 $\left(\dfrac{-y^{3/2}}{y^{-1/3}}\right)^3 = \dfrac{-y^{9/2}}{y^{-1}} = -y^{11/2}$

43 $\left(\dfrac{x^6}{16y^{-4}}\right)^{-1/2} = \dfrac{x^{-3}}{16^{-1/2}y^2} = \dfrac{16^{1/2}}{x^3y^2} = \dfrac{4}{x^3y^2}$

44 $\left(\dfrac{c^{-4}}{81d^8}\right)^{3/4} = \dfrac{c^{-3}}{\left(\sqrt[4]{81}\right)^3 d^6} = \dfrac{c^{-3}}{3^3d^6} = \dfrac{1}{27c^3d^6}$

45 $\dfrac{(x^6y^3)^{-1/3}}{(x^4y^2)^{-1/2}} = \dfrac{(x^6)^{-1/3}(y^3)^{-1/3}}{(x^4)^{-1/2}(y^2)^{-1/2}} = \dfrac{x^{-2}y^{-1}}{x^{-2}y^{-1}} = 1$

46 $a^{4/3}a^{-3/2}a^{1/6} = a^{(8/6)-(9/6)+(1/6)} = a^{0/6} = a^0 = 1$

47 $\sqrt[4]{x^4+y} = \left(x^4+y\right)^{1/4}$

48 $\sqrt[3]{x^3+y^2} = (x^3+y^2)^{1/3}$

49 $\sqrt[3]{(a+b)^2} = \left[(a+b)^2\right]^{1/3} = (a+b)^{2/3}$

50 $\sqrt{a+\sqrt{b}} = \left(a+b^{1/2}\right)^{1/2}$

51 $\sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$ ***Note:*** $\sqrt{x^2+y^2} \neq x+y$

52 $\sqrt[3]{r^3-s^3} = (r^3-s^3)^{1/3}$

53 (a) $4x^{3/2} = 4x^1x^{1/2} = 4x\sqrt{x}$ (b) $(4x)^{3/2} = (4x)^1(4x)^{1/2} = (4x)^1\,4^{1/2}x^{1/2} = 4x \cdot 2 \cdot x^{1/2} = 8x\sqrt{x}$

54 (a) $4+x^{3/2} = 4+x^1x^{1/2} = 4+x\sqrt{x}$ (b) $(4+x)^{3/2} = (4+x)^1(4+x)^{1/2} = (4+x)\sqrt{4+x}$

55 (a) $8-y^{1/3} = 8-\sqrt[3]{y}$ (b) $(8-y)^{1/3} = \sqrt[3]{8-y}$

56 (a) $64y^{1/3} = 64\sqrt[3]{y}$ (b) $(64y)^{1/3} = 64^{1/3}y^{1/3} = 4\sqrt[3]{y}$

57 $\sqrt{81} = \sqrt{9^2} = 9$

58 $\sqrt[3]{-216} = \sqrt[3]{(-6)^3} = -6$

59 $\sqrt[5]{-64} = \sqrt[5]{-32}\,\sqrt[5]{2} = \sqrt[5]{(-2)^5}\,\sqrt[5]{2} = -2\,\sqrt[5]{2}$

60 $\sqrt[4]{512} = \sqrt[4]{256}\,\sqrt[4]{2} = \sqrt[4]{4^4}\,\sqrt[4]{2} = 4\sqrt[4]{2}$

61 In the denominator, you would like to have $\sqrt[3]{2^3}$. How do you get it? Multiply by $\sqrt[3]{2^2}$, or, equivalently, $\sqrt[3]{4}$. Of course, we have to multiply the numerator by the same value so that we don't change the value of the given fraction.

$$\frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2 \cdot 4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2} = \frac{1}{2}\sqrt[3]{4}$$

62 $\sqrt{\dfrac{1}{5}} = \dfrac{\sqrt{1}}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{1}{5}\sqrt{5}$

63 $\sqrt{9x^{-4}y^6} = \left(9x^{-4}y^6\right)^{1/2} = 9^{1/2}\left(x^{-4}\right)^{1/2}\left(y^6\right)^{1/2} = 3x^{-2}y^3 = \dfrac{3y^3}{x^2}$

64 $\sqrt{16a^8b^{-2}} = 4a^4b^{-1} = \dfrac{4a^4}{b}$

65 $\sqrt[3]{8a^6b^{-3}} = 2a^2b^{-1} = \dfrac{2a^2}{b}$

66 $\sqrt[4]{81r^5s^8} = \sqrt[4]{3^4r^4s^8}\,\sqrt[4]{r} = 3rs^2\sqrt[4]{r}$

Note: For exercises similar to numbers 67–74, pick a multiplier that will make all of the exponents of the terms in the denominator a multiple of the index.

67 The index is 2. Choose the multiplier to be $\sqrt{2y}$ so that the denominator contains only terms with even exponents.
$\sqrt{\dfrac{3x}{2y^3}} = \sqrt{\dfrac{3x}{2y^3}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{\sqrt{6xy}}{\sqrt{4y^4}} = \dfrac{\sqrt{6xy}}{2y^2}$, or $\dfrac{1}{2y^2}\sqrt{6xy}$

68 $\sqrt{\dfrac{1}{3x^3y}} = \sqrt{\dfrac{1}{3x^3y}} \cdot \dfrac{\sqrt{3xy}}{\sqrt{3xy}} = \dfrac{\sqrt{3xy}}{\sqrt{9x^4y^2}} = \dfrac{1}{3x^2y}\sqrt{3xy}$

69 The index is 3. Choose the multiplier to be $\sqrt[3]{3x^2}$ so that the denominator contains only terms with exponents that are multiples of 3.

$$\sqrt[3]{\frac{2x^4y^4}{9x}} = \sqrt[3]{\frac{2x^4y^4}{9x}} \cdot \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{\sqrt[3]{6x^6y^4}}{\sqrt[3]{27x^3}} = \frac{\sqrt[3]{x^6y^3}\,\sqrt[3]{6y}}{3x} = \frac{x^2y\,\sqrt[3]{6y}}{3x} = \frac{xy}{3}\sqrt[3]{6y}$$

70 $\sqrt[3]{\dfrac{3x^2y^5}{4x}} = \sqrt[3]{\dfrac{3x^2y^5}{4x}} \cdot \dfrac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \dfrac{\sqrt[3]{6x^4y^5}}{\sqrt[3]{8x^3}} = \dfrac{\sqrt[3]{x^3y^3}\,\sqrt[3]{6xy^2}}{2x} = \dfrac{xy\,\sqrt[3]{6xy^2}}{2x} = \dfrac{y}{2}\sqrt[3]{6xy^2}$

71 The index is 4. Choose the multiplier to be $\sqrt[4]{3x^2}$ so that the denominator contains only terms with exponents that are multiples of 4.

$$\sqrt[4]{\frac{5x^8y^3}{27x^2}} = \sqrt[4]{\frac{5x^8y^3}{27x^2}} \cdot \frac{\sqrt[4]{3x^2}}{\sqrt[4]{3x^2}} = \frac{\sqrt[4]{15x^{10}y^3}}{\sqrt[4]{81x^4}} = \frac{\sqrt[4]{x^8}\,\sqrt[4]{15x^2y^3}}{3x} = \frac{x^2\,\sqrt[4]{15x^2y^3}}{3x} = \frac{x}{3}\sqrt[4]{15x^2y^3}$$

72 $\sqrt[4]{\dfrac{x^7y^{12}}{125x}} = \sqrt[4]{\dfrac{x^7y^{12}}{125x}} \cdot \dfrac{\sqrt[4]{5x^3}}{\sqrt[4]{5x^3}} = \dfrac{\sqrt[4]{5x^{10}y^{12}}}{\sqrt[4]{625x^4}} = \dfrac{\sqrt[4]{x^8y^{12}}\,\sqrt[4]{5x^2}}{5x} = \dfrac{x^2y^3\,\sqrt[4]{5x^2}}{5x} = \dfrac{xy^3}{5}\sqrt[4]{5x^2}$

73 The index is 5. Choose the multiplier to be $\sqrt[5]{4x^2}$ so that the denominator contains only terms with exponents that are multiples of 5.

$$\sqrt[5]{\frac{5x^7y^2}{8x^3}} = \sqrt[5]{\frac{5x^7y^2}{8x^3}} \cdot \frac{\sqrt[5]{4x^2}}{\sqrt[5]{4x^2}} = \frac{\sqrt[5]{20x^9y^2}}{\sqrt[5]{32x^5}} = \frac{\sqrt[5]{x^5}\,\sqrt[5]{20x^4y^2}}{2x} = \frac{x\,\sqrt[5]{20x^4y^2}}{2x} = \frac{1}{2}\sqrt[5]{20x^4y^2}$$

74 $\sqrt[5]{\dfrac{3x^{11}y^3}{9x^2}} = \sqrt[5]{\dfrac{3x^{11}y^3}{9x^2}} \cdot \dfrac{\sqrt[5]{27x^3}}{\sqrt[5]{27x^3}} = \dfrac{\sqrt[5]{81x^{14}y^3}}{\sqrt[5]{243x^5}} = \dfrac{\sqrt[5]{x^{10}}\,\sqrt[5]{81x^4y^3}}{3x} = \dfrac{x^2\,\sqrt[5]{81x^4y^3}}{3x} = \dfrac{x}{3}\sqrt[5]{81x^4y^3}$

75 $\sqrt[4]{(5x^5y^{-2})^4} = 5x^5y^{-2} = \dfrac{5x^5}{y^2}$

76 $\sqrt[6]{(7u^{-3}v^4)^6} = 7u^{-3}v^4 = \dfrac{7v^4}{u^3}$

77 $\sqrt[5]{\dfrac{8x^3}{y^4}}\sqrt[5]{\dfrac{4x^4}{y^2}} = \sqrt[5]{\dfrac{8x^3}{y^4}}\sqrt[5]{\dfrac{4x^4}{y^2}} \cdot \dfrac{\sqrt[5]{y^4}}{\sqrt[5]{y^4}} = \dfrac{\sqrt[5]{32x^5}\,\sqrt[5]{x^2y^4}}{\sqrt[5]{y^{10}}} = \dfrac{\sqrt[5]{32x^5}\,\sqrt[5]{x^2y^4}}{y^2} = \dfrac{2x}{y^2}\sqrt[5]{x^2y^4}$

78 $\sqrt{5xy^7}\,\sqrt{15x^3y^3} = \sqrt{25x^4y^{10}}\,\sqrt{3} = 5x^2y^5\sqrt{3}$

79 $\sqrt[3]{3t^4v^2}\sqrt[3]{-9t^{-1}v^4} = \sqrt[3]{-27t^3v^6} = -3tv^2$

80 $\sqrt[3]{(2r-s)^3} = 2r - s$

81 $\sqrt{x^6y^4} = \sqrt{(x^3)^2(y^2)^2} = \sqrt{(x^3)^2}\sqrt{(y^2)^2} = |x^3||y^2| = |x^3|y^2$ since y^2 is always nonnegative.

Note: $|x^3|$ could be written as $x^2|x|$.

82 $\sqrt{x^4y^{10}} = \sqrt{(x^2)^2(y^5)^2} = |x^2||y^5| = x^2|y^5|$

83 $\sqrt[4]{x^8(y-3)^{12}} = \sqrt[4]{(x^2)^4\left((y-3)^3\right)^4} = |x^2|\left|(y-3)^3\right| = x^2\left|(y-3)^3\right|$, or $x^2(y-3)^2|(y-3)|$

84 $\sqrt[4]{(x+2)^{12}y^4} = \sqrt[4]{\left((x+2)^3\right)^4 y^4} = \left|(x+2)^3\right||y|$, or $(x+2)^2|(x+2)y|$

85 $(a^r)^2 = a^{2r} \boxed{\neq} a^{(r^2)}$ since $2r \neq r^2$ for all values of r; for example, let $r = 1$.

86 Squaring the right side gives us $(a+1)^2 = a^2 + 2a + 1$. Squaring the left side gives us $a^2 + 1$.
$a^2 + 2a + 1 \boxed{\neq} a^2 + 1$ for all values of a; for example, let $a = 1$.

87 $(ab)^{xy} = a^{xy}b^{xy} \boxed{\neq} a^x b^y$ for all values of x and y; for example, let $x = 1$ and $y = 2$.

88 $\sqrt{a^r} = (a^r)^{1/2} = \left(a^{1/2}\right)^r \boxed{=} \left(\sqrt{a}\right)^r$

89 $\sqrt[n]{\dfrac{1}{c}} = \left(\dfrac{1}{c}\right)^{1/n} = \dfrac{1^{1/n}}{c^{1/n}} \boxed{=} \dfrac{1}{\sqrt[n]{c}}$

90 $\dfrac{1}{a^k} = a^{-k} \boxed{\neq} a^{1/k}$ since $-k \neq 1/k$ for all values of k; for example, let $k = 1$.

91 (a) $(-3)^{2/5} = \left[(-3)^2\right]^{1/5} = 9^{1/5} \approx 1.5518$ (b) $(-7)^{4/3} = \left[(-7)^4\right]^{1/3} = 2401^{1/3} \approx 13.3905$

92 (a) $(-1.2)^{3/7} = \left[(-1.2)^3\right]^{1/7} = (-1.728)^{1/7} \approx -1.0813$

(b) $(-5.08)^{7/3} = \left[(-5.08)^7\right]^{1/3} \approx (-87{,}306.38)^{1/3} \approx -44.3624$

93 (a) $\sqrt{\pi + 1} \approx 2.0351$ (b) $\sqrt[3]{17.1} + 5^{1/4} \approx 4.0717$

94 (a) $(2.6 - 1.3)^{-2} \approx 0.5917$ (b) $5^{\sqrt{7}} \approx 70.6807$

95 $\$200(1.04)^{180} \approx \$232{,}825.78$

96 $h = 1454$ ft $\Rightarrow$ $d = 1.2\sqrt{h} = 1.2\sqrt{1454} \approx 45.8$ mi

97 $W = 230$ kg $\Rightarrow$ $L = 0.46\sqrt[3]{W} = 0.46\sqrt[3]{230} \approx 2.82$ m

98 $L = 25$ ft $\Rightarrow$ $W = 0.0016L^{2.43} = 0.0016(25)^{2.43} \approx 3.99$ tons

99 $b = 75$ and $w = 180$ $\Rightarrow$ $W = \dfrac{w}{\sqrt[3]{b-35}} = \dfrac{180}{\sqrt[3]{75-35}} \approx 52.6.$

$b = 120$ and $w = 250$ $\Rightarrow$ $W = \dfrac{w}{\sqrt[3]{b-35}} = \dfrac{250}{\sqrt[3]{120-35}} \approx 56.9.$

It is interesting to note that the 75-kg lifter can lift 2.4 times his/her body weight and the 120-kg lifter can lift approximately 2.08 times his/her body weight, but the formula ranks the 120-kg lifter as the superior lifter.

100 (a) $h = 72$ in. and $w = 175$ lb $\Rightarrow$ $S = (0.1091)w^{0.425}h^{0.725} = (0.1091)(175)^{0.425}(72)^{0.725} \approx 21.76$ ft^2.

(b) $h = 66$ in. $\Rightarrow$ $S_1 = (0.1091)w^{0.425}(66)^{0.725}$. A 10% increase in weight would be represented by $1.1w$ and thus $S_2 = (0.1091)(1.1w)^{0.425}(66)^{0.725}$. $S_2/S_1 = (1.1)^{0.425} \approx 1.04$, which represents a 4% increase in S.

101 $W = 0.1166h^{1.7}$

Height	64	65	66	67	68	69	70	71
Weight	137	141	145	148	152	156	160	164
Height	72	73	74	75	76	77	78	79
Weight	168	172	176	180	184	188	192	196

102 $W = 0.1049h^{1.7}$

Height	60	61	62	63	64	65	66	67
Weight	111	114	117	120	123	127	130	133
Height	68	69	70	71	72	73	74	75
Weight	137	140	144	147	151	154	158	162

1.3 Exercises

1 $(2u+3)(u-4)+4u(u-2) = (2u^2-5u-12)+(4u^2-8u) = 6u^2-13u-12$

2 $(3u-1)(u+2)+7u(u+1) = (3u^2+5u-2)+(7u^2+7u) = 10u^2+12u-2$

3 $\dfrac{8x^2y^3-6x^3y}{2x^2y} = \dfrac{8x^2y^3}{2x^2y} - \dfrac{6x^3y}{2x^2y} = 4y^2-3x$

4 $\dfrac{6x^2yz^3-xy^2z}{xyz} = \dfrac{6x^2yz^3}{xyz} - \dfrac{xy^2z}{xyz} = 6xz^2-y$

5 We recognize this product as the difference of two squares.

$$(2x+7y)(2x-7y) = (2x)^2-(7y)^2 = 4x^2-49y^2$$

6 $(5x+3y)(5x-3y) = (5x)^2-(3y)^2 = 25x^2-9y^2$

7 $(3x+2y)^2 = (3x)^2+2(3x)(2y)+(2y)^2 = 9x^2+12xy+4y^2$

8 $(5x-4y)^2 = (5x)^2-2(5x)(4y)+(4y)^2 = 25x^2-40xy+16y^2$

9 $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2-(\sqrt{y})^2 = x-y$

10 $(\sqrt{x}+\sqrt{y})^2(\sqrt{x}-\sqrt{y})^2 = [(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})]^2 = (x-y)^2 = x^2-2xy+y^2$

11 Use Product Formula (3) on page 30 of the text.

$$(x-2y)^3 = (x)^3-3(x)^2(2y)+3(x)(2y)^2-(2y)^3 = x^3-6x^2y+12xy^2-8y^3$$

12 $(x+3y)^3 = (x)^3+3(x)^2(3y)+3(x)(3y)^2+(3y)^3 = x^3+9x^2y+27xy^2+27y^3$

13 We recognize $8x^2-17x-21$ as a trinomial that may be able to be factored into the product of two binomials. Using trial and error, we obtain $8x^2-17x-21 = (8x+7)(x-3)$. If you are interested in a sure-fire method for factoring trinomials, see Example 6 on page 48 of the text.

14 Using trial and error, we obtain $7x^2+10x-8 = (7x-4)(x+2)$.

15 The factors for x^2+4x+5 would have to be of the form $(x+\_\_)$ and $(x+\_\_)$.
The factors of 5 are 1 and 5, but their sum is 6 (*not* 4). Thus, x^2+4x+5 is irreducible.

16 $3x^2-4x+2$ is irreducible.

17 $36x^2-60x+25 = (6x-5)(6x-5) = (6x-5)^2$

18 $9x^2+24x+16 = (3x+4)(3x+4) = (3x+4)^2$

19 $x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x^2 - 2^2) = x^2(x+2)(x-2)$

20 $x^3 - 16x = x(x^2 - 16) = x(x^2 - 4^2) = x(x+4)(x-4)$

21 We recognize $8x^3 - y^6$ as the difference of two cubes.

$$\begin{aligned} 8x^3 - y^6 = (2x)^3 - \left(y^2\right)^3 &= \left(2x - y^2\right)\left[(2x)^2 + (2x)\left(y^2\right) + \left(y^2\right)^2\right] \\ &= \left(2x - y^2\right)\left(4x^2 + 2xy^2 + y^4\right) \end{aligned}$$

22 $x^6 - 27y^3 = \left(x^2\right)^3 - (3y)^3 = \left(x^2 - 3y\right)\left[\left(x^2\right)^2 + \left(x^2\right)(3y) + (3y)^2\right] = \left(x^2 - 3y\right)\left(x^4 + 3x^2y + 9y^2\right)$

23 We recognize $343x^3 + y^9$ as the sum of two cubes.

$$\begin{aligned} 343x^3 + y^9 = (7x)^3 + \left(y^3\right)^3 &= \left(7x + y^3\right)\left[(7x)^2 - (7x)\left(y^3\right) + \left(y^3\right)^2\right] \\ &= \left(7x + y^3\right)\left(49x^2 - 7xy^3 + y^6\right) \end{aligned}$$

24 $x^3 + 64 = (x)^3 + (4)^3 = (x+4)\left[(x)^2 - (x)(4) + (4)^2\right] = (x+4)(x^2 - 4x + 16)$

25
$$\begin{aligned} 3x^3 + 3x^2 - 27x - 27 &= 3\left(x^3 + x^2 - 9x - 9\right) && \{\text{gcf} = 3\} \\ &= 3\left[x^2(x+1) - 9(x+1)\right] && \{\text{factor by grouping}\} \\ &= 3\left(x^2 - 9\right)(x+1) && \{\text{factor out } (x+1)\} \\ &= 3(x+3)(x-3)(x+1) && \{\text{difference of two squares}\} \end{aligned}$$

26
$$\begin{aligned} 5x^3 + 10x^2 - 20x - 40 &= 5\left(x^3 + 2x^2 - 4x - 8\right) = 5\left[x^2(x+2) - 4(x+2)\right] \\ &= 5\left(x^2 - 4\right)(x+2) = 5(x+2)(x-2)(x+2) = 5(x+2)^2(x-2) \end{aligned}$$

27 We could treat $a^6 - b^6$ as the difference of two squares or the difference of two cubes. Factoring $a^6 - b^6$ as the difference of two squares and then factoring as the sum and difference of two cubes leads to the following:

$$\begin{aligned} a^6 - b^6 = \left(a^3\right)^2 - \left(b^3\right)^2 &= \left(a^3 + b^3\right)\left(a^3 - b^3\right) \\ &= (a+b)(a-b)\left(a^2 - ab + b^2\right)\left(a^2 + ab + b^2\right) \end{aligned}$$

28 $x^8 - 16 = \left(x^4\right)^2 - 4^2 = \left(x^4 + 4\right)\left(x^4 - 4\right) = \left(x^4 + 4\right)(x^2 + 2)(x^2 - 2)$

29 We might first try to factor $x^2 + 4x + 4 - 9y^2$ by grouping since it has more than 3 terms, but this would prove to be unsuccessful. Instead, we will group the terms containing x and the constant term together, and then proceed as in Example 2(c).

$$x^2 + 4x + 4 - 9y^2 = (x+2)^2 - (3y)^2 = (x + 2 + 3y)(x + 2 - 3y)$$

30 $x^2 - 4y^2 - 6x + 9 = (x^2 - 6x + 9) - 4y^2 = (x-3)^2 - (2y)^2 = (x - 3 + 2y)(x - 3 - 2y)$

31 $\dfrac{y^2 - 25}{y^3 - 125} = \dfrac{(y+5)(y-5)}{(y-5)(y^2 + 5y + 25)} = \dfrac{y+5}{y^2 + 5y + 25}$

32 $\dfrac{12 + r - r^2}{r^3 + 3r^2} = \dfrac{(3+r)(4-r)}{r^2(r+3)} = \dfrac{4-r}{r^2}$

33
$$\begin{aligned} \frac{9x^2 - 4}{3x^2 - 5x + 2} \cdot \frac{9x^4 - 6x^3 + 4x^2}{27x^4 + 8x} &= \frac{(3x+2)(3x-2)}{(3x-2)(x-1)} \cdot \frac{x^2(9x^2 - 6x + 4)}{x(27x^3 + 8)} \\ &= \frac{(3x+2)(3x-2)x^2(9x^2 - 6x + 4)}{(3x-2)(x-1)x(3x+2)(9x^2 - 6x + 4)} = \frac{x}{x-1} \end{aligned}$$

34 $\dfrac{5a^2 + 12a + 4}{a^4 - 16} \div \dfrac{25a^2 + 20a + 4}{a^2 - 2a} = \dfrac{(5a+2)(a+2)}{(a^2+4)(a+2)(a-2)} \cdot \dfrac{a(a-2)}{(5a+2)(5a+2)} = \dfrac{a}{(a^2+4)(5a+2)}$

35 $\dfrac{4}{3s+1}-\dfrac{11}{(3s+1)^2}=\dfrac{4(3s+1)}{(3s+1)^2}-\dfrac{11}{(3s+1)^2}=\dfrac{12s+4-11}{(3s+1)^2}=\dfrac{12s-7}{(3s+1)^2}$

36 $\dfrac{4}{(5s-2)^2}+\dfrac{s}{5s-2}=\dfrac{4}{(5s-2)^2}+\dfrac{s(5s-2)}{(5s-2)^2}=\dfrac{4+5s^2-2s}{(5s-2)^2}=\dfrac{5s^2-2s+4}{(5s-2)^2}$

37 $\dfrac{2}{x}+\dfrac{3x+1}{x^2}-\dfrac{x-2}{x^3}=\dfrac{2x^2}{x^3}+\dfrac{(3x+1)x}{x^3}-\dfrac{(x-2)}{x^3}=\dfrac{2x^2+3x^2+x-x+2}{x^3}=\dfrac{5x^2+2}{x^3}$

38 $\dfrac{5}{x}-\dfrac{2x-1}{x^2}+\dfrac{x+7}{x^3}=\dfrac{5x^2}{x^3}-\dfrac{(2x-1)x}{x^3}+\dfrac{x+7}{x^3}=\dfrac{5x^2-2x^2+x+x+7}{x^3}=\dfrac{3x^2+2x+7}{x^3}$

39 $\dfrac{3t}{t+2}+\dfrac{5t}{t-2}-\dfrac{40}{t^2-4}=\dfrac{3t}{t+2}+\dfrac{5t}{t-2}-\dfrac{40}{(t+2)(t-2)}$

$=\dfrac{3t(t-2)}{(t+2)(t-2)}+\dfrac{5t(t+2)}{(t+2)(t-2)}-\dfrac{40}{(t+2)(t-2)}$

$=\dfrac{3t^2-6t+5t^2+10t-40}{(t+2)(t-2)}$

$=\dfrac{8t^2+4t-40}{(t+2)(t-2)}=\dfrac{4(2t^2+t-10)}{(t+2)(t-2)}=\dfrac{4(2t+5)(t-2)}{(t+2)(t-2)}=\dfrac{4(2t+5)}{t+2}$

40 $\dfrac{t}{t+3}+\dfrac{4t}{t-3}-\dfrac{18}{t^2-9}=\dfrac{t(t-3)+4t(t+3)-18}{t^2-9}=\dfrac{5t^2+9t-18}{t^2-9}=\dfrac{(5t-6)(t+3)}{(t+3)(t-3)}=\dfrac{5t-6}{t-3}$

41 $\dfrac{4x}{3x-4}+\dfrac{8}{3x^2-4x}+\dfrac{2}{x}=\dfrac{4x(x)+8+2(3x-4)}{x(3x-4)}=\dfrac{4x^2+6x}{x(3x-4)}=\dfrac{2x(2x+3)}{x(3x-4)}=\dfrac{2(2x+3)}{3x-4}$

42 $\dfrac{12x}{2x+1}-\dfrac{3}{2x^2+x}+\dfrac{5}{x}=\dfrac{12x(x)-3+5(2x+1)}{x(2x+1)}=\dfrac{12x^2+10x+2}{x(2x+1)}=\dfrac{2(6x^2+5x+1)}{x(2x+1)}$

$=\dfrac{2(2x+1)(3x+1)}{x(2x+1)}=\dfrac{2(3x+1)}{x}$

43 $\dfrac{2x}{x+2}-\dfrac{8}{x^2+2x}+\dfrac{3}{x}=\dfrac{2x(x)-8+3(x+2)}{x(x+2)}=\dfrac{2x^2+3x-2}{x(x+2)}=\dfrac{(2x-1)(x+2)}{x(x+2)}=\dfrac{2x-1}{x}$

44 $\dfrac{5x}{2x+3}-\dfrac{6}{2x^2+3x}+\dfrac{2}{x}=\dfrac{5x(x)-6+2(2x+3)}{x(2x+3)}=\dfrac{5x^2+4x}{x(2x+3)}=\dfrac{x(5x+4)}{x(2x+3)}=\dfrac{5x+4}{2x+3}$

45 $3+\dfrac{5}{u}+\dfrac{2u}{3u+1}=\dfrac{3u(3u+1)+5(3u+1)+2u(u)}{u(3u+1)}$ {common denominator}

$=\dfrac{9u^2+3u+15u+5+2u^2}{u(3u+1)}$ {multiply terms}

$=\dfrac{11u^2+18u+5}{u(3u+1)}$ {add like terms}

46 $6+\dfrac{2}{u}-\dfrac{3u}{u+5}=\dfrac{6u(u+5)+2(u+5)-3u(u)}{u(u+5)}=\dfrac{3u^2+32u+10}{u(u+5)}$

47 $\dfrac{2x+1}{x^2+4x+4}-\dfrac{6x}{x^2-4}+\dfrac{3}{x-2}=\dfrac{2x+1}{(x+2)^2}-\dfrac{6x}{(x+2)(x-2)}+\dfrac{3}{x-2}$

$=\dfrac{(2x+1)(x-2)-6x(x+2)+3(x^2+4x+4)}{(x+2)^2(x-2)}$

$=\dfrac{2x^2-3x-2-6x^2-12x+3x^2+12x+12}{(x+2)^2(x-2)}$

$=\dfrac{-x^2-3x+10}{(x+2)^2(x-2)}=-\dfrac{x^2+3x-10}{(x+2)^2(x-2)}=-\dfrac{(x+5)(x-2)}{(x+2)^2(x-2)}=-\dfrac{x+5}{(x+2)^2}$

48 $\dfrac{4x+12}{x^2+6x+9}+\dfrac{5x}{x^2-9}+\dfrac{7}{x-3}=\dfrac{4}{x+3}+\dfrac{5x}{x^2-9}+\dfrac{7}{x-3}=\dfrac{4(x-3)+5x+7(x+3)}{x^2-9}=\dfrac{16x+9}{x^2-9}$

49 The lcd of the entire expression is ab. Thus, we will multiply both the numerator and denominator by ab.

$$\frac{\dfrac{b}{a}-\dfrac{a}{b}}{\dfrac{1}{a}-\dfrac{1}{b}}=\frac{\left(\dfrac{b}{a}-\dfrac{a}{b}\right)\cdot ab}{\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\cdot ab}=\frac{b^2-a^2}{b-a}=\frac{(b+a)(b-a)}{b-a}=a+b$$

50 The lcd of the entire expression is x^2y^2. Thus, we will multiply both the numerator and denominator by x^2y^2.

$$\frac{\dfrac{x}{y^2}-\dfrac{y}{x^2}}{\dfrac{1}{y^2}-\dfrac{1}{x^2}}=\frac{\left(\dfrac{x}{y^2}-\dfrac{y}{x^2}\right)\cdot x^2y^2}{\left(\dfrac{1}{y^2}-\dfrac{1}{x^2}\right)\cdot x^2y^2}=\frac{x^3-y^3}{x^2-y^2}=\frac{(x-y)(x^2+xy+y^2)}{(x+y)(x-y)}=\frac{x^2+xy+y^2}{x+y}$$

51 The lcd of the entire expression is xy. Thus, we will multiply both the numerator and denominator by xy.

$$\frac{y^{-1}+x^{-1}}{(xy)^{-1}}=\frac{\dfrac{1}{y}+\dfrac{1}{x}}{\dfrac{1}{xy}}=\frac{\left(\dfrac{1}{y}+\dfrac{1}{x}\right)\cdot xy}{\left(\dfrac{1}{xy}\right)\cdot xy}=\frac{x+y}{1}=x+y$$

52 $\dfrac{y^{-2}-x^{-2}}{y^{-2}+x^{-2}}=\dfrac{\dfrac{1}{y^2}-\dfrac{1}{x^2}}{\dfrac{1}{y^2}+\dfrac{1}{x^2}}=\dfrac{\left(\dfrac{1}{y^2}-\dfrac{1}{x^2}\right)\cdot x^2y^2}{\left(\dfrac{1}{y^2}+\dfrac{1}{x^2}\right)\cdot x^2y^2}=\dfrac{x^2-y^2}{x^2+y^2}$

53 $\dfrac{\dfrac{r}{s}+\dfrac{s}{r}}{\dfrac{r^2}{s^2}-\dfrac{s^2}{r^2}}=\dfrac{\left(\dfrac{r}{s}+\dfrac{s}{r}\right)\cdot r^2s^2}{\left(\dfrac{r^2}{s^2}-\dfrac{s^2}{r^2}\right)\cdot r^2s^2}=\dfrac{r^3s+rs^3}{r^4-s^4}=\dfrac{rs(r^2+s^2)}{(r^2+s^2)(r^2-s^2)}=\dfrac{rs}{r^2-s^2}$

54 $\dfrac{\dfrac{2}{w}-\dfrac{4}{2w+1}}{\dfrac{5}{w}+\dfrac{8}{2w+1}}=\dfrac{\dfrac{2(2w+1)-4w}{w(2w+1)}}{\dfrac{5(2w+1)+8w}{w(2w+1)}}=\dfrac{4w+2-4w}{10w+5+8w}=\dfrac{2}{18w+5}$

55
$$\begin{aligned}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}&=\frac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}\\&=\frac{2xh+h^2-3h}{h}=\frac{h(2x+h-3)}{h}=2x+h-3\end{aligned}$$

56
$$\begin{aligned}\frac{(x+h)^3+5(x+h)-(x^3+5x)}{h}&=\frac{x^3+3x^2h+3xh^2+h^3+5x+5h-x^3-5x}{h}\\&=\frac{3x^2h+3xh^2+h^3+5h}{h}\\&=\frac{h(3x^2+3xh+h^2+5)}{h}=3x^2+3xh+h^2+5\end{aligned}$$

57
$$\begin{aligned}\frac{\dfrac{5}{x-1}-\dfrac{5}{a-1}}{x-a}=\frac{\dfrac{5(a-1)-5(x-1)}{(x-1)(a-1)}}{x-a}=\frac{5a-5x}{(x-1)(a-1)(x-a)}&=\frac{5(a-x)}{(x-1)(a-1)(x-a)}\\&=-\frac{5}{(x-1)(a-1)}\end{aligned}$$

58 $$\dfrac{\dfrac{x+2}{x}-\dfrac{a+2}{a}}{x-a}=\dfrac{\dfrac{a(x+2)-x(a+2)}{ax}}{x-a}=\dfrac{2a-2x}{ax(x-a)}=\dfrac{2(a-x)}{ax(x-a)}=-\dfrac{2}{ax}$$

59 $$\dfrac{\dfrac{1}{(x+h)^3}-\dfrac{1}{x^3}}{h}=\dfrac{\dfrac{x^3-(x+h)^3}{(x+h)^3x^3}}{h}$$
$$=\dfrac{x^3-(x+h)^3}{hx^3(x+h)^3}=\dfrac{[x-(x+h)]\left[x^2+x(x+h)+(x+h)^2\right]}{hx^3(x+h)^3}\ \{\text{difference of two cubes}\}$$
$$=\dfrac{-h[x^2+x^2+xh+x^2+2xh+h^2]}{hx^3(x+h)^3}=\dfrac{-h(3x^2+3xh+h^2)}{hx^3(x+h)^3}=-\dfrac{3x^2+3xh+h^2}{x^3(x+h)^3}$$

60 $$\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}=\dfrac{\dfrac{x-(x+h)}{(x+h)x}}{h}=\dfrac{-h}{hx(x+h)}=\dfrac{-1}{x(x+h)}$$

61 The conjugate of $\sqrt{t}-5$ is $\sqrt{t}+5$. Multiply the numerator and the denominator by the conjugate of the denominator. This will eliminate the radical in the denominator.

$$\dfrac{\sqrt{t}+5}{\sqrt{t}-5}=\dfrac{\sqrt{t}+5}{\sqrt{t}-5}\cdot\dfrac{\sqrt{t}+5}{\sqrt{t}+5}=\dfrac{\left(\sqrt{t}\right)^2+2\cdot5\sqrt{t}+5^2}{\left(\sqrt{t}\right)^2-5^2}=\dfrac{t+10\sqrt{t}+25}{t-25}$$

62 $$\dfrac{16x^2-y^2}{2\sqrt{x}-\sqrt{y}}=\dfrac{16x^2-y^2}{2\sqrt{x}-\sqrt{y}}\cdot\dfrac{2\sqrt{x}+\sqrt{y}}{2\sqrt{x}+\sqrt{y}}=\dfrac{(4x+y)(4x-y)\left(2\sqrt{x}+\sqrt{y}\right)}{4x-y}=(4x+y)\left(2\sqrt{x}+\sqrt{y}\right)$$

63 We must recognize $\sqrt[3]{a}-\sqrt[3]{b}$ as the first factor of the product formula for the difference of two cubes, $x^3-y^3=(x-y)(x^2+xy+y^2)$. The second factor is then

$$\left(\sqrt[3]{a}\right)^2+\left(\sqrt[3]{a}\right)\left(\sqrt[3]{b}\right)+\left(\sqrt[3]{b}\right)^2=\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}.$$

$$\dfrac{1}{\sqrt[3]{a}-\sqrt[3]{b}}=\dfrac{1}{\sqrt[3]{a}-\sqrt[3]{b}}\cdot\dfrac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}=\dfrac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{a-b}$$

64 $$\dfrac{1}{\sqrt[3]{x}+\sqrt[3]{y}}=\dfrac{1}{\sqrt[3]{x}+\sqrt[3]{y}}\cdot\dfrac{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}=\dfrac{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}{x+y}$$

65 $$\dfrac{\sqrt{a}-\sqrt{b}}{a^2-b^2}=\dfrac{\sqrt{a}-\sqrt{b}}{a^2-b^2}\cdot\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\dfrac{a-b}{(a+b)(a-b)\left(\sqrt{a}+\sqrt{b}\right)}=\dfrac{1}{(a+b)\left(\sqrt{a}+\sqrt{b}\right)}$$

66 $$\dfrac{\sqrt{b}+\sqrt{c}}{b^2-c^2}=\dfrac{\sqrt{b}+\sqrt{c}}{b^2-c^2}\cdot\dfrac{\sqrt{b}-\sqrt{c}}{\sqrt{b}-\sqrt{c}}=\dfrac{b-c}{(b+c)(b-c)\left(\sqrt{b}-\sqrt{c}\right)}=\dfrac{1}{(b+c)\left(\sqrt{b}-\sqrt{c}\right)}$$

67 $$\dfrac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}=\dfrac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\dfrac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$
$$=\dfrac{(2x+2h+1)-(2x+1)}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$$
$$=\dfrac{2h}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}=\dfrac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

68 $\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} = \dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \cdot \dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} = \dfrac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

$= \dfrac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)} = \dfrac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

69 $\dfrac{3x^2-x+7}{x^{2/3}} = \dfrac{3x^2}{x^{2/3}} - \dfrac{x}{x^{2/3}} + \dfrac{7}{x^{2/3}} = 3x^{4/3} - x^{1/3} + 7x^{-2/3}$

70 $\dfrac{x^2+4x-6}{\sqrt{x}} = \dfrac{x^2}{\sqrt{x}} + \dfrac{4x}{\sqrt{x}} - \dfrac{6}{\sqrt{x}} = x^{3/2} + 4x^{1/2} - 6x^{-1/2}$

71 $\dfrac{(x^2+2)^2}{x^5} = \dfrac{x^4+4x^2+4}{x^5} = \dfrac{x^4}{x^5} + \dfrac{4x^2}{x^5} + \dfrac{4}{x^5} = x^{-1} + 4x^{-3} + 4x^{-5}$

72 $\dfrac{\left(\sqrt{x}-3\right)^2}{x^3} = \dfrac{x-6\sqrt{x}+9}{x^3} = \dfrac{x}{x^3} - \dfrac{6\sqrt{x}}{x^3} + \dfrac{9}{x^3} = x^{-2} - 6x^{-5/2} + 9x^{-3}$

Note: You may wish to demonstrate the three techniques shown in Example 7 with one of these simpler expressions in 73–76.

Note: Exercises 73–90 are worked using the factoring concept given as the third method of simplification in Example 7.

73 The smallest exponent that appears on the variable x is -3.

$$x^{-3} + x^2 \text{ \{factor out } x^{-3}\text{\}} = x^{-3}\left(1 + x^{2-(-3)}\right) = x^{-3}\left(1+x^5\right) = \frac{1+x^5}{x^3}$$

74 $x^{-5} - x$ {factor out x^{-5}} $= x^{-5}\left(1 - x^{1-(-5)}\right) = x^{-5}(1-x^6) = \dfrac{1-x^6}{x^5}$

75 $x^{-1/2} - x^{3/2}$ {factor out $x^{-1/2}$} $= x^{-1/2}\left(1 - x^{3/2-(-1/2)}\right) = x^{-1/2}(1-x^2) = \dfrac{1-x^2}{x^{1/2}}$

76 $x^{-2/3} + x^{7/3}$ {factor out $x^{-2/3}$} $= x^{-2/3}\left(1 + x^{7/3-(-2/3)}\right) = x^{-2/3}(1+x^3) = \dfrac{1+x^3}{x^{2/3}}$

77 $\left(2x^2-3x+1\right)(4)(3x+2)^3(3) + (3x+2)^4(4x-3)$

$= (3x+2)^3\left[12\left(2x^2-3x+1\right) + (3x+2)(4x-3)\right]$ $\left\{\text{factor out the gcf of } (3x+2)^3\right\}$

$= (3x+2)^3\left(24x^2-36x+12+12x^2-x-6\right)$

$= (3x+2)^3\left(36x^2-37x+6\right)$

78 $(6x-5)^3(2)\left(x^2+4\right)(2x) + \left(x^2+4\right)^2(3)(6x-5)^2(6) = 2(6x-5)^2\left(x^2+4\right)\left[2x(6x-5) + 9\left(x^2+4\right)\right]$

$= 2(6x-5)^2\left(x^2+4\right)\left(12x^2-10x+9x^2+36\right)$

$= 2\left(x^2+4\right)(6x-5)^2\left(21x^2-10x+36\right)$

79 The smallest exponent that appears on the factor (x^2-4) is $-\frac{1}{2}$ and the smallest exponent that appears on the factor $(2x+1)$ is 2. Thus, we will factor out $(x^2-4)^{-1/2}(2x+1)^2$.

$(x^2-4)^{1/2}(3)(2x+1)^2(2) + (2x+1)^3\left(\frac{1}{2}\right)(x^2-4)^{-1/2}(2x) = (x^2-4)^{-1/2}(2x+1)^2[6(x^2-4) + x(2x+1)]$

If you are unsure of this factoring, it is easy to visually check at this stage by merely multiplying the expression—that is, we mentally add the exponents on the factor (x^2-4), $-\frac{1}{2}$ and 1, and we get $\frac{1}{2}$, which is the exponent we started with.

Proceeding: $\left(x^2-4\right)^{-1/2}(2x+1)^2\left[6\left(x^2-4\right) + x(2x+1)\right] = \left(x^2-4\right)^{-1/2}(2x+1)^2\left(6x^2-24+2x^2+x\right)$

$= \dfrac{(2x+1)^2(8x^2+x-24)}{(x^2-4)^{1/2}}$

80 $(3x+2)^{1/3}(2)(4x-5)(4)+(4x-5)^2\left(\frac{1}{3}\right)(3x+2)^{-2/3}(3)=(3x+2)^{-2/3}(4x-5)[8(3x+2)+(4x-5)]$

$$=\frac{(4x-5)(28x+11)}{(3x+2)^{2/3}}$$

81 $(3x+1)^6\left(\frac{1}{2}\right)(2x-5)^{-1/2}(2)+(2x-5)^{1/2}(6)(3x+1)^5(3)$

$=(3x+1)^5(2x-5)^{-1/2}[(3x+1)+18(2x-5)]$ $\left\{\text{factor out } (3x+1)^5(2x-5)^{-1/2}\right\}$

$$=\frac{(3x+1)^5(3x+1+36x-90)}{(2x-5)^{1/2}}=\frac{(3x+1)^5(39x-89)}{(2x-5)^{1/2}}$$

82 $\left(x^2+9\right)^4\left(-\frac{1}{3}\right)(x+6)^{-4/3}+(x+6)^{-1/3}(4)\left(x^2+9\right)^3(2x)$

$$=\left(\tfrac{1}{3}\right)\left(x^2+9\right)^3(x+6)^{-4/3}\left[-\left(x^2+9\right)+24x(x+6)\right]=\frac{\left(x^2+9\right)^3(23x^2+144x-9)}{3(x+6)^{4/3}}$$

83 $$\frac{(6x+1)^3(27x^2+2)-(9x^3+2x)(3)(6x+1)^2(6)}{(6x+1)^6}=\frac{(6x+1)^2[(6x+1)(27x^2+2)-18(9x^3+2x)]}{(6x+1)^6}$$

$$=\frac{(6x+1)^2(162x^3+27x^2+12x+2-162x^3-36x)}{(6x+1)^6}$$

$$=\frac{27x^2-24x+2}{(6x+1)^4}$$

84 $$\frac{(x^2-1)^4(2x)-x^2(4)(x^2-1)^3(2x)}{(x^2-1)^8}=\frac{(2x)(x^2-1)^3[(x^2-1)-4x^2]}{(x^2-1)^8}=\frac{2x(-3x^2-1)}{(x^2-1)^5}=\frac{-2x(3x^2+1)}{(x^2-1)^5}$$

85 $$\frac{(x^2+2)^3(2x)-x^2(3)(x^2+2)^2(2x)}{\left[(x^2+2)^3\right]^2}=\frac{(x^2+2)^2(2x)\left[(x^2+2)^1-x^2(3)\right]}{(x^2+2)^6}=$$

$$\frac{2x(x^2+2-3x^2)}{(x^2+2)^4}=\frac{2x(2-2x^2)}{(x^2+2)^4}=\frac{4x(1-x^2)}{(x^2+2)^4}$$

86 $$\frac{(x^2-5)^4(3x^2)-x^3(4)(x^2-5)^3(2x)}{\left[(x^2-5)^4\right]^2}=\frac{(x^2-5)^3(x^2)\left[(x^2-5)^1(3)-(x)(4)(2x)\right]}{(x^2-5)^8}=$$

$$\frac{x^2(3x^2-15-8x^2)}{(x^2-5)^5}=\frac{x^2(-5x^2-15)}{(x^2-5)^5}=-\frac{5x^2(x^2+3)}{(x^2-5)^5}$$

87 $$\frac{(x^2+4)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)(x^2+4)^{-2/3}(2x)}{\left[(x^2+4)^{1/3}\right]^2}=\frac{(x^2+4)^{-2/3}[3(x^2+4)-2x^2]}{(x^2+4)^{2/3}}=\frac{3x^2+12-2x^2}{(x^2+4)^{4/3}}=\frac{x^2+12}{(x^2+4)^{4/3}}$$

88 $$\frac{(1-x^2)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x)}{\left[(1-x^2)^{1/2}\right]^2}=\frac{x(1-x^2)^{-1/2}[2(1-x^2)+x^2]}{(1-x^2)^1}=\frac{x(2-2x^2+x^2)}{(1-x^2)^{3/2}}=\frac{x(2-x^2)}{(1-x^2)^{3/2}}$$

89 $$\frac{(4x^2+9)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)(4x^2+9)^{-1/2}(8x)}{\left[(4x^2+9)^{1/2}\right]^2}=\frac{(4x^2+9)^{-1/2}[2(4x^2+9)-4x(2x+3)]}{(4x^2+9)^1}$$

$$=\frac{8x^2+18-8x^2-12x}{(4x^2+9)^{3/2}}=\frac{18-12x}{(4x^2+9)^{3/2}}=\frac{6(3-2x)}{(4x^2+9)^{3/2}}$$

90 $\dfrac{(3x+2)^{1/2}\left(\frac{1}{3}\right)(2x+3)^{-2/3}(2)-(2x+3)^{1/3}\left(\frac{1}{2}\right)(3x+2)^{-1/2}(3)}{\left[(3x+2)^{1/2}\right]^2}$

$$= \frac{\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)(3x+2)^{-1/2}(2x+3)^{-2/3}[4(3x+2)-9(2x+3)]}{(3x+2)^1}$$

$$= \frac{\left(\frac{1}{6}\right)(12x+8-18x-27)}{(3x+2)^{3/2}(2x+3)^{2/3}} = \frac{\left(\frac{1}{6}\right)(-6x-19)}{(3x+2)^{3/2}(2x+3)^{2/3}} = -\frac{6x+19}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

91 Table $Y_1 = \dfrac{113x^3+280x^2-150x}{22x^3+77x^2-100x-350}$ and $Y_2 = \dfrac{3x}{2x+7} + \dfrac{4x^2}{1.1x^2-5}$.

x	Y_1	Y_2
1	−0.6923	−0.6923
2	−26.12	−26.12
3	8.0392	8.0392
4	5.8794	5.8794
5	5.3268	5.3268

The values for Y_1 and Y_2 agree. Therefore, the two expressions might be equal.

92 Table $Y_1 = \dfrac{20x^2+41x+31}{10x^3+10x^2}$ and $Y_2 = \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{3.2}{x^2}$.

x	Y_1	Y_2
1	4.6	4.7
2	1.6083	1.6333
3	0.92778	0.93889
4	0.64375	0.65
5	0.49067	0.49467

The values for Y_1 and Y_2 do not agree. Therefore, the two expressions are not equal.

93 In the second figure, the dimensions of area I are (x) and $(x-y)$. The area of I is $(x-y)x$, and the area of II is $(x-y)y$. The area $A = \underline{x^2-y^2}$ {in the first figure}

$= (x-y)x+(x-y)y$ {in the second figure}

$= \underline{(x-y)(x+y)}.$ {in the third figure}

94 Volume of I is $x^2(x-y)$, volume of II is $xy(x-y)$, and volume of III is $y^2(x-y)$.

$$V = \underline{x^3-y^3} = x^2(x-y)+xy(x-y)+y^2(x-y) = \underline{(x-y)(x^2+xy+y^2)}.$$

95 (a) For the 25-year-old female, use

$C_f = 66.5 + 13.8w + 5h - 6.8y$ with $w = 59$, $h = 163$, and $y = 25$.

$C_f = 66.5 + 13.8(59) + 5(163) - 6.8(25) = 1525.7$ calories

For the 55-year-old male, use

$C_m = 655 + 9.6w + 1.9h - 4.7y$ with $w = 75$, $h = 178$, and $y = 55$.

$C_m = 655 + 9.6(75) + 1.9(178) - 4.7(55) = 1454.7$ calories

(b) As people age they require fewer calories. The coefficients of w and h are positive because large people require more calories.

1.4 Exercises

1 $4x-3=-5x+6 \Rightarrow 4x+5x=6+3 \Rightarrow 9x=9 \Rightarrow x=1$

2 $5x-4=2(x-2) \Rightarrow 5x-4=2x-4 \Rightarrow 3x=0 \Rightarrow x=0$

3 $(3x-2)^2=(x-5)(9x+4) \Rightarrow 9x^2-12x+4=9x^2-41x-20 \Rightarrow 29x=-24 \Rightarrow x=-\frac{24}{29}$

4 $(x+5)^2+3=(x-2)^2 \Rightarrow x^2+10x+25+3=x^2-4x+4 \Rightarrow 14x=-24 \Rightarrow x=-\frac{12}{7}$

5 $\left[\frac{3x+1}{6x-2}=\frac{2x+5}{4x-13}\right]\cdot(6x-2)(4x-13) \Rightarrow (3x+1)(4x-13)=(2x+5)(6x-2) \Rightarrow$

$12x^2-35x-13=12x^2+26x-10 \Rightarrow -3=61x \Rightarrow x=-\frac{3}{61}$ $\left\{\text{note that } x\neq\frac{1}{3},\frac{13}{4}\right\}$

6 $\left[\frac{7x+2}{14x-3}=\frac{x-8}{2x+3}\right]\cdot(14x-3)(2x+3) \Rightarrow (7x+2)(2x+3)=(x-8)(14x-3) \Rightarrow$

$14x^2+25x+6=14x^2-115x+24 \Rightarrow 140x=18 \Rightarrow x=\frac{9}{70}$

7 $\left[\frac{4}{x+2}+\frac{1}{x-2}=\frac{5x-6}{x^2-4}\right]\cdot(x+2)(x-2) \Rightarrow 4(x-2)+1(x+2)=5x-6 \Rightarrow$

$4x-8+x+2=5x-6 \Rightarrow 5x-6=5x-6$ {or $0=0$}, indicating an identity. The solution is $\mathbb{R}-\{\pm 2\}$.

8 $\left[\frac{2}{2x+5}+\frac{3}{2x-5}=\frac{10x+5}{4x^2-25}\right]\cdot(2x+5)(2x-5) \Rightarrow 2(2x-5)+3(2x+5)=10x+5 \Rightarrow$

$4x-10+6x+15=10x+5 \Rightarrow 10x+5=10x+5$ {or $0=0$}, indicating an identity.

The solution is $\mathbb{R}-\left\{\pm\frac{5}{2}\right\}$.

9 $\left[\frac{5}{2x+3}+\frac{4}{2x-3}=\frac{14x+3}{4x^2-9}\right]\cdot(2x+3)(2x-3) \Rightarrow 5(2x-3)+4(2x+3)=14x+3 \Rightarrow$

$10x-15+8x+12=14x+3 \Rightarrow 18x-3=14x+3 \Rightarrow 4x=6 \Rightarrow$

$x=\frac{3}{2}$, which is not in the domain of the given expressions. **No solution**

10 $\left[\frac{-3}{x+4}+\frac{7}{x-4}=\frac{-5x+4}{x^2-16}\right]\cdot(x+4)(x-4) \Rightarrow -3(x-4)+7(x+4)=-5x+4 \Rightarrow$

$-3x+12+7x+28=-5x+4 \Rightarrow 4x+40=-5x+4 \Rightarrow 9x=-36 \Rightarrow$

$x=-4$, which is not in the domain of the given expressions. **No solution**

11 Divide both sides by a nonzero constant whenever possible. In this case, 5 divides evenly into both sides.

$75x^2+35x-10=0$ {divide by 5} $\Rightarrow 15x^2+7x-2=0$ {factor} $\Rightarrow$

$(3x+2)(5x-1)=0$ {zero factor theorem} $\Rightarrow x=-\frac{2}{3},\frac{1}{5}$

12 $48x^2+12x-90=0$ {divide by 6} $\Rightarrow 8x^2+2x-15=0 \Rightarrow (2x+3)(4x-5)=0 \Rightarrow x=-\frac{3}{2},\frac{5}{4}$

13 Here, the lcd is $x(x+3)$ and we need to remember that $x\neq 0,-3$.

$\left[\frac{x}{x+3}+\frac{1}{x}-4=\frac{9}{x^2+3x}\right]\cdot x(x+3) \Rightarrow x(x)+1(x+3)-4(x^2+3x)=9 \Rightarrow$

$x^2+x+3-4x^2-12x=9 \Rightarrow 0=3x^2+11x+6 \Rightarrow (3x+2)(x+3)=0 \Rightarrow$

$x=-\frac{2}{3}$ {-3 is not in the domain of the given expressions}

14 $\left[\frac{3x}{x-2}+\frac{1}{x+2}=\frac{-4}{x^2-4}\right]\cdot(x+2)(x-2) \Rightarrow 3x(x+2)+1(x-2)=-4 \Rightarrow$

$3x^2+6x+x-2=-4 \Rightarrow 3x^2+7x+2=0 \Rightarrow (3x+1)(x+2)=0 \Rightarrow$

$x=-\frac{1}{3}$ {-2 is not in the domain of the given expressions}

15 $25x^2=9 \Rightarrow x^2=\frac{9}{25} \Rightarrow x=\pm\sqrt{\frac{9}{25}} \Rightarrow x=\pm\frac{3}{5}$

16 $64x^2=49 \Rightarrow x^2=\frac{49}{64} \Rightarrow x=\pm\sqrt{\frac{49}{64}} \Rightarrow x=\pm\frac{7}{8}$

17 $(x-3)^2=17 \Rightarrow x-3=\pm\sqrt{17} \Rightarrow x=3\pm\sqrt{17}$

18 $(x+5)^2=29 \Rightarrow x+5=\pm\sqrt{29} \Rightarrow x=-5\pm\sqrt{29}$

19 $x^2+6x+3=0 \Rightarrow x=\frac{-6\pm\sqrt{36-12}}{2(1)}=\frac{-6\pm\sqrt{24}}{2}=\frac{-6\pm2\sqrt{6}}{2}=-3\pm\sqrt{6}$

20 $x^2-4x-2=0 \Rightarrow x=\frac{4\pm\sqrt{16+8}}{2(1)}=\frac{4\pm\sqrt{24}}{2}=\frac{4\pm2\sqrt{6}}{2}=2\pm\sqrt{6}$

21 The *expression* is x^2+x-30. The associated quadratic *equation* is $x^2+x-30=0$.

Using the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, to solve for x with $a=1$, $b=1$, and $c=-30$ gives us:

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-30)}}{2(1)}=\frac{-1\pm\sqrt{1+120}}{2}=\frac{-1\pm\sqrt{121}}{2}=\frac{-1\pm11}{2}=\frac{10}{2},\frac{-12}{2}=5,-6$$

Write the equation as a product of linear factors: $[x-(5)]\,[x-(-6)]=0$

Now simplify: $(x-5)(x+6)=0$

So the final factored form of x^2+x-30 is $(x-5)(x+6)$.

22 $x^2-11x=0$ $\{a=1, b=-11, c=0\}$, so $x=\frac{11\pm\sqrt{121-0}}{2}=\frac{11\pm11}{2}=11,0.$

Thus, $x^2-11x=(x-11)(x-0)=x(x-11)$.

23 $12x^2-16x-3=0$ $\{a=12, b=-16, c=-3\}$, so $x=\frac{16\pm\sqrt{256+144}}{24}=\frac{16\pm20}{24}=\frac{3}{2},-\frac{1}{6}.$

Write the equation as a product of linear factors: $\left[x-\left(\frac{3}{2}\right)\right]\left[x-\left(-\frac{1}{6}\right)\right]=0$

Now multiply the first factor by 2 and the second factor by 6. $(2x-3)(6x+1)=0$

So the final factored form of $12x^2-16x-3$ is $(2x-3)(6x+1)$.

24 $15x^2+34x-16=0$ $\{a=15, b=34, c=-16\}$, so $x=\frac{-34\pm\sqrt{1156+960}}{30}=\frac{-34\pm46}{30}=\frac{2}{5},-\frac{8}{3}.$

Thus, $15x^2+34x-16=5\left[x-\left(\frac{2}{5}\right)\right]\cdot3\left[x-\left(-\frac{8}{3}\right)\right]=(5x-2)(3x+8)$.

25 We must first isolate the absolute value term before proceeding.

$|3x-2|+3=7 \Rightarrow |3x-2|=4 \Rightarrow 3x-2=4$ or $3x-2=-4 \Rightarrow$

$3x=6$ or $3x=-2 \Rightarrow x=2$ or $x=-\frac{2}{3}$

26 $2|5x+2|-1=5 \Rightarrow 2|5x+2|=6 \Rightarrow |5x+2|=3 \Rightarrow$

$5x+2=3$ or $5x+2=-3 \Rightarrow 5x=1$ or $5x=-5 \Rightarrow x=\frac{1}{5}$ or $x=-1$

27 $3|x+1|-5=-11 \Rightarrow 3|x+1|=-6 \Rightarrow |x+1|=-2.$

Since the absolute value of an expression is nonnegative, $|x+1|=-2$ has no solution.

28 $|x-3|+6=6 \quad \Rightarrow \quad |x-3|=0$. Since the absolute value of an expression can only equal 0 if the expression itself is 0, $|x-3|=0 \quad \Rightarrow \quad x-3=0 \quad \Rightarrow \quad x=3$.

29 Since there are four terms, we first try factoring by grouping.

$9x^3-18x^2-4x+8=0 \quad \Rightarrow \quad 9x^2(x-2)-4(x-2)=0 \quad \Rightarrow$

$(9x^2-4)(x-2)=0 \quad \Rightarrow \quad x^2=\frac{4}{9}$ or $x=2 \quad \Rightarrow \quad x=\pm\frac{2}{3}, 2$

30 Notice that we can factor an x out of each term, and then factor by grouping.

$4x^4+10x^3=6x^2+15x \quad \Rightarrow \quad 4x^4+10x^3-6x^2-15x=0 \quad \Rightarrow \quad x(4x^3+10x^2-6x-15)=0 \quad \Rightarrow$

$x\left[2x^2(2x+5)-3(2x+5)\right]=0 \quad \Rightarrow \quad x(2x^2-3)(2x+5)=0 \quad \Rightarrow$

$x=0$ or $x^2=\frac{3}{2}$ or $x=-\frac{5}{2} \quad \Rightarrow \quad x=0, \pm\frac{1}{2}\sqrt{6}, -\frac{5}{2}$

Note: $x^2=\frac{3}{2} \quad \Rightarrow \quad x=\pm\sqrt{\frac{3}{2}}=\pm\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\pm\frac{\sqrt{6}}{2}=\pm\frac{1}{2}\sqrt{6}$.

There are several ways to write this answer—your professor may have a preference.

31 $y^{3/2}=5y \quad \Rightarrow \quad y^{3/2}-5y=0 \quad \Rightarrow \quad y\left(y^{1/2}-5\right)=0 \quad \Rightarrow \quad y=0$ or $y^{1/2}=5$.

$y^{1/2}=5 \quad \Rightarrow \quad \left(y^{1/2}\right)^2=5^2 \quad \Rightarrow \quad y=25$. The solutions are $y=0$ and $y=25$.

Note: The following guidelines may be helpful when solving radical equations.

Guidelines for Solving a Radical Equation

(1) Isolate the radical. If we cannot get the radical isolated on one side of the equals sign because there is more than one radical, then we will split up the radical terms as evenly as possible on each side of the equals sign. For example, if there are two radicals, we put one on each side; if there are three radicals, we put two on one side and one on the other.

(2) Raise both sides to the same power as the root index. ***Note:*** Remember here that

$$\boxed{\left(a+b\sqrt{n}\right)^2=a^2+2ab\sqrt{n}+b^2n}$$

and that $\left(a+b\sqrt{n}\right)^2$ is **not** a^2+b^2n.

(3) If your equation contains no radicals, proceed to part (4). If there are still radicals in the equation, go back to part (1).

(4) Solve the resulting equation.

(5) Check the answers found in part (4) in the original equation to determine the valid solutions.
Note: You may check the solutions in any equivalent equation of the original equation, that is, an equation which occurs prior to raising both sides to a power. Also, extraneous real number solutions are introduced when raising both sides to an even power. Hence, all solutions *must* be checked in this case. Checking solutions when raising each side to an odd power is up to the individual professor.

32 $y^{4/3}=-4y \quad \Rightarrow \quad y^{4/3}+4y=0 \quad \Rightarrow \quad y\left(y^{1/3}+4\right)=0 \quad \Rightarrow \quad y=0$ or $y^{1/3}=-4$.

$y^{1/3}=-4 \quad \Rightarrow \quad \left(y^{1/3}\right)^3=(-4)^3 \quad \Rightarrow \quad y=-64$. $y=0, -64$

33 $\sqrt{7-5x}=8 \quad \Rightarrow \quad \left(\sqrt{7-5x}\right)^2=8^2 \quad \Rightarrow \quad 7-5x=64 \quad \Rightarrow \quad -57=5x \quad \Rightarrow \quad x=-\frac{57}{5}$

34 $\sqrt{3-x}-x=3 \quad \Rightarrow \quad \left(\sqrt{3-x}\right)^2=(x+3)^2 \quad \Rightarrow \quad 3-x=x^2+6x+9 \quad \Rightarrow \quad x^2+7x+6=0 \quad \Rightarrow$

$(x+1)(x+6)=0 \quad \Rightarrow \quad x=-1$ and -6 is an extraneous solution.

35 $x = 3 + \sqrt{5x - 9} \Rightarrow x - 3 = \sqrt{5x - 9} \Rightarrow x^2 - 6x + 9 = 5x - 9 \Rightarrow$
$x^2 - 11x + 18 = 0 \Rightarrow (x-2)(x-9) = 0 \Rightarrow x = 9$ and 2 is an extraneous solution.

36 $x + \sqrt{5x + 19} = -1 \Rightarrow \sqrt{5x + 19} = -x - 1 \Rightarrow 5x + 19 = x^2 + 2x + 1 \Rightarrow$
$x^2 - 3x - 18 = 0 \Rightarrow (x-6)(x+3) = 0 \Rightarrow x = -3, 6.$

Check $x = -3$: LS $= -3 + 2 = -1 =$ RS $\Rightarrow x = -3$ is a solution.

Check $x = 6$: LS $= 6 + 7 = 13 \neq$ RS $\Rightarrow x = 6$ is an extraneous solution.

Note: Substitution could be used instead of factoring for the following exercises.

37 We recognize this equation as a quadratic equation in y^2 and apply the quadratic formula, solving for y^2, *not* y.

$5y^4 - 7y^2 + 1.5 = 0 \Rightarrow y^2 = \frac{7 \pm \sqrt{19}}{10} \cdot \frac{10}{10} = \frac{70 \pm 10\sqrt{19}}{100} \Rightarrow y = \pm\frac{1}{10}\sqrt{70 \pm 10\sqrt{19}}$

Alternatively, let $u = y^2$ and solve $5u^2 - 7u + 1.5 = 0$.

38 $3y^4 - 5y^2 + 1.5 = 0 \Rightarrow y^2 = \frac{5 \pm \sqrt{7}}{6} \cdot \frac{6}{6} = \frac{30 \pm 6\sqrt{7}}{36} \Rightarrow y = \pm\frac{1}{6}\sqrt{30 \pm 6\sqrt{7}}$

39 $36x^{-4} - 13x^{-2} + 1 = 0 \Rightarrow (4x^{-2} - 1)(9x^{-2} - 1) = 0 \Rightarrow x^{-2} = \frac{1}{4}, \frac{1}{9} \Rightarrow x^2 = 4, 9 \Rightarrow x = \pm 2, \pm 3$

Alternatively, let $u = x^{-2}$ and solve $36u^2 - 13u + 1 = 0$.

40 $x^{-2} - 2x^{-1} - 35 = 0 \Rightarrow (x^{-1} - 7)(x^{-1} + 5) = 0 \Rightarrow x^{-1} = 7, -5 \Rightarrow x = \frac{1}{7}, -\frac{1}{5}$

41 $3x^{2/3} + 4x^{1/3} - 4 = 0 \Rightarrow \left(3x^{1/3} - 2\right)\left(x^{1/3} + 2\right) = 0 \Rightarrow$
$\sqrt[3]{x} = \frac{2}{3}, -2 \Rightarrow x = \left(\frac{2}{3}\right)^3, (-2)^3 \Rightarrow x = \frac{8}{27}, -8$

Alternatively, let $u = x^{1/3}$ and solve $3u^2 + 4u - 4 = 0$.

42 $2y^{1/3} - 3y^{1/6} + 1 = 0 \Rightarrow \left(2y^{1/6} - 1\right)\left(y^{1/6} - 1\right) = 0 \Rightarrow \sqrt[6]{y} = \frac{1}{2}, 1 \Rightarrow y = \frac{1}{64}, 1$

43 See the illustrations and discussion on text page 51 for help on solving equations by raising both sides to a reciprocal power. Note that if $x^{m/n}$ is in the given equation and m is even, we have to use the $\pm$ symbol for the solutions. Here are a few more examples:

Equation	**Solution**
$x^{1/2} = 4$	$\left(x^{1/2}\right)^{2/1} = 4^{2/1} \Rightarrow x = 16$
$x^{-1/2} = 5$	$\left(x^{-1/2}\right)^{-2/1} = 5^{-2/1} \Rightarrow x = \frac{1}{25}$
$x^{3/4} = 8$	$\left(x^{3/4}\right)^{4/3} = 8^{4/3} \Rightarrow x = 16$
$x^{4/3} = 81$	$\left(x^{4/3}\right)^{3/4} = 81^{3/4} \Rightarrow x = \pm 27$ { $\pm$ since 4 is even}

(a) $x^{5/3} = 32 \Rightarrow \left(x^{5/3}\right)^{3/5} = (32)^{3/5} \Rightarrow x = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

(b) $x^{4/3} = 16 \Rightarrow \left(x^{4/3}\right)^{3/4} = \pm(16)^{3/4} \Rightarrow x = \pm\left(\sqrt[4]{16}\right)^3 = \pm 2^3 = \pm 8$

(c) $x^{2/3} = -64 \Rightarrow \left(x^{2/3}\right)^{3/2} = \pm(-64)^{3/2} \Rightarrow x = \pm\left(\sqrt{-64}\right)^3$, which are not real numbers.

No real solutions

(d) $x^{3/4} = 125 \Rightarrow \left(x^{3/4}\right)^{4/3} = (125)^{4/3} \Rightarrow x = \left(\sqrt[3]{125}\right)^4 = 5^4 = 625$

(e) $x^{3/2} = -27 \Rightarrow \left(x^{3/2}\right)^{2/3} = (-27)^{2/3} \Rightarrow x = \left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9,$

which is an extraneous solution. **No real solutions**

44 (a) $x^{3/5} = -27 \quad \Rightarrow \quad \left(x^{3/5}\right)^{5/3} = (-27)^{5/3} \quad \Rightarrow \quad x = \left(\sqrt[3]{-27}\right)^5 = (-3)^5 = -243$

(b) $x^{2/3} = 25 \quad \Rightarrow \quad \left(x^{2/3}\right)^{3/2} = \pm(25)^{3/2} \quad \Rightarrow \quad x = \pm\left(\sqrt{25}\right)^3 = \pm 5^3 = \pm 125$

(c) $x^{4/3} = -49 \quad \Rightarrow \quad \left(x^{4/3}\right)^{3/4} = \pm(-49)^{3/4} \quad \Rightarrow \quad x = \pm\left(\sqrt[4]{-49}\right)^3$, which are not real numbers.

No real solutions

(d) $x^{3/2} = 64 \quad \Rightarrow \quad \left(x^{3/2}\right)^{2/3} = (64)^{2/3} \quad \Rightarrow \quad x = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

(e) $x^{3/4} = -8 \quad \Rightarrow \quad \left(x^{3/4}\right)^{4/3} = (-8)^{4/3} \quad \Rightarrow \quad x = \left(\sqrt[3]{-8}\right)^4 = (-2)^4 = 16$, which is an extraneous solution.

No real solutions

45 (a) For this exercise, we must recognize the equation as a quadratic in x, that is,

$$Ax^2 + Bx + C = 0$$

where A is the coefficient of x^2, B is the coefficient of x, and C is the collection of all terms that do not contain x^2 or x.

$4x^2 - 4xy + 1 - y^2 = 0 \quad \Rightarrow \quad (4)x^2 + (-4y)x + (1 - y^2) = 0 \quad \Rightarrow$

$$x = \frac{4y \pm \sqrt{16y^2 - 16(1 - y^2)}}{2(4)} = \frac{4y \pm \sqrt{16\left[y^2 - (1 - y^2)\right]}}{2(4)} = \frac{4y \pm 4\sqrt{2y^2 - 1}}{2(4)} = \frac{y \pm \sqrt{2y^2 - 1}}{2}$$

(b) Similar to part (a), we must now recognize the equation as a quadratic equation in y.

$4x^2 - 4xy + 1 - y^2 = 0 \quad \Rightarrow \quad (-1)y^2 + (-4x)y + (4x^2 + 1) = 0 \quad \Rightarrow$

$$y = \frac{4x \pm \sqrt{16x^2 + 4(4x^2 + 1)}}{2(-1)} = \frac{4x \pm \sqrt{4[4x^2 + (4x^2 + 1)]}}{-2} = \frac{4x \pm 2\sqrt{8x^2 + 1}}{-2} = -2x \pm \sqrt{8x^2 + 1}$$

46 (a) $2x^2 - xy = 3y^2 + 1 \quad \Rightarrow \quad (2)x^2 + (-y)x + (-3y^2 - 1) = 0 \quad \Rightarrow$

$$x = \frac{y \pm \sqrt{y^2 - 8(-3y^2 - 1)}}{2(2)} = \frac{y \pm \sqrt{25y^2 + 8}}{4}$$

(b) $2x^2 - xy = 3y^2 + 1 \quad \Rightarrow \quad (-3)y^2 + (-x)y + (2x^2 - 1) = 0 \quad \Rightarrow$

$$y = \frac{x \pm \sqrt{x^2 + 12(2x^2 - 1)}}{2(-3)} = \frac{x \pm \sqrt{25x^2 - 12}}{-6}$$

47 (a) $x = \dfrac{-4{,}500{,}000 \pm \sqrt{4{,}500{,}000^2 - 4(1)(-0.96)}}{2} \approx 0$ and $-4{,}500{,}000$

(b) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \cdot \frac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} = \frac{b^2 - (b^2 - 4ac)}{2a\left(-b \mp \sqrt{b^2 - 4ac}\right)}$$

$$= \frac{4ac}{2a\left(-b \mp \sqrt{b^2 - 4ac}\right)} = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

The root near zero was obtained in part (a) using the plus sign. In the second formula, it corresponds to the minus sign. $x = \dfrac{2(-0.96)}{-4{,}500{,}000 - \sqrt{4{,}500{,}000^2 - 4(1)(-0.96)}} \approx 2.13 \times 10^{-7}$

48 (a) $x = \dfrac{73{,}000{,}000 \pm \sqrt{(-73{,}000{,}000)^2 - 4(1)(2.01)}}{2} \approx 73{,}000{,}000$ and 0

(b) The root near zero was obtained in part (a) using the minus sign. In the second formula, it corresponds to the plus sign. $x = \dfrac{2(2.01)}{73{,}000{,}000 + \sqrt{(-73{,}000{,}000)^2 - 4(1)(2.01)}} \approx 2.75 \times 10^{-8}$

49
$$\begin{aligned} EK + L &= D - TK && \{\text{given equation, solve for } K\} \\ EK + TK &= D - L && \{\text{get } K\text{-terms on one side, everything else on the other}\} \\ K(E+T) &= D - L && \{\text{factor out } K\} \\ K &= \frac{D-L}{E+T} && \{\text{divide by } (E+T) \text{ to solve for } K\} \end{aligned}$$

50 $CD + C = PC + R \;\Rightarrow\; CD + C - PC = R \;\Rightarrow\; C(D + 1 - P) = R \;\Rightarrow\; C = \dfrac{R}{D+1-P}$

51 $N = \dfrac{Q+1}{Q} \;\Rightarrow\; NQ = Q + 1 \;\Rightarrow\; NQ - Q = 1 \;\Rightarrow\; Q(N-1) = 1 \;\Rightarrow\; Q = \dfrac{1}{N-1}$

52 $\beta = \dfrac{\alpha}{1-\alpha} \;\Rightarrow\; \beta(1-\alpha) = \alpha \;\Rightarrow\; \beta - \beta\alpha = \alpha \;\Rightarrow\; \beta = \alpha + \beta\alpha \;\Rightarrow\; \beta = \alpha(1+\beta) \;\Rightarrow\; \alpha = \dfrac{\beta}{1+\beta}$

53 $A = P + Prt \;\Rightarrow\; A - P = Prt \;\Rightarrow\; r = \dfrac{A-P}{Pt}$

54 $s = \dfrac{1}{2}gt^2 + v_0t \;\Rightarrow\; 2s = gt^2 + 2v_0t \;\Rightarrow\; 2s - gt^2 = 2v_0t \;\Rightarrow\; v_0 = \dfrac{2s - gt^2}{2t}$

55
$$\begin{aligned} S &= \frac{p}{q + p(1-q)} && \{\text{given equation, solve for } q\} \\ S[q + p(1-q)] &= p && \{\text{eliminate the fraction}\} \\ Sq + Sp(1-q) &= p && \{\text{multiply terms}\} \\ Sq + Sp - Spq &= p && \{\text{multiply terms}\} \\ Sq - Spq &= p - Sp && \{\text{isolate terms containing } q\} \\ Sq(1-p) &= p(1-S) && \{\text{factor our } Sq\} \\ q &= \frac{p(1-S)}{S(1-p)} && \{\text{divide by } S(1-p) \text{ to solve for } q\} \end{aligned}$$

56 $S = 2(lw + hw + hl) \;\Rightarrow\; S = 2lw + 2hw + 2hl \;\Rightarrow\; S - 2lw = 2h(w + l) \;\Rightarrow\; h = \dfrac{S - 2lw}{2(w+l)}$

57 $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$ {multiply by the lcd, fpq} $\Rightarrow$

$pq = fq + fp \;\Rightarrow\; pq - fq = fp \;\Rightarrow\; q(p - f) = fp \;\Rightarrow\; q = \dfrac{fp}{p-f}$

58 $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ {multiply by the lcd, $RR_1R_2R_3$} $\Rightarrow\; R_1R_2R_3 = RR_2R_3 + RR_1R_3 + RR_1R_2 \;\Rightarrow$

$R_1R_2R_3 - RR_2R_3 - RR_1R_2 = RR_1R_3 \;\Rightarrow\; R_2(R_1R_3 - RR_3 - RR_1) = RR_1R_3 \;\Rightarrow$

$R_2 = \dfrac{RR_1R_3}{R_1R_3 - RR_3 - RR_1}$

59 $K = \dfrac{1}{2}mv^2 \;\Rightarrow\; v^2 = \dfrac{2K}{m} \;\Rightarrow\; v = \pm\sqrt{\dfrac{2K}{m}} \;\Rightarrow\; v = \sqrt{\dfrac{2K}{m}}$ since $v > 0$.

60 $F = g\dfrac{mM}{d^2} \;\Rightarrow\; d^2 = \dfrac{gmM}{F} \;\Rightarrow\; d = \pm\sqrt{\dfrac{gmM}{F}} \;\Rightarrow\; d = \sqrt{\dfrac{gmM}{F}}$ since $d > 0$.

61 $A = 2\pi r(r + h) \;\Rightarrow\; A = 2\pi r^2 + 2\pi rh \;\Rightarrow\; (2\pi)r^2 + (2\pi h)r - A = 0$ {a quadratic equation in r} $\Rightarrow$

$$\begin{aligned} r = \frac{-(2\pi h) \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)} &= \frac{-2\pi h \pm \sqrt{4\pi^2h^2 + 8\pi A}}{2(2\pi)} \\ &= \frac{-2\pi h \pm 2\sqrt{\pi^2h^2 + 2\pi A}}{2(2\pi)} = \frac{-\pi h \pm \sqrt{\pi^2h^2 + 2\pi A}}{2\pi} \end{aligned}$$

Since $r > 0$, we must use the plus sign, and $r = \dfrac{-\pi h + \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$.

62 $s = \frac{1}{2}gt^2 + v_0 t \;\Rightarrow\; \left(\frac{1}{2}g\right)t^2 + (v_0)t - s = 0 \;\Rightarrow\; t = \dfrac{-v_0 \pm \sqrt{v_0^2 + 2gs}}{g}$.

Since $t > 0$, we must use the plus sign, and $t = \dfrac{-v_0 + \sqrt{v_0^2 + 2gs}}{g}$.

63 $d = \frac{1}{2}\sqrt{4R^2 - C^2} \;\Rightarrow\; 2d = \sqrt{4R^2 - C^2} \;\Rightarrow\; 4d^2 = 4R^2 - C^2 \;\Rightarrow\; C^2 = 4R^2 - 4d^2 \;\Rightarrow$
$C^2 = 4(R^2 - d^2) \;\Rightarrow\; C = \pm 2\sqrt{R^2 - d^2} \;\Rightarrow\; C = 2\sqrt{R^2 - d^2}$ since $C > 0$

64 $S = \pi r\sqrt{r^2 + h^2} \;\Rightarrow\; \dfrac{S}{\pi r} = \sqrt{r^2 + h^2} \;\Rightarrow\; \left(\dfrac{S}{\pi r}\right)^2 = \left(\sqrt{r^2 + h^2}\right)^2 \;\Rightarrow\; \dfrac{S^2}{\pi^2 r^2} = r^2 + h^2 \;\Rightarrow$
$\dfrac{S^2}{\pi^2 r^2} - r^2 = h^2 \;\Rightarrow\; \dfrac{S^2}{\pi^2 r^2} - \dfrac{\pi^2 r^4}{\pi^2 r^2} = h^2 \;\Rightarrow\; h^2 = \dfrac{1}{\pi^2 r^2}\left(S^2 - \pi^2 r^4\right) \;\Rightarrow\; h = \pm\dfrac{1}{\pi r}\sqrt{S^2 - \pi^2 r^4} \;\Rightarrow$
$h = \dfrac{1}{\pi r}\sqrt{S^2 - \pi^2 r^4}$ since $h > 0$

65 Let x denote the number of months needed to recover the cost of the insulation. The savings in one month is 10% of \$200 $=$ \$20, so the savings in x months is $20x$. $20x = 2400 \;\Rightarrow\; x = 120$ months (or 10 yr).

66 Let x denote the amount (in millions) invested in bonds.
$x(0.06) + (800 - x)(0.05) = 42 \;\Rightarrow\; 0.06x + 40 - 0.05x = 42 \;\Rightarrow\; 0.01x = 2 \;\Rightarrow\; x = 200.$
The arena should be financed by selling \$200 million in bonds and borrowing \$600 million.

67 (a) They will meet when the sum of their distances is 224. Let t denote the desired number of seconds.
Using distance $=$ rate $\times$ time, we have $1.5t + 2t = 224 \;\Rightarrow\; 3.5t = 224 \;\Rightarrow\; t = 64$ sec.

(b) The children will have walked $64(1.5) = 96$ m and $64(2) = 128$ m, respectively.

68 Let l denote the length of the side parallel to the river bank. $P = 2w + l$

(a) $l = 2w \;\Rightarrow\; P = 2w + 2w = 4w$. $4w = 180 \;\Rightarrow\; w = 45$ ft and $A = (45)(90) = 4050$ ft^2.

(b) $l = \frac{1}{2}w \;\Rightarrow\; P = 2w + \frac{1}{2}w = \frac{5}{2}w$. $\frac{5}{2}w = 180 \;\Rightarrow\; w = 72$ ft and $A = (72)(36) = 2592$ ft^2.

(c) $l = w \;\Rightarrow\; P = 2w + w = 3w$. $3w = 180 \;\Rightarrow\; w = 60$ ft and $A = (60)(60) = 3600$ ft^2.

69 Let x denote the distance to the target. We know the total time involved and need a formula for time. Solving $d = rt$ for t gives us $t = d/r$.
$\text{Time}_{\text{to target}} + \text{Time}_{\text{from target}} = \text{Time}_{\text{total}} \;\Rightarrow\; \dfrac{x}{3300} + \dfrac{x}{1100} = 1.5$ {multiply by the lcd, 3300} $\Rightarrow$
$x + 3x = 1.5(3300) \;\Rightarrow\; 4x = 4950 \;\Rightarrow\; x = 1237.5$ ft.

70 Let x denote the miles in one direction. A 6-minute-mile pace is equivalent to a rate of $\frac{1}{6}$ mile/min. Solving $d = rt$ for t gives us $t = d/r$. $\text{Minutes}_{\text{north}} + \text{Minutes}_{\text{south}} = \text{Minutes}_{\text{total}} \;\Rightarrow\; \dfrac{x}{1/6} + \dfrac{x}{1/7} = 47 \;\Rightarrow$
$6x + 7x = 47 \;\Rightarrow\; x = \frac{47}{13}$. The total distance is $2 \cdot \frac{47}{13} = \frac{94}{13}$, or $7\frac{3}{13}$ mi.

71 Let b_2 denote the larger base. $A = \frac{1}{2}(b_1 + b_2)h \;\Rightarrow\; 5 = \frac{1}{2}(3 + b_2)(1) \;\Rightarrow\; 10 = 3 + b_2 \;\Rightarrow\; b_2 = 7$ ft.

72 Let h_1 denote the height of the cylinder. $V = \frac{2}{3}\pi r^3 + \pi r^2 h_1 = 11{,}250\pi$ and $r = 15 \;\Rightarrow$
$2250\pi + 225\pi h_1 = 11{,}250\pi \;\Rightarrow\; 225\pi h_1 = 9000\pi \;\Rightarrow\; h_1 = 40$. The total height is 40 ft $+$ 15 ft $=$ 55 ft.

73 (a) $T = T_0 - \left(\frac{5.5}{1000}\right)h$ • $h = 5280$ ft and $T_0 = 70$°F $\Rightarrow\; T = 70 - \left(\frac{5.5}{1000}\right)5280 = 40.96$°F.

(b) $T = 32$°F $\Rightarrow\; 32 = 70 - \left(\frac{5.5}{1000}\right)h \;\Rightarrow\; \left(\frac{5.5}{1000}\right)h = 38 \;\Rightarrow\; h = 38\left(\frac{1000}{5.5}\right) \approx 6909$ ft.

74 (a) $h = 227(T - D)$ • $T = 70°\text{F}$ and $D = 55°\text{F}$ $\Rightarrow$ $h = 227(70 - 55) = 3405$ ft.

(b) $h = 3500$ ft and $D = 65°\text{F}$ $\Rightarrow$ $3500 = 227(T - 65)$ $\Rightarrow$ $T = \frac{3500}{227} + 65 \approx 80.4°\text{F}$

75 $T = B - \left(\frac{3}{1000}\right)h$ • $B = 55°\text{F}$ and $h = 10{,}000 - 4000 = 6000$ ft $\Rightarrow$ $T = 55 - \left(\frac{3}{1000}\right)(6000) = 37°\text{F}$.

76 (a) $h = 65 + 3.14x$ • $x = 30$ cm $\Rightarrow$ $h = 65 + 3.14(30) = 159.2$ cm.

(b) $h = 73.6 + 3.0x$ • $x = 34$ $\Rightarrow$ $h = 73.6 + 3(34) = 175.6$ cm. The height of the skeleton has decreased by $175.6 - 174 = 1.6$ cm due to aging after age 30. $\frac{1.6}{0.06} \approx 27$ years. The male was approximately $30 + 27 = 57$ years old at death.

77 (a) $v = 55$ $\Rightarrow$ $d = v + (v^2/20) = 55 + (55^2/20) = 206.25$ ft

(b) $d = 120$ $\Rightarrow$ $120 = v + (v^2/20)$ $\Rightarrow$ $2400 = 20v + v^2$ $\Rightarrow$ $v^2 + 20v - 2400 = 0$ $\Rightarrow$
$(v + 60)(v - 40) = 0$ $\Rightarrow$ $v = 40$ mi/hr

78 (a) $T = 98$ $\Rightarrow$ $h = 1000(100 - T) + 580(100 - T)^2 = 1000(2) + 580(2)^2 = 4320$ m.

(b) If $x = 100 - T$ and $h = 8840$, then $8840 = 1000x + 580x^2$ $\Rightarrow$

$29x^2 + 50x - 442 = 0$ $\Rightarrow$ $x = \dfrac{-50 \pm \sqrt{2500 + 51{,}272}}{2(29)} = \dfrac{-25 \pm \sqrt{13{,}443}}{29} \approx -4.86, 3.14.$

$x = -4.86$ $\Rightarrow$ $T = 100 - x = 104.86°\text{C}$, which is outside the allowable range of T.

$x = 3.14$ $\Rightarrow$ $T = 100 - x = 96.86°\text{C}$ for $95 \le T \le 100$.

79 (a) The northbound plane travels $\frac{1}{2} \cdot 200 = 100$ miles from 2 P.M. to 2:30 P.M., so the distances of the northbound and eastbound planes are $100 + 200t$ and $400t$, respectively. Using the Pythagorean theorem,

$$d = \sqrt{(100 + 200t)^2 + (400t)^2} = \sqrt{100^2(1 + 2t)^2 + 100^2(4t)^2} = \sqrt{100^2\left[(1 + 2t)^2 + (4t)^2\right]}$$
$$= 100\sqrt{1 + 4t + 4t^2 + 16t^2} = 100\sqrt{20t^2 + 4t + 1}.$$

(b) $d = 500$ $\Rightarrow$ $500 = 100\sqrt{20t^2 + 4t + 1}$ $\Rightarrow$ $5 = \sqrt{20t^2 + 4t + 1}$ $\Rightarrow$ $5^2 = 20t^2 + 4t + 1$ $\Rightarrow$
$20t^2 + 4t - 24 = 0$ $\Rightarrow$ $5t^2 + t - 6 = 0$ $\Rightarrow$ $(5t + 6)(t - 1) = 0$ $\Rightarrow$
$t = 1$ hour after 2:30 P.M., or 3:30 P.M.

80 Let t denote the number of seconds the rock falls, so that $4 - t$ is the number of seconds for the sound to travel.
$\text{Distance}_{\text{down}} = \text{Distance}_{\text{up}}$ $\Rightarrow$ $16t^2 = 1100(4 - t)$ $\{d = rt\}$ $\Rightarrow$ $4t^2 = 275(4 - t)$ $\Rightarrow$
$4t^2 + 275t - 1100 = 0$ $\Rightarrow$ $t = \dfrac{-275 \pm \sqrt{93{,}225}}{2(4)} = \dfrac{-275 + 5\sqrt{3729}}{8} \approx 3.79.$

The height is $16t^2 \approx 229.94$, or 230 ft.

81 Let x denote the number of \$10 reductions in price.
Revenue = (unit price) × (# of units) $\Rightarrow$ $7000 = (300 - 10x)(15 + 2x)$ $\Rightarrow$
$7000 = 10(30 - x)(15 + 2x)$ $\Rightarrow$ $700 = -2x^2 + 45x + 450$ $\Rightarrow$ $2x^2 - 45x + 250 = 0$ $\Rightarrow$
$(2x - 25)(x - 10) = 0$ $\Rightarrow$ $x = 10$ or 12.5.

The selling price is \$300 − \$10(10) = \$200, or \$300 − \$10(12.5) = \$175.

82 The total surface area is the sum of the surface area of the cylinder and that of the top and bottom.
$S = 2\pi rh + 2\pi r^2$ and $h = 4$ $\Rightarrow$ $10\pi = 8\pi r + 2\pi r^2$ {divide by 2π} $\Rightarrow$ $5 = 4r + r^2$ $\Rightarrow$
$r^2 + 4r - 5 = 0$ $\Rightarrow$ $(r + 5)(r - 1) = 0$ $\Rightarrow$ $r = 1$, and the diameter is 2 ft.

83 (a) $\textbf{Area}_{\textbf{capsule}} = \text{Area}_{\text{sphere}}$ {the two ends are hemispheres} $+ \text{Area}_{\text{cylinder}}$

$$= 4\pi r^2 + 2\pi rh = 4\pi\left(\tfrac{1}{4}\right)^2 + 2\pi\left(\tfrac{1}{4}\right)\left(2 - \tfrac{1}{2}\right) = \tfrac{\pi}{4} + \tfrac{3\pi}{4} = \pi \text{ cm}^2.$$

$\textbf{Area}_{\textbf{tablet}} = \text{Area}_{\text{top and bottom}} + \text{Area}_{\text{cylinder}} = 2\pi r^2 + 2\pi r\left(\tfrac{1}{2}\right) = 2\pi r^2 + \pi r.$

Equating the two surface areas yields $2\pi r^2 + \pi r = \pi \quad \Rightarrow$

$$2r^2 + r - 1 = 0 \quad \Rightarrow \quad (2r-1)(r+1) = 0 \quad \Rightarrow \quad r = \tfrac{1}{2}, \text{ and the diameter is 1 cm.}$$

(b) $\textbf{Volume}_{\textbf{capsule}} = \text{Volume}_{\text{sphere}} + \text{Volume}_{\text{cylinder}}$

$$= \tfrac{4}{3}\pi r^3 + \pi r^2 h = \tfrac{4}{3}\pi\left(\tfrac{1}{4}\right)^3 + \pi\left(\tfrac{1}{4}\right)^2 \tfrac{3}{2} = \tfrac{\pi}{48} + \tfrac{3\pi}{32} = \tfrac{11\pi}{96} \approx 0.360 \text{ cm}^3.$$

$\textbf{Volume}_{\textbf{tablet}} = \text{Volume}_{\text{cylinder}} = \pi r^2 h = \pi\left(\tfrac{1}{2}\right)^2 \tfrac{1}{2} = \tfrac{\pi}{8} \approx 0.393 \text{ cm}^3.$

84 $P = 15{,}700 S^{5/2} RD \quad \Rightarrow \quad S^{5/2} = \dfrac{P}{15{,}700RD} \quad \Rightarrow$

$$S = \left(\frac{P}{15{,}700RD}\right)^{2/5} = \left[\frac{380}{(15{,}700)(0.113/2)(2)}\right]^{2/5} \approx 0.54$$

85 From the Pythagorean theorem, $d^2 + h^2 = L^2$. Since d is to be 25% of L, we have

$d = \tfrac{1}{4}L$, so $\left(\tfrac{1}{4}L\right)^2 + h^2 = L^2 \quad \Rightarrow \quad h^2 = L^2 - \left(\tfrac{1}{4}L\right)^2 \quad \Rightarrow \quad h^2 = 1L^2 - \tfrac{1}{16}L^2 \quad \Rightarrow$

$h^2 = \tfrac{15}{16}L^2 \quad \Rightarrow \quad h = \sqrt{\tfrac{15}{16}L^2}$ {since $h > 0$} $= \tfrac{\sqrt{15}}{4}L \approx 0.97L$. Thus, $h \approx 97\%L$.

86 $T = 0.25P^{1/4}/\sqrt{v} \quad \Rightarrow \quad P^{1/4} = 4T\sqrt{v} \quad \Rightarrow \quad P = (4T)^4 v^2 = 4^4 3^4 5^2 = 518{,}400$

87 $\text{Cost}_{\text{underwater}} + \text{Cost}_{\text{overland}} = \text{Cost}_{\text{total}} \quad \Rightarrow \quad 7500 \cdot (\text{underwater miles}) + 6000 \cdot (\text{overland miles}) = 35{,}000 \quad \Rightarrow$

$7500\sqrt{x^2+1} + 6000(5-x) = 35{,}000 \quad \Rightarrow \quad 7500\sqrt{x^2+1} + 30{,}000 - 6000x = 35{,}000 \quad \Rightarrow$

$7500\sqrt{x^2+1} = 6000x + 5{,}000 \quad \Rightarrow \quad 15\sqrt{x^2+1} = 12x + 10$ {divide by 500} $\Rightarrow$

$225(x^2+1) = 144x^2 + 240x + 100 \quad \Rightarrow \quad 225x^2 + 225 = 144x^2 + 240x + 100 \quad \Rightarrow$

$81x^2 - 240x + 125 = 0 \quad \Rightarrow \quad x = \dfrac{240 \pm \sqrt{17{,}100}}{162} = \dfrac{6 \cdot 40 \pm \sqrt{900 \cdot 19}}{6 \cdot 27} = \dfrac{6 \cdot 40 \pm 30\sqrt{19}}{6 \cdot 27} = \dfrac{40 \pm 5\sqrt{19}}{27} \approx$

$2.2887, 0.6743$ mi. There are two possible routes.

88 The y-value decreases 1.2 units for each 1 unit increase in the x-value. The data are best described by equation (1), $y = -1.2x + 2$.

89 The y-values are increasing rapidly and can best be described by equation (4), $y = x^3 - x^2 + x - 10$.

90 (a) Let $Y_1 = T_1 = -1.09L + 96.01$ and $Y_2 = T_2 = -0.011L^2 - 0.126L + 81.45$. Table each equation and compare them to the actual temperatures.

x (L)	85	75	65	55	45	35	25	15	5
Y_1	3.36	14.26	25.16	36.06	46.96	57.86	68.76	75.66	90.56
Y_2	−8.74	10.13	26.79	41.25	53.51	63.57	71.43	77.09	80.55
S. Hem.	−5	10	27	42	53	65	75	78	79

Comparing Y_1 (T_1) with Y_2 (T_2), we can see that the linear equation T_1 is not as accurate as the quadratic equation T_2.

(b) $L = 50 \quad \Rightarrow \quad T_2 = -0.011(50)^2 - 0.126(50) + 81.45 = 47.65$°F.

91 (a) Let $Y_1 = D_1 = 6.096L + 685.7$ and $Y_2 = D_2 = 0.00178L^3 - 0.072L^2 + 4.37L + 719$.

Table each equation and compare them to the actual values.

x (L)	0	10	20	30	40	50	60
Y_1	686	747	808	869	930	991	1051
Y_2	719	757	792	833	893	980	1106
Summer	720	755	792	836	892	978	1107

Comparing Y_1 (D_1) with Y_2 (D_2),

we see that the linear equation D_1 is not as accurate as the cubic equation D_2.

(b) $L = 35 \quad \Rightarrow \quad D_2 = 0.00178(35)^3 - 0.072(35)^2 + 4.37(35) + 719 \approx 860$ min.

1.5 Exercises

1 $(5-2i)+(-3+6i) = [5+(-3)]+(-2+6)i = 2+4i$

2 $(-5+4i)+(3+9i) = (-5+3)+(4+9)i = -2+13i$

3 $(7-8i)-(-5-3i) = (7+5)+(-8+3)i = 12-5i$

4 $(-3+8i)-(2+3i) = (-3-2)+(8-3)i = -5+5i$

5 $(3+5i)(2-7i) = (3+5i)2+(3+5i)(-7i)$ {distributive property}

$= 6+10i-21i-35i^2$ {multiply terms}

$= 6-11i-35(-1)$ {combine i-terms, $i^2=-1$}

$= 6-11i+35$

$= 41-11i$

6 $(-2+3i)(8-i) = (-16-3i^2)+(2+24)i = (-16+3)+26i = -13+26i$

7 $(4-3i)(2+7i) = (8-21i^2)+(28-6)i = (8+21)+22i = 29+22i$

8 $(8+2i)(7-3i) = (56-6i^2)+(-24+14)i = (56+6)-10i = 62-10i$

9 Use the *special product formula* for $(x-y)^2$ on the inside front cover of the text.

$(5-2i)^2 = 5^2-2(5)(2i)+(2i)^2 = 25-20i+4i^2 = (25-4)-20i = 21-20i$

10 $(6+7i)^2 = 6^2+2(6)(7i)+(7i)^2 = (36-49)+84i = -13+84i$

11 $i(3+4i)^2 = i\left[3^2+2(3)(4i)+(4i)^2\right] = i\left[(9-16)+24i\right] = i(-7+24i) = -24-7i$

12 $i(2-7i)^2 = i\left[2^2+2(2)(-7i)+(-7i)^2\right] = i\left[(4-49)-28i\right] = i(-45-28i) = 28-45i$

13 $(3+4i)(3-4i)$ {note that this difference of squares ...}

$= 3^2-(4i)^2 = 9-(-16) =$ {... becomes a "sum of squares"} $9+16=25$

14 $(4+7i)(4-7i) = 4^2-(7i)^2 = 16-(-49) = 16+49 = 65$

15 (a) Since $i^k = 1$ if k is a multiple of 4, we will write i^{43} as $i^{40}i^3$, knowing that i^{40} will reduce to 1.

$i^{43} = i^{40}i^3 = \left(i^4\right)^{10}(-i) = 1^{10}(-i) = -i$

(b) As in Example 3(e), choose $b=20$. $i^{-20}\cdot i^{20} = i^0 = 1$.

16 (a) $i^{68} = \left(i^4\right)^{17} = 1^{17} = 1$

(b) As in Example 3(e), choose $b=36$. $i^{-33}\cdot i^{36} = i^3 = -i$.

17 (a) Since $i^k = 1$ if k is a multiple of 4, we will write i^{73} as $i^{72}i^1$, knowing that i^{72} will reduce to 1.

$$i^{73} = i^{72}i = \left(i^4\right)^{18} i = 1^{18} i = i$$

(b) As in Example 3(e), choose $b = 48$. $i^{-46} \cdot i^{48} = i^2 = -1$.

18 (a) $i^{66} = i^{64}i^2 = \left(i^4\right)^{16}(-1) = 1^{16}(-1) = -1$

(b) As in Example 3(e), choose $b = 56$. $i^{-55} \cdot i^{56} = i^1 = i$.

19 Multiply both the numerator and the denominator by the conjugate of the denominator to eliminate all i's in the denominator. The new denominator is the sum of the squares of the coefficients—in this case, 2^2 and 4^2.

$$\frac{3}{2+4i} \cdot \frac{2-4i}{2-4i} = \frac{3(2-4i)}{4-(-16)} = \frac{6-12i}{20} = \frac{6}{20} - \frac{12}{20}i = \frac{3}{10} - \frac{3}{5}i$$

20 $\dfrac{5}{3-7i} \cdot \dfrac{3+7i}{3+7i} = \dfrac{5(3+7i)}{9-(-49)} = \dfrac{15+35i}{58} = \dfrac{15}{58} + \dfrac{35}{58}i$

21 Multiply both the numerator and the denominator by the conjugate of the denominator to eliminate all i's in the denominator. The new denominator is the sum of the squares of the coefficients—in this case, 6^2 and 2^2.

$$\frac{1-7i}{6-2i} \cdot \frac{6+2i}{6+2i} = \frac{(6+14)+(2-42)i}{36-(-4)} = \frac{20-40i}{40} = \frac{20}{40} - \frac{40}{40}i = \frac{1}{2} - i$$

22 $\dfrac{2+9i}{-3-i} \cdot \dfrac{-3+i}{-3+i} = \dfrac{(-6-9)+(2-27)i}{9-(-1)} = \dfrac{-15-25i}{10} = -\dfrac{3}{2} - \dfrac{5}{2}i$

23 $\dfrac{-4+6i}{2+7i} \cdot \dfrac{2-7i}{2-7i} = \dfrac{(-8+42)+(28+12)i}{4-(-49)} = \dfrac{34+40i}{53} = \dfrac{34}{53} + \dfrac{40}{53}i$

24 $\dfrac{-3-2i}{5+2i} \cdot \dfrac{5-2i}{5-2i} = \dfrac{(-15-4)+(6-10)i}{25-(-4)} = \dfrac{-19-4i}{29} = -\dfrac{19}{29} - \dfrac{4}{29}i$

25 Multiplying the denominator by i will eliminate the i's in the denominator.

$$\frac{4-2i}{-7i} = \frac{4-2i}{-7i} \cdot \frac{i}{i} = \frac{4i-2i^2}{-7i^2} = \frac{2+4i}{7} = \frac{2}{7} + \frac{4}{7}i$$

26 $\dfrac{-2+6i}{3i} = \dfrac{-2+6i}{3i} \cdot \dfrac{-i}{-i} = \dfrac{2i-6i^2}{-3i^2} = \dfrac{6+2i}{3} = 2 + \dfrac{2}{3}i$

27 Use the *special product formula* for $(x+y)^3$ on the inside front cover of the text.

$$\begin{aligned}(2+5i)^3 &= (2)^3 + 3(2)^2(5i) + 3(2)(5i)^2 + (5i)^3 \\ &= 8 + 60i + 6\left(25i^2\right) + 125i^3 \\ &= \left(8 + 150i^2\right) + \left(60i + 125i^3\right) = (8-150) + (60-125)i = -142 - 65i\end{aligned}$$

28 $(3-2i)^3 = (3)^3 + 3(3)^2(-2i) + 3(3)(-2i)^2 + (-2i)^3 = 27 - 54i + 9\left(4i^2\right) - 8i^3$
$= \left(27 + 36i^2\right) + \left(-54i - 8i^3\right) = (27-36) + (-54+8)i = -9 - 46i$

29 A common mistake is to multiply $\sqrt{-4}\,\sqrt{-16}$ and obtain $\sqrt{64}$, or 8.
The correct procedure is $\sqrt{-4}\,\sqrt{-16} = \sqrt{4}\,i \cdot \sqrt{16}\,i = (2i)(4i) = 8i^2 = -8$.

$$\left(2 - \sqrt{-4}\right)\left(3 - \sqrt{-16}\right) = (2-2i)(3-4i) = (6-8) + (-6i-8i) = -2 - 14i$$

30 $\left(-3 + \sqrt{-25}\right)\left(8 - \sqrt{-36}\right) = (-3+5i)(8-6i) = (-24+30) + (40i+18i) = 6 + 58i$

31 $\dfrac{4+\sqrt{-81}}{2-\sqrt{-9}} = \dfrac{4+9i}{2-3i}\cdot\dfrac{2+3i}{2+3i} = \dfrac{(8-27)+(12+18)i}{4-(-9)} = \dfrac{-19+30i}{13} = -\dfrac{19}{13}+\dfrac{30}{13}i$

32 $\dfrac{5-\sqrt{-121}}{1+\sqrt{-25}} = \dfrac{5-11i}{1+5i}\cdot\dfrac{1-5i}{1-5i} = \dfrac{(5-55)+(-25-11)i}{1-(-25)} = \dfrac{-50-36i}{26} = -\dfrac{25}{13}-\dfrac{18}{13}i$

33 $\dfrac{\sqrt{-36}\,\sqrt{-49}}{\sqrt{-16}} = \dfrac{(6i)(7i)}{4i} = \dfrac{42i^2}{4i} = \dfrac{-21}{2i} = \dfrac{-21}{2i}\cdot\dfrac{-i}{-i} = \dfrac{21i}{-2i^2} = \dfrac{21i}{2} = \dfrac{21}{2}i$

34 $\dfrac{\sqrt{-25}}{\sqrt{-16}\,\sqrt{-81}} = \dfrac{5i}{(4i)(9i)} = \dfrac{5i}{36i^2} = \dfrac{5i}{-36} = -\dfrac{5}{36}i$

35 We need to equate the real parts and the imaginary parts on each side of " $=$ ".
$4+(x+2y)i = x+2i \;\Rightarrow\; 4 = x$ and $x+2y = 2 \;\Rightarrow$
$x = 4$ and $4+2y = 2 \;\Rightarrow\; 2y = -2 \;\Rightarrow\; y = -1$, so $x = 4$ and $y = -1$.

36 $(x-y)+3i = 4+yi \;\Rightarrow\; 3 = y$ and $x-y = 4 \;\Rightarrow\; x-3 = 4 \;\Rightarrow\; x = 7, y = 3$

37 $(2x-y)-16i = 10+4yi \;\Rightarrow\; 2x-y = 10$ and $-16 = 4y \;\Rightarrow\; y = -4$ and $2x-(-4) = 10 \;\Rightarrow$
$2x+4 = 10 \;\Rightarrow\; 2x = 6 \;\Rightarrow\; x = 3$, so $x = 3$ and $y = -4$.

38 $8+(3x+y)i = 2x-4i \;\Rightarrow\; 2x = 8$ and $3x+y = -4 \;\Rightarrow\; x = 4$ and $12+y = -4 \;\Rightarrow\; x = 4, y = -16$

39 $x^2-6x+13 = 0 \;\Rightarrow\; x = \dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)} = \dfrac{6\pm\sqrt{36-52}}{2} = \dfrac{6\pm\sqrt{-16}}{2} = \dfrac{6\pm 4i}{2} = 3\pm 2i$

40 $x^2-2x+26 = 0 \;\Rightarrow$

$$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(26)}}{2(1)} = \frac{2\pm\sqrt{4-104}}{2} = \frac{2\pm\sqrt{-100}}{2} = \frac{2\pm 10i}{2} = 1\pm 5i$$

41 $x^2+12x+37 = 0 \;\Rightarrow$

$$x = \frac{-12\pm\sqrt{(-12)^2-4(1)(37)}}{2(1)} = \frac{-12\pm\sqrt{144-148}}{2} = \frac{-12\pm\sqrt{-4}}{2} = \frac{-12\pm 2i}{2} = -6\pm i$$

42 $x^2+8x+17 = 0 \;\Rightarrow\; x = \dfrac{-8\pm\sqrt{64-68}}{2(1)} = \dfrac{-8\pm\sqrt{-4}}{2} = \dfrac{-8\pm 2i}{2} = -4\pm i$

43 $x^2-5x+20 = 0 \;\Rightarrow\; x = \dfrac{-(-5)\pm\sqrt{(-5)^2-4(1)(20)}}{2(1)} = \dfrac{5\pm\sqrt{25-80}}{2} = \dfrac{5\pm\sqrt{-55}}{2} = \dfrac{5}{2}\pm\dfrac{1}{2}\sqrt{55}\,i$

44 $x^2+3x+6 = 0 \;\Rightarrow\; x = \dfrac{-3\pm\sqrt{9-24}}{2(1)} = \dfrac{-3\pm\sqrt{-15}}{2} = -\dfrac{3}{2}\pm\dfrac{1}{2}\sqrt{15}\,i$

45 $4x^2+x+3 = 0 \;\Rightarrow\; x = \dfrac{-1\pm\sqrt{1^2-4(4)(3)}}{2(4)} = \dfrac{-1\pm\sqrt{1-48}}{8} = \dfrac{-1\pm\sqrt{-47}}{8} = -\dfrac{1}{8}\pm\dfrac{1}{8}\sqrt{47}\,i$

46 $-3x^2+x-5 = 0 \;\Rightarrow\; x = \dfrac{-1\pm\sqrt{1^2-4(-3)(-5)}}{2(-3)} = \dfrac{-1\pm\sqrt{1-60}}{-6} = \dfrac{1}{6}\pm\dfrac{1}{6}\sqrt{59}\,i$

47 Solving $x^3 = -64$ by taking the cube root of both sides would only give us the solution $x = -4$, so we need to factor x^3+64 as the sum of cubes. $x^3+64 = 0 \;\Rightarrow\; (x+4)(x^2-4x+16) = 0 \;\Rightarrow$
$x = -4$ or $x = \dfrac{4\pm\sqrt{16-64}}{2} = \dfrac{4\pm 4\sqrt{3}\,i}{2}$. The three solutions are $-4, 2\pm 2\sqrt{3}\,i$.

48 $x^3 - 27 = 0 \quad \Rightarrow \quad (x-3)(x^2+3x+9) = 0 \quad \Rightarrow$

$$x = 3 \text{ or } x = \frac{-3 \pm \sqrt{9-36}}{2} = \frac{-3 \pm 3\sqrt{3}\,i}{2}.$$ The three solutions are $3, -\frac{3}{2} \pm \frac{3}{2}\sqrt{3}\,i$.

49 $27x^3 = (x+5)^3 \quad \Rightarrow \quad (3x)^3 - (x+5)^3 = 0 \quad \Rightarrow$

{difference of cubes} $[3x - (x+5)]\left[(3x)^2 + 3x(x+5) + (x+5)^2\right] = 0 \quad \Rightarrow$

$(3x - x - 5)(9x^2 + 3x^2 + 15x + x^2 + 10x + 25) = 0 \quad \Rightarrow$

$(2x-5)(13x^2 + 25x + 25) = 0 \quad \Rightarrow \quad x = \frac{5}{2} \text{ or } x = \frac{-25 \pm \sqrt{625 - 1300}}{2(13)} = \frac{-25 \pm 15\sqrt{3}\,i}{26}.$

The three solutions are $\frac{5}{2}, -\frac{25}{26} \pm \frac{15}{26}\sqrt{3}\,i$.

50 $16x^4 = (x-4)^4 \quad \Rightarrow \quad (4x^2)^2 - \left[(x-4)^2\right]^2 = 0 \quad \Rightarrow$ {difference of squares}

$\left[4x^2 + (x-4)^2\right]\left[4x^2 - (x-4)^2\right] = 0 \quad \Rightarrow \quad (5x^2 - 8x + 16)(3x^2 + 8x - 16) = 0 \quad \Rightarrow$

$5x^2 - 8x + 16 = 0$ or $3x^2 + 8x - 16 = (x+4)(3x-4) = 0$.

$5x^2 - 8x + 16 = 0 \quad \Rightarrow \quad x = \frac{8 \pm \sqrt{64 - 320}}{10} = \frac{8 \pm 16i}{10} = \frac{4 \pm 8i}{5}$. The four solutions are $-4, \frac{4}{3}, \frac{4}{5} \pm \frac{8}{5}i$.

51 $x^4 = 625 \quad \Rightarrow \quad x^4 - 625 = 0 \quad \Rightarrow \quad (x^2 - 25)(x^2 + 25) = 0 \quad \Rightarrow$

$$x^2 = 25, -25 \quad \Rightarrow \quad x = \pm\sqrt{25}, \pm\sqrt{-25} \quad \Rightarrow \quad x = \pm 5, \pm 5i$$

52 $x^4 = 81 \quad \Rightarrow \quad x^4 - 81 = 0 \quad \Rightarrow \quad (x^2 - 9)(x^2 + 9) = 0 \quad \Rightarrow$

$$x^2 = 9, -9 \quad \Rightarrow \quad x = \pm\sqrt{9}, \pm\sqrt{-9} \quad \Rightarrow \quad x = \pm 3, \pm 3i$$

53 $4x^4 + 25x^2 + 36 = 0 \quad \Rightarrow \quad (x^2 + 4)(4x^2 + 9) = 0 \quad \Rightarrow$

$$x^2 = -4, -\tfrac{9}{4} \quad \Rightarrow \quad x = \pm\sqrt{-4}, \pm\sqrt{-\tfrac{9}{4}} \quad \Rightarrow \quad x = \pm 2i, \pm\tfrac{3}{2}i$$

54 $27x^4 + 21x^2 + 4 = 0 \quad \Rightarrow \quad (9x^2 + 4)(3x^2 + 1) = 0 \quad \Rightarrow$

$$x^2 = -\tfrac{4}{9}, -\tfrac{1}{3} \quad \Rightarrow \quad x = \pm\sqrt{-\tfrac{4}{9}}, \pm\sqrt{-\tfrac{1}{3}} \quad \Rightarrow \quad x = \pm\tfrac{2}{3}i, \pm\tfrac{1}{3}\sqrt{3}\,i$$

55 $x^3 + 3x^2 + 4x = 0 \quad \Rightarrow \quad x(x^2 + 3x + 4) = 0 \quad \Rightarrow$

$$x = 0 \text{ or } x = \frac{-3 \pm \sqrt{9-16}}{2} = \frac{-3 \pm \sqrt{7}\,i}{2}.$$ The three solutions are $0, -\frac{3}{2} \pm \frac{1}{2}\sqrt{7}\,i$.

56 $8x^3 - 12x^2 + 2x - 3 = 0 \quad \Rightarrow \quad 4x^2(2x-3) + 1(2x-3) = 0 \quad \Rightarrow \quad (4x^2+1)(2x-3) = 0 \quad \Rightarrow$

$$x^2 = -\tfrac{1}{4} \text{ or } x = \tfrac{3}{2} \quad \Rightarrow \quad x = \pm\sqrt{-\tfrac{1}{4}} \text{ or } x = \tfrac{3}{2} \quad \Rightarrow \quad x = \tfrac{3}{2}, \pm\tfrac{1}{2}i$$

Note: In Exercises 57–62: Let $z = a + bi$ and $w = c + di$.

57

$\overline{z+w} = \overline{(a+bi)+(c+di)}$	{definition of z and w}
$= \overline{(a+c)+(b+d)i}$	{write in complex number form}
$= (a+c) - (b+d)i$	{definition of conjugate}
$= (a-bi) + (c-di)$	{rearrange terms (*)}
$= \overline{z} + \overline{w}$	{definition of conjugates of z and w}

(*) We are really looking ahead to the terms we want to obtain, $\overline{z}$ and $\overline{w}$.

58 $\overline{z-w} = \overline{(a+bi)-(c+di)} = \overline{(a-c)+(b-d)i} = (a-c) - (b-d)i = (a-bi) - (c-di) = \overline{z} - \overline{w}$.

59 $\overline{z \cdot w} = \overline{(a+bi)\cdot(c+di)} = \overline{(ac-bd)+(ad+bc)i}$

$= (ac-bd) - (ad+bc)i = ac - adi - bd - bci = a(c-di) - bi(c-di) = (a-bi)\cdot(c-di) = \overline{z}\cdot\overline{w}$

60 $\overline{\left(\frac{z}{w}\right)} = \overline{\left(\frac{a+bi}{c+di}\right)} = \overline{\left(\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}\right)} = \overline{\left(\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}\right)} = \overline{\left(\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i\right)}$

$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}i = \frac{(ac+bd)+(ad-bc)i}{c^2+d^2} = \frac{a-bi}{c-di}\cdot\frac{c+di}{c+di} = \frac{a-bi}{c-di} = \frac{\overline{(a+bi)}}{\overline{(c+di)}} = \frac{\overline{z}}{\overline{w}}$

61 **(1)** If $\overline{z} = z$, then $a - bi = a + bi$ and hence $-bi = bi$, or $2bi = 0$. Thus, $b = 0$ and $z = a$ is real.

(2) Conversely, if z is real, then $b = 0$ and hence $\overline{z} = \overline{a + 0i} = a - 0i = a + 0i = z$.

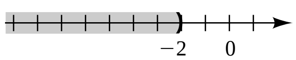
Thus, by **(1)** and **(2)**, $\overline{z} = z$ if and only if z is real.

62 $\overline{z^2} = \overline{(a+bi)^2} = \overline{a^2+2abi-b^2} = \overline{(a^2-b^2)+2abi} = (a^2-b^2)-2abi = a^2-2abi-b^2 = (a-bi)^2 = (\overline{z})^2$

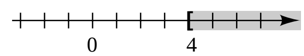
1.6 Exercises

Note: Brackets, "[" and "]", are used with $\le$ or $\ge$ and indicate that the endpoint of the interval is part of the solution. Parentheses, "(" and ")", are used with $<$ or $>$ and indicate that the endpoint is *not* part of the solution.

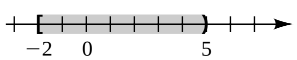
1 $x < -2 \;\Leftrightarrow\; (-\infty, -2)$



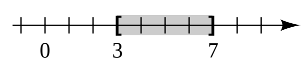
2 $x \ge 4 \;\Leftrightarrow\; [4, \infty)$



3 $5 > x \ge -2 \;\Leftrightarrow\; -2 \le x < 5 \;\Leftrightarrow\; [-2, 5)$



4 $3 \le x \le 7 \;\Leftrightarrow\; [3, 7]$



5 $(-5, 4] \;\Leftrightarrow\; -5 < x \le 4$

6 $(-6, \infty) \;\Leftrightarrow\; x > -6$

7 $2x + 5 < 3x - 7 \;\Rightarrow\; -x < -12 \;\Rightarrow\; x > 12$ {change inequality} $\Leftrightarrow\; (12, \infty)$

8 $x - 6 > 5x + 3 \;\Rightarrow\; -4x > 9 \;\Rightarrow\; x < -\frac{9}{4}$ {change inequality} $\Leftrightarrow\; \left(-\infty, -\frac{9}{4}\right)$

9 $\left[3 \le \frac{2x-9}{5} < 7\right]\cdot 5$ {multiply by the lcd, 5} $\Rightarrow\; 15 \le 2x - 9 < 35 \;\Rightarrow$

$24 \le 2x < 44$ {add 9 to all three parts} $\Rightarrow\; 12 \le x < 22$ {divide all three parts by 2} $\Leftrightarrow$

$[12, 22)$ {equivalent interval notation}

10 $\left[-2 < \frac{4x+1}{3} \le 0\right]\cdot 3 \;\Rightarrow\; -6 < 4x + 1 \le 0 \;\Rightarrow\; -7 < 4x \le -1 \;\Rightarrow\; -\frac{7}{4} < x \le -\frac{1}{4} \;\Leftrightarrow\; \left(-\frac{7}{4}, -\frac{1}{4}\right]$

11 By the law of signs, a quotient is positive if the sign of the numerator and the sign of the denominator are the same. Since the numerator is positive, $\frac{4}{3x+2} > 0 \;\Rightarrow\; 3x + 2 > 0 \;\Rightarrow\; x > -\frac{2}{3} \;\Leftrightarrow\; \left(-\frac{2}{3}, \infty\right)$.

The expression is never equal to 0 since the numerator is never 0. Thus, the solution of $\frac{4}{3x+2} \ge 0$ is $\left(-\frac{2}{3}, \infty\right)$.

12 $\frac{3}{2x+5} \le 0 \;\Rightarrow\; 2x + 5 < 0$ {denominator must be negative} $\Rightarrow\; 2x < -5 \;\Rightarrow\; x < -\frac{5}{2} \;\Leftrightarrow\; \left(-\infty, -\frac{5}{2}\right)$

13 $\frac{-7}{4-3x} > 0 \;\Rightarrow\; 4 - 3x < 0$ {denominator must also be negative} $\Rightarrow$

$4 < 3x \;\Rightarrow\; 3x > 4 \;\Rightarrow\; x > \frac{4}{3} \;\Leftrightarrow\; \left(\frac{4}{3}, \infty\right)$

14 $\frac{-3}{2-x} < 0 \;\Rightarrow\; 2 - x > 0 \;\Rightarrow\; 2 > x \;\Rightarrow\; x < 2 \;\Leftrightarrow\; (-\infty, 2)$

15 $(1-x)^2 > 0\ \forall x$ *except* 1. Thus, $\frac{5}{(1-x)^2} > 0$ has solution $\mathbb{R} - \{1\}$.

16 $x^2+4>0\ \forall x$. Hence, $\dfrac{3}{x^2+4}>0\ \forall x$, and $\dfrac{3}{x^2+4}<0$ has no solution.

17 $|x+3|<0.01 \quad\Rightarrow\quad -0.01<x+3<0.01 \quad\Rightarrow\quad -3.01<x<-2.99 \quad\Leftrightarrow\quad (-3.01,-2.99)$

18 $|x-4|\le 0.03 \quad\Rightarrow\quad -0.03\le x-4\le 0.03 \quad\Rightarrow\quad 3.97\le x\le 4.03 \quad\Leftrightarrow\quad [3.97,4.03]$

19 $|3x-7|\ge 5 \quad\Rightarrow\quad 3x-7\ge 5$ or $3x-7\le -5 \quad\Rightarrow\quad 3x\ge 12$ or $3x\le 2 \quad\Rightarrow$
$x\ge 4$ or $x\le \frac{2}{3} \quad\Leftrightarrow\quad \left(-\infty,\frac{2}{3}\right]\cup[4,\infty)$

20 $2|-11-7x|-2>10 \quad\Rightarrow\quad 2|-11-7x|>12 \quad\Rightarrow\quad |-11-7x|>6 \quad\Rightarrow$
$-11-7x>6$ or $-11-7x<-6 \quad\Rightarrow\quad -7x>17$ or $-7x<5 \quad\Rightarrow$
$x<-\frac{17}{7}$ or $x>-\frac{5}{7} \quad\Leftrightarrow\quad \left(-\infty,-\frac{17}{7}\right)\cup\left(-\frac{5}{7},\infty\right)$

21 Since $|7x+2|\ge 0\ \forall x$, $|7x+2|>-2$ has solution $(-\infty,\infty)$.

22 Since $|6x-5|\ge 0\ \forall x$, $|6x-5|\le -2$ has no solution.

23 $|3x-9|>0\ \forall x$ except when $3x-9=0$, or $x=3$. The solution is $(-\infty,3)\cup(3,\infty)$.

24 $|5x+2|=0$ if $x=-\frac{2}{5}$, but is never less than 0. Thus, $|5x+2|\le 0$ has solution $x=-\frac{2}{5}$.

25 $-2<|x|<4 \quad\Rightarrow\quad -2<|x|$ *and* $|x|<4$. Since -2 is always less than $|x|$ {because $|x|\ge 0$},
we only need to consider $|x|<4$. $|x|<4 \quad\Rightarrow\quad -4<x<4 \quad\Leftrightarrow\quad (-4,4)$

26 $1<|x|<5 \quad\Rightarrow\quad 1<x<5$ *or* $1<-x<5 \quad\Rightarrow\quad 1<x<5$ or $-1>x>-5 \quad\Rightarrow$
$1<x<5$ or $-5<x<-1 \quad\Leftrightarrow\quad (-5,-1)\cup(1,5)$

27 $(3x+1)(5-10x)>0$ • See the sign chart for details concerning the signs of the individual factors and the resulting sign. The given inequality has solutions in the interval $\left(-\frac{1}{3},\frac{1}{2}\right)$, which corresponds to the positive values for the **Resulting sign**.

Interval	$\left(-\infty,-\frac{1}{3}\right)$	$\left(-\frac{1}{3},\frac{1}{2}\right)$	$\left(\frac{1}{2},\infty\right)$
Sign of $5-10x$	$+$	$+$	$-$
Sign of $3x+1$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

28 $(x+2)(x-1)(4-x)\le 0$ • From the chart, we see the product is negative for $x\in(-2,1)\cup(4,\infty)$. Since we want to also include the values that make the product equal to zero {-2, 1, and 4}, the solution is $[-2,1]\cup[4,\infty)$.

Interval	$(-\infty,-2)$	$(-2,1)$	$(1,4)$	$(4,\infty)$
Sign of $4-x$	$+$	$+$	$+$	$-$
Sign of $x-1$	$-$	$-$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$

29 $x^2-x-6<0 \quad\Rightarrow\quad (x-3)(x+2)<0$ ★ $(-2,3)$

Interval	$(-\infty,-2)$	$(-2,3)$	$(3,\infty)$
Sign of $x-3$	$-$	$-$	$+$
Sign of $x+2$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

30 $x^2 + 4x + 3 \geq 0 \quad \Rightarrow \quad (x+1)(x+3) \geq 0$ ★ $(-\infty, -3] \cup [-1, \infty)$

Interval	$(-\infty, -3)$	$(-3, -1)$	$(-1, \infty)$
Sign of $x+1$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

31 $x(2x+3) \geq 5 \quad \Rightarrow \quad 2x^2 + 3x - 5 \geq 0 \quad \Rightarrow \quad (2x+5)(x-1) \geq 0$ ★ $\left(-\infty, -\frac{5}{2}\right] \cup \left[1, \infty\right)$

Interval	$\left(-\infty, -\frac{5}{2}\right)$	$\left(-\frac{5}{2}, 1\right)$	$(1, \infty)$
Sign of $x-1$	$-$	$-$	$+$
Sign of $2x+5$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

32 $8x - 15 > x^2 \quad \Rightarrow \quad x^2 - 8x + 15 < 0 \quad \Rightarrow \quad (x-3)(x-5) < 0$ ★ $(3, 5)$

Interval	$(-\infty, 3)$	$(3, 5)$	$(5, \infty)$
Sign of $x-5$	$-$	$-$	$+$
Sign of $x-3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

Note: Solving $x^2 < a^2$ or $x^2 > a^2$ for $a > 0$ may be achieved using factoring, that is, $x^2 - a^2 < 0 \quad \Rightarrow \quad (x+a)(x-a) < 0 \quad \Rightarrow \quad -a < x < a$; or by taking the square root of each side, that is, $\sqrt{x^2} < \sqrt{a^2} \quad \Rightarrow \quad |x| < a \quad \Rightarrow \quad -a < x < a$. The most common mistake is forgetting that $\sqrt{x^2} = |x|$.

33 $25x^2 - 16 < 0 \quad \Rightarrow \quad x^2 < \frac{16}{25} \quad \Rightarrow \quad |x| < \frac{4}{5} \quad \Rightarrow \quad -\frac{4}{5} < x < \frac{4}{5} \quad \Leftrightarrow \quad \left(-\frac{4}{5}, \frac{4}{5}\right)$

34 $25x^2 - 16x < 0 \quad \Rightarrow \quad x(25x - 16) < 0$ ★ $\left(0, \frac{16}{25}\right)$

Interval	$(-\infty, 0)$	$\left(0, \frac{16}{25}\right)$	$\left(\frac{16}{25}, \infty\right)$
Sign of $25x - 16$	$-$	$-$	$+$
Sign of x	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

35 $\dfrac{x^2(x+2)}{(x+2)(x+1)} \leq 0 \quad \Rightarrow \quad \dfrac{x^2}{x+1} \leq 0$ {we will exclude $x = -2$ since it makes the original expression undefined} $\Rightarrow \quad \dfrac{1}{x+1} \leq 0$ {we can divide by x^2 since $x^2 \geq 0$ and we will include $x = 0$ since it makes x^2 equal to zero and we want all solutions less than *or equal to* zero} $\Rightarrow \quad x + 1 < 0$ {the fraction cannot equal zero and $x + 1$ must be negative so that the fraction is negative} $\Rightarrow \quad x < -1$ ★ $(-\infty, -2) \cup (-2, -1) \cup \{0\}$

36 $\dfrac{(x^2+1)(x-3)}{x^2 - 9} \geq 0 \quad \Rightarrow \quad \dfrac{x-3}{(x+3)(x-3)} \geq 0$ $\{x^2 + 1 > 0\}$ $\Rightarrow \quad \dfrac{1}{x+3} \geq 0$ {exclude 3} $\Rightarrow$ $x + 3 > 0$ {exclude -3} $\Rightarrow \quad x > -3$ ★ $(-3, 3) \cup (3, \infty)$

37 $\dfrac{x^2 - x}{x^2 + 2x} \leq 0 \quad \Rightarrow \quad \dfrac{x(x-1)}{x(x+2)} \leq 0 \quad \Rightarrow \quad \dfrac{x-1}{x+2} \leq 0$ {we will exclude $x = 0$ from the solution} ★ $(-2, 0) \cup (0, 1]$

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $x-1$	$-$	$-$	$+$
Sign of $x+2$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

38 $\dfrac{(x+3)^2(2-x)}{(x+4)(x^2-4)} \le 0 \quad \Rightarrow \quad \dfrac{2-x}{(x+4)(x+2)(x-2)} \le 0$ {include -3} $\quad \Rightarrow \quad \dfrac{1}{(x+4)(x+2)} \ge 0$ {cancel, change inequality, exclude 2} ★ $(-\infty,-4) \cup \{-3\} \cup (-2,2) \cup (2,\infty)$

Interval	$(-\infty,-4)$	$(-4,-2)$	$(-2,\infty)$
Sign of $x+2$	$-$	$-$	$+$
Sign of $x+4$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

39 $\dfrac{x-2}{x^2-3x-10} \ge 0 \quad \Rightarrow \quad \dfrac{x-2}{(x-5)(x+2)} \ge 0$ {$x=2$ is a solution since it makes the fraction equal to zero, $x=5$ and $x=-2$ are excluded since these values make the fraction undefined} ★ $[-2,2) \cup (5,\infty)$

Interval	$(-\infty,-2)$	$(-2,2)$	$(2,5)$	$(5,\infty)$
Sign of $x-5$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

40 $\dfrac{x+6}{x^2-7x+12} \le 0 \quad \Rightarrow \quad \dfrac{x+6}{(x-3)(x-4)} \le 0$ ★ $(-\infty,-6] \cup (3,4)$

Interval	$(-\infty,-6)$	$(-6,3)$	$(3,4)$	$(4,\infty)$
Sign of $x-4$	$-$	$-$	$-$	$+$
Sign of $x-3$	$-$	$-$	$+$	$+$
Sign of $x+6$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

41 $\dfrac{-3x}{x^2-9} > 0 \quad \Rightarrow \quad \dfrac{x}{(x+3)(x-3)} < 0$ {divide by -3} ★ $(-\infty,-3) \cup (0,3)$

Interval	$(-\infty,-3)$	$(-3,0)$	$(0,3)$	$(3,\infty)$
Sign of $x-3$	$-$	$-$	$-$	$+$
Sign of x	$-$	$-$	$+$	$+$
Sign of $x+3$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

42 $\dfrac{5x}{16-x^2} < 0 \quad \Rightarrow \quad \dfrac{x}{(4+x)(4-x)} < 0$ {divide by 5} ★ $(-4,0) \cup (4,\infty)$

Interval	$(-\infty,-4)$	$(-4,0)$	$(0,4)$	$(4,\infty)$
Sign of $4-x$	$+$	$+$	$+$	$-$
Sign of x	$-$	$-$	$+$	$+$
Sign of $4+x$	$-$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$

43 $\dfrac{x+1}{2x-3} > 2 \quad \Rightarrow \quad \dfrac{x+1}{2x-3} - 2 > 0 \quad \Rightarrow \quad \dfrac{x+1-2(2x-3)}{2x-3} > 0 \quad \Rightarrow \quad \dfrac{x+1-4x+6}{2x-3} > 0 \quad \Rightarrow$ $\dfrac{-3x+7}{2x-3} > 0$. From the sign chart, the solution is $\left(\frac{3}{2},\frac{7}{3}\right)$. Note that you should *not* multiply by the factor $2x-3$ as we did with rational *equations* because $2x-3$ may be positive or negative, and multiplying by it would require solving two inequalities. This method of solution tends to be more difficult than the sign chart method.

Interval	$\left(-\infty,\frac{3}{2}\right)$	$\left(\frac{3}{2},\frac{7}{3}\right)$	$\left(\frac{7}{3},\infty\right)$
Sign of $-3x+7$	$+$	$+$	$-$
Sign of $2x-3$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

44 $\dfrac{x-2}{3x+5} \le 4 \;\Rightarrow\; \dfrac{x-2-4(3x+5)}{3x+5} \le 0 \;\Rightarrow\; \dfrac{-11x-22}{3x+5} \le 0$ ★ $(-\infty,-2] \cup \left(-\frac{5}{3},\infty\right)$

Interval	$(-\infty,-2)$	$\left(-2,-\frac{5}{3}\right)$	$\left(-\frac{5}{3},\infty\right)$
Sign of $3x+5$	$-$	$-$	$+$
Sign of $-11x-22$	$+$	$-$	$-$
Resulting sign	$-$	$+$	$-$

45 $\dfrac{1}{x-2} \ge \dfrac{3}{x+1} \;\Rightarrow\; \dfrac{1}{x-2} - \dfrac{3}{x+1} \ge 0 \;\Rightarrow\; \dfrac{1(x+1)-3(x-2)}{(x-2)(x+1)} \ge 0 \;\Rightarrow$
$\dfrac{x+1-3x+6}{(x-2)(x+1)} \ge 0 \;\Rightarrow\; \dfrac{-2x+7}{(x-2)(x+1)} \ge 0$ ★ $(-\infty,-1) \cup \left(2,\frac{7}{2}\right]$

Interval	$(-\infty,-1)$	$(-1,2)$	$\left(2,\frac{7}{2}\right)$	$\left(\frac{7}{2},\infty\right)$
Sign of $-2x+7$	$+$	$+$	$+$	$-$
Sign of $x-2$	$-$	$-$	$+$	$+$
Sign of $x+1$	$-$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$

46 $\dfrac{2}{2x+3} \le \dfrac{2}{x-5} \;\Rightarrow\; \dfrac{2(x-5)-2(2x+3)}{(2x+3)(x-5)} \le 0 \;\Rightarrow\; \dfrac{-2x-16}{(2x+3)(x-5)} \le 0$ ★ $\left[-8,-\frac{3}{2}\right) \cup (5,\infty)$

Interval	$(-\infty,-8)$	$\left(-8,-\frac{3}{2}\right)$	$\left(-\frac{3}{2},5\right)$	$(5,\infty)$
Sign of $x-5$	$-$	$-$	$-$	$+$
Sign of $2x+3$	$-$	$-$	$+$	$+$
Sign of $-2x-16$	$+$	$-$	$-$	$-$
Resulting sign	$+$	$-$	$+$	$-$

47 $\dfrac{x}{3x-5} \le \dfrac{2}{x-1} \;\Rightarrow\; \dfrac{x}{3x-5} - \dfrac{2}{x-1} \le 0 \;\Rightarrow\; \dfrac{x(x-1)-2(3x-5)}{(3x-5)(x-1)} \le 0 \;\Rightarrow$
$\dfrac{x^2-x-6x+10}{(3x-5)(x-1)} \le 0 \;\Rightarrow\; \dfrac{x^2-7x+10}{(3x-5)(x-1)} \le 0 \;\Rightarrow\; \dfrac{(x-2)(x-5)}{(3x-5)(x-1)} \le 0$ ★ $\left(1,\frac{5}{3}\right) \cup [2,5]$

Interval	$(-\infty,1)$	$\left(1,\frac{5}{3}\right)$	$\left(\frac{5}{3},2\right)$	$(2,5)$	$(5,\infty)$
Sign of $x-5$	$-$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$-$	$+$	$+$
Sign of $3x-5$	$-$	$-$	$+$	$+$	$+$
Sign of $x-1$	$-$	$+$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$	$+$

48 $\dfrac{x}{2x-1} \ge \dfrac{3}{x+2} \;\Rightarrow\; \dfrac{x}{2x-1} - \dfrac{3}{x+2} \ge 0 \;\Rightarrow\; \dfrac{x(x+2)-3(2x-1)}{(2x-1)(x+2)} \ge 0 \;\Rightarrow$
$\dfrac{x^2+2x-6x+3}{(2x-1)(x+2)} \ge 0 \;\Rightarrow\; \dfrac{x^2-4x+3}{(2x-1)(x+2)} \ge 0 \;\Rightarrow\; \dfrac{(x-1)(x-3)}{(2x-1)(x+2)} \ge 0$ ★ $(-\infty,-2) \cup \left(\frac{1}{2},1\right] \cup [3,\infty)$

Interval	$(-\infty,-2)$	$\left(-2,\frac{1}{2}\right)$	$\left(\frac{1}{2},1\right)$	$(1,3)$	$(3,\infty)$
Sign of $x-3$	$-$	$-$	$-$	$-$	$+$
Sign of $x-1$	$-$	$-$	$-$	$+$	$+$
Sign of $2x-1$	$-$	$-$	$+$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$	$+$

49 $x^3 > x \Rightarrow x^3 - x > 0 \Rightarrow x(x^2-1) > 0 \Rightarrow x(x+1)(x-1) > 0$ ★ $(-1,0) \cup (1,\infty)$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
Sign of $x-1$	$-$	$-$	$-$	$+$
Sign of x	$-$	$-$	$+$	$+$
Sign of $x+1$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

50 $x^4 \geq x^2 \Rightarrow x^4 - x^2 \geq 0 \Rightarrow x^2(x^2-1) \geq 0 \Rightarrow x^2(x+1)(x-1) \geq 0$. Since $x^2 \geq 0$, x^2 does not need to be included in the sign chart, but 0 must be included in the answer because of the equality.

★ $(-\infty,-1] \cup \{0\} \cup [1,\infty)$

Interval	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
Sign of $x-1$	$-$	$-$	$+$
Sign of $x+1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

51 (a) $|x+5| = 3 \Rightarrow x+5 = 3$ or $x+5 = -3 \Rightarrow x = -2$ or $x = -8$.

(b) $|x+5| < 3$ has solutions between the values found in part (a), that is, $(-8,-2)$.

(c) The solutions of $|x+5| > 3$ are the portions of the real line that are not in parts (a) and (b), that is, $(-\infty,-8) \cup (-2,\infty)$.

52 (a) $|x-4| < 3 \Rightarrow -3 < x-4 < 3 \Rightarrow 1 < x < 7 \Leftrightarrow (1,7)$.

(b) $|x-4| = 3$ has solutions at the endpoints of the interval in part (a); that is, at $x = 1$ and $x = 7$.

(c) As in Exercise 51(c), $|x-4| > 3$ has solutions in $(-\infty,1) \cup (7,\infty)$.

53 We could think of this statement as "the difference between w and 141 is at most 2." In symbols, we have $|w-141| \leq 2$. Intuitively, we know that this inequality must describe the weights from 139 to 143.

54 "r must be within 0.01 centimeter of 1 centimeter" is written as $|r-1| \leq 0.01$.

55 $M = \dfrac{f}{f-p}$ • We want to know what condition will assure us that an object's image is at least 3 times as large as the object, or, equivalently, when $M \geq 3$. $M \geq 3$ {and $f = 6$} $\Rightarrow \dfrac{6}{6-p} \geq 3 \Rightarrow 6 \geq 18 - 3p$ {since $6 - p > 0$, we can multiply by $6-p$ and not change the direction of the inequality} $\Rightarrow 3p \geq 12 \Rightarrow p \geq 4$, but $p < 6$ since $p < f$. Thus, $4 \leq p < 6$.

56 $c = \dfrac{3.5t}{t+1}$ • $c > 1.5 \Rightarrow \dfrac{3.5t}{t+1} > 1.5 \Rightarrow \{t+1>0\}\ 3.5t > 1.5t + 1.5 \Rightarrow 2t > \dfrac{3}{2} \Rightarrow t > \dfrac{3}{4}$ hr

57 Let x denote the number of years before A becomes more economical than B.
The costs are the initial costs plus the yearly costs times the number of years.
$\text{Cost}_A < \text{Cost}_B \Rightarrow 100{,}000 + 8000x < 80{,}000 + 11{,}000x \Rightarrow 20{,}000 < 3000x \Rightarrow x > \frac{20}{3}$, or $6\frac{2}{3}$ yr.

58 Let t denote the time in years from the present. $\text{Cost}_B < \text{Cost}_A \Rightarrow$
$\text{Purchase}_B + \text{Insurance}_B + \text{Gas}_B < \text{Purchase}_A + \text{Insurance}_A + \text{Gas}_A \Rightarrow$
$24{,}000 + 1200t + \dfrac{15{,}000}{50} \cdot 3t < 20{,}000 + 1000t + \dfrac{15{,}000}{30} \cdot 3t \Rightarrow$
$24{,}000 + 2100t < 20{,}000 + 2500t \Rightarrow 4000 < 400t \Rightarrow t > 10$ yr.

59 $s > 9 \Rightarrow -16t^2 + 24t + 1 > 9 \Rightarrow -16t^2 + 24t - 8 > 0 \Rightarrow 2t^2 - 3t + 1 < 0$ {divide by -8} $\Rightarrow$ $(2t-1)(t-1) < 0$ {use a sign chart} $\Rightarrow \frac{1}{2} < t < 1$.

The dog is more than 9 ft off the ground for $1 - \frac{1}{2} = \frac{1}{2}$ sec.

60 $s \geq 1536 \Rightarrow -16t^2 + 320t \geq 1536 \Rightarrow -16t^2 + 320t - 1536 \geq 0 \Rightarrow$ $t^2 - 20t + 96 \leq 0$ {divide by -16} $\Rightarrow (t-8)(t-12) \leq 0$ {use a sign chart} $\Leftrightarrow 8 \leq t \leq 12$

61 $d < 75 \Rightarrow v + \frac{1}{20}v^2 < 75$ {multiply by 20} $\Rightarrow 20v + v^2 < 1500 \Rightarrow v^2 + 20v - 1500 < 0 \Rightarrow$ $(v+50)(v-30) < 0$ {use a sign chart} $\Rightarrow -50 < v < 30 \Rightarrow 0 \leq v < 30$ {since $v \geq 0$}

62 $M \geq 45 \Rightarrow -\frac{1}{30}v^2 + \frac{5}{2}v \geq 45$ {multiply by -30} $\Rightarrow v^2 - 75v \leq -1350 \Rightarrow$ $v^2 - 75v + 1350 \leq 0 \Rightarrow (v-30)(v-45) \leq 0$ {use a sign chart} $\Leftrightarrow 30 \leq v \leq 45$

63 (a) 5 ft 9 in $= 69$ in. In a 40 year period, a person's height will decrease by $40 \times 0.024 = 0.96$ in ≈ 1 in. The person will be approximately one inch shorter, or 5 ft 8 in. at age 70.

(b) 5 ft 6 in $= 66$ in. In 20 years, a person's height $(h = 66)$ will change by $0.024 \times 20 = 0.48$ in. Thus, $66 - 0.48 \leq h \leq 66 + 0.48 \Rightarrow 65.52 \leq h \leq 66.48$.

64 $7500 \leq W \leq 10{,}000 \Rightarrow 7500 \leq 0.00334V^2S \leq 10{,}000 \Rightarrow 7500 \leq 0.00334(210)V^2 \leq 10{,}000 \Rightarrow$ $\frac{7500}{0.7014} \leq V^2 \leq \frac{10{,}000}{0.7014} \Rightarrow \sqrt{\frac{7500}{0.7014}} \leq V \leq \sqrt{\frac{10{,}000}{0.7014}} \Rightarrow 103.4 \leq V \leq 119.4$ {in ft/sec}. To convert ft/sec to mi/hr, multiply by $\frac{60}{88}$ or $\frac{15}{22}$, which are reduced forms of $\frac{1 \text{ foot}}{1 \text{ second}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}}$. Using the approximations in ft/sec, we get $103.4 \leq V \leq 119.4$ {in ft/sec} $\Rightarrow 70.5 \leq V \leq 81.4$ {in mi/hr}.

65 The numerator is equal to zero when $x = 2, 3$ and the denominator is equal to zero when $x = \pm 1$. From the table, the expression $Y_1 = \frac{(2-x)(3x-9)}{(1-x)(x+1)}$ is positive when $x \in [-2, -1) \cup (1, 2) \cup (3, 3.5]$. See the table on the left.

x	Y_1	x	Y_1
-2.0	20	1.0	ERROR
-1.5	37.8	1.5	1.8
-1.0	ERROR	2.0	0
-0.5	-35	2.5	-0.1429
0.0	-18	3.0	0
0.5	-15	3.5	0.2

x	Y_1	x	Y_1
-3.5	30.938	1.0	36
-3.0	0	1.5	19.688
-2.5	-7.313	2.0	0
-2.0	0	2.5	-18.56
-1.5	14.438	3.0	-30
-1.0	30	3.5	-26.81
-0.5	42.188	4.0	0
0.0	48	4.5	60.938
0.5	45.938	5.0	168

66 By using a table it can be shown that the expression is equal to zero when $x = -3, -2, 2, 4$.
The expression $Y_1 = x^4 - x^3 - 16x^2 + 4x + 48$ is negative when $x \in (-3, -2) \cup (2, 4)$. See the table on the right.

Chapter 1 Review Exercises

1 If $x \leq -3$, then $x + 3 \leq 0$, and $|x+3| = -(x+3) = -x - 3$.

2 If $2 < x < 3$, then $x - 2 > 0$ $\{x - 2$ is positive$\}$ and $x - 3 < 0$ $\{x - 3$ is negative$\}$. Thus, $(x-2)(x-3) < 0$ {positive times negative is negative}, and since the absolute value of an expression that is negative is the negative of the expression, $|(x-2)(x-3)| = -(x-2)(x-3)$, or, equivalently, $(2-x)(x-3)$.

3 $\left(\dfrac{a^{2/3}b^{3/2}}{a^2b}\right)^6 = \dfrac{a^4b^9}{a^{12}b^6} = \dfrac{b^3}{a^8}$

4 $(-2p^2q)^3\left(\dfrac{p}{4q^2}\right)^2 = (-8p^6q^3)\left(\dfrac{p^2}{16q^4}\right) = -\dfrac{p^8}{2q}$

5 $\left(\dfrac{xy^{-1}}{\sqrt{z}}\right)^4 \div \left(\dfrac{x^{1/3}y^2}{z}\right)^3 = \dfrac{x^4y^{-4}}{z^2}\cdot\dfrac{z^3}{xy^6} = \dfrac{x^3z}{y^{10}}$

6 $\left(\dfrac{-64x^3}{z^6y^9}\right)^{2/3} = \dfrac{\left(\sqrt[3]{-64}\right)^2x^2}{z^4y^6} = \dfrac{16x^2}{z^4y^6}$

7 $\left[\left(a^{2/3}b^{-2}\right)^3\right]^{-1} = (a^2b^{-6})^{-1} = a^{-2}b^6 = \dfrac{b^6}{a^2}$

8 $x^{-2} - y^{-1} = \dfrac{1}{x^2} - \dfrac{1}{y} = \dfrac{y - x^2}{x^2y}$

9 $\sqrt[3]{27x^5y^3z^4} = \sqrt[3]{27x^3y^3z^3}\,\sqrt[3]{x^2z} = 3xyz\sqrt[3]{x^2z}$

10 $\sqrt[4]{(-4a^3b^2c)^2} = \sqrt[4]{16a^6b^4c^2} = \sqrt[4]{2^4a^4b^4}\,\sqrt[4]{a^2c^2} = 2ab\,\sqrt[4]{(ac)^2} = 2ab\,\sqrt{ac}$

11 $\dfrac{1}{\sqrt{t}}\left(\dfrac{1}{\sqrt{t}} - 1\right) = \dfrac{1}{\sqrt{t}}\left(\dfrac{1}{\sqrt{t}} - \dfrac{\sqrt{t}}{\sqrt{t}}\right) = \dfrac{1}{\sqrt{t}}\left(\dfrac{1-\sqrt{t}}{\sqrt{t}}\right) = \dfrac{1-\sqrt{t}}{t}$

12 $\sqrt{\sqrt[3]{(c^3d^6)^4}} = \sqrt[6]{c^{12}d^{24}} = c^2d^4$

13 $\dfrac{\sqrt{12x^4y}}{\sqrt{3x^2y^7}} = \sqrt{\dfrac{12x^4y}{3x^2y^7}} = \sqrt{\dfrac{4x^2}{y^6}} = \dfrac{2x}{y^3}$

14 $\dfrac{3+\sqrt{x}}{3-\sqrt{x}} = \dfrac{3+\sqrt{x}}{3-\sqrt{x}}\cdot\dfrac{3+\sqrt{x}}{3+\sqrt{x}} = \dfrac{x + 6\sqrt{x} + 9}{9 - x}$

15 $(3x^3 - 4x^2 + x - 6) + (x^4 - 2x^3 + 3x^2 + 5) = x^4 + x^3 - x^2 + x - 1$

16 $(x+4)(x+3) - (2x-3)(x-5) = (x^2 + 7x + 12) - (2x^2 - 13x + 15) = -x^2 + 20x - 3$

17 $(3a - 5b)(4a + 7b) = 12a^2 + 21ab - 20ab - 35b^2 = 12a^2 + ab - 35b^2$

18 $(4r^2 - 3s)^2 = (4r^2)^2 - 2(4r^2)(3s) + (3s)^2 = 16r^4 - 24r^2s + 9s^2$

19 $(13a^2 + 5b)(13a^2 - 5b) = (13a^2)^2 - (5b)^2 = 169a^4 - 25b^2$

20 $(2a + b)^3 = (2a)^3 + 3(2a)^2(b) + 3(2a)(b)^2 + (b)^3 = 8a^3 + 12a^2b + 6ab^2 + b^3$

21 $(3x + 2y)^2(3x - 2y)^2 = [(3x + 2y)(3x - 2y)]^2 = (9x^2 - 4y^2)^2 = 81x^4 - 72x^2y^2 + 16y^4$

22 $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$

23 $60xw + 50w = 10w(6x + 5)$

24 $16a^4 + 24a^2b^2 + 9b^4 = (4a^2 + 3b^2)(4a^2 + 3b^2) = (4a^2 + 3b^2)^2$

25 $8x^3 + 64y^3 = 8(x^3 + 8y^3) = 8\left[(x)^3 + (2y)^3\right] = 8(x + 2y)(x^2 - 2xy + 4y^2)$

26 $u^3v^4 - u^6v = u^3v(v^3 - u^3) = u^3v(v - u)(v^2 + uv + u^2)$

27 $p^8 - q^8 = (p^4)^2 - (q^4)^2 = (p^4 + q^4)(p^4 - q^4) = (p^4 + q^4)(p^2 + q^2)(p^2 - q^2)$
$= (p^4 + q^4)(p^2 + q^2)(p + q)(p - q)$

28 $x^4 - 12x^3 + 36x^2 = x^2(x^2 - 12x + 36) = x^2(x - 6)(x - 6) = x^2(x - 6)^2$

29 $x^2 - 49y^2 - 14x + 49 = (x^2 - 14x + 49) - 49y^2 = (x - 7)^2 - (7y)^2 = (x - 7 + 7y)(x - 7 - 7y)$

30 $x^5 - 4x^3 + 8x^2 - 32 = x^3(x^2 - 4) + 8(x^2 - 4) = (x^3 + 8)(x^2 - 4)$
$= \left[(x + 2)(x^2 - 2x + 4)\right]\left[(x + 2)(x - 2)\right] = (x - 2)(x + 2)^2(x^2 - 2x + 4)$

31 $\dfrac{6}{4x-5}-\dfrac{15}{10x+1}=\dfrac{6(10x+1)-15(4x-5)}{(4x-5)(10x+1)}=\dfrac{60x+6-60x+75}{(4x-5)(10x+1)}=\dfrac{81}{(4x-5)(10x+1)}$

32 $\dfrac{7}{x+2}+\dfrac{3x}{(x+2)^2}-\dfrac{5}{x}=\dfrac{7(x)(x+2)+3x(x)-5(x+2)^2}{x(x+2)^2}=\dfrac{7x^2+14x+3x^2-5x^2-20x-20}{x(x+2)^2}=$

$$\dfrac{5x^2-6x-20}{x(x+2)^2}$$

33 $\dfrac{x+x^{-2}}{1+x^{-2}}=\dfrac{x+\dfrac{1}{x^2}}{1+\dfrac{1}{x^2}}=\dfrac{\left(x+\dfrac{1}{x^2}\right)\cdot x^2}{\left(1+\dfrac{1}{x^2}\right)\cdot x^2}=\dfrac{x^3+1}{x^2+1}$. We could factor the numerator,

but since it doesn't lead to a reduction of the fraction, we leave it in this form.

34 $(a^{-1}+b^{-1})^{-1}=\left(\dfrac{1}{a}+\dfrac{1}{b}\right)^{-1}=\left(\dfrac{b+a}{ab}\right)^{-1}=\left(\dfrac{ab}{a+b}\right)^{1}=\dfrac{ab}{a+b}$

35 $\dfrac{\dfrac{x}{x+2}-\dfrac{4}{x+2}}{x-3-\dfrac{6}{x+2}}=\dfrac{\dfrac{x-4}{x+2}}{\dfrac{(x-3)(x+2)-6}{x+2}}=\dfrac{x-4}{(x^2-x-6)-6}=\dfrac{x-4}{x^2-x-12}=\dfrac{x-4}{(x+3)(x-4)}=\dfrac{1}{x+3}$

36 $\dfrac{(4-x^2)\left(\frac{1}{3}\right)(6x+1)^{-2/3}(6)-(6x+1)^{1/3}(-2x)}{(4-x^2)^2}=\dfrac{2(6x+1)^{-2/3}[(4-x^2)+x(6x+1)]}{(4-x^2)^2}$

$$=\dfrac{2(4-x^2+6x^2+x)}{(6x+1)^{2/3}(4-x^2)^2}=\dfrac{2(5x^2+x+4)}{(6x+1)^{2/3}(4-x^2)^2}$$

37 $\left[\dfrac{3x+1}{5x+7}=\dfrac{6x+11}{10x-3}\right]\cdot(5x+7)(10x-3)\quad\Rightarrow\quad(3x+1)(10x-3)=(6x+11)(5x+7)\quad\Rightarrow$

$$30x^2+x-3=30x^2+97x+77\quad\Rightarrow\quad-96x=80\quad\Rightarrow\quad x=-\tfrac{5}{6}$$

38 $2x^2+7x-15=0\quad\Rightarrow\quad(x+5)(2x-3)=0\quad\Rightarrow\quad x=-5,\tfrac{3}{2}$

39 $x(3x+4)=2\quad\Rightarrow\quad3x^2+4x-2=0\quad\Rightarrow\quad x=\dfrac{-4\pm\sqrt{16+24}}{6}=\dfrac{-4\pm2\sqrt{10}}{6}=-\dfrac{2}{3}\pm\dfrac{1}{3}\sqrt{10}$

40 $4x^4-37x^2+75=0\quad\Rightarrow\quad(4x^2-25)(x^2-3)\quad\Rightarrow\quad x^2=\tfrac{25}{4},3\quad\Rightarrow\quad x=\pm\tfrac{5}{2},\pm\sqrt{3}$

41 $20x^3+8x^2-55x-22=0\quad\Rightarrow\quad4x^2(5x+2)-11(5x+2)=0\quad\Rightarrow$

$$(4x^2-11)(5x+2)=0\quad\Rightarrow\quad x^2=\tfrac{11}{4}\text{ or }x=-\tfrac{2}{5}\quad\Rightarrow\quad x=\pm\tfrac{1}{2}\sqrt{11},-\tfrac{2}{5}$$

42 $|4x-1|=7\quad\Rightarrow\quad4x-1=7$ or $4x-1=-7\quad\Rightarrow\quad4x=8$ or $4x=-6\quad\Rightarrow\quad x=2$ or $x=-\tfrac{3}{2}$

43 $2|2x+1|+1=15\quad\Rightarrow\quad2|2x+1|=14\quad\Rightarrow\quad|2x+1|=7\quad\Rightarrow$

$2x+1=7$ or $2x+1=-7\quad\Rightarrow\quad2x=6$ or $2x=-8\quad\Rightarrow\quad x=3$ or $x=-4$

44 $\left[\dfrac{1}{x}+6=\dfrac{5}{\sqrt{x}}\right]\cdot x\quad\Rightarrow\quad1+6x=5\sqrt{x}\quad\Rightarrow$

$6x-5\sqrt{x}+1=0$ {factoring or substituting would be appropriate} $\Rightarrow$

$$\left(2\sqrt{x}-1\right)\left(3\sqrt{x}-1\right)=0\quad\Rightarrow\quad\sqrt{x}=\tfrac{1}{2},\tfrac{1}{3}\quad\Rightarrow\quad x=\left(\tfrac{1}{2}\right)^2,\left(\tfrac{1}{3}\right)^2\quad\Rightarrow\quad x=\tfrac{1}{4},\tfrac{1}{9}$$

Check $x=\frac{1}{4}$: LS $=4+6=10$; RS $=5/\frac{1}{2}=10\quad\Rightarrow\quad x=\frac{1}{4}$ is a solution.

Check $x=\frac{1}{9}$: LS $=9+6=15$; RS $=5/\frac{1}{3}=15\quad\Rightarrow\quad x=\frac{1}{9}$ is a solution.

45 $\sqrt{7x+2}+x=6\quad\Rightarrow\quad\left(\sqrt{7x+2}\right)^2=(6-x)^2\quad\Rightarrow\quad7x+2=36-12x+x^2\quad\Rightarrow$

$x^2-19x+34=0\quad\Rightarrow\quad(x-2)(x-17)=0\quad\Rightarrow\quad x=2$ and 17 is an extraneous solution.

46 $\sqrt{3x+1} - \sqrt{x+4} = 1 \Rightarrow \sqrt{3x+1} = 1 + \sqrt{x+4} \Rightarrow \left(\sqrt{3x+1}\right)^2 = \left(1+\sqrt{x+4}\right)^2 \Rightarrow$
$3x+1 = 1 + 2\sqrt{x+4} + x + 4 \Rightarrow 2\sqrt{x+4} = 2x-4 \Rightarrow \sqrt{x+4} = x-2 \Rightarrow$
$\left(\sqrt{x+4}\right)^2 = (x-2)^2 \Rightarrow x+4 = x^2-4x+4 \Rightarrow x^2-5x=0 \Rightarrow x(x-5)=0 \Rightarrow x = 0, 5.$

Check $x=0$: LS $= 1-2 = -1 \neq$ RS $\Rightarrow$ $x=0$ is an extraneous solution.

Check $x=5$: LS $= 4-3 = 1 =$ RS $\Rightarrow$ $x=5$ is a solution.

47 $10-7x < 4+8x \Rightarrow -15x < -6 \Rightarrow x > \frac{2}{5} \Leftrightarrow \left(\frac{2}{5}, \infty\right)$

48 $\left[-\frac{1}{2} < \frac{2x+3}{5} < \frac{3}{2}\right] \cdot 10 \Rightarrow -5 < 4x+6 < 15 \Rightarrow -11 < 4x < 9 \Rightarrow -\frac{11}{4} < x < \frac{9}{4} \Leftrightarrow \left(-\frac{11}{4}, \frac{9}{4}\right)$

49 $\frac{7}{10x+3} < 0 \Rightarrow 10x+3 < 0$ {since $7 > 0$} $\Rightarrow x < -\frac{3}{10} \Leftrightarrow \left(-\infty, -\frac{3}{10}\right)$

50 $|4x+7| < 21 \Rightarrow -21 < 4x+7 < 21 \Rightarrow -28 < 4x < 14 \Rightarrow -7 < x < \frac{7}{2} \Leftrightarrow \left(-7, \frac{7}{2}\right)$

51 $2|3-x|+1 > 5 \Rightarrow 2|3-x| > 4 \Rightarrow |3-x| > 2 \Rightarrow 3-x > 2$ or $3-x < -2 \Rightarrow$
$1 > x$ or $5 < x \Rightarrow x < 1$ or $x > 5 \Leftrightarrow (-\infty, 1) \cup (5, \infty)$

52 $|16-3x| \geq 5 \Rightarrow 16-3x \geq 5$ or $16-3x \leq -5 \Rightarrow -3x \geq -11$ or $-3x \leq -21 \Rightarrow$
$x \leq \frac{11}{3}$ or $x \geq 7 \Leftrightarrow \left(-\infty, \frac{11}{3}\right] \cup [7, \infty)$

53 $10x^2 + 11x > 6 \Rightarrow 10x^2 + 11x - 6 > 0 \Rightarrow (2x+3)(5x-2) > 0$ ★ $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{2}{5}, \infty\right)$

Interval	$\left(-\infty, -\frac{3}{2}\right)$	$\left(-\frac{3}{2}, \frac{2}{5}\right)$	$\left(\frac{2}{5}, \infty\right)$
Sign of $5x-2$	$-$	$-$	$+$
Sign of $2x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

54 $x(x-3) \leq 18 \Rightarrow x^2 - 3x - 18 \leq 0 \Rightarrow (x-6)(x+3) \leq 0$ ★ $[-3, 6]$

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x-6$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

55 $\frac{x^2(3-x)}{x+2} \leq 0 \Rightarrow \frac{3-x}{x+2} \leq 0$ $\{x^2 \geq 0$, include $0\}$ ★ $(-\infty, -2) \cup \{0\} \cup [3, \infty)$

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Sign of $3-x$	$+$	$+$	$-$
Sign of $x+2$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

56 $\frac{x^2-x-2}{x^2+4x+3} \leq 0 \Rightarrow \frac{(x-2)(x+1)}{(x+1)(x+3)} \leq 0 \Rightarrow \frac{x-2}{x+3} \leq 0$ {exclude -1} ★ $(-3, -1) \cup (-1, 2]$

Interval	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
Sign of $x-2$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

57 $\dfrac{3}{2x+3} < \dfrac{1}{x-2} \Rightarrow \dfrac{3}{2x+3} - \dfrac{1}{x-2} < 0 \Rightarrow \dfrac{3(x-2)-1(2x+3)}{(2x+3)(x-2)} < 0 \Rightarrow$
$\dfrac{3x-6-2x-3}{(2x+3)(x-2)} < 0 \Rightarrow \dfrac{x-9}{(2x+3)(x-2)} < 0$ ★ $\left(-\infty, -\frac{3}{2}\right) \cup (2, 9)$

Interval	$\left(-\infty, -\frac{3}{2}\right)$	$\left(-\frac{3}{2}, 2\right)$	$(2, 9)$	$(9, \infty)$
Sign of $x-9$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$+$	$+$
Sign of $2x+3$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

58 $\dfrac{x+2}{x^2-25} \leq 0 \Rightarrow \dfrac{x+2}{(x+5)(x-5)} \leq 0$ ★ $(-\infty, -5) \cup [-2, 5)$

Interval	$(-\infty, -5)$	$(-5, -2)$	$(-2, 5)$	$(5, \infty)$
Sign of $x-5$	$-$	$-$	$-$	$+$
Sign of $x+2$	$-$	$-$	$+$	$+$
Sign of $x+5$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

59 $x^3 > x^2 \Rightarrow x^2(x-1) > 0 \; \{x^2 \geq 0\} \Rightarrow x-1 > 0 \Rightarrow x > 1 \Leftrightarrow (1, \infty)$

60 $(x^2-x)(x^2-5x+6) < 0 \Rightarrow x(x-1)(x-2)(x-3) < 0$ ★ $(0, 1) \cup (2, 3)$

Interval	$(-\infty, 0)$	$(0, 1)$	$(1, 2)$	$(2, 3)$	$(3, \infty)$
Sign of $x-3$	$-$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$-$	$+$	$+$
Sign of $x-1$	$-$	$-$	$+$	$+$	$+$
Sign of x	$-$	$+$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$	$+$

61 $P + N = \dfrac{C+2}{C} \Rightarrow C(P+N) = C+2 \Rightarrow CP + CN = C + 2 \Rightarrow$
$CP + CN - C = 2 \Rightarrow C(P+N-1) = 2 \Rightarrow C = \dfrac{2}{P+N-1}$

62 $A = B\sqrt[3]{\dfrac{C}{D}} - E \Rightarrow A + E = B\sqrt[3]{\dfrac{C}{D}} \Rightarrow \dfrac{A+E}{B} = \sqrt[3]{\dfrac{C}{D}} \Rightarrow \left(\dfrac{A+E}{B}\right)^3 = \dfrac{C}{D} \Rightarrow$
$D \cdot \dfrac{(A+E)^3}{B^3} = C \Rightarrow D(A+E)^3 = C \cdot B^3 \Rightarrow D = \dfrac{CB^3}{(A+E)^3}$

63 $F = \dfrac{\pi P R^4}{8VL} \Rightarrow R^4 = \dfrac{8FVL}{\pi P} \Rightarrow R = \pm\sqrt[4]{\dfrac{8FVL}{\pi P}} \Rightarrow R = \sqrt[4]{\dfrac{8FVL}{\pi P}}$ since $R > 0$

64 $V = \frac{1}{3}\pi h(r^2 + R^2 + rR) \Rightarrow 3V = \pi h(r^2 + R^2 + rR) \Rightarrow (\pi h)r^2 + (\pi hR)r + (\pi hR^2 - 3V) = 0 \Rightarrow$
$r = \dfrac{-(\pi hR) \pm \sqrt{(\pi hR)^2 - 4(\pi h)(\pi hR^2 - 3V)}}{2(\pi h)} = \dfrac{-\pi hR \pm \sqrt{12\pi hV - 3\pi^2h^2R^2}}{2\pi h}.$
Since $r > 0$, we must use the plus sign, and $r = \dfrac{-\pi hR + \sqrt{12\pi hV - 3\pi^2h^2R^2}}{2\pi h}$.

65 $(5+8i)^2 = 5^2 + 2(5)(8i) + (8i)^2 = (25-64) + 80i = -39 + 80i$

66 $\dfrac{1}{9-\sqrt{-4}} = \dfrac{1}{9-2i} = \dfrac{1}{9-2i} \cdot \dfrac{9+2i}{9+2i} = \dfrac{9+2i}{81+4} = \dfrac{9}{85} + \dfrac{2}{85}i$

67 $\dfrac{6-3i}{2+7i}=\dfrac{6-3i}{2+7i}\cdot\dfrac{2-7i}{2-7i}=\dfrac{(12-21)+(-42-6)i}{4+49}=-\dfrac{9}{53}-\dfrac{48}{53}i$

68 $\dfrac{24-8i}{4i}=\dfrac{4(6-2i)}{4i}=\dfrac{6-2i}{i}\cdot\dfrac{-i}{-i}=\dfrac{-6i+2i^2}{-i^2}=\dfrac{-2-6i}{1}=-2-6i$

69 Let P denote the principal that will be invested, and r the yield rate of the stock fund.
$\text{Income}_{\text{stocks}}-28\%\text{ federal tax}-7\%\text{ state tax}=\text{Income}_{\text{bonds}}\Rightarrow$
$(Pr)-0.28(Pr)-0.07(Pr)=0.07186P$ {divide by P} $\Rightarrow 1r-0.28r-0.07r=0.07186\Rightarrow$
$0.65r=0.07186\Rightarrow r=\frac{0.07186}{0.65}\Rightarrow r\approx 0.11055$, or, 11.055%.

70 Let x denote the number of cm^3 of gold. $\text{Grams}_{\text{gold}}+\text{Grams}_{\text{silver}}=\text{Grams}_{\text{total}}\Rightarrow$
$x(19.3)+(5-x)(10.5)=80\Rightarrow 19.3x+52.5-10.5x=80\Rightarrow 8.8x=27.5\Rightarrow x=3.125$.
The number of grams of gold is $19.3x=60.3125\approx 60.3$.

71 Let x denote the number of ounces of the vegetable portion, $10-x$ the number of ounces of meat.
$\text{Protein}_{\text{vegetable}}+\text{Protein}_{\text{meat}}=\text{Protein}_{\text{total}}\Rightarrow \frac{1}{2}(x)+1(10-x)=7\Rightarrow \frac{1}{2}x+10-x=7\Rightarrow$
$-\frac{1}{2}x=-3\Rightarrow x=6$. Use 6 oz of vegetables and 4 oz of meat.

72 Let x denote the number of gallons of 20% solution, $120-x$ the number of gallons of 50% solution.
$20(x)+50(120-x)=30(120)$ {all in %} $\Rightarrow 20x+6000-50x=3600\Rightarrow 2400=30x\Rightarrow x=80$.
Use 80 gal of the 20% solution and 40 gal of the 50% solution.

73 Let $x=$ the amount of copper they have to mix with 140 kg of zinc to make brass.
$\text{Copper}_{\text{amount put in}}=\text{Copper}_{\text{amount in final product}}\Rightarrow$
$x=0.65(x+140)\Rightarrow x=0.65x+91\Rightarrow 0.35x=91\Rightarrow x=\frac{91}{0.35}=260$ kg

74 Let x denote the number of hours needed to fill an empty bin.
Using the hourly rates, $\left[\dfrac{1}{2}-\dfrac{1}{5}=\dfrac{1}{x}\right]\cdot 10x\Rightarrow 5x-2x=10\Rightarrow 3x=10\Rightarrow$
$x=\frac{10}{3}$ hr. Since the bin was half-full at the start, $\frac{1}{2}x=\frac{1}{2}\cdot\frac{10}{3}=\frac{5}{3}$ hr, or, 1 hr 40 min.

75 (a) The eastbound car has distance $20t$ and the southbound car has distance $(-2+50t)$.
$d^2=(20t)^2+(-2+50t)^2\Rightarrow d^2=400t^2+4-200t+2500t^2\Rightarrow d=\sqrt{2900t^2-200t+4}$

(b) $104=\sqrt{2900t^2-200t+4}\Rightarrow 10{,}816=2900t^2-200t+4\Rightarrow 2900t^2-200t-10{,}812=0\Rightarrow$
$725t^2-50t-2703=0$ {divide by 4} $\Rightarrow$
$t=\dfrac{50\pm\sqrt{7{,}841{,}200}}{1450}\ \{t>0\}=\dfrac{5+2\sqrt{19{,}603}}{145}\approx 1.97$, or approximately 11:58 A.M.

76 Let l and w denote the length and width, respectively. $3l+6w=270\Rightarrow 6w=270-3l\Rightarrow w=45-\frac{1}{2}l$.
The total area is to be $10\cdot 100=1000\text{ ft}^2$. Area $=lw\Rightarrow 1000=l\left(45-\frac{1}{2}l\right)\Rightarrow 1000=45l-\frac{1}{2}l^2\Rightarrow$
$l^2-90l+2000=0\Rightarrow (l-40)(l-50)=0\Rightarrow l=40,50$ and $w=25,20$.
There are two arrangements: 40 ft $\times$ 25 ft and 50 ft $\times$ 20 ft.

77 Let x denote the length of one side of an end.

(a) $V=lwh\Rightarrow 48=6\cdot x\cdot x\Rightarrow x^2=8\Rightarrow x=2\sqrt{2}$ ft

(b) $S=lw+2wh+2lh\Rightarrow 44=6x+2(x^2)+2(6x)\Rightarrow 44=2x^2+18x\Rightarrow$
$x^2+9x-22=0\Rightarrow (x+11)(x-2)=0\Rightarrow x=2$ ft

78 $pv = 200 \quad \Rightarrow \quad v = \dfrac{200}{p}$. $25 \le v \le 50 \quad \Rightarrow \quad 25 \le \dfrac{200}{p} \le 50 \quad \Rightarrow \quad \dfrac{1}{25} \ge \dfrac{p}{200} \ge \dfrac{1}{50} \quad \Rightarrow \quad 8 \ge p \ge 4 \quad \Rightarrow$

$4 \le p \le 8$

79 Let x denote the amount of yearly business. $\text{Pay}_\text{B} > \text{Pay}_\text{A} \quad \Rightarrow \quad \$40{,}000 + 0.20x > \$50{,}000 + 0.10x \quad \Rightarrow$

$0.10x > \$10{,}000 \quad \Rightarrow \quad x > \$100{,}000$

80 $v > 1100 \quad \Rightarrow \quad 1087\sqrt{\dfrac{T}{273}} > 1100 \quad \Rightarrow \quad \sqrt{\dfrac{T}{273}} > \dfrac{1100}{1087} \quad \Rightarrow \quad \dfrac{T}{273} > \dfrac{1100^2}{1087^2} \quad \Rightarrow \quad T > \dfrac{273 \cdot 1100^2}{1087^2} \quad \Rightarrow$

$T > 279.57$ K

81 Let x denote the number of trees *over* 24. Then $24 + x$ represents the total number of trees planted per acre, and $600 - 12x$ represents the number of apples per tree.

$$\textbf{Total apples} = (\text{number of trees})(\text{number of apples per tree})$$
$$= (24 + x)(600 - 12x) = -12x^2 + 312x + 14{,}400$$

Apples $\ge 16{,}416 \quad \Rightarrow \quad -12x^2 + 312x + 14{,}400 \ge 16{,}416 \quad \Rightarrow \quad -12x^2 + 312x - 2016 \ge 0 \quad \Rightarrow$
$x^2 - 26x + 168 \le 0 \quad \Rightarrow \quad (x - 12)(x - 14) \le 0 \quad \Rightarrow \quad 12 \le x \le 14 \quad \Rightarrow \quad 36 \le 24 + x \le 38$

Hence, 36 to 38 trees per acre should be planted.

82 Let x denote the number of \$25 increases in rent. Then the number of occupied apartments is $218 - 5x$ and the rent per apartment is $940 + 25x$.

$$\textbf{Total income} = (\text{number of occupied apartments})(\text{rent per apartment})$$
$$= (218 - 5x)(940 + 25x) = -125x^2 + 750x + 204{,}920$$

Income $\ge 205{,}920 \quad \Rightarrow \quad -125x^2 + 750x + 204{,}920 \ge 205{,}920 \quad \Rightarrow \quad -125x^2 + 750x - 1000 \ge 0 \quad \Rightarrow$
$x^2 - 6x + 8 \le 0 \quad \Rightarrow \quad (x - 2)(x - 4) \le 0 \quad \Rightarrow \quad 2 \le x \le 4 \quad \Rightarrow \quad 990 \le 940 + 25x \le 1040$

Hence, the rent charged should be \$990 to \$1040.

83 The y-values are increasing slowly and can best be described by equation (3), $y = 3\sqrt{x - 0.5}$.

Chapter 1 Discussion Exercises

1 $\dfrac{\$1 \text{ in cash back}}{100 \text{ points}} \times \dfrac{1 \text{ point}}{\$10 \text{ charged}} = \dfrac{\$1 \text{ in cash back}}{\$1000 \text{ charged}} = 0.001$, or 0.1%.

2 Squaring the right side gives us $(a + b)^2 = a^2 + 2ab + b^2$. Squaring the left side gives us $a^2 + b^2$. Now $a^2 + 2ab + b^2$ will equal $a^2 + b^2$ only if $2ab = 0$. The expression $2ab$ equals zero only if either $a = 0$ or $b = 0$.

3 We first need to determine the term that needs to be added and subtracted. Since $25 = \underline{5}^2$, it makes sense to add and subtract $2 \cdot \underline{5}x = 10x$. Then we will obtain the square of a binomial—i.e.,
$(x^2 + 10x + 25) - 10x = (x + 5)^2 - 10x$. We can now factor this expression as the difference of two squares,

$$(x + 5)^2 - 10x = (x + 5)^2 - \left(\sqrt{10x}\right)^2 = \left(x + 5 + \sqrt{10x}\right)\left(x + 5 - \sqrt{10x}\right).$$

4 The expression $\dfrac{1}{x + 1}$ can be evaluated at $x = 1$, whereas the expression $\dfrac{x - 1}{x^2 - 1}$ is undefined at $x = 1$.

5 Try $\dfrac{3x^2-4x+7}{8x^2+9x-100}$ with $x=10^3$, 10^4, and 10^5. You get approximately 0.374, 0.3749, and 0.37499. The numbers seem to be getting closer to 0.375, which is the decimal representation for $\frac{3}{8}$, which is the ratio of the coefficients of the x^2 terms. In general, the quotients of this form get close to the ratio of leading coefficients as x gets larger.

6 $\dfrac{3x^2-5x-2}{x^2-4}=\dfrac{(3x+1)(x-2)}{(x+2)(x-2)}=\dfrac{3x+1}{x+2}$. Evaluating the original expression and the simplified expression with any $x \neq \pm 2$ gives us the same value. This evaluation does not prove that the expressions are equal for any value of x other than the one selected. The simplification proves that the expressions are equal for all values of x except $x=2$.

7 Follow the algebraic simplification given.

1) Write down his/her age.	Denote the age by x.
2) Multiply it by 2.	$2x$
3) Add 5.	$2x+5$
4) Multiply this sum by 50.	$50(2x+5)=100x+250$
5) Subtract 365.	$(100x+250)-365=100x-115$
6) Add his/her height (in inches).	$100x-115+y$, where y is the height
7) Add 115.	$100x-115+y+115=100x+y$

As a specific example, suppose the age is 21 and the height is 68. The number obtained by following the steps is $100x+y=2168$ and we can see that the first two digits of the result equal the age and the last two digits equal the height.

8
$$\begin{aligned}
V_{\text{out}} &= I_{\text{in}}\left(-\frac{RXi}{R-Xi}\right)=\frac{V_{\text{in}}}{Z_{\text{in}}}\left(-\frac{RXi}{R-Xi}\right) && \{\text{definition of } I_{\text{in}}\}\\
&= \frac{V_{\text{in}}}{\dfrac{R^2-X^2-3RXi}{R-Xi}}\left(-\frac{RXi}{R-Xi}\right) && \{\text{definition of } Z_{\text{in}}\}\\
&= \frac{V_{\text{in}}(R-Xi)}{R^2-X^2-3RXi}\left(-\frac{RXi}{R-Xi}\right)\\
&= -\frac{RXi}{R^2-X^2-3RXi}(V_{\text{in}})\\
&= -\frac{RRi}{R^2-R^2-3RRi}(V_{\text{in}}) && \{\text{let } X=R\}\\
&= -\frac{R^2i}{-3R^2i}(V_{\text{in}})=\tfrac{1}{3}V_{\text{in}}
\end{aligned}$$

9 We need to solve the equation $x^2-xy+y^2=0$ for x.

Use the quadratic formula with $a=1$, $b=-y$, and $c=y^2$.

$$x=\frac{-(-y)\pm\sqrt{(-y)^2-4(1)(y^2)}}{2(1)}=\frac{y\pm\sqrt{y^2-4y^2}}{2}=\frac{y\pm\sqrt{-3y^2}}{2}=\frac{y\pm|y|\sqrt{3}\,i}{2}.$$

Since this equation has imaginary solutions, x^2-xy+y^2 is not factorable over the reals.

A similar argument holds for x^2+xy+y^2.

10 The solutions are $x_1=\dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $x_2=\dfrac{-b-\sqrt{b^2-4ac}}{2a}$.

The average is $\dfrac{x_1+x_2}{2}=\dfrac{-2b/2a}{2}=-\dfrac{b}{2a}$. Suppose you solve the equation $-x^2+4x+7=0$ and obtain the solutions $x_1\approx-1.32$ and $x_2\approx 5.32$. Averaging these numbers gives us the value 2, which we can easily see is equal to $-b/(2a)$.

11 (a) $\dfrac{1}{\dfrac{a+bi}{c+di}} = \dfrac{c+di}{a+bi} \cdot \dfrac{a-bi}{a-bi} = \dfrac{ac+bd+(ad-bc)i}{a^2+b^2} = \dfrac{ac+bd}{a^2+b^2} + \dfrac{ad-bc}{a^2+b^2}\,i = p+qi$

(b) Yes, try an example such as $\frac{3}{4}$. Let $a=3$, $b=0$, $c=4$, and $d=0$. Then, from part (a),
$p+qi = \frac{12}{9} + \frac{0}{9}i = \frac{12}{9} = \frac{4}{3}$, which is the multiplicative inverse of $\frac{3}{4}$.

(c) a and b cannot both be 0 because then the denominator would be 0.

12 Since we don't know the value of x, we don't know the sign of $x-2$, and hence we are unsure of whether or not to reverse the direction of the inequality sign.

13 *Hint*: Try these examples to help you get to the general solution.

(1) $x^2+1 \geq 0$ {In this case, $a>0$, $D=-4<0$, and by examining a sign chart with x^2+1 as the only factor, we see that the solution is $x \in \mathbb{R}$.}

(2) $x^2-2x-3 \geq 0$

(3) $-x^2-4 \geq 0$

(4) $-x^2-2x-1 \geq 0$

(5) $-x^2+2x+3 \geq 0$

General solutions categorized by a and D:

(1) $a>0$, $D \leq 0$: solution is $x \in \mathbb{R}$

(2) $a>0$, $D>0$: let $x_1 = \dfrac{-b-\sqrt{D}}{2a}$ and $x_2 = \dfrac{-b+\sqrt{D}}{2a}$ $\Rightarrow$ solution is $(-\infty, x_1] \cup [x_2, \infty)$

(3) $a<0$, $D<0$: no solution

(4) $a<0$, $D=0$: solution is $x = -\dfrac{b}{2a}$

(5) $a<0$, $D>0$: solution is $[x_1, x_2]$

14 (a) This problem is solved in three steps.

(i) First, we must determine the height of the cloud base using the formula in Exercise 74 in Section 1.4, $h = 227(T-D) = 227(80-68) = 2724$ ft.

(ii) Next, we must determine the temperature T at the cloud base. From (i), the height of the cloud base is $h = 2724$ and $T = T_0 - \frac{5.5}{1000}\,h = 80 - \frac{5.5}{1000} \cdot 2724 = 65.018$°F.

(iii) Finally, we must solve the equation $T = B - \frac{3}{1000}\,h$ for h, when $T = 32$°F and $B = 65.018$°F.
$32 = 65.018 - \frac{3}{1000}\,h \quad \Rightarrow \quad h = (65.018-32)\frac{1000}{3} = 11{,}006$ ft.

(b) Following the procedure in part (a) and changing a few variable names, we obtain the following:

(i) $H = 227(G-D) = 227G - 227D$

(ii) $T = T_0 - \frac{5.5}{1000}h \quad \Rightarrow \quad B = G - \frac{11}{2000}H \quad \Rightarrow \quad B = G - \frac{11}{2000}(227G - 227D) \quad \Rightarrow$
$B = G - \frac{2497}{2000}G + \frac{2497}{2000}D = \frac{2497}{2000}D - \frac{497}{2000}G$

(iii) $h = (B-32)\frac{1000}{3} \quad \Rightarrow \quad h = \left(\frac{2497}{2000}D - \frac{497}{2000}G - 32\right)\frac{1000}{3} \quad \Rightarrow \quad h = \frac{2497}{6}D - \frac{497}{6}G - \frac{32{,}000}{3} \quad \Rightarrow$
$h = \frac{2497}{6}D - \frac{497}{6}G - \frac{64{,}000}{6} \quad \Rightarrow \quad h = \frac{1}{6}(2497D - 497G - 64{,}000)$

15 The first equation, $\sqrt{2x-3}+\sqrt{x+5}=0$, is a sum of square roots that is equal to 0. The only way this could be true is if both radicals are actually equal to 0. It is easy to see that $\sqrt{x+5}$ is equal to 0 only if $x=-5$, but -5 will not make $\sqrt{2x-3}$ equal to 0, so there is no reason to try to solve the first equation.

On the other hand, the second equation, $\sqrt[3]{2x-3}+\sqrt[3]{x+5}=0$, can be written as $\sqrt[3]{2x-3}=-\sqrt[3]{x+5}$. This just says that one cube root is equal to the negative of another cube root, which could happen since a cube root can be negative. Solving this equation gives us $2x-3=-(x+5) \;\Rightarrow\; 3x=-2 \;\Rightarrow\; x=-\frac{2}{3}$.

16 $\sqrt{x}=cx-2/c \;\Rightarrow\; c\sqrt{x}=c^2x-2 \;\Rightarrow\; c^2x=c^4x^2-4c^2x+4 \;\Rightarrow\; 0=c^4x^2-5c^2x+4 \;\Rightarrow$
$0=(c^2x-1)(c^2x-4) \;\Rightarrow\; x_{1,2}=\dfrac{1}{c^2},\dfrac{4}{c^2}$.

Check $x_1=\dfrac{1}{c^2}=\dfrac{1}{(2\times10^{500})^2}=\dfrac{1}{4\times10^{1000}}$.

LS $=\sqrt{x_1}=\dfrac{1}{2\times10^{500}}$

RS $=cx_1-\dfrac{2}{c}=\dfrac{2\times10^{500}}{4\times10^{1000}}-\dfrac{2}{2\times10^{500}}=\dfrac{1}{2\times10^{500}}-\dfrac{2}{2\times10^{500}}=-\dfrac{1}{2\times10^{500}}$

Check $x_2=\dfrac{4}{c^2}=\dfrac{4}{(2\times10^{500})^2}=\dfrac{4}{4\times10^{1000}}=\dfrac{1}{10^{1000}}$.

LS $=\sqrt{x_2}=\dfrac{1}{10^{500}}$

RS $=cx_2-\dfrac{2}{c}=\dfrac{2\times10^{500}}{10^{1000}}-\dfrac{2}{2\times10^{500}}=\dfrac{2}{10^{500}}-\dfrac{1}{10^{500}}=\dfrac{1}{10^{500}}$

So x_2 is a valid solution. The right side of the original equation, $cx-2/c$, must be nonnegative since it is equal to a square root. Note that the right side equals a negative number when $x=x_1$.

17 (a) $S=975$, $A=599$, and $x=1.83 \;\Rightarrow\;$ winning percentage $=\dfrac{S^x}{S^x+A^x}\approx 0.709\,206$. Since they played 154 games $(110+44)$, the number of wins using the estimated winning percentage would be $0.709(154)\approx 109$. Hence, the Pythagorean win-loss record of the 1927 Yankees is 109–45 (only one game off their actual record).

(b) The actual winning percentage is $\frac{110}{154}\approx 0.714\,286$. For an estimate of x, we'll assign $\dfrac{975^x}{975^x+599^x}$ to Y_1 and look at a table of values of x starting with $x=1.80$ and incrementing by 0.01. From the table, we see that $x=1.88$ corresponds to $Y_1\approx 0.714\,204$, which is the closest value to the actual winning percentage. Thus, the value of x is 1.88.

18 1 gallon ≈ 0.13368 ft^3 is a conversion factor that would help.

The volume of the tank is 10,000 gallons ≈ 1336.8 ft^3. Use $V=\frac{4}{3}\pi r^3$ to determine the radius.

$1336.8=\dfrac{4}{3}\pi r^3 \;\Rightarrow\; r^3=\dfrac{1002.6}{\pi} \;\Rightarrow\; r\approx 6.83375$ ft. Then use $S=4\pi r^2$ to find the surface area.

$S=4\pi(6.83375)^2\approx 586.85$ ft^2.

Chapter 1 Test

1 y^{99} is negative since it is a negative number raised to an odd power. $y-x$ is negative since it is a negative number made even more negative by subtracting a positive number. The quotient of two negatives is a *positive* number.

2 The quotient of x and y is not greater than 5 $\;\Leftrightarrow\; \dfrac{x}{y}\le 5$.

3 Since $-x^2 - 3 < 0$ for every x (it doesn't matter that x is negative), $|-x^2 - 3| = -(-x^2 - 3) = x^2 + 3$.

4 Using distance = rate × time, we get $t = \dfrac{d}{r} = \dfrac{\text{91,500,000 miles}}{\text{186,000 miles per second}} \approx 492$ seconds.

5 $$\frac{x^2y^{-3}}{z}\left(\frac{3x^0}{zy^2}\right)^{-2} = \frac{x^2}{y^3z}\left(\frac{zy^2}{3x^0}\right)^2 = \frac{x^2}{y^3z}\cdot\frac{z^2y^4}{3^2} = \frac{x^2yz}{9}$$

6 $x^{-2/3}x^{3/4} = x^{-8/12}x^{9/12} = x^{(-8/12)+(9/12)} = x^{1/12} = \sqrt[12]{x}$

7 $$\sqrt[3]{\frac{x^2y}{3}} = \frac{\sqrt[3]{x^2y}}{\sqrt[3]{3}}\cdot\frac{\sqrt[3]{xy^2}}{\sqrt[3]{xy^2}} = \frac{\sqrt[3]{x^3y^3}}{\sqrt[3]{3xy^2}} = \frac{xy}{\sqrt[3]{3xy^2}}$$

8 $$\begin{aligned}(x+2)(x^2-3x+5) &= x(x^2) + x(-3x) + x(5) + 2(x^2) + 2(-3x) + 2(5)\\ &= x^3 - 3x^2 + 5x + 2x^2 - 6x + 10\\ &= x^3 - x^2 - x + 10\end{aligned}$$

9 The leading term of $2x^2(2x+3)^4$ will be determined by multiplying $2x^2$ times $(2x)^4$. The "$+3$" will affect other terms, but not the leading term. Hence, $2x^2(2x)^4 = 2x^2(16x^4) = 32x^6$.

10 By trial and error, $2x^2 + 7x - 15 = (2x-3)(x+5)$.

11 $3x^3 - 27x = 3x(x^2 - 9) = 3x(x+3)(x-3)$

12 Recognizing this polynomial as a sum of cubes, we get

$$64x^3 + 1 = (4x)^3 + 1^3 = (4x+1)[(4x)^2 - (4x)(1) + 1^2] = (4x+1)(16x^2 - 4x + 1).$$

13 We must recognize that $(\sqrt[3]{x})^3 = x$, and then factor as we would any other difference of cubes.

$$x - 5 = (\sqrt[3]{x})^3 - (\sqrt[3]{5})^3 = (\sqrt[3]{x} - \sqrt[3]{5})[(\sqrt[3]{x})^2 + (\sqrt[3]{x})(\sqrt[3]{5}) + (\sqrt[3]{5})^2] = (\sqrt[3]{x} - \sqrt[3]{5})(\sqrt[3]{x^2} + \sqrt[3]{5x} + \sqrt[3]{25})$$

14 Factor by grouping. $2x^2 + 4x - 3xy - 6y = 2x(x+2) - 3y(x+2) = (2x - 3y)(x+2)$

15 Recognizing this polynomial as a difference of cubes, we get

$$x^{93} - 1 = (x^{31})^3 - 1^3 = (x^{31} - 1)[(x^{31})^2 + (x^{31})(1) + 1^2] = (x^{31} - 1)(x^{62} + x^{31} + 1).$$

16 $$\begin{aligned}\frac{3x}{x-2} + \frac{5}{x} - \frac{12}{x^2-2x} &= \frac{3x(x) + 5(x-2) - 12}{x(x-2)} = \frac{3x^2 + 5x - 10 - 12}{x(x-2)}\\ &= \frac{3x^2+5x-22}{x(x-2)} = \frac{(3x+11)(x-2)}{x(x-2)} = \frac{3x+11}{x}\end{aligned}$$

17 Multiply numerator and denominator by xy.

$$\frac{\dfrac{x^2}{y} - \dfrac{y^2}{x}}{\dfrac{x}{y} + 1 + \dfrac{y}{x}} = \frac{\left(\dfrac{x^2}{y} - \dfrac{y^2}{x}\right)\cdot xy}{\left(\dfrac{x}{y} + 1 + \dfrac{y}{x}\right)\cdot xy} = \frac{x^3 - y^3}{x^2 + xy + y^2} = \frac{(x-y)(x^2+xy+y^2)}{x^2+xy+y^2} = x - y$$

18 $$\begin{aligned}\frac{(x+h)^2 + 7(x+h) - (x^2+7x)}{h} &= \frac{x^2 + 2xh + h^2 + 7x + 7h - x^2 - 7x}{h} = \frac{2xh + h^2 + 7h}{h}\\ &= \frac{h(2x+h+7)}{h} = 2x + h + 7\end{aligned}$$

19 $$\begin{aligned}\frac{6h^2}{\sqrt{x+h} - \sqrt{x}} &= \frac{6h^2}{\sqrt{x+h} - \sqrt{x}}\cdot\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{6h^2(\sqrt{x+h} + \sqrt{x})}{(x+h) - x} = \frac{6h^2(\sqrt{x+h} + \sqrt{x})}{h}\\ &= 6h(\sqrt{x+h} + \sqrt{x})\end{aligned}$$

20 $(x+2)^3(4)(x-3)^3+(x-3)^4(3)(x+2)^2=(x+2)^2(x-3)^3[4(x+2)+3(x-3)]$
$=(x+2)^2(x-3)^3(4x+8+3x-9)=(x+2)^2(x-3)^3(7x-1)$

21 $\dfrac{(x^2-3)^2(2x)-x^2(2)(x^2-3)(2x)}{[(x^2-3)^2]^2}=\dfrac{(x^2-3)(2x)[(x^2-3)-2x^2]}{(x^2-3)^4}=\dfrac{2x(-3-x^2)}{(x^2-3)^3}$

22 $\left[\dfrac{5x}{x-3}+\dfrac{7}{x}=\dfrac{45}{x^2-3x}\right]\cdot x(x-3) \quad\Rightarrow\quad 5x^2+7(x-3)=45 \quad\Rightarrow\quad 5x^2+7x-66=0 \quad\Rightarrow$
$(5x+22)(x-3)=0 \quad\Rightarrow\quad x=-\frac{22}{5}, 3$. But x cannot equal 3 since it would make denominators in the original equation equal to 0, so $x=-\frac{22}{5}$.

23 $A=\dfrac{3B}{2B-5} \quad\Rightarrow\quad A(2B-5)=3B \quad\Rightarrow\quad 2AB-5A=3B \quad\Rightarrow\quad 2AB-3B=5A \quad\Rightarrow$
$B(2A-3)=5A \quad\Rightarrow\quad B=\dfrac{5A}{2A-3}$

24 Let x denote the original value of the stock. Then $x+0.2x$ is the value after the first year and $x+0.3(x+0.2x)$ is the value after the next year, so an equation that describes the problem is $x+0.3(x+0.2x)=2720$. Solving gives us $x+0.3(x+0.2x)=2720 \quad\Rightarrow\quad x+0.3x+0.06x=2720 \quad\Rightarrow\quad 1.36x=2720 \quad\Rightarrow\quad x=\frac{2720}{1.36}=2000$.
The original value was \$2000.

25 $3x^2+\sqrt{60}xy+5y^2=0 \quad\Rightarrow\quad 3x^2+\left(\sqrt{60}y\right)x+5y^2=0 \quad\Rightarrow$

$$x=\frac{-\sqrt{60}y\pm\sqrt{\left(\sqrt{60}y\right)^2-4(3)(5y^2)}}{2(3)}=\frac{-\sqrt{4}\sqrt{15}y\pm\sqrt{60y^2-60y^2}}{2(3)}=\frac{-2\sqrt{15}y\pm 0}{2(3)}=\frac{-\sqrt{15}y}{3}$$

26 $(x-y+z)^2=9 \quad\Rightarrow\quad x-y+z=\pm 3 \quad\Rightarrow\quad x=y-z\pm 3$

27 $h=1584 \quad\Rightarrow\quad -16t^2+320t=1584 \quad\Rightarrow\quad -16t^2+320t-1584=0 \quad\Rightarrow$
$t^2-20t+99=0$ {divide by -16} $\quad\Rightarrow\quad (t-9)(t-11)=0 \quad\Rightarrow\quad t=9$ or 11.
Thus, the object is 1584 feet above the ground after 9 seconds and after 11 seconds.

28 $i^{4x+3}=\left(i^{4x}\right)\left(i^3\right)=\left(i^4\right)^x\left(i^3\right)=(1)^x(-i)=(1)(-i)=-i=0-i$, so $a=0$ and $b=-1$.

29 $x^3-64=0 \quad\Rightarrow\quad (x-4)(x^2+4x+16)=0 \quad\Rightarrow\quad x=4$ or $x^2+4x+16=0$.
By the quadratic formula, $x=\dfrac{-4\pm\sqrt{4^2-4(1)(16)}}{2(1)}=\dfrac{-4\pm\sqrt{16-64}}{2}=\dfrac{-4\pm\sqrt{-48}}{2}$ ★ $4, -2\pm 2\sqrt{3}i$
$=\dfrac{-4\pm\sqrt{16}\sqrt{-3}}{2}=\dfrac{-4\pm 4\sqrt{3}i}{2}=-2\pm 2\sqrt{3}i$.

30 $A=B\sqrt{x^2+r^2} \quad\Rightarrow\quad \dfrac{A}{B}=\sqrt{x^2+r^2} \quad\Rightarrow\quad \left(\dfrac{A}{B}\right)^2=x^2+r^2 \quad\Rightarrow\quad \dfrac{A^2}{B^2}-r^2=x^2 \quad\Rightarrow$
$x^2=\dfrac{1}{B^2}(A^2-B^2r^2) \quad\Rightarrow\quad x=\pm\dfrac{1}{B}\sqrt{A^2-B^2r^2}$

31 $3x^{32}(x+2)^{65}(x-5)^{13}\left(x^{2/3}-4\right)=0 \quad\Rightarrow\quad x^{32}=0$ or $(x+2)^{65}=0$ or $(x-5)^{13}=0$ or $x^{2/3}-4=0 \quad\Rightarrow$
$x=0$ or $x=-2$ or $x=5$ or $x^{2/3}=4$. Now $x^{2/3}=4 \quad\Rightarrow\quad \left(x^{2/3}\right)^{3/2}=\pm(4)^{3/2} \quad\Rightarrow$
$x=\pm\left(\sqrt{4}\right)^3=\pm 2^3=\pm 8$. Thus, the solutions of the equation are $0, -2, 5$, and ± 8.

32 $20{,}000=\frac{4}{3}\pi r_1^3 \quad\Rightarrow\quad r_1^3=15{,}000/\pi \quad\Rightarrow\quad r_1=\sqrt[3]{15{,}000/\pi}$.
Similarly, $25{,}000=\frac{4}{3}\pi r_2^3 \quad\Rightarrow\quad r_2=\sqrt[3]{18{,}750/\pi}$. The radius increased $(r_2-r_1)/r_1\approx 0.077$, or about 7.7%.

33 Plan A pays out \$3300 per month for 10 years before plan B starts, so its total payout is $(10)(12)(3300) + 3300x$, where x is the number of months that plan B has paid out. Plan B's total payout is $4200x$.

Plan B $\geq$ Plan A $\Rightarrow$ $4200x \geq 396{,}000 + 3300x$ $\Rightarrow$ $900x \geq 396{,}000$ $\Rightarrow$ $x \geq 440$.

It will take plan B 440 months (36 years, 8 months) to have a total payout at least as large as plan A.

34 $-\frac{1}{4}|3-2x| + 6 \geq 2 \Rightarrow -\frac{1}{4}|3-2x| \geq -4 \Rightarrow |3-2x| \leq 16 \Rightarrow -16 \leq 3-2x \leq 16 \Rightarrow$ $-19 \leq -2x \leq 13 \Rightarrow \frac{19}{2} \geq x \geq -\frac{13}{2}$. The solution in interval notation is $\left[-\frac{13}{2}, \frac{19}{2}\right]$.

35 $x(2x+1) \geq 3 \Rightarrow 2x^2 + x - 3 \geq 0 \Rightarrow (2x+3)(x-1) \geq 0 \Rightarrow$ the solution is $(-\infty, -\frac{3}{2}] \cup [1, \infty)$.

36 $\dfrac{(x+1)^2(x-7)}{(7-x)(x-4)} \leq 0 \Rightarrow \dfrac{x-7}{(7-x)(x-4)} \leq 0$ {include -1} $\Rightarrow$

$\dfrac{1}{x-4} \geq 0$ {cancel, change inequality, exclude 7} $\Rightarrow x - 4 > 0$ {exclude 4} $\Rightarrow x > 4 \Rightarrow$

the solution is $\{-1\} \cup (4, 7) \cup (7, \infty)$.

37 $\dfrac{2}{x-3} \leq \dfrac{2}{x+1} \Rightarrow \dfrac{2}{x-3} - \dfrac{2}{x+1} \leq 0 \Rightarrow \dfrac{2(x+1) - 2(x-3)}{(x-3)(x+1)} \leq 0 \Rightarrow$

$\dfrac{2x+2-2x+6}{(x-3)(x+1)} \leq 0 \Rightarrow \dfrac{8}{(x-3)(x+1)} \leq 0 \Rightarrow (x-3)(x+1) < 0$ ★ $(-1, 3)$

Interval	$(-\infty, -1)$	$(-1, 3)$	$(3, \infty)$
Sign of $x-3$	$-$	$-$	$+$
Sign of $x+1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

38 Let L and W denote the length and width of the rectangle. Then $L + W = 14$, so $L = 14 - W$ and the area is $A = LW = (14-W)W$. Since $A \geq 45$, we have $(14-W)W \geq 45 \Rightarrow -W^2 + 14W \geq 45 \Rightarrow$ $-W^2 + 14W - 45 \geq 0 \Rightarrow W^2 - 14W + 45 \leq 0 \Rightarrow (W-5)(W-9) \leq 0$.

Interval	$(-\infty, 5)$	$(5, 9)$	$(9, \infty)$
Sign of $W-5$	$-$	$+$	$+$
Sign of $W-9$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, we see that the inequality is satisfied for $5 \leq W \leq 9$. Of course, once the width passes 7, it becomes the length, but that's not the point of the problem.