2 FUNCTIONS

2.1 FUNCTIONS

1. If $f(x) = x^3 + 1$, then
(a) the value of f at $x = -1$ is $f(-1) = (-1)^3 + 1 = 0$.
(b) the value of f at $x = 2$ is $f(2) = 2^3 + 1 = 9$.
(c) the net change in the value of f between $x = -1$ and $x = 2$ is $f(2) - f(-1) = 9 - 0 = 9$.

2. For a function f, the set of all possible inputs is called the *domain* of f, and the set of all possible outputs is called the *range* of f.

3. (a) $f(x) = x^2 - 3x$ and $g(x) = \dfrac{x-5}{x}$ have 5 in their domain because they are defined when $x = 5$. However, $h(x) = \sqrt{x-10}$ is undefined when $x = 5$ because $\sqrt{5-10} = \sqrt{-5}$, so 5 is not in the domain of h.
(b) $f(5) = 5^2 - 3(5) = 25 - 15 = 10$ and $g(5) = \dfrac{5-5}{5} = \dfrac{0}{5} = 0$.

4. (a) *Verbal:* "Subtract 4, then *square* and *add* 3."
(b) *Numerical*:

x	$f(x)$
0	19
2	7
4	3
6	7

5. A function f is a rule that assigns to each element x in a set A exactly *one* element called $f(x)$ in a set B. Table (i) defines y as a function of x, but table (ii) does not, because $f(1)$ is not uniquely defined.

6. (a) Yes, it is possible that $f(1) = f(2) = 5$. [For instance, let $f(x) = 5$ for all x.]
(b) No, it is not possible to have $f(1) = 5$ and $f(1) = 6$. A function assigns each value of x in its domain exactly one value of $f(x)$.

7. Multiplying x by 3 gives $3x$, then subtracting 5 gives $f(x) = 3x - 5$.

8. Squaring x gives x^2, then adding two gives $f(x) = x^2 + 2$.

9. Subtracting 1 gives $x - 1$, then squaring gives $f(x) = (x-1)^2$.

10. Adding 1 gives $x + 1$, taking the square root gives $\sqrt{x+1}$, then dividing by 6 gives $f(x) = \dfrac{\sqrt{x+1}}{6}$.

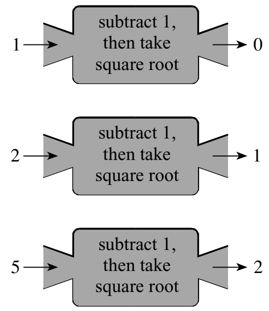
11. $f(x) = 2x + 3$: Multiply by 2, then add 3.

12. $g(x) = \dfrac{x+2}{3}$: Add 2, then divide by 3.

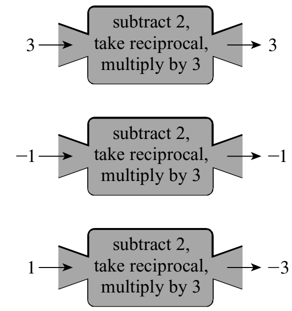
13. $h(x) = 5(x+1)$: Add 1, then multiply by 5.

14. $k(x) = \dfrac{x^2-4}{3}$: Square, then subtract 4, then divide by 3.

15. Machine diagram for $f(x) = \sqrt{x-1}$.



16. Machine diagram for $f(x) = \dfrac{3}{x-2}$.



17. $f(x) = 2(x-1)^2$

x	$f(x)$
-1	$2(-1-1)^2 = 8$
0	$2(-1)^2 = 2$
1	$2(1-1)^2 = 0$
2	$2(2-1)^2 = 2$
3	$2(3-1)^2 = 8$

18. $g(x) = |2x+3|$

x	$g(x)$
-3	$\|2(-3)+3\| = 3$
-2	$\|2(-2)+3\| = 1$
0	$\|2(0)+3\| = 3$
1	$\|2(1)+3\| = 5$
3	$\|2(3)+3\| = 9$

19. $f(x) = x^2 - 6$; $f(-3) = (-3)^2 - 6 = 9 - 6 = 3$; $f(3) = 3^2 - 6 = 9 - 6 = 3$; $f(0) = 0^2 - 6 = -6$; $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 6 = \frac{1}{4} - 6 = -\frac{23}{4}$.

20. $f(x) = x^3 + 2x$; $f(-2) = (-2)^3 + 2(-2) = -8 - 4 = -12$; $f(-1) = (-1)^3 + 2(-1) = -1 - 2 = -3$; $f(0) = 0^3 + 2(0) = 0$; $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right) = \frac{1}{8} + 1 = \frac{9}{8}$.

21. $f(x) = \dfrac{1-2x}{3}$; $f(2) = \dfrac{1-2(2)}{3} = -1$; $f(-2) = \dfrac{1-2(-2)}{3} = \dfrac{5}{3}$; $f\left(\frac{1}{2}\right) = \dfrac{1-2\left(\frac{1}{2}\right)}{3} = 0$; $f(a) = \dfrac{1-2a}{3}$; $f(-a) = \dfrac{1-2(-a)}{3} = \dfrac{1+2a}{3}$; $f(a-1) = \dfrac{1-2(a-1)}{3} = \dfrac{3-2a}{3}$.

22. $h(x) = \dfrac{x^2+4}{5}$; $h(2) = \dfrac{2^2+4}{5} = \dfrac{8}{5}$; $h(-2) = \dfrac{(-2)^2+4}{5} = \dfrac{8}{5}$; $h(a) = \dfrac{a^2+4}{5}$; $h(-x) = \dfrac{(-x)^2+4}{5} = \dfrac{x^2+4}{5}$; $h(a-2) = \dfrac{(a-2)^2+4}{5} = \dfrac{a^2-4a+8}{5}$; $h\left(\sqrt{x}\right) = \dfrac{\left(\sqrt{x}\right)^2+4}{5} = \dfrac{x+4}{5}$.

23. $f(x) = x^2 + 2x$; $f(0) = 0^2 + 2(0) = 0$; $f(3) = 3^2 + 2(3) = 9 + 6 = 15$; $f(-3) = (-3)^2 + 2(-3) = 9 - 6 = 3$; $f(a) = a^2 + 2(a) = a^2 + 2a$; $f(-x) = (-x)^2 + 2(-x) = x^2 - 2x$; $f\left(\dfrac{1}{a}\right) = \left(\dfrac{1}{a}\right)^2 + 2\left(\dfrac{1}{a}\right) = \dfrac{1}{a^2} + \dfrac{2}{a}$.

24. $h(x) = x + \dfrac{1}{x}$; $h(-1) = (-1) + \frac{1}{-1} = -1 - 1 = -2$; $h(2) = 2 + \frac{1}{2} = \frac{5}{2}$; $h\left(\frac{1}{2}\right) = \dfrac{1}{2} + \dfrac{1}{\frac{1}{2}} = \dfrac{1}{2} + 2 = \dfrac{5}{2}$; $h(x-1) = x - 1 + \dfrac{1}{x-1}$; $h\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + \dfrac{1}{\frac{1}{x}} = \dfrac{1}{x} + x$.

25. $g(x) = \frac{1-x}{1+x}$; $g(2) = \frac{1-(2)}{1+(2)} = \frac{-1}{3} = -\frac{1}{3}$; $g(-1) = \frac{1-(-1)}{1+(-1)}$, which is undefined; $g\left(\frac{1}{2}\right) = \frac{1-\left(\frac{1}{2}\right)}{1+\left(\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$;

$g(a) = \frac{1-(a)}{1+(a)} = \frac{1-a}{1+a}$; $g(a-1) = \frac{1-(a-1)}{1+(a-1)} = \frac{1-a+1}{1+a-1} = \frac{2-a}{a}$; $g\left(x^2-1\right) = \frac{1-\left(x^2-1\right)}{1+\left(x^2-1\right)} = \frac{2-x^2}{x^2}$.

26. $g(t) = \frac{t+2}{t-2}$; $g(-2) = \frac{-2+2}{-2-2} = 0$; $g(2) = \frac{2+2}{2-2}$, which is undefined; $g(0) = \frac{0+2}{0-2} = -1$; $g(a) = \frac{a+2}{a-2}$;

$g\left(a^2-2\right) = \frac{a^2-2+2}{a^2-2-2} = \frac{a^2}{a^2-4}$; $g(a+1) = \frac{a+1+2}{a+1-2} = \frac{a+3}{a-1}$.

27. $k(x) = -x^2-2x+3$; $k(0) = -0^2-2(0)+3 = 3$; $k(2) = -2^2-2(2)+3 = -5$; $k(-2) = -(-2)^2-2(-2)+3 = 3$;

$k\left(\sqrt{2}\right) = -\left(\sqrt{2}\right)^2 - 2\left(\sqrt{2}\right) + 3 = 1-2\sqrt{2}$; $k(a+2) = -(a+2)^2-2(a+2)+3 = -a^2-6a-5$;

$k(-x) = -(-x)^2-2(-x)+3 = -x^2+2x+3$; $k\left(x^2\right) = -\left(x^2\right)^2 - 2\left(x^2\right) + 3 = -x^4-2x^2+3$.

28. $k(x) = 2x^3-3x^2$; $k(0) = 2(0)^3-3(0)^2 = 0$; $k(3) = 2(3)^3-3(3)^2 = 27$; $k(-3) = 2(-3)^3-3(-3)^2 = -81$;

$k\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 = -\frac{1}{2}$; $k\left(\frac{a}{2}\right) = 2\left(\frac{a}{2}\right)^3 - 3\left(\frac{a}{2}\right)^2 = \frac{a^3-3a^2}{4}$; $k(-x) = 2(-x)^3-3(-x)^2 = -2x^3-3x^2$;

$k\left(x^3\right) = 2\left(x^3\right)^3 - 3\left(x^3\right)^2 = 2x^9-3x^6$.

29. $f(x) = 2|x-1|$; $f(-2) = 2|-2-1| = 2(3) = 6$; $f(0) = 2|0-1| = 2(1) = 2$;

$f\left(\frac{1}{2}\right) = 2\left|\frac{1}{2}-1\right| = 2\left(\frac{1}{2}\right) = 1$; $f(2) = 2|2-1| = 2(1) = 2$; $f(x+1) = 2|(x+1)-1| = 2|x|$;

$f\left(x^2+2\right) = 2\left|\left(x^2+2\right)-1\right| = 2\left|x^2+1\right| = 2x^2+2$ (since $x^2+1>0$).

30. $f(x) = \frac{|x|}{x}$; $f(-2) = \frac{|-2|}{-2} = \frac{2}{-2} = -1$; $f(-1) = \frac{|-1|}{-1} = \frac{1}{-1} = -1$; $f(x)$ is not defined at $x=0$;

$f(5) = \frac{|5|}{5} = \frac{5}{5} = 1$; $f\left(x^2\right) = \frac{\left|x^2\right|}{x^2} = \frac{x^2}{x^2} = 1$ since $x^2>0$, $x \neq 0$; $f\left(\frac{1}{x}\right) = \frac{|1/x|}{1/x} = \frac{x}{|x|}$.

31. Since $-2<0$, we have $f(-2) = (-2)^2 = 4$. Since $-1<0$, we have $f(-1) = (-1)^2 = 1$. Since $0 \geq 0$, we have $f(0) = 0+1 = 1$. Since $1 \geq 0$, we have $f(1) = 1+1 = 2$. Since $2 \geq 0$, we have $f(2) = 2+1 = 3$.

32. Since $-3 \leq 2$, we have $f(-3) = 5$. Since $0 \leq 2$, we have $f(0) = 5$. Since $2 \leq 2$, we have $f(2) = 5$. Since $3>2$, we have $f(3) = 2(3)-3 = 3$. Since $5>2$, we have $f(5) = 2(5)-3 = 7$.

33. Since $-4 \leq -1$, we have $f(-4) = (-4)^2+2(-4) = 16-8 = 8$. Since $-\frac{3}{2} \leq -1$, we have $f\left(-\frac{3}{2}\right) = \left(-\frac{3}{2}\right)^2 + 2\left(-\frac{3}{2}\right) = \frac{9}{4}-3 = -\frac{3}{4}$. Since $-1 \leq -1$, we have $f(-1) = (-1)^2+2(-1) = 1-2 = -1$. Since $-1<0 \leq 1$, we have $f(0) = 0$. Since $25>1$, we have $f(25) = -1$.

34. Since $-5<0$, we have $f(-5) = 3(-5) = -15$. Since $0 \leq 0 \leq 2$, we have $f(0) = 0+1 = 1$. Since $0 \leq 1 \leq 2$, we have $f(1) = 1+1 = 2$. Since $0 \leq 2 \leq 2$, we have $f(2) = 2+1 = 3$. Since $5>2$, we have $f(5) = (5-2)^2 = 9$.

35. $f(x+2) = (x+2)^2+1 = x^2+4x+4+1 = x^2+4x+5$; $f(x)+f(2) = x^2+1+(2)^2+1 = x^2+1+4+1 = x^2+6$.

36. $f(2x) = 3(2x)-1 = 6x-1$; $2f(x) = 2(3x-1) = 6x-2$.

37. $f\left(x^2\right) = x^2+4$; $[f(x)]^2 = [x+4]^2 = x^2+8x+16$.

38. $f\left(\frac{x}{3}\right) = 6\left(\frac{x}{3}\right) - 18 = 2x-18$; $\frac{f(x)}{3} = \frac{6x-18}{3} = \frac{3(2x-6)}{3} = 2x-6$.

39. $f(x) = 3x-2$, so $f(1) = 3(1)-2 = 1$ and $f(5) = 3(5)-2 = 13$. Thus, the net change is $f(5)-f(1) = 13-1 = 12$.

40. $f(x) = 4-5x$, so $f(3) = 4-5(3) = -11$ and $f(5) = 4-5(5) = -21$. Thus, the net change is $f(5)-f(3) = -21-(-11) = -10$.

41. $g(t) = 1 - t^2$, so $g(-2) = 1 - (-2)^2 = 1 - 4 = -3$ and $g(5) = 1 - 5^2 = -24$. Thus, the net change is $g(5) - g(-2) = -24 - (-3) = -21$.

42. $h(t) = t^2 + 5$, so $h(-3) = (-3)^2 + 5 = 14$ and $h(6) = 6^2 + 5 = 41$. Thus, the net change is $h(6) - h(-3) = 41 - 14 = 27$.

43. $f(a) = 5 - 2a$; $f(a+h) = 5 - 2(a+h) = 5 - 2a - 2h$;

$$\frac{f(a+h) - f(a)}{h} = \frac{5 - 2a - 2h - (5 - 2a)}{h} = \frac{5 - 2a - 2h - 5 + 2a}{h} = \frac{-2h}{h} = -2.$$

44. $f(a) = 3a^2 + 2$; $f(a+h) = 3(a+h)^2 + 2 = 3a^2 + 6ah + 3h^2 + 2$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\left(3a^2 + 6ah + 3h^2 + 2\right) - \left(3a^2 + 2\right)}{h} = \frac{6ah + 3h^2}{h} = 6a + 3h.$$

45. $f(a) = 5$; $f(a+h) = 5$; $\dfrac{f(a+h) - f(a)}{h} = \dfrac{5 - 5}{h} = 0$.

46. $f(a) = \dfrac{1}{a+1}$; $f(a+h) = \dfrac{1}{a+h+1}$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{1}{a+h+1} - \dfrac{1}{a+1}}{h} = \frac{\dfrac{a+1}{(a+1)(a+h+1)} - \dfrac{a+h+1}{(a+1)(a+h+1)}}{h}$$

$$= \frac{\dfrac{-h}{(a+1)(a+h+1)}}{h} = \frac{-1}{(a+1)(a+h+1)}.$$

47. $f(a) = \dfrac{a}{a+1}$; $f(a+h) = \dfrac{a+h}{a+h+1}$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{a+h}{a+h+1} - \dfrac{a}{a+1}}{h} = \frac{\dfrac{(a+h)(a+1)}{(a+h+1)(a+1)} - \dfrac{a(a+h+1)}{(a+h+1)(a+1)}}{h}$$

$$= \frac{\dfrac{(a+h)(a+1) - a(a+h+1)}{(a+h+1)(a+1)}}{h} = \frac{a^2 + a + ah + h - \left(a^2 + ah + a\right)}{h(a+h+1)(a+1)}$$

$$= \frac{1}{(a+h+1)(a+1)}$$

48. $f(a) = \dfrac{2a}{a-1}$; $f(a+h) = \dfrac{2(a+h)}{a+h-1}$;

$$\frac{f(a+h) - f(a)}{h} = \frac{\dfrac{2(a+h)}{a+h-1} - \dfrac{2a}{a-1}}{h} = \frac{\dfrac{(2a+2h)(a-1)}{(a+h-1)(a-1)} - \dfrac{2a(a+h-1)}{(a+h-1)(a-1)}}{h}$$

$$= \frac{\dfrac{2(a+h)(a-1) - 2a(a+h-1)}{(a+h-1)(a-1)}}{h} = \frac{2a^2 + 2ah - 2a - 2h - 2a^2 - 2ah + 2a}{h(a+h-1)(a-1)}$$

$$= \frac{-2h}{h(a+h-1)(a-1)} = -\frac{2}{(a+h-1)(a-1)}$$

49. $f(a) = 3 - 5a + 4a^2$;

$$f(a+h) = 3 - 5(a+h) + 4(a+h)^2 = 3 - 5a - 5h + 4\left(a^2 + 2ah + h^2\right)$$
$$= 3 - 5a - 5h + 4a^2 + 8ah + 4h^2;$$

$$\frac{f(a+h) - f(a)}{h} = \frac{\left(3 - 5a - 5h + 4a^2 + 8ah + 4h^2\right) - \left(3 - 5a + 4a^2\right)}{h}$$

$$= \frac{3 - 5a - 5h + 4a^2 + 8ah + 4h^2 - 3 + 5a - 4a^2}{h} = \frac{-5h + 8ah + 4h^2}{h}$$

$$= \frac{h(-5 + 8a + 4h)}{h} = -5 + 8a + 4h.$$

50. $f(a) = a^3$; $f(a+h) = (a+h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$;

$$\frac{f(a+h)-f(a)}{h} = \frac{\left(a^3+3a^2h+3ah^2+h^3\right)-\left(a^3\right)}{h} = \frac{3a^2h+3ah^2+h^3}{h}$$
$$= \frac{h\left(3a^2+3ah+h^2\right)}{h} = 3a^2+3ah+h^2.$$

51. $f(x) = 3x$. Since there is no restriction, the domain is all real numbers, $(-\infty, \infty)$. Since every real number y is three times the real number $\frac{1}{3}y$, the range is all real numbers $(-\infty, \infty)$.

52. $f(x) = 5x^2 + 4$. Since there is no restriction, the domain is all real numbers, $(-\infty, \infty)$. Since $5x^2 \geq 0$ for all x, $5x^2 + 4 \geq 4$ for all x, so the range is $[4, \infty)$.

53. $f(x) = 3x$, $-2 \leq x \leq 6$. The domain is $[-2, 6]$, $f(-2) = 3(-2) = -6$, and $f(6) = 3(6) = 18$, so the range is $[-6, 18]$.

54. $f(x) = 5x^2 + 4$, $0 \leq x \leq 2$. The domain is $[0, 2]$, $f(0) = 5(0)^2 + 4 = 4$, and $f(2) = 5(2)^2 + 4 = 24$, so the range is $[4, 24]$.

55. $f(x) = \dfrac{1}{x-3}$. Since the denominator cannot equal 0 we have $x - 3 \neq 0 \Leftrightarrow x \neq 3$. Thus the domain is $\{x \mid x \neq 3\}$. In interval notation, the domain is $(-\infty, 3) \cup (3, \infty)$.

56. $f(x) = \dfrac{1}{3x-6}$. Since the denominator cannot equal 0, we have $3x - 6 \neq 0 \Leftrightarrow 3x \neq 6 \Leftrightarrow x \neq 2$. In interval notation, the domain is $(-\infty, 2) \cup (2, \infty)$.

57. $f(x) = \dfrac{x+2}{x^2-1}$. Since the denominator cannot equal 0 we have $x^2 - 1 \neq 0 \Leftrightarrow x^2 \neq 1 \Rightarrow x \neq \pm 1$. Thus the domain is $\{x \mid x \neq \pm 1\}$. In interval notation, the domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

58. $f(x) = \dfrac{x^4}{x^2+x-6}$. Since the denominator cannot equal 0, $x^2 + x - 6 \neq 0 \Leftrightarrow (x+3)(x-2) \neq 0 \Rightarrow x \neq -3$ or $x \neq 2$. In interval notation, the domain is $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$.

59. $f(x) = \sqrt{x+1}$. We must have $x + 1 \geq 0 \Leftrightarrow x \geq -1$. Thus, the domain is $[-1, \infty)$.

60. $g(x) = \sqrt{x^2+9}$. The argument of the square root is positive for all x, so the domain is $(-\infty, \infty)$.

61. $f(t) = \sqrt[3]{t-1}$. Since the odd root is defined for all real numbers, the domain is the set of real numbers, $(-\infty, \infty)$.

62. $g(x) = \sqrt{7-3x}$. For the square root to be defined, we must have $7 - 3x \geq 0 \Leftrightarrow 7 \geq 3x \Leftrightarrow \frac{7}{3} \geq x$. Thus the domain is $\left(-\infty, \frac{7}{3}\right]$.

63. $f(x) = \sqrt{1-2x}$. Since the square root is defined as a real number only for nonnegative numbers, we require that $1 - 2x \geq 0 \Leftrightarrow x \leq \frac{1}{2}$. So the domain is $\{x \mid x \leq \frac{1}{2}\}$. In interval notation, the domain is $\left(-\infty, \frac{1}{2}\right]$.

64. $g(x) = \sqrt{x^2-4}$. We must have $x^2 - 4 \geq 0 \Leftrightarrow (x-2)(x+2) \geq 0$. We make a table:

	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Sign of $x - 2$	$-$	$-$	$+$
Sign of $x + 2$	$-$	$+$	$+$
Sign of $(x-2)(x+2)$	$+$	$-$	$+$

Thus the domain is $(-\infty, -2] \cup [2, \infty)$.

65. $g(x) = \dfrac{\sqrt{2+x}}{3-x}$. We require $2 + x \geq 0$, and the denominator cannot equal 0. Now $2 + x \geq 0 \Leftrightarrow x \geq -2$, and $3 - x \neq 0 \Leftrightarrow x \neq 3$. Thus the domain is $\{x \mid x \geq -2 \text{ and } x \neq 3\}$, which can be expressed in interval notation as $[-2, 3) \cup (3, \infty)$.

66. $g(x) = \dfrac{\sqrt{x}}{2x^2 + x - 1}$. We must have $x \geq 0$ for the numerator and $2x^2 + x - 1 \neq 0$ for the denominator. So $2x^2 + x - 1 \neq 0$ $\Leftrightarrow (2x - 1)(x + 1) \neq 0 \Rightarrow 2x - 1 \neq 0$ or $x + 1 \neq 0 \Leftrightarrow x \neq \frac{1}{2}$ or $x \neq -1$. Thus the domain is $\left[0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

67. $g(x) = \sqrt[4]{x^2 - 6x}$. Since the input to an even root must be nonnegative, we have $x^2 - 6x \geq 0 \Leftrightarrow x(x - 6) \geq 0$. We make a table:

	$(-\infty, 0)$	$(0, 6)$	$(6, \infty)$
Sign of x	$-$	$+$	$+$
Sign of $x - 6$	$-$	$-$	$+$
Sign of $x(x - 6)$	$+$	$-$	$+$

Thus the domain is $(-\infty, 0] \cup [6, \infty)$.

68. $g(x) = \sqrt{x^2 - 2x - 8}$. We must have $x^2 - 2x - 8 \geq 0 \Leftrightarrow (x - 4)(x + 2) \geq 0$. We make a table:

	$(-\infty, -2)$	$(-2, 4)$	$(4, \infty)$
Sign of $x - 4$	$-$	$-$	$+$
Sign of $x + 2$	$-$	$+$	$+$
Sign of $(x - 4)(x + 2)$	$+$	$-$	$+$

Thus the domain is $(-\infty, -2] \cup [4, \infty)$.

69. $f(x) = \dfrac{3}{\sqrt{x - 4}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $x - 4 > 0 \Leftrightarrow x > 4$. Thus the domain is $(4, \infty)$.

70. $f(x) = \dfrac{x^2}{\sqrt{6 - x}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $6 - x > 0 \Leftrightarrow 6 > x$. Thus the domain is $(-\infty, 6)$.

71. $f(x) = \dfrac{(x + 1)^2}{\sqrt{2x - 1}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $2x - 1 > 0 \Leftrightarrow x > \frac{1}{2}$. Thus the domain is $\left(\frac{1}{2}, \infty\right)$.

72. $f(x) = \dfrac{x}{\sqrt[4]{9 - x^2}}$. Since the input to an even root must be nonnegative and the denominator cannot equal 0, we have $9 - x^2 > 0 \Leftrightarrow (3 - x)(3 + x) > 0$. We make a table:

Interval	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
Sign of $3 - x$	$+$	$+$	$-$
Sign of $3 + x$	$-$	$+$	$+$
Sign of $(x - 4)(x + 2)$	$-$	$+$	$-$

Thus the domain is $(-3, 3)$.

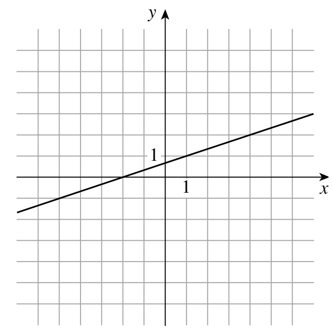
73. To evaluate $f(x)$, divide the input by 3 and add $\frac{2}{3}$ to the result.

(a) $f(x) = \dfrac{x}{3} + \dfrac{2}{3}$

(b)

x	$f(x)$
2	$\frac{4}{3}$
4	2
6	$\frac{8}{3}$
8	$\frac{10}{3}$

(c)



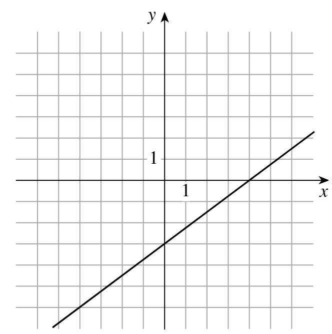
74. To evaluate $g(x)$, subtract 4 from the input and multiply the result by $\frac{3}{4}$.

(a) $g(x) = (x-4) \cdot \frac{3}{4} = \frac{3}{4}(x-4)$

(b)

x	$g(x)$
2	$-\frac{3}{2}$
4	0
6	$\frac{3}{2}$
8	3

(c)



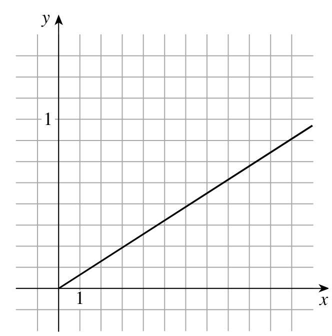
75. Let $T(x)$ be the amount of sales tax charged in Lemon County on a purchase of x dollars. To find the tax, take 8% of the purchase price.

(a) $T(x) = 0.08x$

(b)

x	$T(x)$
2	0.16
4	0.32
6	0.48
8	0.64

(c)



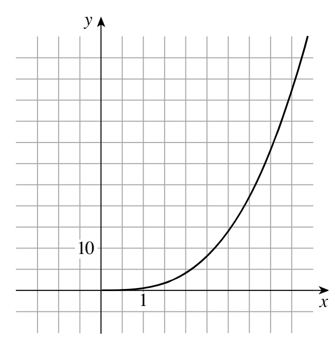
76. Let $V(d)$ be the volume of a sphere of diameter d. To find the volume, take the cube of the diameter, then multiply by π and divide by 6.

(a) $V(d) = d^3 \cdot \pi/6 = \frac{\pi}{6}d^3$

(b)

x	$f(x)$
2	$\frac{4\pi}{3} \approx 4.2$
4	$\frac{32\pi}{3} \approx 33.5$
6	$36\pi \approx 113$
8	$\frac{256\pi}{3} \approx 268$

(c)



77. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 5 & \text{if } x \text{ is irrational} \end{cases}$ The domain of f is all real numbers, since every real number is either rational or irrational; and the range of f is $\{1, 5\}$.

78. $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 5x & \text{if } x \text{ is irrational} \end{cases}$ The domain of f is all real numbers, since every real number is either rational or irrational. If x is irrational, then $5x$ is also irrational, and so the range of f is $\{x \mid x = 1 \text{ or } x \text{ is irrational}\}$.

79. (a) $V(0) = 50\left(1 - \frac{0}{20}\right)^2 = 50$ and $V(20) = 50\left(1 - \frac{20}{20}\right)^2 = 0$.

(b) $V(0) = 50$ represents the volume of the full tank at time $t = 0$, and $V(20) = 0$ represents the volume of the empty tank twenty minutes later.

(c)

x	$V(x)$
0	50
5	28.125
10	12.5
15	3.125
20	0

(d) The net change in V as t changes from 0 minutes to 20 minutes is $V(20) - V(0) = 0 - 50 = -50$ liters.

80. (a) $S(2) = 4\pi(2)^2 = 16\pi \approx 50.27$, $S(3) = 4\pi(3)^2 = 36\pi \approx 113.10$.

(b) $S(2)$ represents the surface area of a sphere of radius 2, and $S(3)$ represents the surface area of a sphere of radius 3.

81. (a) $L(0.5c) = 10\sqrt{1 - \frac{(0.5c)^2}{c^2}} \approx 8.66$ m, $L(0.75c) = 10\sqrt{1 - \frac{(0.75c)^2}{c^2}} \approx 6.61$ m, and $L(0.9c) = 10\sqrt{1 - \frac{(0.9c)^2}{c^2}} \approx 4.36$ m.

(b) It will appear to get shorter.

82. (a) $R(1) = \sqrt{\frac{13 + 7(1)^{0.4}}{1 + 4(1)^{0.4}}} = \sqrt{\frac{20}{5}} = 2$ mm,

$R(10) = \sqrt{\frac{13 + 7(10)^{0.4}}{1 + 4(10)^{0.4}}} \approx 1.66$ mm, and

$R(100) = \sqrt{\frac{13 + 7(100)^{0.4}}{1 + 4(100)^{0.4}}} \approx 1.48$ mm.

(b)

x	$R(x)$
1	2
10	1.66
100	1.48
200	1.44
500	1.41
1000	1.39

(c) The net change in R as x changes from 10 to 100 is $R(100) - R(10) \approx 1.48 - 1.66 = -0.18$ mm.

83. (a) $v(0.1) = 18500\left(0.25 - 0.1^2\right) = 4440$,

$v(0.4) = 18500\left(0.25 - 0.4^2\right) = 1665$.

(b) They tell us that the blood flows much faster (about 2.75 times faster) 0.1 cm from the center than 0.1 cm from the edge.

(c)

r	$v(r)$
0	4625
0.1	4440
0.2	3885
0.3	2960
0.4	1665
0.5	0

(d) The net change in V as r changes from 0.1 cm to 0.5 cm is $V(0.5) - V(0.1) = 0 - 4440 = -4440$ cm/s.

84. (a) $D(0.1) = \sqrt{2(6340)(0.1) + (0.1)^2} = \sqrt{1268.01} \approx 35.6$ kilometers

$D(0.2) = \sqrt{2(6340)(0.2) + (0.2)^2} = \sqrt{2536.04} \approx 50.4$ kilometers

(b) 346 meters $= \frac{346}{1000}$ meters ≈ 0.346 meters. $D(0.346) = \sqrt{2\,(6340)\,(0.346) + (0.346)^2} = \sqrt{4387.4} \approx 66.2$ kilometers

(c) $D\,(11) = \sqrt{2\,(6340)\,(11) + (11)^2} = \sqrt{139601} \approx 373.6$ kilometers

(d) The net change in D as h changes from 346 m (or 0.346 km) to 11 km is $D(11) - D\,(0.346) \approx 373.6 - 66.2 = 307.4$ kilometers.

85. **(a)** Since $0 \le 5{,}000 \le 10{,}000$ we have $T\,(5{,}000) = 0$. Since $10{,}000 < 12{,}000 \le 20{,}000$ we have $T\,(12{,}000) = 0.08\,(12{,}000) = 960$. Since $20{,}000 < 25{,}000$ we have $T\,(25{,}000) = 1600 + 0.15\,(25{,}000) = 5350$.

(b) There is no tax on \$5000, a tax of \$960 on \$12,000 income, and a tax of \$5350 on \$25,000.

86. **(a)** $C\,(75) = 75 + 15 = \$90$; $C\,(90) = 90 + 15 = \$105$; $C\,(100) = \$100$; and $C\,(105) = \$105$.

(b) The total price of the books purchased, including shipping.

87. **(a)** $T\,(x) = \begin{cases} 75x & \text{if } 0 \le x \le 2 \\ 150 + 50\,(x - 2) & \text{if } x > 2 \end{cases}$

(b) $T\,(2) = 75\,(2) = 150$; $T\,(3) = 150 + 50\,(3 - 2) = 200$; and $T\,(5) = 150 + 50\,(5 - 2) = 300$.

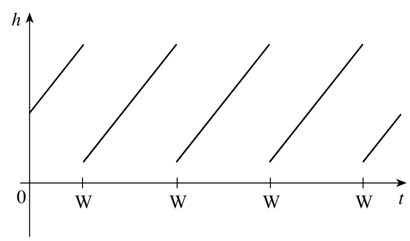
(c) The total cost of the lodgings.

88. **(a)** $F\,(x) = \begin{cases} 15\,(65 - x) & \text{if } 0 < x < 65 \\ 0 & \text{if } 65 \le x \le 105 \\ 15\,(x - 105) & \text{if } x > 105 \end{cases}$

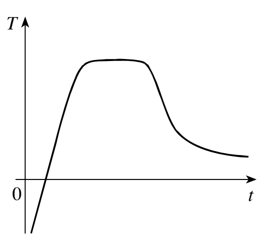
(b) $F\,(50) = 15\,(65 - 50) = 15 \cdot 15 = \$225$; $F\,(80) = \$0$; and $F\,(120) = 15\,(120 - 105)\; 15 \cdot 15 = \$225$.

(c) The fines for violating the speed limits on the freeway.

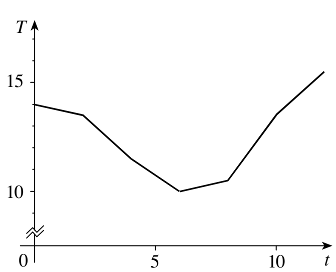
89. We assume the grass grows linearly.



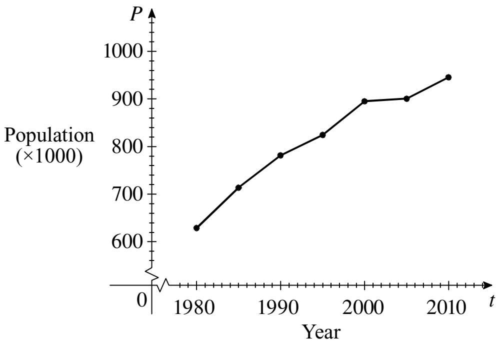
90.



91.



92.



93. Answers will vary.

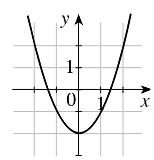
94. Answers will vary.

95. Answers will vary.

2.2 GRAPHS OF FUNCTIONS

1. To graph the function f we plot the points $(x, f(x))$ in a coordinate plane. To graph $f(x) = x^2 - 2$, we plot the points $\left(x, x^2 - 2\right)$. So, the point $\left(3, 3^2 - 2\right) = (3, 7)$ is on the graph of f. The height of the graph of f above the x-axis when $x = 3$ is 7.

x	$f(x)$	(x, y)
-2	2	$(-2, 2)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
2	2	$(2, 2)$



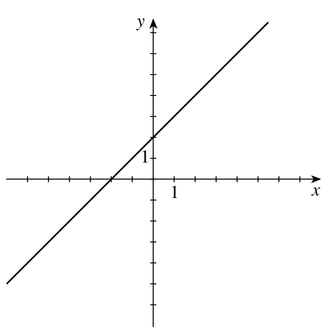
2. If $f(4) = 10$ then the point $(4, 10)$ is on the graph of f.

3. If the point $(3, 7)$ is on the graph of f, then $f(3) = 7$.

4. **(a)** $f(x) = x^2$ is a power function with an even exponent. It has graph IV.
(b) $f(x) = x^3$ is a power function with an odd exponent. It has graph II.
(c) $f(x) = \sqrt{x}$ is a root function. It has graph I.
(d) $f(x) = |x|$ is an absolute value function. It has graph III.

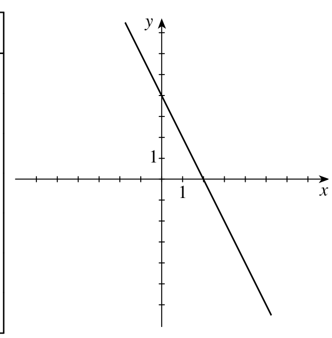
5.

x	$f(x) = x + 2$
-6	-4
-4	-2
-2	0
0	2
2	4
4	6
6	8



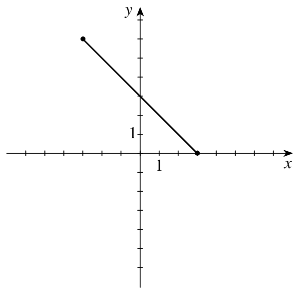
6.

x	$f(x) = 4 - 2x$
-2	8
-1	6
0	4
1	2
2	0
3	-2
4	-4



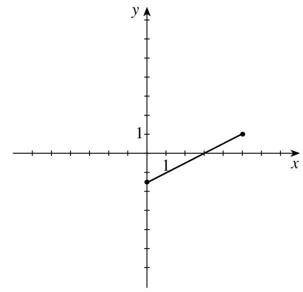
7.

x	$f(x) = -x + 3$, $-3 \le x \le 3$
-3	6
-2	5
0	3
1	2
2	1
3	0



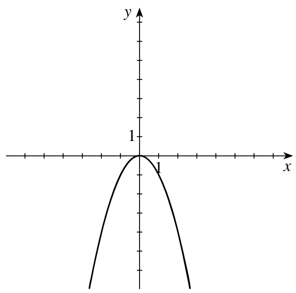
8.

x	$f(x) = \dfrac{x-3}{2}$, $0 \le x \le 5$
0	-1.5
1	-1
2	-0.5
3	0
4	0.5
5	1



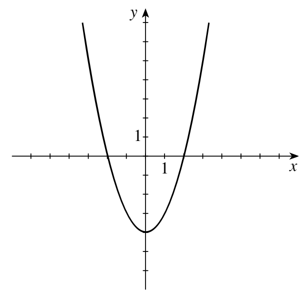
9.

x	$f(x) = -x^2$
± 4	-16
± 3	-9
± 2	-4
± 1	-1
0	0



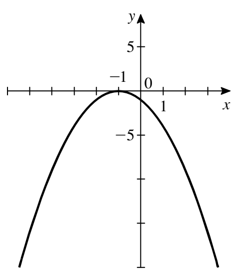
10.

x	$f(x) = x^2 - 4$
± 5	21
± 4	12
± 3	5
± 2	0
± 1	-3
0	-4



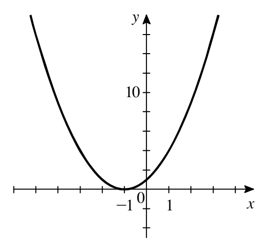
11.

x	$g(x) = -(x+1)^2$
-5	-16
-3	-4
-2	-1
-1	0
0	-1
1	-4
3	-16



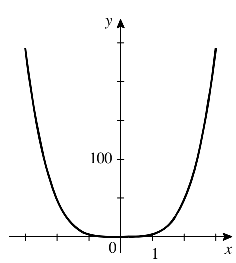
12.

x	$g(x) = x^2 + 2x + 1$
-5	16
-3	4
-2	1
-1	0
0	1
1	4
3	16



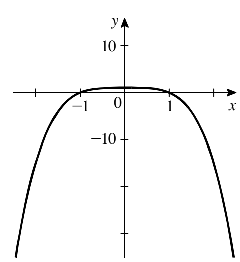
13.

x	$r(x) = 3x^4$
-3	243
-2	48
-1	3
0	0
1	3
2	48
3	243



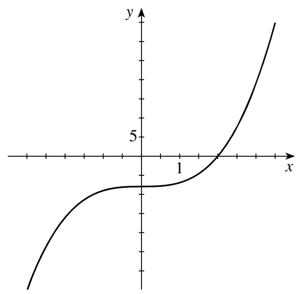
14.

x	$r(x) = 1 - x^4$
-3	-80
-2	-15
-1	0
0	1
1	0
2	-15
3	-80



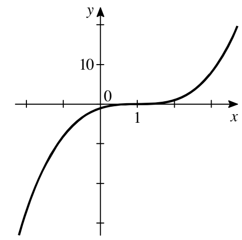
15.

x	$g(x) = x^3 - 8$
-3	-35
-2	-16
-1	-9
0	-8
1	-7
2	0
3	19



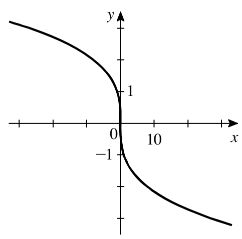
16.

x	$g(x) = (x-1)^3$
-2	-27
-1	-8
0	-1
1	0
2	1
3	8
4	27



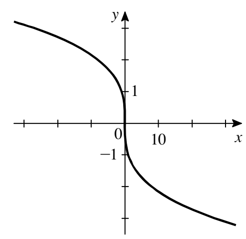
17.

x	$k(x) = \sqrt[3]{-x}$
-27	3
-8	2
-1	1
0	0
1	-1
8	-2
27	-3



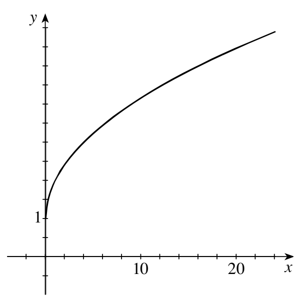
18.

x	$k(x) = -\sqrt[3]{x}$
-27	3
-8	2
-1	1
0	0
1	-1
8	-2
27	-3



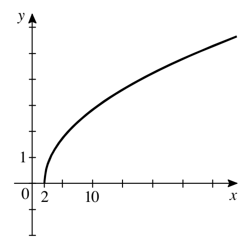
19.

x	$f(x) = 1 + \sqrt{x}$
0	1
1	2
4	3
9	4
16	5
25	6



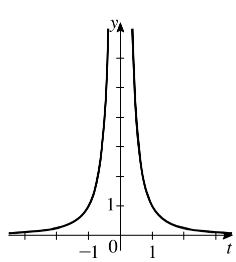
20.

x	$f(x) = \sqrt{x-2}$
2	0
3	1
6	2
11	3
18	4
27	5
38	6



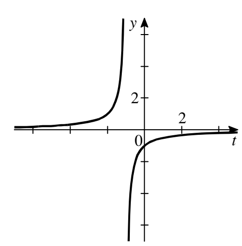
21.

x	$C(t) = \frac{1}{t^2}$
-2	$\frac{1}{4}$
-1	1
$-\frac{1}{2}$	4
$-\frac{1}{4}$	16
0	$-$
$\frac{1}{4}$	16
$\frac{1}{2}$	4
1	1
2	$\frac{1}{4}$



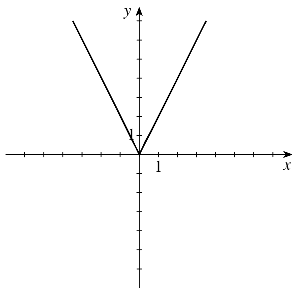
22.

x	$C(t) = -\frac{1}{t+1}$
-3	$\frac{1}{2}$
-2	1
$-\frac{3}{2}$	2
-1	$-$
$-\frac{1}{2}$	-2
0	-1
1	$-\frac{1}{2}$
2	$-\frac{1}{3}$



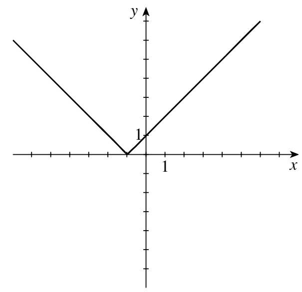
23.

x	$H(x) = \lvert 2x \rvert$
± 5	10
± 4	8
± 3	6
± 2	4
± 1	2
0	0



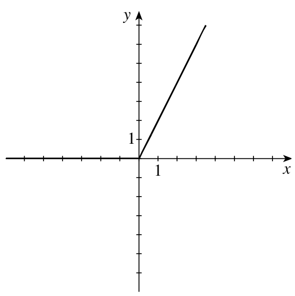
24.

x	$H(x) = \lvert x+1 \rvert$
-5	4
-4	3
-3	2
-2	1
-1	0
0	1
1	2



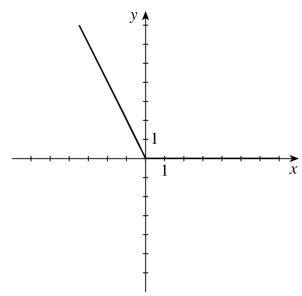
25.

x	$G(x) = \lvert x \rvert + x$
-5	0
-2	0
0	0
1	2
2	4
5	10



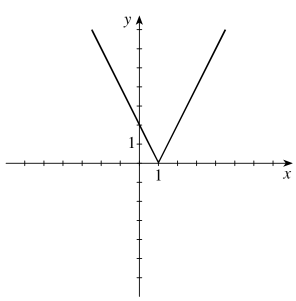
26.

x	$G(x) = \lvert x \rvert - x$
-5	10
-2	4
-1	2
0	0
1	0
3	0



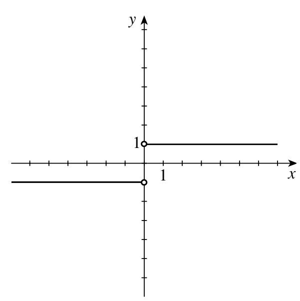
27.

x	$f(x) = \lvert 2x-2 \rvert$
-5	12
-2	8
0	2
1	0
2	2
5	8



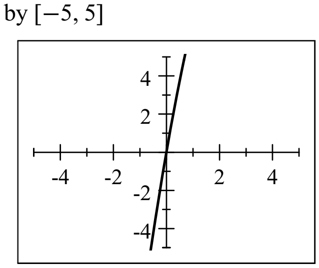
28.

x	$f(x) = \dfrac{x}{\lvert x \rvert}$
-3	-1
-2	-1
-1	-1
0	undefined
1	1
2	1
3	1

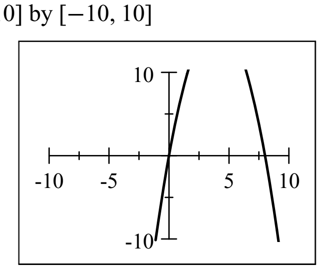


29. $f(x) = 8x - x^2$

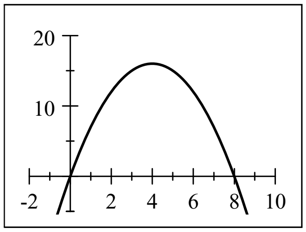
(a) $[-5, 5]$ by $[-5, 5]$



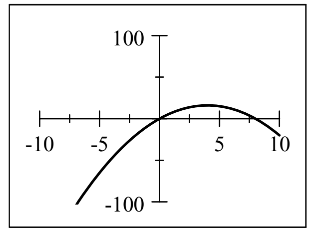
(b) $[-10, 10]$ by $[-10, 10]$



(c) $[-2, 10]$ by $[-5, 20]$



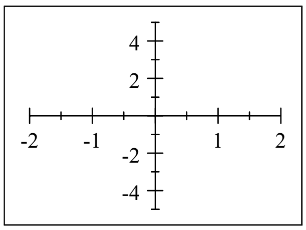
(d) $[-10, 10]$ by $[-100, 100]$



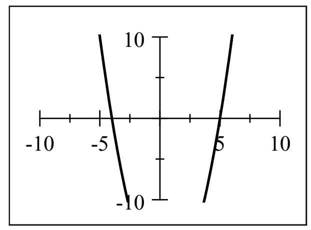
The viewing rectangle in part (c) produces the most appropriate graph of the equation.

30. $g(x) = x^2 - x - 20$

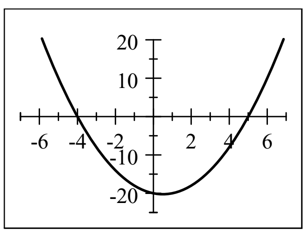
(a) $[-2, 2]$ by $[-5, 5]$



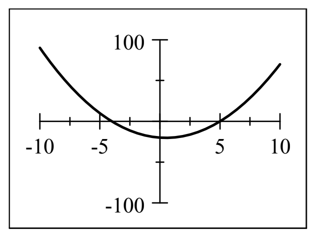
(b) $[-10, 10]$ by $[-10, 10]$



(c) $[-7, 7]$ by $[-25, 20]$



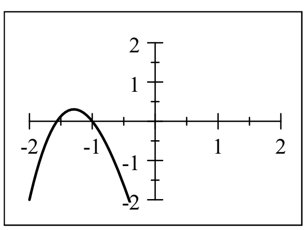
(d) $[-10, 10]$ by $[-100, 100]$



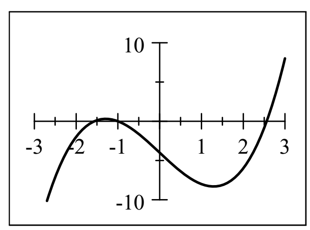
The viewing rectangle in part (c) produces the most appropriate graph of the equation.

31. $h(x) = x^3 - 5x - 4$

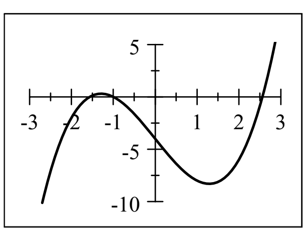
(a) $[-2, 2]$ by $[-2, 2]$



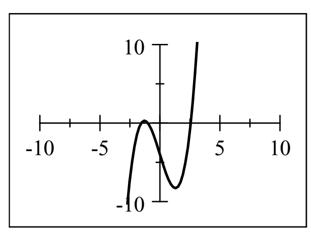
(b) $[-3, 3]$ by $[-10, 10]$



(c) $[-3, 3]$ by $[-10, 5]$



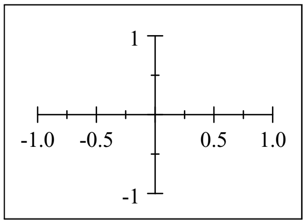
(d) $[-10, 10]$ by $[-10, 10]$



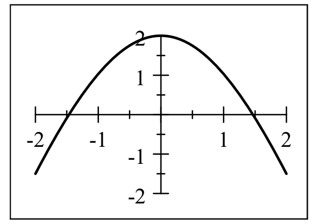
The viewing rectangle in part (c) produces the most appropriate graph of the equation.

32. $k(x) = \frac{1}{32}x^4 - x^2 + 2$

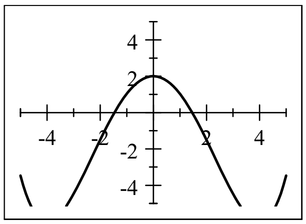
(a) $[-1, 1]$ by $[-1, 1]$



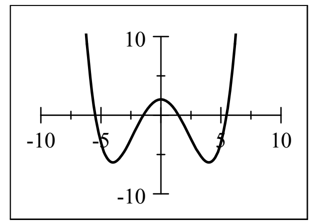
(b) $[-2, 2]$ by $[-2, 2]$



(c) $[-5, 5]$ by $[-5, 5]$

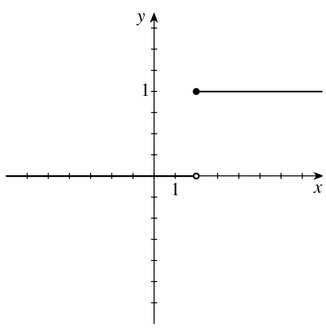


(d) $[-10, 10]$ by $[-10, 10]$

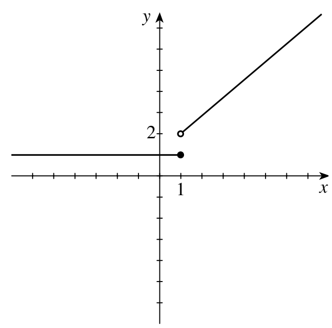


The viewing rectangle in part (d) produces the most appropriate graph of the equation.

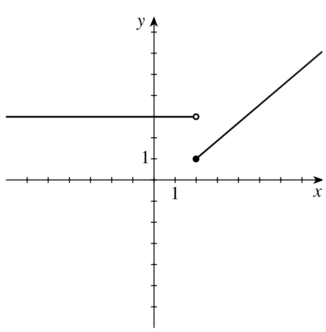
33. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$



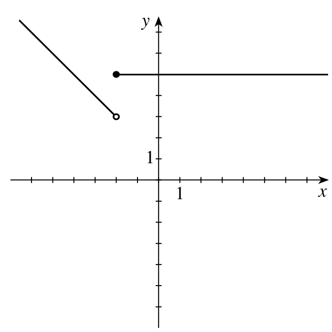
34. $f(x) = \begin{cases} 1 & \text{if } x \leq 1 \\ x + 1 & \text{if } x > 1 \end{cases}$



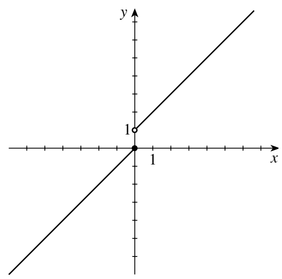
35. $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ x - 1 & \text{if } x \geq 2 \end{cases}$



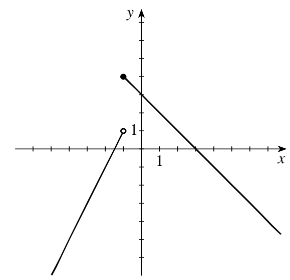
36. $f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ 5 & \text{if } x \geq -2 \end{cases}$



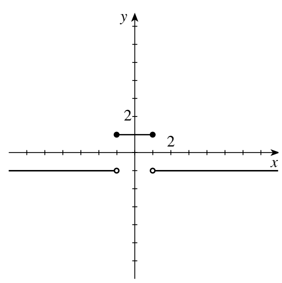
37. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x+1 & \text{if } x > 0 \end{cases}$



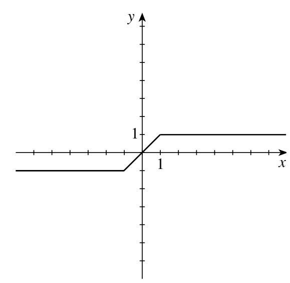
38. $f(x) = \begin{cases} 2x+3 & \text{if } x < -1 \\ 3-x & \text{if } x \ge -1 \end{cases}$



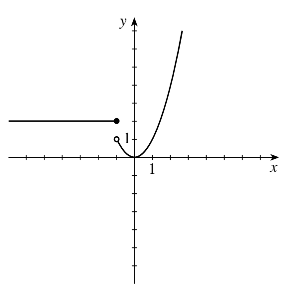
39. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$



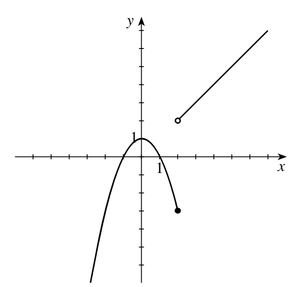
40. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$



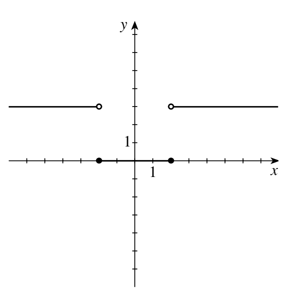
41. $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$



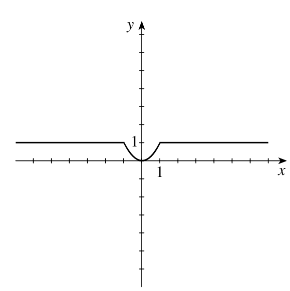
42. $f(x) = \begin{cases} 1-x^2 & \text{if } x \le 2 \\ x & \text{if } x > 2 \end{cases}$



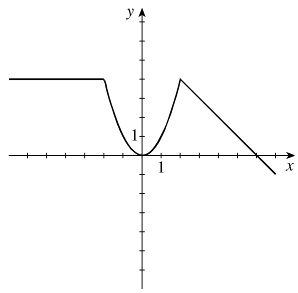
43. $f(x) = \begin{cases} 0 & \text{if } |x| \le 2 \\ 3 & \text{if } |x| > 2 \end{cases}$



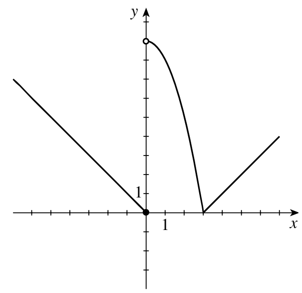
44. $f(x) = \begin{cases} x^2 & \text{if } |x| \le 1 \\ 1 & \text{if } |x| > 1 \end{cases}$



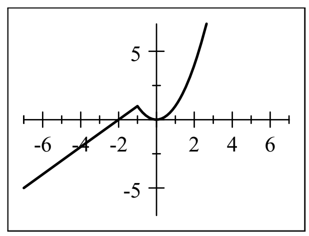
45. $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -x + 6 & \text{if } x > 2 \end{cases}$



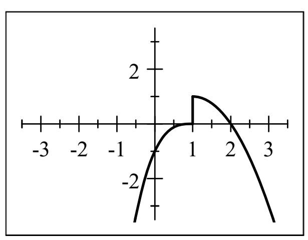
46. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ 9 - x^2 & \text{if } 0 < x \le 3 \\ x - 3 & \text{if } x > 3 \end{cases}$

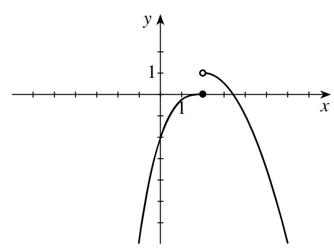


47. $f(x) = \begin{cases} x + 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$



48. $f(x) = \begin{cases} 2x - x^2 & \text{if } x > 1 \\ (x-1)^3 & \text{if } x \le 1 \end{cases}$ The first graph shows the output of a typical graphing device. However, the actual graph of this function is also shown, and its difference from the graphing device's version should be noted.





49. $f(x) = \begin{cases} -2 & \text{if } x < -2 \\ x & \text{if } -2 \le x \le 2 \\ 2 & \text{if } x > 2 \end{cases}$

50. $f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ 1 - x & \text{if } -1 < x \le 2 \\ -2 & \text{if } x > 2 \end{cases}$

51. The curves in parts (a) and (c) are graphs of a function of x, by the Vertical Line Test.

52. The curves in parts (b) and (c) are graphs of functions of x, by the Vertical Line Test.

53. The given curve is the graph of a function of x, by the Vertical Line Test. Domain: $[-3, 2]$. Range: $[-2, 2]$.

54. No, the given curve is not the graph of a function of x, by the Vertical Line Test.

55. No, the given curve is not the graph of a function of x, by the Vertical Line Test.

56. The given curve is the graph of a function of x, by the Vertical Line Test. Domain: $[-3, 2]$. Range: $\{-2\} \cup (0, 3]$.

57. Solving for y in terms of x gives $3x - 5y = 7 \Leftrightarrow y = \frac{3}{5}x - \frac{7}{5}$. This defines y as a function of x.

58. Solving for y in terms of x gives $3x^2 - y = 5 \Leftrightarrow y = 3x^2 - 5$. This defines y as a function of x.

59. Solving for y in terms of x gives $x = y^2 \Leftrightarrow y = \pm\sqrt{x}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

60. Solving for y in terms of x gives $x^2 + (y-1)^2 = 4 \Leftrightarrow (y-1)^2 = 4 - x^2 \Leftrightarrow y - 1 = \pm\sqrt{4-x^2} \Leftrightarrow y = 1 \pm \sqrt{4-x^2}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

61. Solving for y in terms of x gives $2x - 4y^2 = 3 \Leftrightarrow 4y^2 = 2x - 3 \Leftrightarrow y = \pm\frac{1}{2}\sqrt{2x-3}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

62. Solving for y in terms of x gives $2x^2 - 4y^2 = 3 \Leftrightarrow 4y^2 = 2x^2 - 3 \Leftrightarrow y = \pm\frac{1}{2}\sqrt{2x^2-3}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

63. Solving for y in terms of x using the Quadratic Formula gives $2xy - 5y^2 = 4 \Leftrightarrow 5y^2 - 2xy + 4 = 0 \Leftrightarrow$
$y = \dfrac{-(-2x) \pm \sqrt{(-2x)^2 - 4(5)(4)}}{2(5)} = \dfrac{2x \pm \sqrt{4x^2-80}}{10} = \dfrac{x \pm \sqrt{x^2-20}}{5}$. The last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

64. Solving for y in terms of x gives $\sqrt{y} - 5 = x \Leftrightarrow y = (x+5)^2$. This defines y as a function of x.

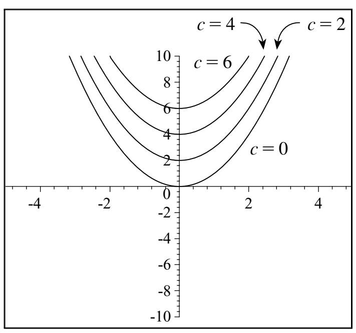
65. Solving for y in terms of x gives $2|x| + y = 0 \Leftrightarrow y = -2|x|$. This defines y as a function of x.

66. Solving for y in terms of x gives $2x + |y| = 0 \Leftrightarrow |y| = -2x$. Since $|a| = |-a|$, the last equation gives two values of y for a given value of x. Thus, this equation does not define y as a function of x.

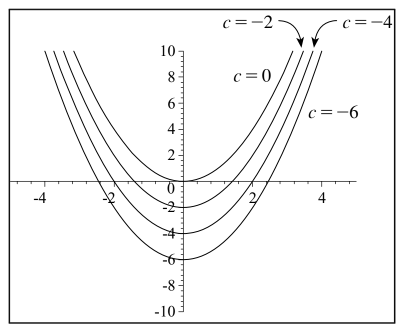
67. Solving for y in terms of x gives $x = y^3 \Leftrightarrow y = \sqrt[3]{x}$. This defines y as a function of x.

68. Solving for y in terms of x gives $x = y^4 \Leftrightarrow y = \pm\sqrt[4]{x}$. The last equation gives two values of y for any positive value of x. Thus, this equation does not define y as a function of x.

69. **(a)** $f(x) = x^2 + c$, for $c = 0, 2, 4$, and 6.

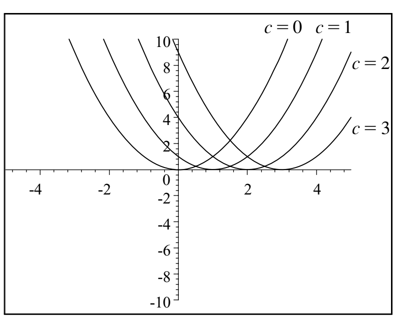


(b) $f(x) = x^2 + c$, for $c = 0, -2, -4$, and -6.

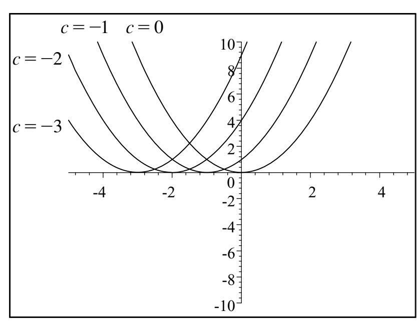


(c) The graphs in part (a) are obtained by shifting the graph of $f(x) = x^2$ upward c units, $c > 0$. The graphs in part (b) are obtained by shifting the graph of $f(x) = x^2$ downward c units.

70. **(a)** $f(x) = (x-c)^2$, for $c = 0, 1, 2$, and 3.

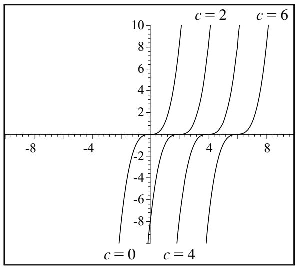


(b) $f(x) = (x-c)^2$, for $c = 0, -1, -2$, and -3.

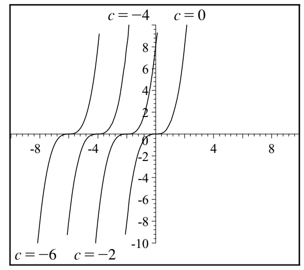


(c) The graphs in part (a) are obtained by shifting the graph of $y = x^2$ to the right 1, 2, and 3 units, while the graphs in part (b) are obtained by shifting the graph of $y = x^2$ to the left 1, 2, and 3 units.

71. **(a)** $f(x) = (x - c)^3$, for $c = 0, 2, 4$, and 6.

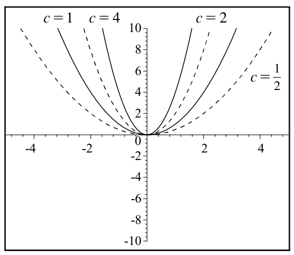


(b) $f(x) = (x - c)^3$, for $c = 0, -2, -4$, and -6.

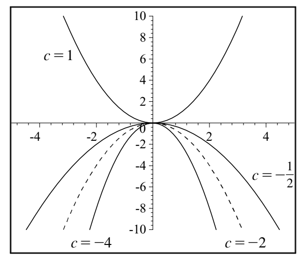


(c) The graphs in part (a) are obtained by shifting the graph of $f(x) = x^3$ to the right c units, $c > 0$. The graphs in part (b) are obtained by shifting the graph of $f(x) = x^3$ to the left $|c|$ units, $c < 0$.

72. **(a)** $f(x) = cx^2$, for $c = 1, \frac{1}{2}, 2$, and 4.

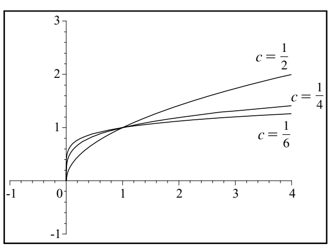


(b) $f(x) = cx^2$, for $c = 1, -1, -\frac{1}{2}$, and -2.

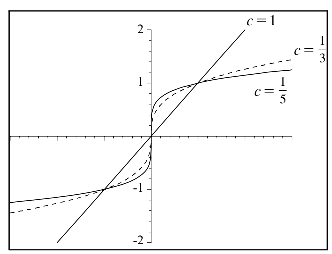


(c) As $|c|$ increases, the graph of $f(x) = cx^2$ is stretched vertically. As $|c|$ decreases, the graph of f is flattened. When $c < 0$, the graph is reflected about the x-axis.

73. **(a)** $f(x) = x^c$, for $c = \frac{1}{2}, \frac{1}{4}$, and $\frac{1}{6}$.

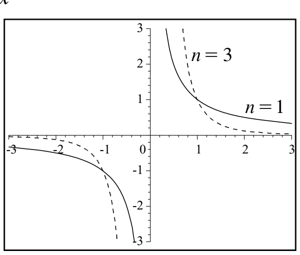


(b) $f(x) = x^c$, for $c = 1, \frac{1}{3}$, and $\frac{1}{5}$.

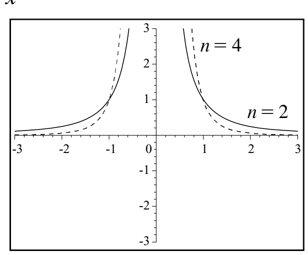


(c) Graphs of even roots are similar to $y = \sqrt{x}$, graphs of odd roots are similar to $y = \sqrt[3]{x}$. As c increases, the graph of $y = \sqrt[c]{x}$ becomes steeper near $x = 0$ and flatter when $x > 1$.

74. **(a)** $f(x) = \dfrac{1}{x^n}$, for $n = 1$ and 3.



(b) $f(x) = \dfrac{1}{x^n}$, for $n = 2$ and 4.



(c) As n increases, the graphs of $y = 1/x^n$ go to zero faster for x large. Also, as n increases and x goes to 0, the graphs of $y = 1/x^n$ go to infinity faster. The graphs of $y = 1/x^n$ for n odd are similar to each other. Likewise, the graphs for n even are similar to each other.

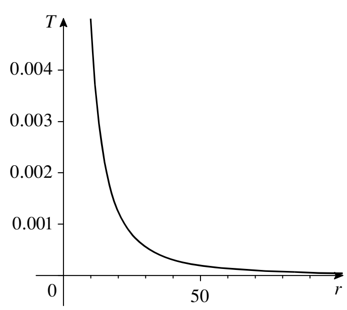
75. The slope of the line segment joining the points $(-2, 1)$ and $(4, -6)$ is $m = \dfrac{-6-1}{4-(-2)} = -\frac{7}{6}$. Using the point-slope form, we have $y - 1 = -\frac{7}{6}(x+2) \Leftrightarrow y = -\frac{7}{6}x - \frac{7}{3} + 1 \Leftrightarrow y = -\frac{7}{6}x - \frac{4}{3}$. Thus the function is $f(x) = -\frac{7}{6}x - \frac{4}{3}$ for $-2 \le x \le 4$.

76. The slope of the line containing the points $(-3, -2)$ and $(6, 3)$ is $m = \dfrac{-2-3}{-3-6} = \dfrac{-5}{-9} = \frac{5}{9}$. Using the point-slope equation of the line, we have $y - 3 = \frac{5}{9}(x-6) \Leftrightarrow y = \frac{5}{9}x - \frac{10}{3} + 3 = \dfrac{5}{9}x - \frac{1}{3}$. Thus the function is $f(x) = \frac{5}{9}x - \frac{1}{3}$, for $-3 \le x \le 6$.

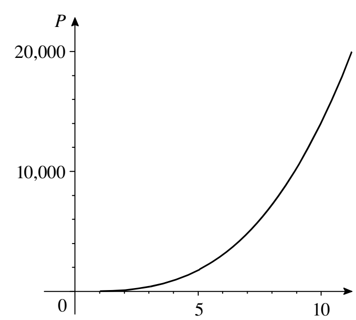
77. First solve the circle for y: $x^2 + y^2 = 9 \Leftrightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9-x^2}$. Since we seek the top half of the circle, we choose $y = \sqrt{9-x^2}$. So the function is $f(x) = \sqrt{9-x^2}$, $-3 \le x \le 3$.

78. First solve the circle for y: $x^2 + y^2 = 9 \Leftrightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9-x^2}$. Since we seek the bottom half of the circle, we choose $y = -\sqrt{9-x^2}$. So the function is $f(x) = -\sqrt{9-x^2}$, $-3 \le x \le 3$.

79. We graph $T(r) = \dfrac{0.5}{r^2}$ for $10 \le r \le 100$. As the balloon is inflated, the skin gets thinner, as we would expect.

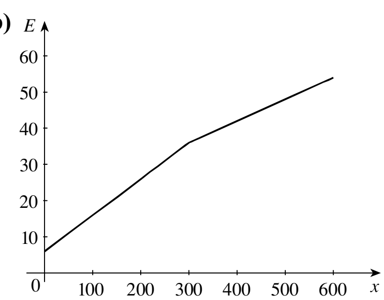


80. We graph $P(v) = 14.1v^3$ for $1 \le v \le 10$. As wind speed increases, so does power output, as expected.

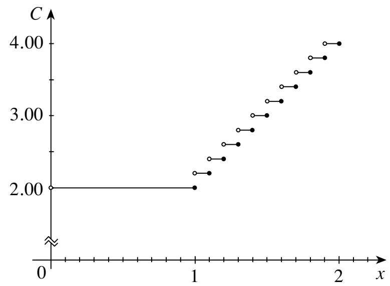


81. (a) $E(x) = \begin{cases} 6.00 + 0.10x & \text{if } 0 \le x \le 300 \\ 36.00 + 0.06(x - 300) & \text{if } 300 < x \end{cases}$

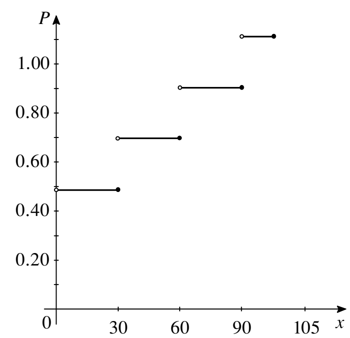
(b)



82. $C(x) = \begin{cases} 2.00 & \text{if } 0 < x \leq 1 \\ 2.20 & \text{if } 1 < x \leq 1.1 \\ 2.40 & \text{if } 1.1 < x \leq 1.2 \\ \vdots & \\ 4.00 & \text{if } 1.9 < x < 2 \end{cases}$



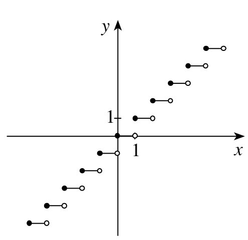
83. $P(x) = \begin{cases} 0.49 & \text{if } 0 < x \leq 30 \\ 0.70 & \text{if } 30 < x \leq 60 \\ 0.91 & \text{if } 60 < x \leq 90 \\ 1.12 & \text{if } 90 < x \leq 105 \end{cases}$



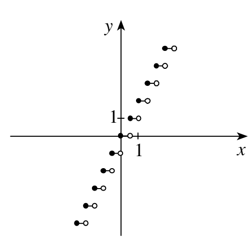
84. The graph of $x = y^2$ is not the graph of a function because both $(1, 1)$ and $(-1, 1)$ satisfy the equation $x = y^2$. The graph of $x = y^3$ is the graph of a function because $x = y^3 \Leftrightarrow x^{1/3} = y$. If n is even, then both $(1, 1)$ and $(-1, 1)$ satisfies the equation $x = y^n$, so the graph of $x = y^n$ is not the graph of a function. When n is odd, $y = x^{1/n}$ is defined for all real numbers, and since $y = x^{1/n} \Leftrightarrow x = y^n$, the graph of $x = y^n$ is the graph of a function.

85. Answers will vary. Some examples are almost anything we purchase based on weight, volume, length, or time, for example gasoline. Although the amount delivered by the pump is continuous, the amount we pay is rounded to the penny. An example involving time would be the cost of a telephone call.

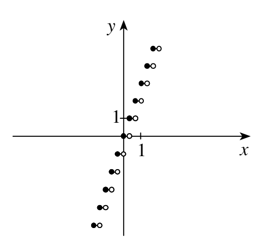
86.



$f(x) = [\![x]\!]$



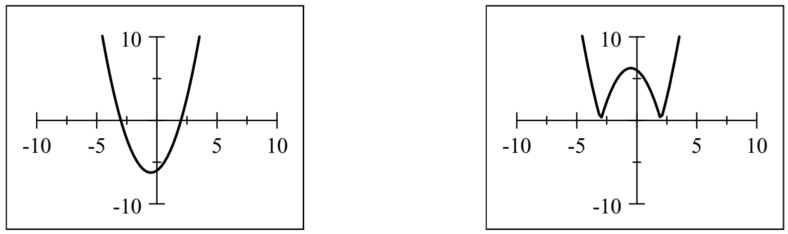
$g(x) = [\![2x]\!]$



$h(x) = [\![3x]\!]$

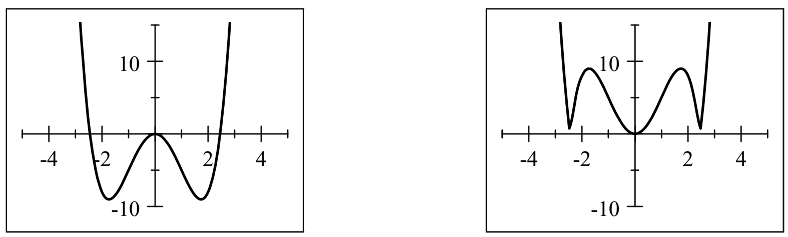
The graph of $k(x) = [\![nx]\!]$ is a step function whose steps are each $\dfrac{1}{n}$ wide.

87. (a) The graphs of $f(x) = x^2 + x - 6$ and $g(x) = \left|x^2 + x - 6\right|$ are shown in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



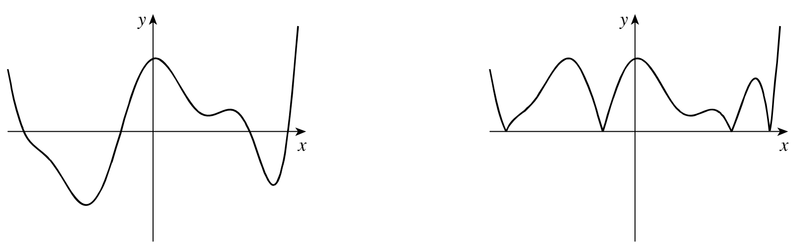
For those values of x where $f(x) \geq 0$, the graphs of f and g coincide, and for those values of x where $f(x) < 0$, the graph of g is obtained from that of f by reflecting the part below the x-axis about the x-axis.

(b) The graphs of $f(x) = x^4 - 6x^2$ and $g(x) = \left|x^4 - 6x^2\right|$ are shown in the viewing rectangle $[-5, 5]$ by $[-10, 15]$.



For those values of x where $f(x) \geq 0$, the graphs of f and g coincide, and for those values of x where $f(x) < 0$, the graph of g is obtained from that of f by reflecting the part below the x-axis above the x-axis.

(c) In general, if $g(x) = |f(x)|$, then for those values of x where $f(x) \geq 0$, the graphs of f and g coincide, and for those values of x where $f(x) < 0$, the graph of g is obtained from that of f by reflecting the part below the x-axis above the x-axis.



$y = f(x)$ $\qquad\qquad$ $y = g(x)$

2.3 GETTING INFORMATION FROM THE GRAPH OF A FUNCTION

1. To find a function value $f(a)$ from the graph of f we find the height of the graph above the x-axis at $x = a$. From the graph of f we see that $f(3) = 4$ and $f(1) = 0$. The net change in f between $x = 1$ and $x = 3$ is $f(3) - f(1) = 4 - 0 = 4$.

2. The domain of the function f is all the x-values of the points on the graph, and the range is all the corresponding y-values. From the graph of f we see that the domain of f is the interval $(-\infty, \infty)$ and the range of f is the interval $(-\infty, 7]$.

3. (a) If f is increasing on an interval, then the y-values of the points on the graph *rise* as the x-values increase. From the graph of f we see that f is increasing on the intervals $(-\infty, 2)$ and $(4, 5)$.

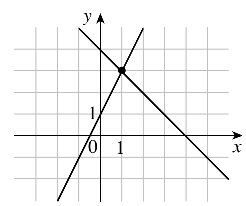
(b) If f is decreasing on an interval, then y-values of the points on the graph *fall* as the x-values increase. From the graph of f we see that f is decreasing on the intervals $(2, 4)$ and $(5, \infty)$.

4. **(a)** A function value $f(a)$ is a local maximum value of f if $f(a)$ is the *largest* value of f on some interval containing a. From the graph of f we see that there are two local maximum values of f: one maximum is 7, and it occurs when $x = 2$; the other maximum is 6, and it occurs when $x = 5$.

(b) A function value $f(a)$ is a local minimum value of f if $f(a)$ is the *smallest* value of f on some interval containing a. From the graph of f we see that there is one local minimum value of f. The minimum value is 2, and it occurs when $x = 4$.

5. The solutions of the equation $f(x) = 0$ are the x-intercepts of the graph of f. The solution of the inequality $f(x) \geq 0$ is the set of x-values at which the graph of f is on or above the x-axis. From the graph of f we find that the solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 7$, and the solution of the inequality $f(x) \geq 0$ is the interval $[1, 7]$.

6. **(a)** To solve the equation $2x + 1 = -x + 4$ graphically we graph the functions $f(x) = 2x + 1$ and $g(x) = -x + 4$ on the same set of axes and determine the values of x at which the graphs of f and g intersect. From the graph, we see that the solution is $x = 1$.



(b) To solve the inequality $2x + 1 < -x + 4$ graphically we graph the functions $f(x) = 2x + 1$ and $g(x) = -x + 4$ on the same set of axes and find the values of x at which the graph of g is *higher* than the graph of f. From the graphs in part (a) we see that the solution of the inequality is $(-\infty, 1)$.

7. **(a)** $h(-2) = 1, h(0) = -1, h(2) = 3$, and $h(3) = 4$.

(b) Domain: $[-3, 4]$. Range: $[-1, 4]$.

(c) $h(-3) = 3$, $h(2) = 3$, and $h(4) = 3$, so $h(x) = 3$ when $x = -3$, $x = 2$, or $x = 4$.

(d) The graph of h lies below or on the horizontal line $y = 3$ when $-3 \leq x \leq 2$ or $x = 4$, so $h(x) \leq 3$ for those values of x.

(e) The net change in h between $x = -3$ and $x = 3$ is $h(3) - h(-3) = 4 - 3 = 1$.

8. **(a)** $g(-4) = 3, g(-2) = 2, g(0) = -2, g(2) = 1$, and $g(4) = 0$.

(b) Domain: $[-4, 4]$. Range: $[-2, 3]$.

(c) $g(-4) = 3$. [Note that $g(2) = 1$ not 3.]

(d) It appears that $g(x) \leq 0$ for $-1 \leq x \leq 1.8$ and for $x = 4$; that is, for $\{x \mid -1 \leq x \leq 1.8\} \cup \{4\}$.

(e) $g(-1) = 0$ and $g(2) = 1$, so the net change between $x = -1$ and $x = 2$ is $1 - 0 = 1$.

9. **(a)** $f(0) = 3 > \frac{1}{2} = g(0)$. So $f(0)$ is larger.

(b) $f(-3) \approx -1 < 2.5 = g(-3)$. So $g(-3)$ is larger.

(c) $f(x) = g(x)$ for $x = -2$ and $x = 2$.

(d) $f(x) \leq g(x)$ for $-4 \leq x \leq -2$ and $2 \leq x \leq 3$; that is, on the intervals $[-4, -2]$ and $[2, 3]$.

(e) $f(x) > g(x)$ for $-2 < x < 2$; that is, on the interval $(-2, 2)$.

10. **(a)** The graph of g is higher than the graph of f at $x = 6$, so $g(6)$ is larger.

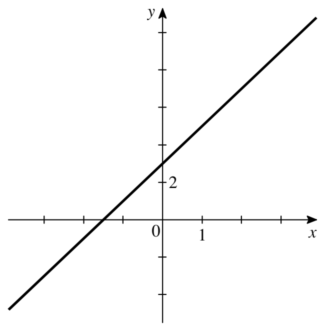
(b) The graph of f is higher than the graph of g at $x = 3$, so $f(3)$ is larger.

(c) The graphs of f and g intersect at $x = 2$, $x = 5$, and $x \approx 7$, so $f(x) = g(x)$ for these values of x.

(d) $f(x) \leq g(x)$ for $1 \leq x \leq 2$ and approximately $5 \leq x \leq 7$; that is, on $[1, 2]$ and $[5, 7]$.

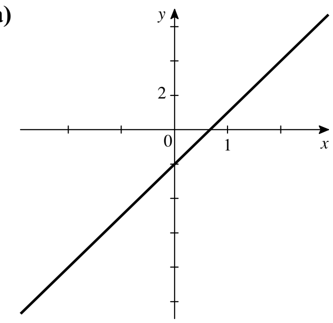
(e) $f(x) > g(x)$ for $2 < x < 5$ and approximately $7 < x \leq 8$; that is, on $[2, 5)$ and $(7, 8]$.

11. (a)



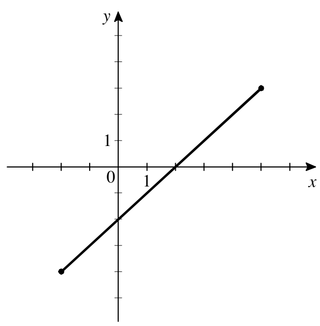
(b) Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

12. (a)



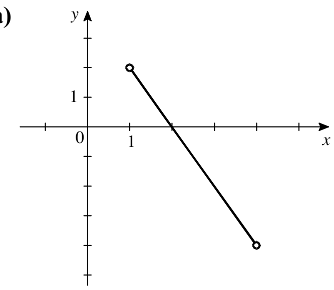
(b) Domain: $(-\infty, \infty)$; Range $(-\infty, \infty)$

13. (a)



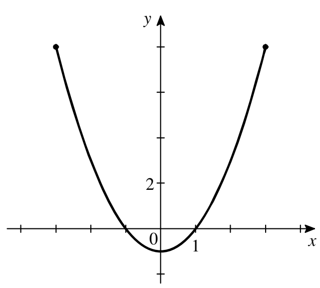
(b) Domain: $[-2, 5]$; Range $[-4, 3]$

14. (a)



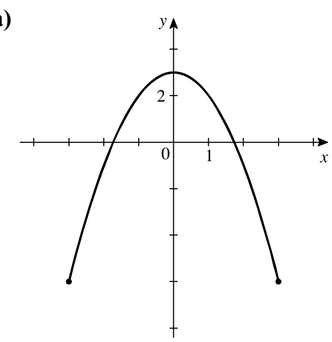
(b) Domain: $(1, 4)$; Range $(-4, 2)$

15. (a)



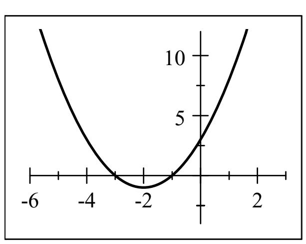
(b) Domain: $[-3, 3]$; Range: $[-1, 8]$

16. (a)



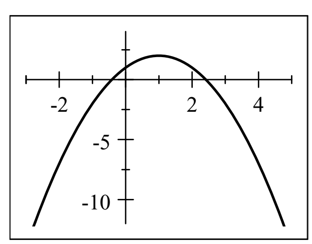
(b) Domain: $[-3, 3]$; Range $[-6, 3]$

17. (a)



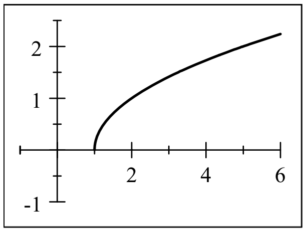
(b) Domain: $(-\infty, \infty)$; Range: $[-1, \infty)$

18. (a)



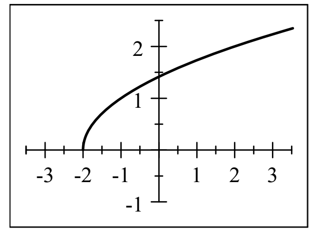
(b) Domain: $(-\infty, \infty)$; Range: $(-\infty, 2]$

19. (a)



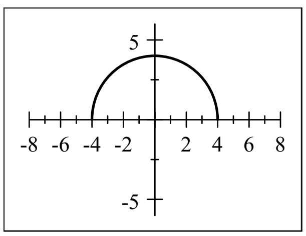
(b) Domain: $[1, \infty)$; Range: $[0, \infty)$

20. (a)



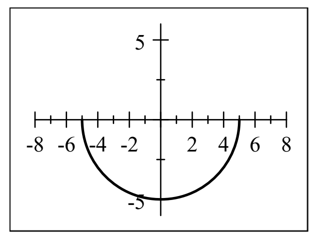
(b) Domain: $[-2, \infty)$; Range: $[0, \infty)$

21. (a)



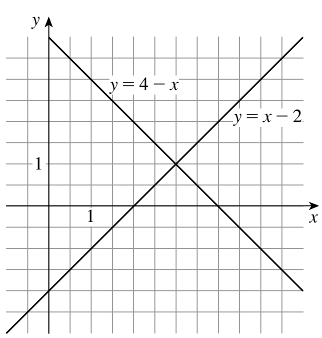
(b) Domain: $[-4, 4]$; Range: $[0, 4]$

22. (a)



(b) Domain: $[-5, 5]$; Range: $[-5, 0]$

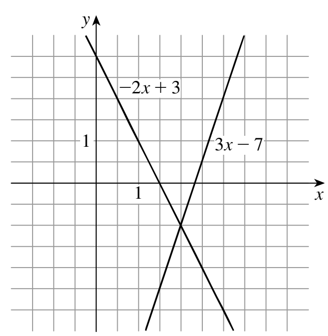
23.



(a) From the graph, we see that $x - 2 = 4 - x$ when $x = 3$.

(b) From the graph, we see $x - 2 > 4 - x$ when $x > 3$.

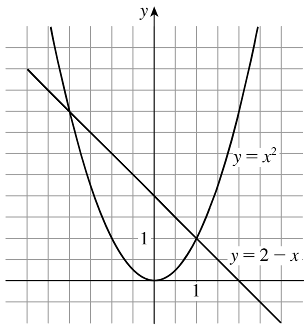
24.



(a) From the graph, we see that $-2x + 3 = 3x - 7$ when $x = 2$.

(b) From the graph, we see that $-2x + 3 \le 3x - 7$ when $x \ge 2$.

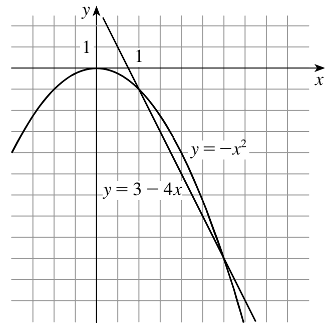
25.



(a) From the graph, we see that $x^2 = 2 - x$ when $x = -2$ or $x = 1$.

(b) From the graph, we see that $x^2 \le 2 - x$ when $-2 \le x \le 1$.

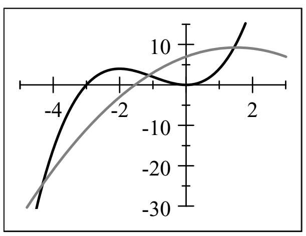
26.



(a) From the graph, we see that $-x^2 = 3 - 4x$ when $x = 1$ or $x = 3$.

(b) From the graph, we see that $-x^2 \ge 3 - 4x$ when $1 \le x \le 3$.

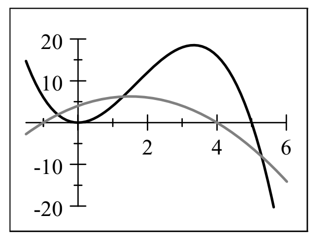
27.



(a) We graph $y = x^3 + 3x^2$ (black) and $y = -x^2 + 3x + 7$ (gray). From the graph, we see that the graphs intersect at $x \approx -4.32$, $x \approx -1.12$, and $x \approx 1.44$.

(b) From the graph, we see that $x^3 + 3x^2 \ge -x^2 + 3x + 7$ on approximately $[-4.32, -1.12]$ and $[1.44, \infty)$.

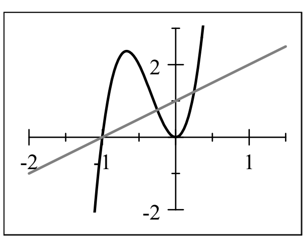
28.



(a) We graph $y = 5x^2 - x^3$ (black) and $y = -x^2 + 3x + 4$ (gray). From the graph, we see that the graphs intersect at $x \approx -0.58$, $x \approx 1.29$, and $x \approx 5.29$.

(b) From the graph, we see that $5x^2 - x^3 \le -x^2 + 3x + 4$ on approximately $[-0.58, 1.29]$ and $[5.29, \infty)$.

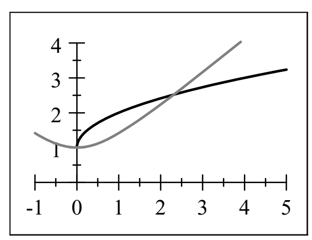
29.



(a) We graph $y = 16x^3 + 16x^2$ (black) and $y = x + 1$ (gray). From the graph, we see that the graphs intersect at $x = -1$, $x = -\frac{1}{4}$, and $x = \frac{1}{4}$.

(b) From the graph, we see that $16x^3 + 16x^2 \ge x + 1$ on $\left[-1, -\frac{1}{4}\right]$ and $\left[\frac{1}{4}, \infty\right)$.

30.



(a) We graph $y = 1 + \sqrt{x}$ (black) and $y = \sqrt{x^2 + 1}$ (gray). From the graph, we see that the solutions are $x = 0$ and $x \approx 2.31$.

(b) From the graph, we see that $1 + \sqrt{x} > \sqrt{x^2 + 1}$ on approximately $(0, 2.31)$.

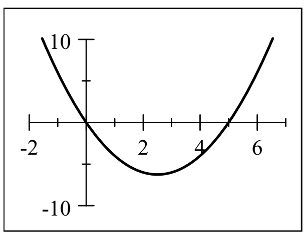
31. (a) The domain is $[-1, 4]$ and the range is $[-1, 3]$.
(b) The function is increasing on $(-1, 1)$ and $(2, 4)$ and decreasing on $(1, 2)$.

32. (a) The domain is $[-2, 3]$ and the range is $[-2, 3]$.
(b) The function is increasing on $(0, 1)$ and decreasing on $(-2, 0)$ and $(1, 3)$.

33. (a) The domain is $[-3, 3]$ and the range is $[-2, 2]$.
(b) The function is increasing on $(-2, -1)$ and $(1, 2)$ and decreasing on $(-3, -2)$, $(-1, 1)$, and $(2, 3)$.

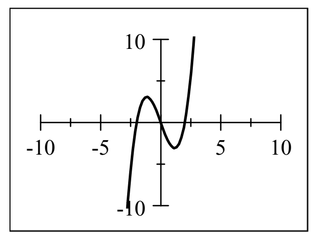
34. (a) The domain is $[-2, 2]$ and the range is $[-2, 2]$.
(b) The function is increasing on $(-1, 1)$ and decreasing on $(-2, -1)$ and $(1, 2)$.

35. (a) $f(x) = x^2 - 5x$ is graphed in the viewing rectangle $[-2, 7]$ by $[-10, 10]$.



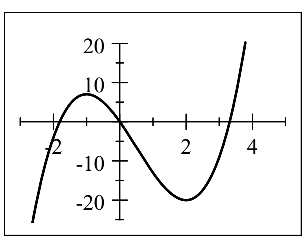
(b) The domain is $(-\infty, \infty)$ and the range is $[-6.25, \infty)$.
(c) The function is increasing on $(2.5, \infty)$. It is decreasing on $(-\infty, 2.5)$.

36. (a) $f(x) = x^3 - 4x$ is graphed in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



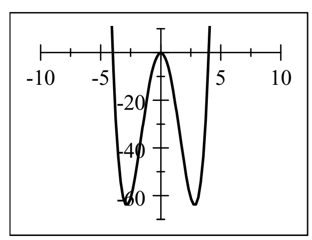
(b) The domain and range are $(-\infty, \infty)$.
(c) The function is increasing on $(-\infty, -1.15)$ and $(1.15, \infty)$. It is decreasing on $(-1.15, 1.15)$.

37. (a) $f(x) = 2x^3 - 3x^2 - 12x$ is graphed in the viewing rectangle $[-3, 5]$ by $[-25, 20]$.



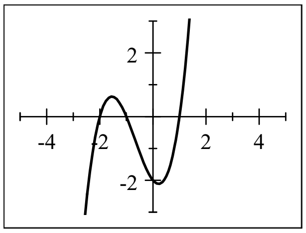
(b) The domain and range are $(-\infty, \infty)$.
(c) The function is increasing on $(-\infty, -1)$ and $(2, \infty)$. It is decreasing on $(-1, 2)$.

38. (a) $f(x) = x^4 - 16x^2$ is graphed in the viewing rectangle $[-10, 10]$ by $[-70, 10]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[-64, \infty)$.
(c) The function is increasing on $(-2.83, 0)$ and $(2.83, \infty)$. It is decreasing on $(-\infty, -2.83)$ and $(0, 2.83)$.

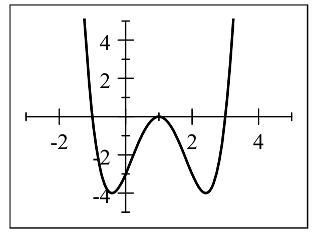
39. (a) $f(x) = x^3 + 2x^2 - x - 2$ is graphed in the viewing rectangle $[-5, 5]$ by $[-3, 3]$.



(b) The domain and range are $(-\infty, \infty)$.

(c) The function is increasing on $(-\infty, -1.55)$ and $(0.22, \infty)$. It is decreasing on $(-1.55, 0.22)$.

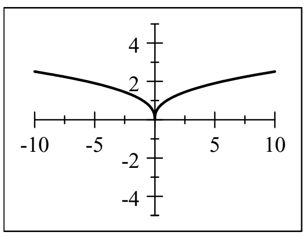
40. (a) $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ is graphed in the viewing rectangle $[-3, 5]$ by $[-5, 5]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[-4, \infty)$.

(c) The function is increasing on $(-0.4, 1)$ and $(2.4, \infty)$. It is decreasing on $(-\infty, -0.4)$ and $(1, 2.4)$.

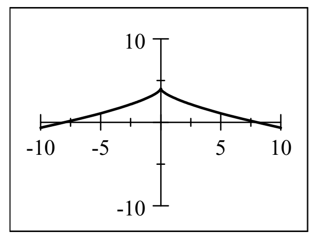
41. (a) $f(x) = x^{2/5}$ is graphed in the viewing rectangle $[-10, 10]$ by $[-5, 5]$.



(b) The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$.

(c) The function is increasing on $(0, \infty)$. It is decreasing on $(-\infty, 0)$.

42. (a) $f(x) = 4 - x^{2/3}$ is graphed in the viewing rectangle $[-10, 10]$ by $[-10, 10]$.



(b) The domain is $(-\infty, \infty)$ and the range is $(-\infty, 4]$.

(c) The function is increasing on $(-\infty, 0)$. It is decreasing on $(0, \infty)$.

43. (a) Local maximum: 2 at $x = 0$. Local minimum: -1 at $x = -2$ and 0 at $x = 2$.

(b) The function is increasing on $(-2, 0)$ and $(2, \infty)$ and decreasing on $(-\infty, -2)$ and $(0, 2)$.

44. (a) Local maximum: 2 at $x = -2$ and 1 at $x = 2$. Local minimum: -1 at $x = 0$.

(b) The function is increasing on $(-\infty, -2)$ and $(0, 2)$ and decreasing on $(-2, 0)$ and $(2, \infty)$.

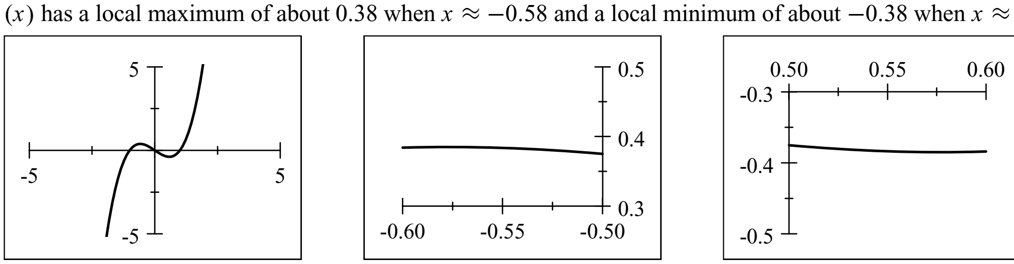
45. (a) Local maximum: 0 at $x = 0$ and 1 at $x = 3$. Local minimum: -2 at $x = -2$ and -1 at $x = 1$.

(b) The function is increasing on $(-2, 0)$ and $(1, 3)$ and decreasing on $(-\infty, -2)$, $(0, 1)$, and $(3, \infty)$.

46. (a) Local maximum: 3 at $x = -2$ and 2 at $x = 1$. Local minimum: 0 at $x = -1$ and -1 at $x = 2$.

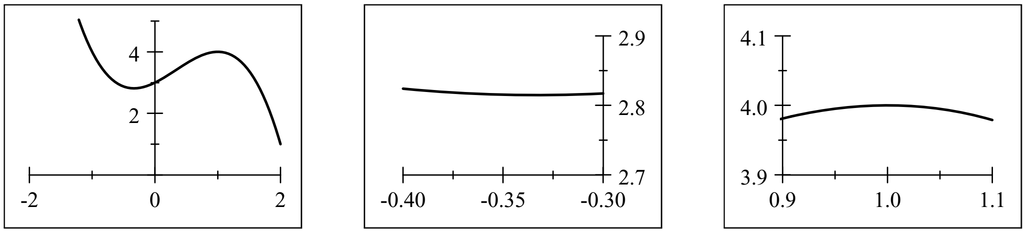
(b) The function is increasing on $(-\infty, -2)$, $(-1, 1)$, and $(2, \infty)$ and decreasing on $(-2, -1)$ and $(1, 2)$.

47. (a) In the first graph, we see that $f(x) = x^3 - x$ has a local minimum and a local maximum. Smaller x- and y-ranges show that $f(x)$ has a local maximum of about 0.38 when $x \approx -0.58$ and a local minimum of about -0.38 when $x \approx 0.58$.



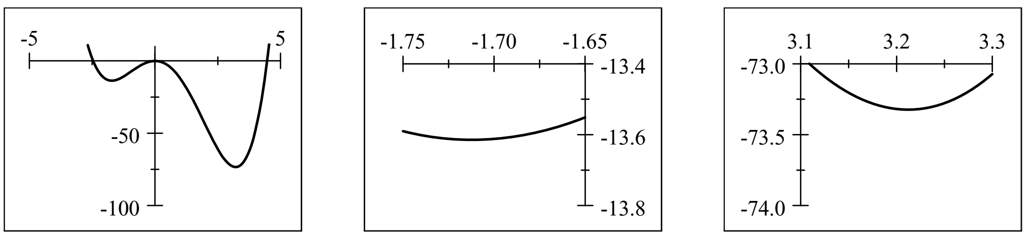
(b) The function is increasing on $(-\infty, -0.58)$ and $(0.58, \infty)$ and decreasing on $(-0.58, 0.58)$.

48. (a) In the first graph, we see that $f(x) = 3 + x + x^2 - x^3$ has a local minimum and a local maximum. Smaller x- and y-ranges show that $f(x)$ has a local maximum of about 4.00 when $x \approx 1.00$ and a local minimum of about 2.81 when $x \approx -0.33$.



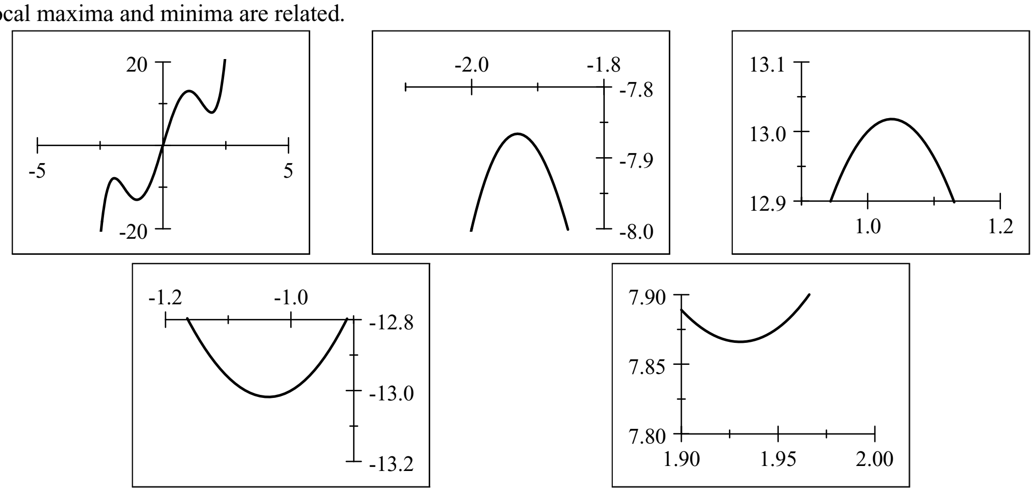
(b) The function is increasing on $(-0.33, 1.00)$ and decreasing on $(-\infty, -0.33)$ and $(1.00, \infty)$.

49. (a) In the first graph, we see that $g(x) = x^4 - 2x^3 - 11x^2$ has two local minimums and a local maximum. The local maximum is $g(x) = 0$ when $x = 0$. Smaller x- and y-ranges show that local minima are $g(x) \approx -13.61$ when $x \approx -1.71$ and $g(x) \approx -73.32$ when $x \approx 3.21$.



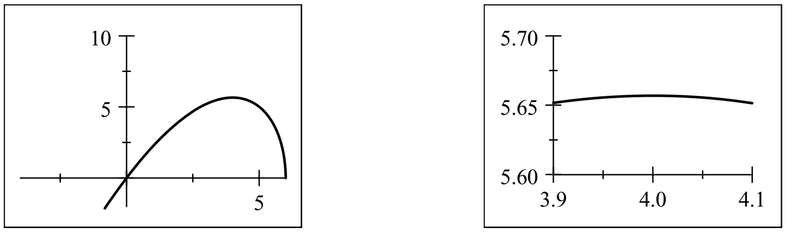
(b) The function is increasing on $(-1.71, 0)$ and $(3.21, \infty)$ and decreasing on $(-\infty, -1.71)$ and $(0, 3.21)$.

50. (a) In the first graph, we see that $g(x) = x^5 - 8x^3 + 20x$ has two local minimums and two local maximums. The local maximums are $g(x) \approx -7.87$ when $x \approx -1.93$ and $g(x) \approx 13.02$ when $x = 1.04$. Smaller x- and y-ranges show that local minimums are $g(x) \approx -13.02$ when $x = -1.04$ and $g(x) \approx 7.87$ when $x \approx 1.93$. Notice that since $g(x)$ is odd, the local maxima and minima are related.



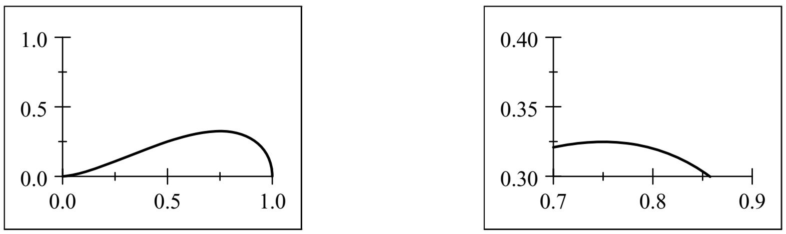
(b) The function is increasing on $(-\infty, -1.93)$, $(-1.04, 1.04)$, and $(1.93, \infty)$ and decreasing on $(-1.93, -1.04)$ and $(1.04, 1.93)$.

51. (a) In the first graph, we see that $U(x) = x\sqrt{6 - x}$ has only a local maximum. Smaller x- and y-ranges show that $U(x)$ has a local maximum of about 5.66 when $x \approx 4.00$.



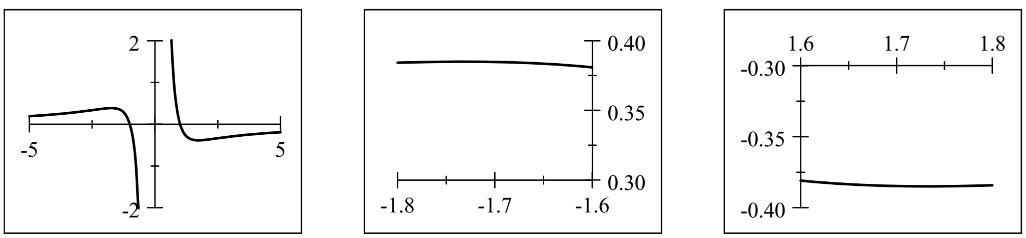
(b) The function is increasing on $(-\infty, 4.00)$ and decreasing on $(4.00, 6)$.

52. (a) In the first viewing rectangle below, we see that $U(x) = x\sqrt{x - x^2}$ has only a local maximum. Smaller x- and y-ranges show that $U(x)$ has a local maximum of about 0.32 when $x \approx 0.75$.



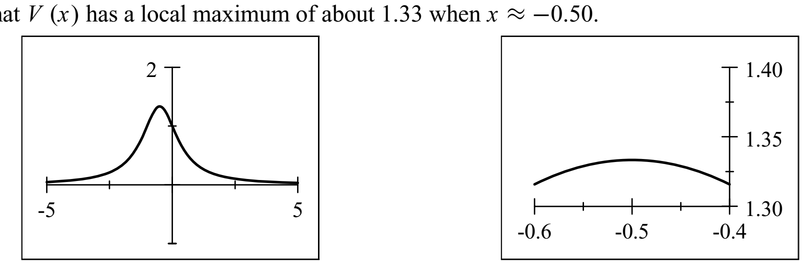
(b) The function is increasing on $(0, 0.75)$ and decreasing on $(0.75, 1)$.

53. (a) In the first graph, we see that $V(x) = \dfrac{1 - x^2}{x^3}$ has a local minimum and a local maximum. Smaller x- and y-ranges show that $V(x)$ has a local maximum of about 0.38 when $x \approx -1.73$ and a local minimum of about -0.38 when $x \approx 1.73$.



(b) The function is increasing on $(-\infty, -1.73)$ and $(1.73, \infty)$ and decreasing on $(-1.73, 0)$ and $(0, 1.73)$.

54. (a) In the first viewing rectangle below, we see that $V(x) = \dfrac{1}{x^2 + x + 1}$ has only a local maximum. Smaller x- and y-ranges show that $V(x)$ has a local maximum of about 1.33 when $x \approx -0.50$.



(b) The function is increasing on $(-\infty, -0.50)$ and decreasing on $(-0.50, \infty)$.

55. (a) At 6 A.M. the graph shows that the power consumption is about 500 megawatts. Since $t = 18$ represents 6 P.M., the graph shows that the power consumption at 6 P.M. is about 725 megawatts.

(b) The power consumption is lowest between 3 A.M. and 4 A.M..

(c) The power consumption is highest just before 12 noon.

(d) The net change in power consumption from 9 A.M. to 7 P.M. is $P(19) - P(9) \approx 690 - 790 \approx -100$ megawatts.

56. (a) The first noticeable movements occurred at time $t = 5$ seconds.

(b) It seemed to end at time $t = 30$ seconds.

(c) Maximum intensity was reached at $t = 17$ seconds.

57. (a) This person appears to be gaining weight steadily until the age of 21 when this person's weight gain slows down. The person continues to gain weight until the age of 30, at which point this person experiences a sudden weight loss. Weight gain resumes around the age of 32, and the person dies at about age 68. Thus, the person's weight W is increasing on $(0, 30)$ and $(32, 68)$ and decreasing on $(30, 32)$.

(b) The sudden weight loss could be due to a number of reasons, among them major illness, a weight loss program, etc.

(c) The net change in the person's weight from age 10 to age 20 is $W(20) - W(10) = 70 - 30 = 40$ kg.

58. (a) Measuring in hours since midnight, the salesman's distance from home D is increasing on $(8, 9)$, $(10, 12)$, and $(15, 17)$, constant on $(9, 10)$, $(12, 13)$, and $(17, 18)$, and decreasing on $(13, 15)$ and $(18, 19)$.

(b) The salesman travels away from home and stops to make a sales call between 9 A.M. and 10 A.M., and then travels further from home for a sales call between 12 noon and 1 P.M. Next he travels along a route that takes him closer to home before taking him further away from home. He then makes a final sales call between 5 P.M. and 6 P.M. and then returns home.

(c) The net change in the distance D from noon to 1 P.M. is $D(1 \text{ P.M.}) - D(\text{noon}) = 0$.

59. (a) The function W is increasing on $(0, 150)$ and $(300, \infty)$ and decreasing on $(150, 300)$.

(b) W has a local maximum at $x = 150$ and a local minimum at $x = 300$.

(c) The net change in the depth W from 100 days to 300 days is $W(300) - W(100) = 8 - 24 = -16$ m.

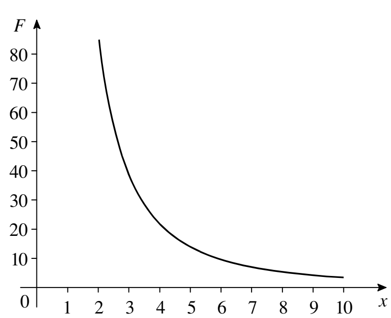
60. (a) The function P is increasing on $(0, 25)$ and decreasing on $(25, 50)$.

(b) The maximum population was 50,000, and it was attained at $x = 25$ years, which represents the year 1975.

(c) The net change in the population P from 1970 to 1990 is $P(40) - P(20) = 40 - 40 = 0$.

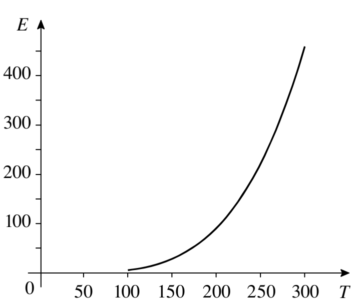
61. Runner A won the race. All runners finished the race. Runner B fell, but got up and finished the race.

62. (a)



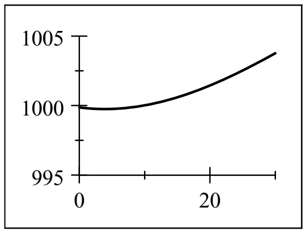
(b) As the distance x increases, the gravitational attraction F decreases. The rate of decrease is rapid at first, and slows as the distance increases.

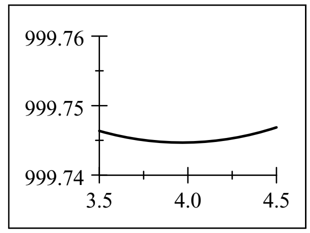
63. (a)



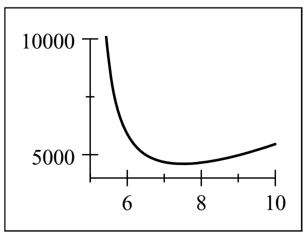
(b) As the temperature T increases, the energy E increases. The rate of increase gets larger as the temperature increases.

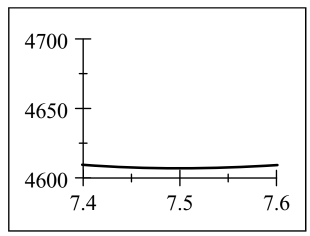
64. In the first graph, we see the general location of the minimum of $V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3$ is around $T = 4$. In the second graph, we isolate the minimum, and from this graph, we see that the minimum volume of 1 kg of water occurs at $T \approx 3.96^\circ$ C.



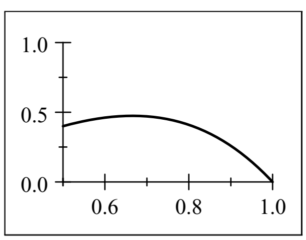


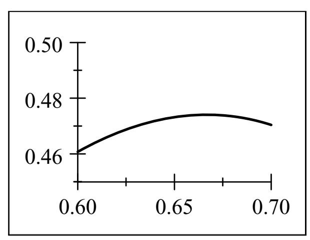
65. In the first graph, we see the general location of the minimum of $E(v) = 2.73v^3 \dfrac{10}{v-5}$. In the second graph, we isolate the minimum, and from this graph, we see that energy is minimized when $v \approx 7.5$ km/h.



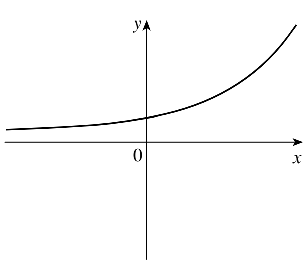


66. In the first graph, we see the general location of the maximum of $v(r) = 3.2(1-r)r^2$ is around $r = 0.7$ cm. In the second graph, we isolate the maximum, and from this graph we see that at the maximum velocity is approximately 0.47 when $r \approx 0.67$ cm.

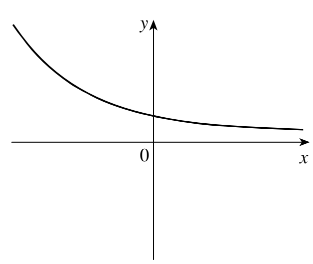




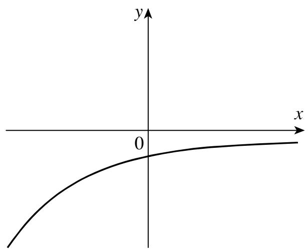
67. **(a)** $f(x)$ is always increasing, and $f(x) > 0$ for all x.



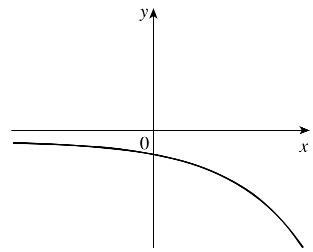
(b) $f(x)$ is always decreasing, and $f(x) > 0$ for all x.



(c) $f(x)$ is always increasing, and $f(x) < 0$ for all x.



(d) $f(x)$ is always decreasing, and $f(x) < 0$ for all x.



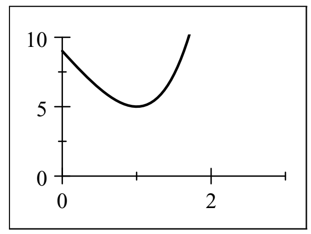
68. Numerous answers are possible.

69. (a) If $x = a$ is a local maximum of $f(x)$ then $f(a) \geq f(x) \geq 0$ for all x around $x = a$. So $[g(a)]^2 \geq [g(x)]^2$ and thus $g(a) \geq g(x)$. Similarly, if $x = b$ is a local minimum of $f(x)$, then $f(x) \geq f(b) \geq 0$ for all x around $x = b$. So $[g(x)]^2 \geq [g(b)]^2$ and thus $g(x) \geq g(b)$.

(b) Using the distance formula,

$$g(x) = \sqrt{(x-3)^2 + \left(x^2 - 0\right)^2} \\ = \sqrt{x^4 + x^2 - 6x + 9}$$

(c) Let $f(x) = x^4 + x^2 - 6x + 9$. From the graph, we see that $f(x)$ has a minimum at $x = 1$. Thus $g(x)$ also has a minimum at $x = 1$ and this minimum value is $g(1) = \sqrt{1^4 + 1^2 - 6(1) + 9} = \sqrt{5}$.



2.4 AVERAGE RATE OF CHANGE OF A FUNCTION

1. If you travel 100 kilometers in two hours then your average speed for the trip is average speed $= \dfrac{100 \text{ kilometers}}{2 \text{ hours}} = 50$ km/h.

2. The average rate of change of a function f between $x = a$ and $x = b$ is average rate of change $= \dfrac{f(b) - f(a)}{b - a}$.

3. The average rate of change of the function $f(x) = x^2$ between $x = 1$ and $x = 5$ is average rate of change $= \frac{f(5) - f(1)}{5 - 1} = \dfrac{5^2 - 1^2}{4} = \dfrac{25 - 1}{4} = \dfrac{24}{4} = 6$.

4. (a) The average rate of change of a function f between $x = a$ and $x = b$ is the slope of the *secant* line between $(a, f(a))$ and $(b, f(b))$.

(b) The average rate of change of the linear function $f(x) = 3x + 5$ between any two points is 3.

5. (a) Yes, the average rate of change of a function between $x = a$ and $x = b$ is the slope of the secant line through $(a, f(a))$ and $(b, f(b))$; that is, $\dfrac{f(b) - f(a)}{b - a}$.

(b) Yes, the average rate of change of a linear function $y = mx + b$ is the same (namely m) for all intervals.

6. (a) No, the average rate of change of an increasing function is positive over any interval.

(b) No, just because the average rate of change of a function between $x = a$ and $x = b$ is negative, it does not follow that the function is decreasing on that interval. For example, $f(x) = x^2$ has negative average rate of change between $x = -2$ and $x = 1$, but f is increasing for $0 < x < 1$.

7. (a) The net change is $f(4) - f(1) = 5 - 3 = 2$.

(b) We use the points $(1, 3)$ and $(4, 5)$, so the average rate of change is $\dfrac{5 - 3}{4 - 1} = \dfrac{2}{3}$.

8. **(a)** The net change is $f(5) - f(1) = 2 - 4 = -2$.

(b) We use the points $(1, 4)$ and $(5, 2)$, so the average rate of change is $\dfrac{2-4}{5-1} = \dfrac{-2}{4} = -\dfrac{1}{2}$.

9. **(a)** The net change is $f(5) - f(0) = 2 - 6 = -4$.

(b) We use the points $(0, 6)$ and $(5, 2)$, so the average rate of change is $\dfrac{2-6}{5-0} = \dfrac{-4}{5}$.

10. **(a)** The net change is $f(5) - f(-1) = 4 - 0 = 4$.

(b) We use the points $(-1, 0)$ and $(5, 4)$, so the average rate of change is $\dfrac{4-0}{5-(-1)} = \dfrac{4}{6} = \dfrac{2}{3}$.

11. **(a)** The net change is $f(3) - f(2) = [3(3) - 2] - [3(2) - 2] = 7 - 4 = 3$.

(b) The average rate of change is $\dfrac{f(3) - f(2)}{3-2} = \dfrac{3}{1} = 3$.

12. **(a)** The net change is $r(6) - r(3) = \left[3 - \frac{1}{3}(6)\right] - \left[3 - \frac{1}{3}(3)\right] = 1 - 2 = -1$.

(b) The average rate of change is $\dfrac{r(6) - r(3)}{6-3} = -\dfrac{1}{3}$.

13. **(a)** The net change is $h(1) - h(-4) = \left[-1 + \frac{3}{2}\right] - \left[-(-4) + \frac{3}{2}\right] = \frac{1}{2} - \frac{11}{2} = -5$.

(b) The average rate of change is $\dfrac{h(1) - h(-4)}{1-(-4)} = \dfrac{-5}{5} = -1$.

14. **(a)** The net change is $g(2) - g(-3) = \left[2 - \frac{2}{3}(2)\right] - \left[2 - \frac{2}{3}(-3)\right] = \frac{2}{3} - 4 = -\frac{10}{3}$.

(b) The average rate of change is $\dfrac{g(2) - g(-3)}{2-(-3)} = \dfrac{-\frac{10}{3}}{5} = -\dfrac{2}{3}$.

15. **(a)** The net change is $h(6) - h(3) = \left[2(6)^2 - 6\right] - \left[2(3)^2 - 3\right] = 66 - 15 = 51$.

(b) The average rate of change is $\dfrac{h(6) - h(3)}{6-3} = \dfrac{51}{3} = 17$.

16. **(a)** The net change is $f(0) - f(-2) = \left[1 - 3(0)^2\right] - \left[1 - 3(-2)^2\right] = 1 - (-11) = 12$.

(b) The average rate of change is $\dfrac{f(0) - f(-2)}{0-(-2)} = \dfrac{12}{2} = 6$.

17. **(a)** The net change is $f(10) - f(0) = \left[10^3 - 4\left(10^2\right)\right] - \left[0^3 - 4\left(0^2\right)\right] = 600 - 0 = 600$.

(b) The average rate of change is $\dfrac{f(10) - f(0)}{10-0} = \dfrac{600}{10} = 60$.

18. **(a)** The net change is $g(2) - g(-2) = \left[2^4 - 2^3 + 2^2\right] - \left[(-2)^4 - (-2)^3 + (-2)^2\right] = 12 - 28 = -16$.

(b) The average rate of change is $\dfrac{g(2) - g(-2)}{2-(-2)} = \dfrac{-16}{4} = -4$.

19. **(a)** The net change is $f(3+h) - f(3) = \left[5(3+h)^2\right] - \left[5(3)^2\right] = 45 + 30h + 5h^2 - 45 = 5h^2 + 30h$.

(b) The average rate of change is $\dfrac{f(3+h) - f(3)}{(3+h)-3} = \dfrac{5h^2 + 30h}{h} = 5h + 30$.

20. **(a)** The net change is $f(2+h) - f(2) = \left[1 - 3(2+h)^2\right] - \left[1 - 3(2)^2\right] = \left(-3h^2 - 12h - 11\right) - (-11) = -3h^2 - 12h$.

(b) The average rate of change is $\dfrac{f(2+h) - f(2)}{(2+h)-2} = \dfrac{-3h^2 - 12h}{h} = -3h - 12$.

21. **(a)** The net change is $g(a) - g(1) = \frac{1}{a} - \frac{1}{1} = \frac{1-a}{a}$.

(b) The average rate of change is $\frac{g(a) - g(1)}{a-1} = \frac{\frac{1-a}{a}}{a-1} = \frac{1-a}{a(a-1)} = -\frac{1}{a}$.

22. **(a)** The net change is $g(h) - g(0) = \frac{2}{h+1} - \frac{2}{0+1} = -\frac{2h}{h+1}$.

(b) The average rate of change is $\frac{g(h) - g(0)}{h-0} = \frac{-\frac{2h}{h+1}}{h} = \frac{-2h}{h(h+1)} = -\frac{2}{h+1}$.

23. **(a)** The net change is $f(a+h) - f(a) = \frac{2}{a+h} - \frac{2}{a} = -\frac{2h}{a(a+h)}$.

(b) The average rate of change is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{-\frac{2h}{a(a+h)}}{h} = -\frac{2h}{ah(a+h)} = -\frac{2}{a(a+h)}.$$

24. **(a)** The net change is $f(a+h) - f(a) = \sqrt{a+h} - \sqrt{a}$.

(b) The average rate of change is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \frac{(a+h) - a}{h\left(\sqrt{a+h} + \sqrt{a}\right)} = \frac{h}{h\left(\sqrt{a+h} + \sqrt{a}\right)} = \frac{1}{\sqrt{a+h} + \sqrt{a}}.$$

25. **(a)** The average rate of change is

$$\frac{f(a+h) - f(a)}{(a+h) - a} = \frac{\left[\frac{1}{2}(a+h) + 3\right] - \left[\frac{1}{2}a + 3\right]}{h} = \frac{\frac{1}{2}a + \frac{1}{2}h + 3 - \frac{1}{2}a - 3}{h} = \frac{\frac{1}{2}h}{h} = \frac{1}{2}.$$

(b) The slope of the line $f(x) = \frac{1}{2}x + 3$ is $\frac{1}{2}$, which is also the average rate of change.

26. **(a)** The average rate of change is

$$\frac{g(a+h) - g(a)}{(a+h) - a} = \frac{[-4(a+h) + 2] - [-4a + 2]}{h} = \frac{-4a - 4h + 2 + 4a - 2}{h} = \frac{-4h}{h} = -4.$$

(b) The slope of the line $g(x) = -4x + 2$ is -4, which is also the average rate of change.

27. The function f has a greater average rate of change between $x = 0$ and $x = 1$. The function g has a greater average rate of change between $x = 1$ and $x = 2$. The functions f and g have the same average rate of change between $x = 0$ and $x = 1.5$.

28. The average rate of change of f is constant, that of g increases, and that of h decreases.

29. The average rate of change is $\frac{W(200) - W(100)}{200 - 100} = \frac{16 - 24}{200 - 100} = \frac{-8}{100} = -\frac{2}{25}$ m/day.

30. **(a)** The average rate of change is $\frac{P(40) - P(20)}{40 - 20} = \frac{40 - 40}{40 - 20} = \frac{0}{20} = 0$.

(b) The population increased and decreased the same amount during the 20 years.

31. **(a)** The average rate of change of population is $\frac{1{,}591 - 856}{2001 - 1998} = \frac{735}{3} = 245$ persons/yr.

(b) The average rate of change of population is $\frac{826 - 1{,}483}{2004 - 2002} = \frac{-657}{2} = -328.5$ persons/yr.

(c) The population was increasing from 1997 to 2001.

(d) The population was decreasing from 2001 to 2006.

32. **(a)** The average speed is $\frac{800-400}{152-68}=\frac{400}{84}=\frac{100}{21}\approx 4.76$ m/s.

(b) The average speed is $\frac{1{,}600-1{,}200}{412-263}=\frac{400}{149}\approx 2.68$ m/s.

(c)

Lap	Length of time to run lap	Average speed of lap.
1	32	6.25 m/s
2	36	5.56 m/s
3	40	5.00 m/s
4	44	4.55 m/s
5	51	3.92 m/s
6	60	3.33 m/s
7	72	2.78 m/s
8	77	2.60 m/s

The man is slowing down throughout the run.

33. **(a)** The average rate of change of sales is $\frac{635-495}{2013-2003}=\frac{140}{10}=14$ players/yr.

(b) The average rate of change of sales is $\frac{513-495}{2004-2003}=\frac{18}{1}=18$ players/yr.

(c) The average rate of change of sales is $\frac{410-513}{2005-2004}=\frac{-103}{1}=-103$ players/yr.

(d)

Year	DVD players sold	Change in sales from previous year
2003	495	—
2004	513	18
2005	410	−103
2006	402	−8
2007	520	118
2008	580	60
2009	631	51
2010	719	88
2011	624	−95
2012	582	−42
2013	635	53

Sales increased most quickly between 2006 and 2007, and decreased most quickly between 2004 and 2005.

34.

Year	Number of books
1980	420
1981	460
1982	500
1985	620
1990	820
1992	900
1995	1020
1997	1100
1998	1140
1999	1180
2000	1220

35. The average rate of change of the temperature of the soup over the first 20 minutes is $\dfrac{T(20)-T(0)}{20-0} = \dfrac{48-93}{20-0} = \dfrac{-45}{20} = -2.25^\circ$ C/min. Over the next 20 minutes, it is $\dfrac{T(40)-T(20)}{40-20} = \dfrac{32-48}{40-20} = -\dfrac{16}{20} = -0.8^\circ$ C/min. The first 20 minutes had a higher average rate of change of temperature (in absolute value).

36. (a) (i) Between 1860 and 1890, the average rate of change was $\dfrac{y(1890)-y(1860)}{1890-1860} \approx \dfrac{4570-2040}{30} \approx 84$, a gain of about 84 farms per year.

(ii) Between 1950 and 1970, the average rate of change was $\dfrac{y(1970)-y(1950)}{1970-1950} \approx \dfrac{2780-5390}{20} \approx -131$, a loss of about 131 farms per year.

(b) From the graph, it appears that the steepest rate of decline was during the period from 1950 to 1960.

37. (a) For all three skiers, the average rate of change is $\dfrac{d(10)-d(0)}{10-0} = \dfrac{100}{10} = 10$.

(b) Skier A gets a great start, but slows at the end of the race. Skier B maintains a steady pace. Runner C is slow at the beginning, but accelerates down the hill.

38. (a) Skater B won the race, because he travels 500 meters before Skater A.

(b) Skater A's average speed during the first 10 seconds is $\dfrac{A(10)-A(0)}{10-0} \approx \dfrac{200-0}{10} = 20$ m/s.

Skater B's average speed during the first 10 seconds is $\dfrac{B(10)-B(0)}{10-0} \approx \dfrac{100-0}{10} = 10$ m/s.

(c) Skater A's average speed during his last 15 seconds is $\dfrac{A(40)-A(25)}{40-25} \approx \dfrac{500-395}{15} = 7$ m/s.

Skater B's average speed during his last 15 seconds is $\dfrac{B(35)-B(20)}{35-20} \approx \dfrac{500-200}{15} = 20$ m/s.

39.

$t=a$	$t=b$	Average Speed $=\dfrac{f(b)-f(a)}{b-a}$
3	3.5	$\dfrac{5(3.5)^2-5(3)^2}{3.5-3}=32.5$
3	3.1	$\dfrac{5(3.1)^2-5(3)^2}{3.1-3}=30.5$
3	3.01	$\dfrac{5(3.01)^2-5(3)^2}{3.01-3}=30.05$
3	3.001	$\dfrac{5(3.001)^2-5(3)^2}{3.001-3}=30.005$
3	3.0001	$\dfrac{5(3.0001)^2-5(3)^2}{3.0001-3}=30.001$

From the table it appears that the average speed approaches 30 m/s as the time intervals get smaller and smaller. It seems reasonable to say that the speed of the object is 30 m/s at the instant $t=3$.

2.5 LINEAR FUNCTIONS AND MODELS

1. If f is a function with constant rate of change, then
(a) f is a linear function of the form $f(x)=ax+b$.
(b) The graph of f is a line.

2. If $f(x)=-5x+7$, then
(a) The rate of change of f is -5.
(b) The graph of f is a line with slope -5 and y-intercept 7.

3. From the graph, we see that $y(2)=50$ and $y(0)=20$, so the slope of the graph is $m=\dfrac{y(2)-y(0)}{2-0}=\dfrac{50-20}{2}=15$ L/min.

4. From Exercise 3, we see that the pool is being filled at the rate of 15 liters per minute.

5. If a linear function has positive rate of change, its graph slopes upward.

6. $f(x)=3$ is a linear function because it is of the form $f(x)=ax+b$, with $a=0$ and $b=3$. Its slope (and hence its rate of change) is 0.

7. $f(x)=3+\frac{1}{3}x=\frac{1}{3}x+3$ is linear with $a=\frac{1}{3}$ and $b=3$.

8. $f(x)=2-4x=-4x+2$ is linear with $a=-4$ and $b=2$.

9. $f(x)=x(4-x)=4x-x^2$ is not of the form $f(x)=ax+b$ for constants a and b, so it is not linear.

10. $f(x)=\sqrt{x}+1$ is not linear.

11. $f(x)=\dfrac{x+1}{5}=\frac{1}{5}x+\frac{1}{5}$ is linear with $a=\frac{1}{5}$ and $b=\frac{1}{5}$.

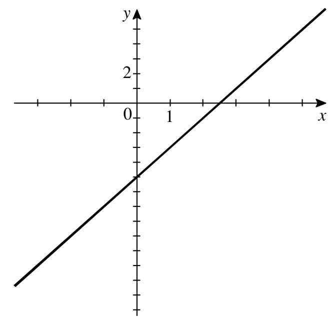
12. $f(x)=\dfrac{2x-3}{x}=2-\dfrac{3}{x}$ is not linear.

13. $f(x)=(x+1)^2=x^2+2x+1$ is not of the form $f(x)=ax+b$ for constants a and b, so it is not linear.

14. $f(x)=\frac{1}{2}(3x-1)=\frac{3}{2}x-\frac{1}{2}$ is linear with $a=\frac{3}{2}$ and $b=-\frac{1}{2}$.

15.

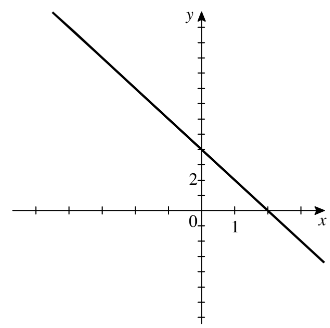
x	$f(x) = 2x - 5$
-1	-7
0	-5
1	-3
2	-1
3	1
4	3



The slope of the graph of $f(x) = 2x - 5$ is 2.

16.

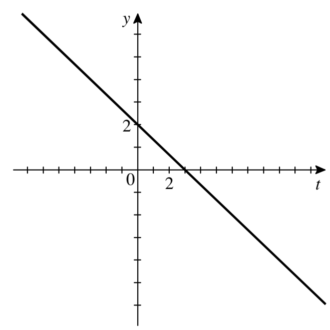
x	$g(x) = 4 - 2x$
-1	6
0	4
1	2
2	0
3	-2
4	-4



The slope of the graph of $g(x) = 4 - 2x = -2x + 4$ is -2.

17.

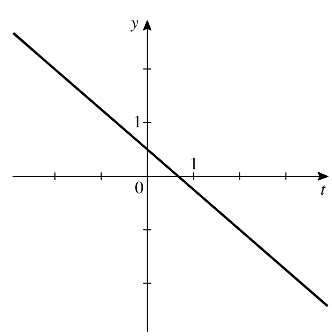
t	$r(t) = -\frac{2}{3}t + 2$
-1	2.67
0	2
1	1.33
2	0.67
3	0
4	-0.67



The slope of the graph of $r(t) = -\frac{2}{3}t + 2$ is $-\frac{2}{3}$.

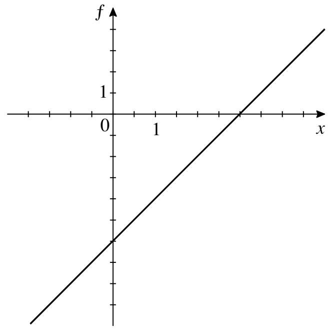
18.

t	$h(t) = \frac{1}{2} - \frac{3}{4}t$
-2	2
-1	1.25
0	0.5
1	-0.25
2	-1
3	-1.75



The slope of the graph of $h(t) = \frac{1}{2} - \frac{3}{4}t$ is $-\frac{3}{4}$.

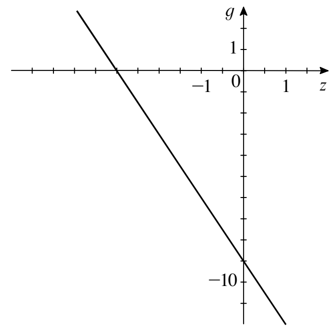
19. (a)



(b) The graph of $f(x) = 2x - 6$ has slope 2.

(c) $f(x) = 2x - 6$ has rate of change 2.

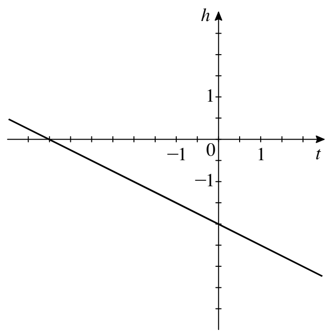
20. (a)



(b) The graph of $g(z) = -3z - 9$ has slope -3.

(c) $g(z) = -3z - 9$ has rate of change -3.

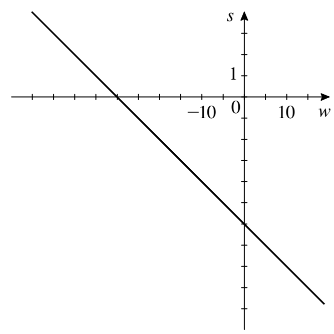
21. (a)



(b) The graph of $h(t) = -0.5t - 2$ has slope -0.5.

(c) $h(t) = -0.5t - 2$ has rate of change -0.5.

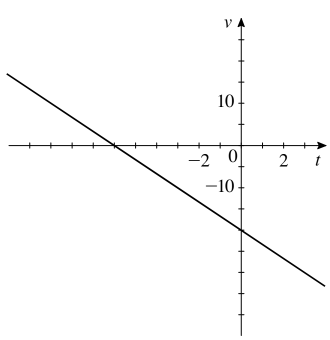
22. (a)



(b) The graph of $s(w) = -0.2w - 6$ has slope -0.2.

(c) $s(w) = -0.2w - 6$ has rate of change -0.2.

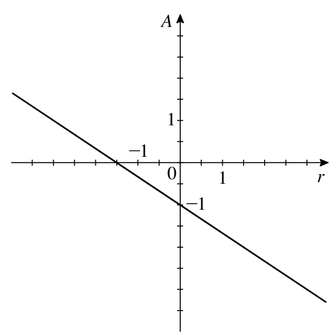
23. (a)



(b) The graph of $v(t) = -\frac{10}{3}t - 20$ has slope $-\frac{10}{3}$.

(c) $v(t) = -\frac{10}{3}t - 20$ has rate of change $-\frac{10}{3}$.

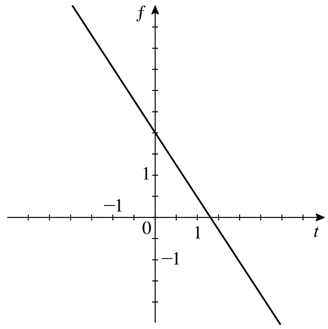
24. (a)



(b) The graph of $A(r) = -\frac{2}{3}r - 1$ has slope $-\frac{2}{3}$.

(c) $A(r) = -\frac{2}{3}r - 1$ has rate of change $-\frac{2}{3}$.

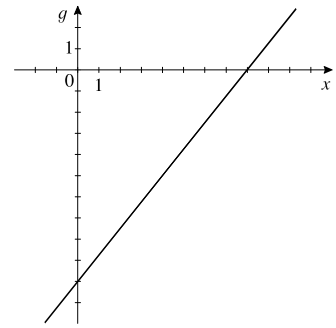
25. (a)



(b) The graph of $f(t) = -\frac{3}{2}t + 2$ has slope $-\frac{3}{2}$.

(c) $f(t) = -\frac{3}{2}t + 2$ has rate of change $-\frac{3}{2}$.

26. (a)



(b) The graph of $g(x) = \frac{5}{4}x - 10$ has slope $\frac{5}{4}$.

(c) $g(x) = \frac{5}{4}x - 10$ has rate of change $\frac{5}{4}$.

27. The linear function f with rate of change 3 and initial value -1 has equation $f(x) = 3x - 1$.

28. The linear function g with rate of change -12 and initial value 100 has equation $g(x) = -12x + 100$.

29. The linear function h with slope $\frac{1}{2}$ and y-intercept 3 has equation $h(x) = \frac{1}{2}x + 3$.

30. The linear function k with slope $-\frac{4}{5}$ and y-intercept -2 has equation $k(x) = -\frac{4}{5}x - 2$.

31. (a) From the table, we see that for every increase of 2 in the value of x, $f(x)$ increases by 3. Thus, the rate of change of f is $\frac{3}{2}$.

(b) When $x = 0$, $f(x) = 7$, so $b = 7$. From part (a), $a = \frac{3}{2}$, and so $f(x) = \frac{3}{2}x + 7$.

32. (a) From the table, we see that $f(-3) = 11$ and $f(0) = 2$. Thus, when x increases by 3, $f(x)$ decreases by 9, and so the rate of change of f is -3.

(b) When $x = 0$, $f(x) = 2$, so $b = 2$. From part (a), $a = -3$, and so $f(x) = -3x + 2$.

33. (a) From the graph, we see that $f(0) = 3$ and $f(1) = 4$, so the rate of change of f is $\dfrac{4-3}{1-0} = 1$.

(b) From part (a), $a = 1$, and $f(0) = b = 3$, so $f(x) = x + 3$.

34. (a) From the graph, we see that $f(0) = 4$ and $f(2) = 0$, so the rate of change of f is $\dfrac{0-4}{2-0} = -2$.

(b) From part (a), $a = -2$, and $f(0) = b = 4$, so $f(x) = -2x + 4$.

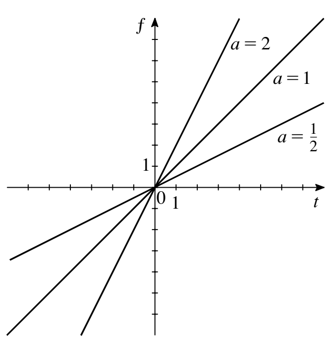
35. (a) From the graph, we see that $f(0) = 2$ and $f(4) = 0$, so the rate of change of f is $\dfrac{0-2}{4-0} = -\dfrac{1}{2}$.

(b) From part (a), $a = -\frac{1}{2}$, and $f(0) = b = 2$, so $f(x) = -\frac{1}{2}x + 2$.

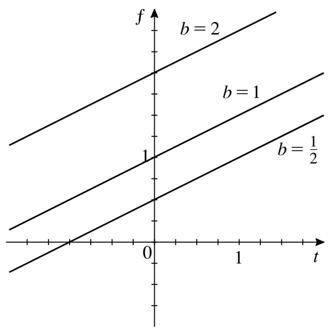
36. (a) From the graph, we see that $f(0) = -1$ and $f(2) = 0$, so the rate of change of f is $\dfrac{0-(-1)}{2-0} = \dfrac{1}{2}$.

(b) From part (a), $a = \frac{1}{2}$, and $f(0) = b = -1$, so $f(x) = \frac{1}{2}x - 1$.

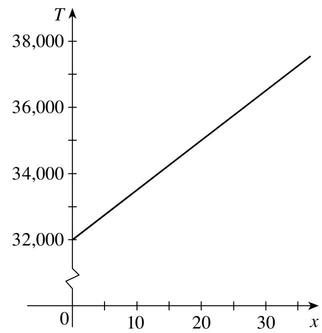
37.



Increasing the value of a makes the graph of f steeper. In other words, it increases the rate of change of f.

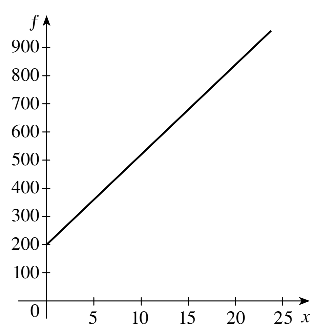
38. 

Increasing the value of b moves the graph of f upward, but does not affect the rate of change of f.

39. (a) 

(b) The slope of $T(x) = 150x + 32{,}000$ is the value of a, 150.

(c) The amount of trash is changing at a rate equal to the slope of the graph, 150 thousand tonnes per year.

40. (a) 

(b) The slope of the graph of $f(x) = 200 + 32x$ is 32.

(c) Ore is being produced at a rate equal to the slope of the graph, 32 thousand tons per year.

41. (a) Let $V(t) = at + b$ represent the volume of hydrogen. The balloon is being filled at the rate of $0.25\ \text{m}^3/\text{s}$, so $a = 0.25$, and initially it contains $1\,\text{m}^3$, so $b = 1$. Thus, $V(t) = 0.25t + 1$.

(b) We solve $V(t) = 7.5 \Leftrightarrow 0.25t + 1 = 7.5 \Leftrightarrow 0.25t = 6.5 \Leftrightarrow t = 26$. Thus, it takes 26 seconds to fill the balloon.

42. (a) Let $V(t) = at + b$ represent the volume of water. The pool is being filled at the rate of 10 L/min, so $a = 10$, and initially it contains 300 L, so $b = 300$. Thus, $V(t) = 10t + 300$.

(b) We solve $V(t) = 1300 \Leftrightarrow 10t + 300 = 1300 \Leftrightarrow 10t = 1000 \Leftrightarrow t = 100$. Thus, it takes 100 minutes to fill the pool.

43. (a) Let $H(x) = ax + b$ represent the height of the ramp. The maximum rise is 2.5 cm per 30 cm, so $a = \frac{1}{12}$. The ramp starts on the ground, so $b = 0$. Thus, $H(x) = \frac{1}{12}x$.

(b) We find $H(375) = \frac{1}{12}(375) = 31.25$. Thus, the ramp reaches a height of 31.25 centimeters.

44. Meilin descends 1200 vertical meters over 15,000 meters, so the grade of her road is $\dfrac{-1200}{15{,}000} = -0.075$, or -7.5%.

Brianna descends 500 vertical meters over 10,000 meters, so the grade of her road is $\dfrac{-500}{10{,}000} = -0.05$, or -5%.

45. (a) From the graph, we see that the slope of Jari's trip is steeper than that of Jade. Thus, Jari is traveling faster.

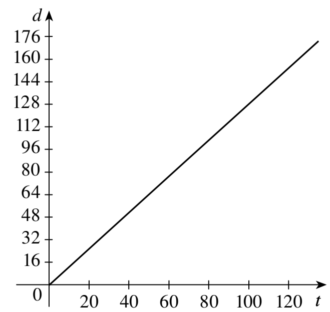
(b) The points (0, 0) and (6, 7) are on Jari's graph, so her speed is $\dfrac{7-0}{6-0} = \dfrac{7}{6}$ kilometers per minute or $60\left(\dfrac{7}{6}\right) = 70$ km/h.

The points (0, 10) and (6, 16) are on Jade's graph, so her speed is $60 \cdot \dfrac{16-10}{6-0} = 60$ km/h.

(c) t is measured in minutes, so Jade's speed is 60 km/h $\cdot \frac{1}{60}$ h/min $=$ 1 km/min and Jari's speed is 70 km/h $\cdot \frac{1}{60}$ h/min $= \frac{7}{6}$ km/min. Thus, Jade's distance is modeled by $f(t) = 1\,(t-0) + 10 = t + 10$ and Jari's distance is modeled by $g\,(t) = \frac{7}{6}\,(t-0) + 0 = \frac{7}{6}t$.

46. **(a)** Let $d\,(t)$ represent the distance traveled. When $t = 0$, $d = 0$, and when $t = 50$, $d = 65$. Thus, the slope of the graph is $\dfrac{65-0}{50-0} = 1.3$. The y-intercept is 0, so $d\,(t) = 1.3t$.

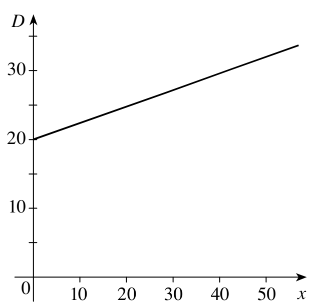
(b)



(c) Jacqueline's speed is equal to the slope of the graph of d, that is, 1.3 km/min or $1.3(60) = 78$ km/h.

47. Let x be the horizontal distance and y the elevation. The slope is $-\frac{6}{100}$, so if we take (0, 0) as the starting point, the elevation is $y = -\frac{6}{100}x$. We have descended 305 m, so we substitute $y = -305$ and solve for x: $-305 = \frac{6}{100}x \Leftrightarrow x \approx 5038$m. Converting to kilometers, the horizontal distance is $\frac{1}{1000}\,(5083) \approx 5.08$ km.

48. **(a)**

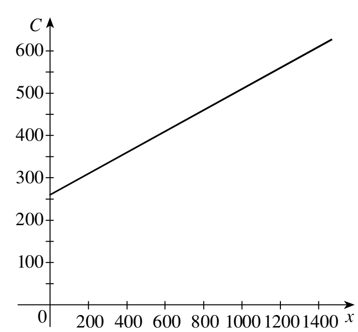


(b) The slope of the graph of $D\,(x) = 20 + 0.24x$ is 0.24.

(c) The rate of sedimentation is equal to the slope of the graph, 0.24 cm/yr or 2.4 mm/yr.

49. **(a)** Let $C\,(x) = ax + b$ be the cost of driving x kilometers. In May Lynn drove 480 kilometers at a cost of \$380, and in June she drove 800 kilometers at a cost of \$460.Thus, the points (480, 380) and (800, 460) are on the graph, so the slope is $a = \dfrac{460-380}{800-480} = \dfrac{1}{4}$. We use the point (480, 380) to find the value of b: $380 = \frac{1}{4}\,(480) + b \Leftrightarrow b = 260$. Thus, $C\,(x) = \frac{1}{4}x + 260$.

(b)

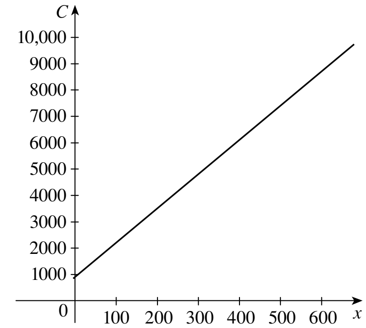


The slope of the graph of $C\,(x) = \frac{1}{4}x + 260$ is the value of a, $\frac{1}{4}$.

(c) The rate at which her cost increases is equal to the slope of the line, that is $\frac{1}{4}$. So her cost increases by \$0.25 for every additional kilometer she drives.

50. (a) Let $C(x) = ax + b$ be the cost of producing x chairs in one day. The first day, it cost \$2200 to produce 100 chairs, and the other day it cost \$4800 to produce 300 chairs.. Thus, the points $(100, 2200)$ and $(300, 4800)$ are on the graph, so the slope is $a = \dfrac{4800 - 2200}{300 - 100} = 13$. We use the point $(100, 2200)$ to find the value of b: $2200 = 13(100) + b \Leftrightarrow b = 900$. Thus, $C(x) = 13x + 900$.

(b)



The slope of the graph of $C(x) = 13x + 900$ is the value of a, 13.

(c) The rate at which the factory's cost increases is equal to the slope of the line, that is \$13/chair.

51. (a) By definition, the average rate of change between x_1 and x_2 is $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{(ax_2 + b) - (ax_1 + b)}{x_2 - x_1} = \dfrac{ax_2 - ax_1}{x_2 - x_1}$.

(b) Factoring the numerator and cancelling, the average rate of change is $\dfrac{ax_2 - ax_1}{x_2 - x_1} = \dfrac{a(x_2 - x_1)}{x_2 - x_1} = a$.

52. (a) The rate of change between any two points is c. In particular, between a and x, the rate of change is $\dfrac{f(x) - f(a)}{x - a} = c$.

(b) Multiplying the equation in part (a) by $x - a$, we obtain $f(x) - f(a) = c(x - a)$. Rearranging and adding $f(a)$ to both sides, we have $f(x) = cx + (f(a) - ca)$, as desired. Because this equation is of the form $f(x) = Ax + B$ with constants $A = c$ and $B = f(a) - ca$, it represents a linear function with slope c and y-intercept $f(a) - ca$.

2.6 TRANSFORMATIONS OF FUNCTIONS

1. (a) The graph of $y = f(x) + 3$ is obtained from the graph of $y = f(x)$ by shifting *upward* 3 units.
(b) The graph of $y = f(x + 3)$ is obtained from the graph of $y = f(x)$ by shifting *left* 3 units.

2. (a) The graph of $y = f(x) - 3$ is obtained from the graph of $y = f(x)$ by shifting *downward* 3 units.
(b) The graph of $y = f(x - 3)$ is obtained from the graph of $y = f(x)$ by shifting *right* 3 units.

3. (a) The graph of $y = -f(x)$ is obtained from the graph of $y = f(x)$ by reflecting in the x-*axis*.
(b) The graph of $y = f(-x)$ is obtained from the graph of $y = f(x)$by reflecting in the y-*axis*.

4. (a) The graph of $f(x) + 2$ is obtained from that of $y = f(x)$ by shifting upward 2 units, so it has graph II.
(b) The graph of $f(x + 3)$ is obtained from that of $y = f(x)$ by shifting to the left 3 units, so it has graph I.
(c) The graph of $f(x - 2)$ is obtained from that of $y = f(x)$ by shifting to the right 2 units, so it has graph III.
(d) The graph of $f(x) - 4$ is obtained from that of $y = f(x)$ by shifting downward 4 units, so it has graph IV.

5. If f is an even function, then $f(-x) = f(x)$ and the graph of f is symmetric about the y-axis.

6. If f is an odd function, then $f(-x) = -f(x)$ and the graph of f is symmetric about the origin.

7. (a) The graph of $y = f(x) - 1$ can be obtained by shifting the graph of $y = f(x)$ downward 1 unit.
(b) The graph of $y = f(x - 2)$ can be obtained by shifting the graph of $y = f(x)$ to the right 2 units.

8. (a) The graph of $y = f(x + 4)$ can be obtained by shifting the graph of $y = f(x)$ to the left 5 units.
(b) The graph of $y = f(x) + 4$ can be obtained by shifting the graph of $y = f(x)$ upward 4 units.

9. (a) The graph of $y = f(-x)$ can be obtained by reflecting the graph of $y = f(x)$ in the y-axis.
(b) The graph of $y = 3f(x)$ can be obtained by stretching the graph of $y = f(x)$ vertically by a factor of 3.

10. **(a)** The graph of $y = -f(x)$ can be obtained by reflecting the graph of $y = f(x)$ about the x-axis.

(b) The graph of $y = \frac{1}{3}f(x)$ can be obtained by shrinking the graph of $y = f(x)$ vertically by a factor of $\frac{1}{3}$.

11. **(a)** The graph of $y = f(x-5)+2$ can be obtained by shifting the graph of $y = f(x)$ to the right 5 units and upward 2 units.

(b) The graph of $y = f(x+1)-1$ can be obtained by shifting the graph of $y = f(x)$ to the left 1 unit and downward 1 unit.

12. **(a)** The graph of $y = f(x+3)+2$ can be obtained by shifting the graph of $y = f(x)$ to the left 3 units and upward 2 units.

(b) The graph of $y = f(x-7)-3$ can be obtained by shifting the graph of $y = f(x)$ to the right 7 units and downward 3 units.

13. **(a)** The graph of $y = -f(x)+5$ can be obtained by reflecting the graph of $y = f(x)$ in the x-axis, then shifting the resulting graph upward 5 units.

(b) The graph of $y = 3f(x)-5$ can be obtained by stretching the graph of $y = f(x)$ vertically by a factor of 3, then shifting the resulting graph downward 5 units.

14. **(a)** The graph of $y = 1-f(-x)$ can be obtained by reflect the graph of $y = f(x)$ about the x-axis, then reflecting about the y-axis, then shifting upward 1 unit.

(b) The graph of $y = 2-\frac{1}{5}f(x)$ can be obtained by shrinking the graph of $y = f(x)$ vertically by a factor of $\frac{1}{5}$, then reflecting about the x-axis, then shifting upward 2 units.

15. **(a)** The graph of $y = 2f(x+5)-1$ can be obtained by shifting the graph of $y = f(x)$ to the left 5 units, stretching vertically by a factor of 2, then shifting downward 1 unit.

(b) The graph of $y = \frac{1}{4}f(x-3)+5$ can be obtained by shifting the graph of $y = f(x)$ to the right 3 units, shrinking vertically by a factor of $\frac{1}{4}$, then shifting upward 5 units.

16. **(a)** The graph of $y = \frac{1}{3}f(x-2)+5$ can be obtained by shifting the graph of $y = f(x)$ to the right 2 units, shrinking vertically by a factor of $\frac{1}{3}$, then shifting upward 5 units.

(b) The graph of $y = 4f(x+1)+3$ can be obtained by shifting the graph of $y = f(x)$ to the left 1 unit, stretching vertically by a factor of 4, then shifting upward 3 units.

17. **(a)** The graph of $y = f(4x)$ can be obtained by shrinking the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{4}$.

(b) The graph of $y = f\left(\frac{1}{4}x\right)$ can be obtained by stretching the graph of $y = f(x)$ horizontally by a factor of 4.

18. **(a)** The graph of $y = f(2x)-1$ can be obtained by shrinking the graph of $y = f(x)$ horizontally by a factor of $\frac{1}{2}$, then shifting it downward 1 unit.

(b) The graph of $y = 2f\left(\frac{1}{2}x\right)$ can be obtained by stretching the graph of $y = f(x)$ horizontally by a factor of 2 and stretching it vertically by a factor of 2.

19. **(a)** The graph of $g(x) = (x+2)^2$ is obtained by shifting the graph of $f(x)$ to the left 2 units.

(b) The graph of $g(x) = x^2+2$ is obtained by shifting the graph of $f(x)$ upward 2 units.

20. **(a)** The graph of $g(x) = (x-4)^3$ is obtained by shifting the graph of $f(x)$ to the right 4 units.

(b) The graph of $g(x) = x^3-4$ is obtained by shifting the graph of $f(x)$ downward 4 units.

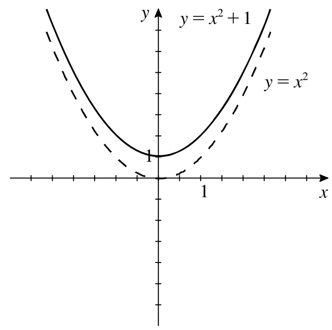
21. **(a)** The graph of $g(x) = |x+2|-2$ is obtained by shifting the graph of $f(x)$ to the left 2 units and downward 2 units.

(b) The graph of $g(x) = g(x) = |x-2|+2$ is obtained from by shifting the graph of $f(x)$ to the right 2 units and upward 2 units.

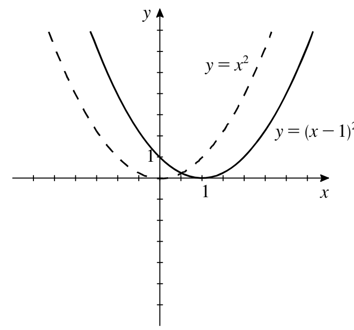
22. **(a)** The graph of $g(x) = -\sqrt{x} + 1$ is obtained by reflecting the graph of $f(x)$ in the x-axis, then shifting the resulting graph upward 1 unit.

(b) The graph of $g(x) = \sqrt{-x} + 1$ is obtained by reflecting the graph of $f(x)$ in the y-axis, then shifting the resulting graph upward 1 unit.

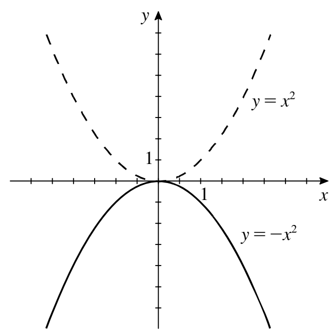
23. **(a)**



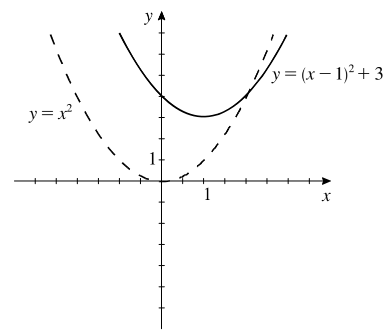
(b)



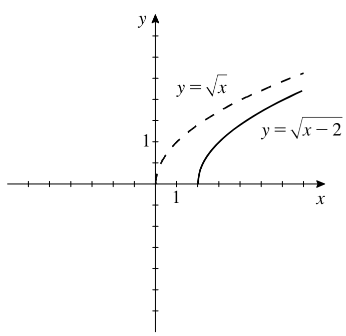
(c)



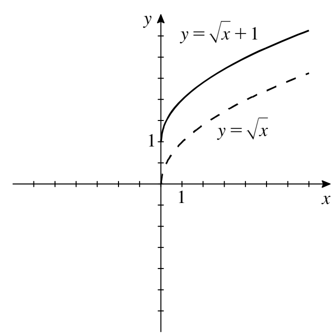
(d)



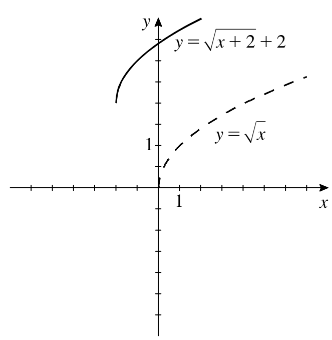
24. **(a)**



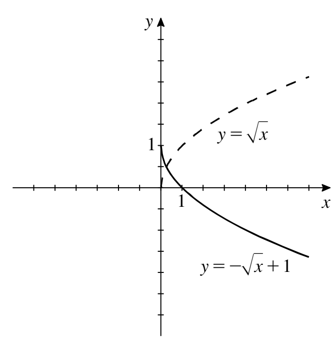
(b)



(c)



(d)



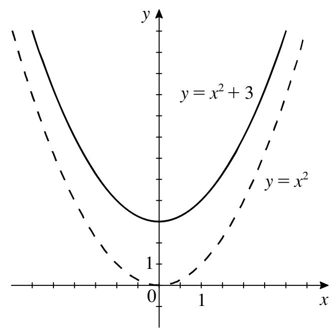
25. The graph of $y = |x + 1|$ is obtained from that of $y = |x|$ by shifting to the left 1 unit, so it has graph II.

26. $y = |x - 1|$ is obtained from that of $y = |x|$ by shifting to the right 1 unit, so it has graph IV.

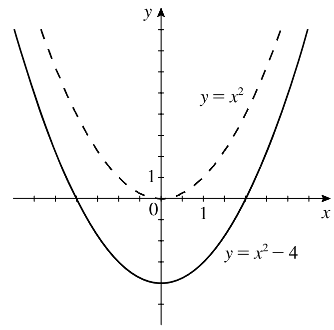
27. The graph of $y = |x| - 1$ is obtained from that of $y = |x|$ by shifting downward 1 unit, so it has graph I.

28. The graph of $y = -|x|$ is obtained from that of $y = |x|$ by reflecting in the x-axis, so it has graph III.

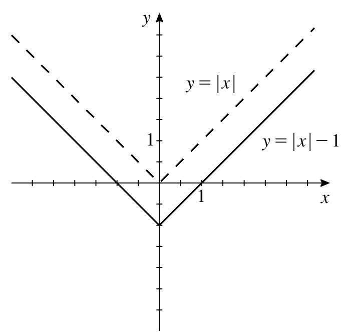
29. $f(x) = x^2 + 3$. Shift the graph of $y = x^2$ upward 3 units.



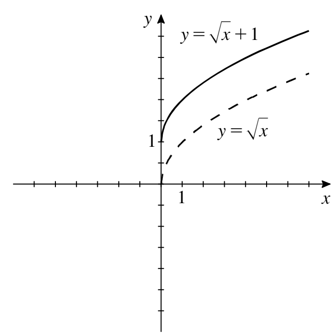
30. $f(x) = x^2 - 4$. Shift the graph of $y = x^2$ downward 4 units.



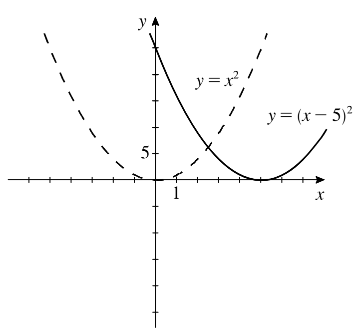
31. $f(x) = |x| - 1$. Shift the graph of $y = |x|$ downward 1 unit.



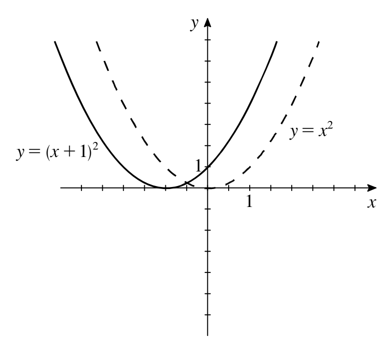
32. $f(x) = \sqrt{x} + 1$. Shift the graph of $y = \sqrt{x}$ upward 1 unit.



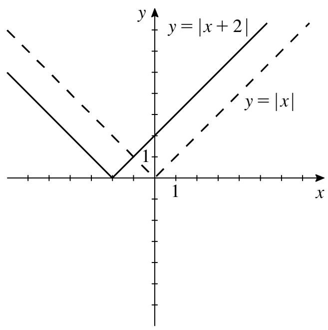
33. $f(x) = (x - 5)^2$. Shift the graph of $y = x^2$ to the right 5 units.



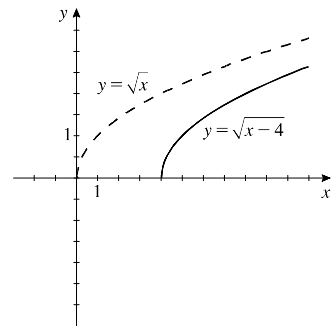
34. $f(x) = (x + 1)^2$. Shift the graph of $y = x^2$ to the left 1 unit.



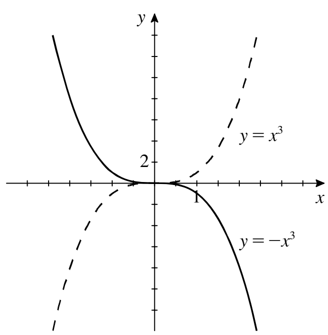
35. $f(x) = |x + 2|$. Shift the graph of $y = |x|$ to the left 2 units.



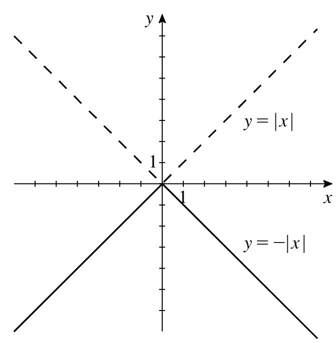
36. $f(x) = \sqrt{x - 4}$. Shift the graph of $y = \sqrt{x}$ to the right 4 units.



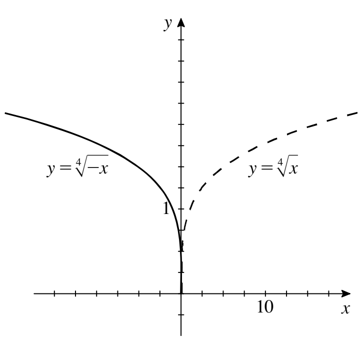
37. $f(x) = -x^3$. Reflect the graph of $y = x^3$ in the x-axis.



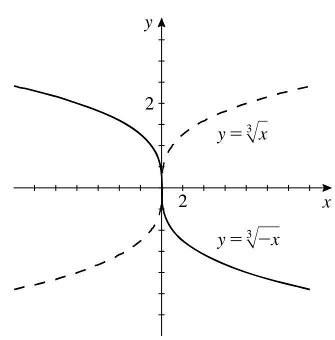
38. $f(x) = -|x|$. Reflect the graph of $y = |x|$ in the x-axis.



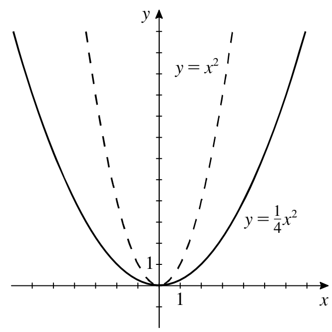
39. $y = \sqrt[4]{-x}$. Reflect the graph of $y = \sqrt[4]{x}$ in the y-axis.



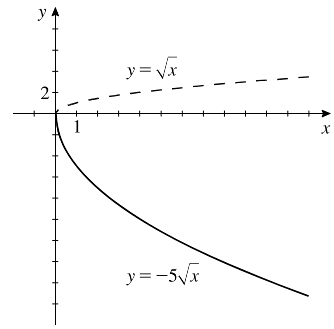
40. $y = \sqrt[3]{-x}$. Reflect the graph of $y = \sqrt[3]{x}$ in the y-axis.



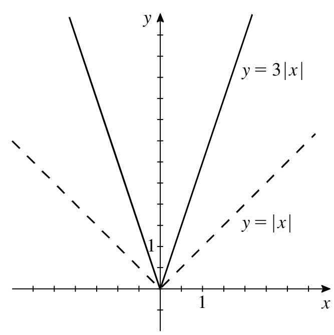
41. $y = \frac{1}{4}x^2$. Shrink the graph of $y = x^2$ vertically by a factor of $\frac{1}{4}$.



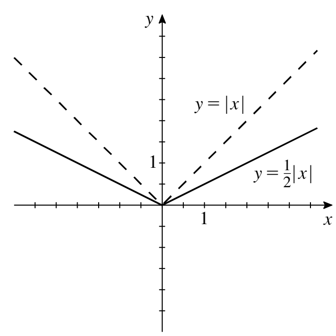
42. $y = -5\sqrt{x}$. Stretch the graph of $y = \sqrt{x}$ vertically by a factor of 5, then reflect it in the x-axis.



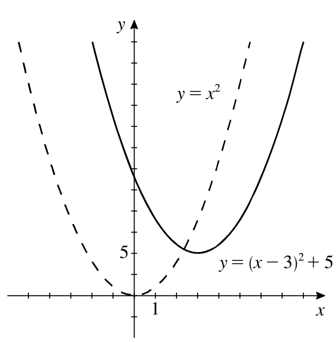
43. $y = 3|x|$. Stretch the graph of $y = |x|$ vertically by a factor of 3.



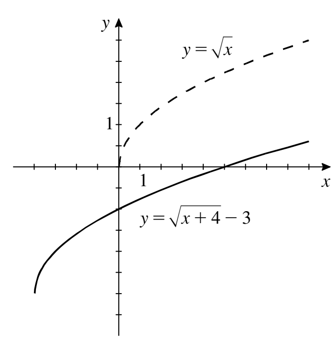
44. $y = \frac{1}{2}|x|$. Shrink the graph of $y = |x|$ vertically by a factor of $\frac{1}{2}$.



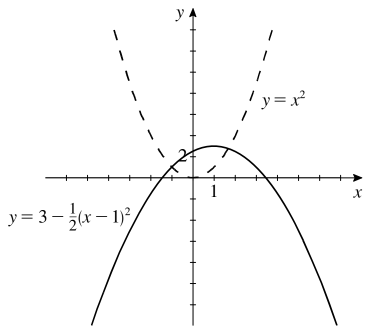
45. $y = (x-3)^2 + 5$. Shift the graph of $y = x^2$ to the right 3 units and upward 5 units.



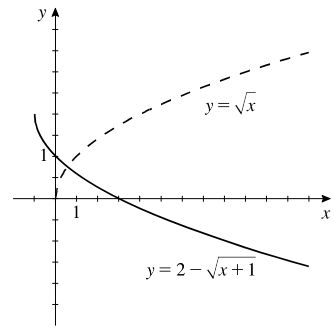
46. $y = \sqrt{x+4} - 3$. Shift the graph of $y = \sqrt{x}$ to the left 4 units and downward 3 units.



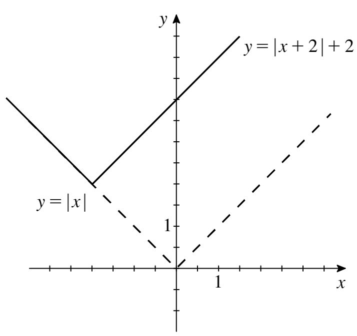
47. $y = 3 - \frac{1}{2}(x-1)^2$. Shift the graph of $y = x^2$ to the right one unit, shrink vertically by a factor of $\frac{1}{2}$, reflect in the x-axis, then shift upward 3 units.



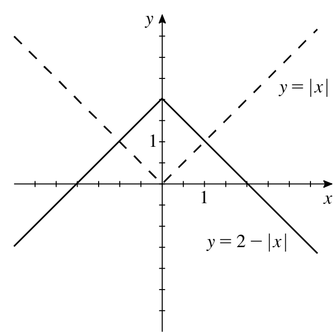
48. $y = 2 - \sqrt{x+1}$. Shift the graph of $y = \sqrt{x}$ to the left 1 unit, reflect the result in the x-axis, then shift upward 2 units.



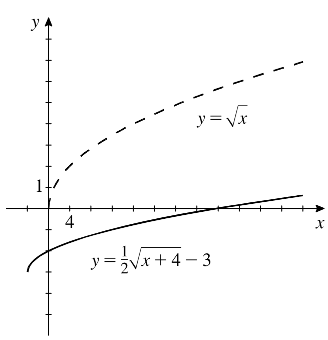
49. $y = |x+2| + 2$. Shift the graph of $y = |x|$ to the left 2 units and upward 2 units.



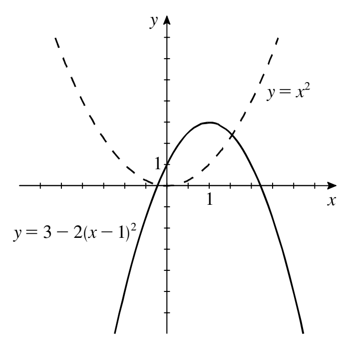
50. $y = 2 - |x|$. Reflect the graph of $y = |x|$ in the x-axis, then shift upward 2 units.



51. $y = \frac{1}{2}\sqrt{x+4} - 3$. Shrink the graph of $y = \sqrt{x}$ vertically by a factor of $\frac{1}{2}$, then shift the result to the left 4 units and downward 3 units.



52. $y = 3 - 2(x-1)^2$. Stretch the graph of $y = x^2$ vertically by a factor of 2, reflect the result in the x-axis, then shift the result to the right 1 unit and upward 3 units.



53. $y = f(x) - 3$. When $f(x) = x^2$, $y = x^2 - 3$.

54. $y = f(x) + 5$. When $f(x) = x^3$, $y = x^3 + 5$.

55. $y = f(x+2)$. When $f(x) = \sqrt{x}$, $y = \sqrt{x+2}$.

56. $y = f(x-1)$. When $f(x) = \sqrt[3]{x}$, $y = \sqrt[3]{x-1}$.

57. $y = f(x+2) - 5$. When $f(x) = |x|$, $y = |x+2| - 5$.

58. $y = -f(x-4) + 3$. When $f(x) = |x|$, $y = -|x-4| + 3$.

59. $y = f(-x) + 1$. When $f(x) = \sqrt[4]{x}$, $y = \sqrt[4]{-x} + 1$.

60. $y = -f(x+2)$. When $f(x) = x^2$, $y = -(x+2)^2$.

61. $y = 2f(x-3) - 2$. When $f(x) = x^2$, $y = 2(x-3)^2 - 2$.

62. $y = \frac{1}{2}f(x+1) + 3$. When $f(x) = |x|$, $y = \frac{1}{2}|x+1| + 3$.

63. $g(x) = f(x-2) = (x-2)^2 = x^2 - 4x + 4$

64. $g(x) = f(x) + 3 = x^3 + 3$

65. $g(x) = f(x+1) + 2 = |x+1| + 2$

66. $g(x) = 2f(x) = 2|x|$

67. $g(x) = -f(x+2) = -\sqrt{x+2}$

68. $g(x) = -f(x-2) + 1 = -(x-2)^2 + 1 = -x^2 + 4x - 3$

69. **(a)** $y = f(x-4)$ is graph #3.

(b) $y = f(x) + 3$ is graph #1.

(c) $y = 2f(x+6)$ is graph #2.

(d) $y = -f(2x)$ is graph #4.

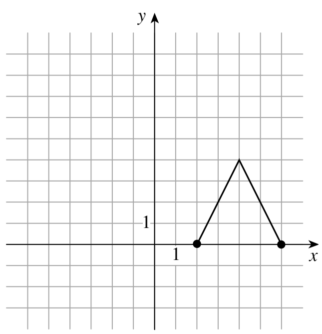
70. **(a)** $y = \frac{1}{3}f(x)$ is graph #2.

(b) $y = -f(x+4)$ is graph #3.

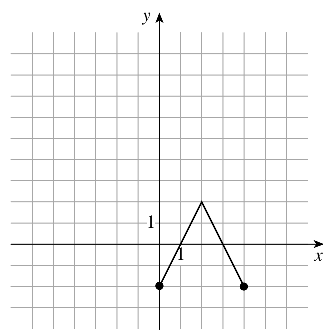
(c) $y = f(x-5) + 3$ is graph #1.

(d) $y = f(-x)$ is graph #4.

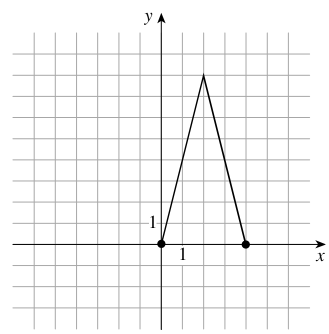
71. **(a)** $y = f(x-2)$



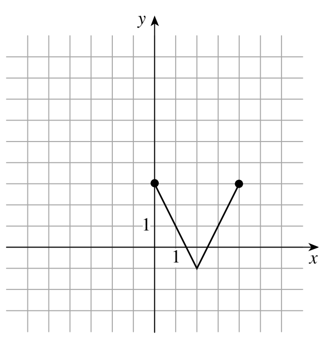
(b) $y = f(x) - 2$



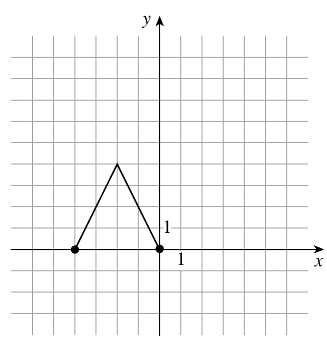
(c) $y = 2f(x)$



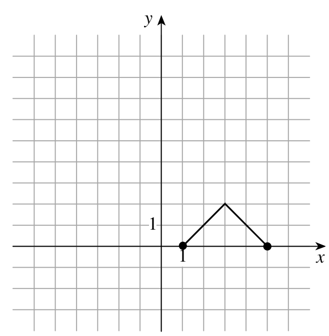
(d) $y = -f(x) + 3$



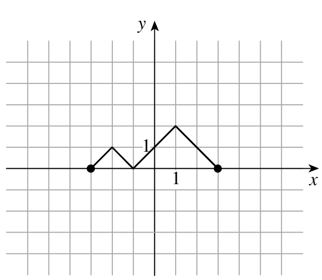
(e) $y = f(-x)$



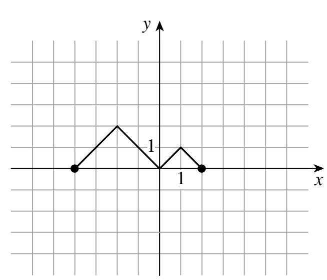
(f) $y = \frac{1}{2}f(x-1)$



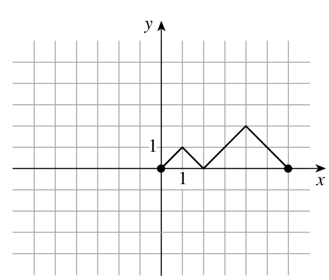
72. **(a)** $y = g(x+1)$



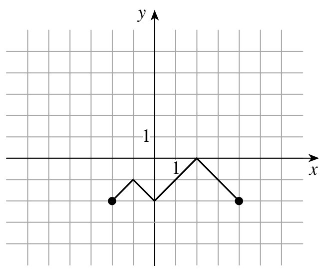
(b) $y = g(-x)$



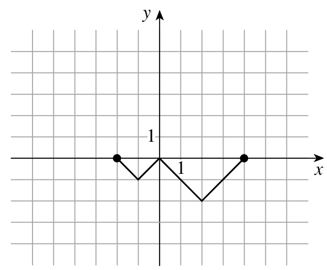
(c) $y = g(x-2)$



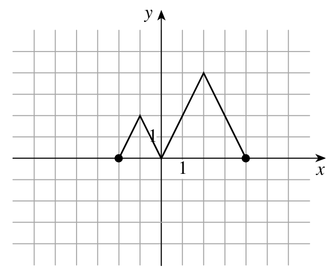
(d) $y = g(x) - 2$



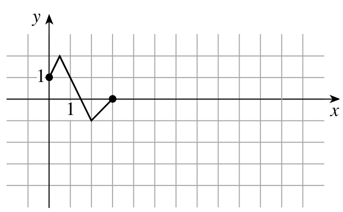
(e) $y = -g(x)$



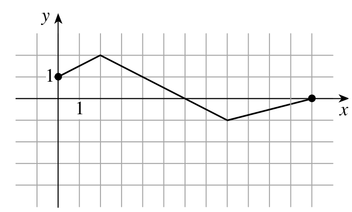
(f) $y = 2g(x)$



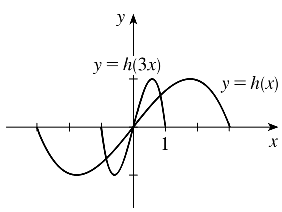
73. (a) $y = g(2x)$



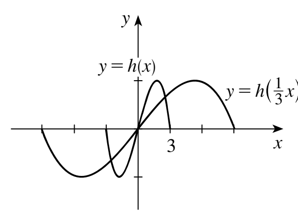
(b) $y = g\left(\frac{1}{2}x\right)$



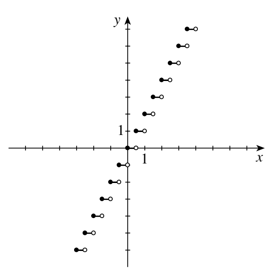
74. (a) $y = h(3x)$



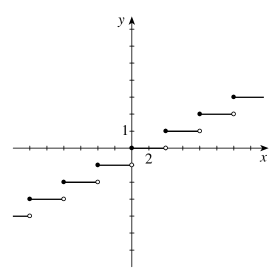
(b) $y = h\left(\frac{1}{3}x\right)$



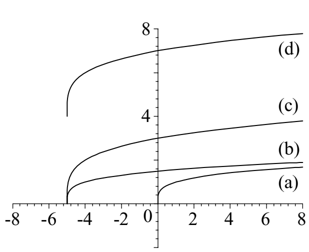
75. $y = [\![2x]\!]$



76. $y = \left[\!\!\left[\frac{1}{4}x\right]\!\!\right]$

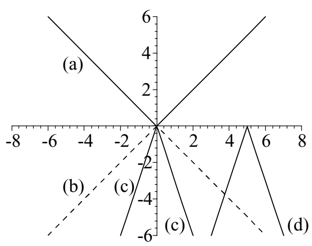


77.



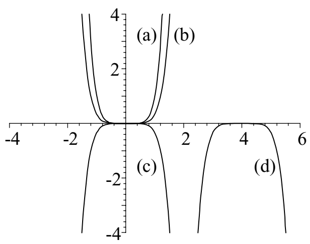
For part (b), shift the graph in (a) to the left 5 units; for part (c), shift the graph in (a) to the left 5 units, and stretch it vertically by a factor of 2; for part (d), shift the graph in (a) to the left 5 units, stretch it vertically by a factor of 2, and then shift it upward 4 units.

78.



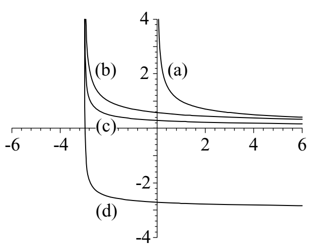
For (b), reflect the graph in (a) in the x-axis; for (c), stretch the graph in (a) vertically by a factor of 3 and reflect in the x-axis; for (d), shift the graph in (a) to the right 5 units, stretch it vertically by a factor of 3, and reflect it in the x-axis. The order in which each operation is applied to the graph in (a) is not important to obtain the graphs in part (c) and (d).

79.



For part (b), shrink the graph in (a) vertically by a factor of $\frac{1}{3}$; for part (c), shrink the graph in (a) vertically by a factor of $\frac{1}{3}$, and reflect it in the x-axis; for part (d), shift the graph in (a) to the right 4 units, shrink vertically by a factor of $\frac{1}{3}$, and then reflect it in the x-axis.

80.

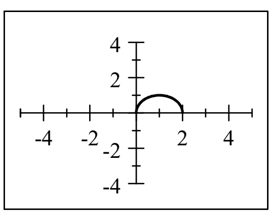


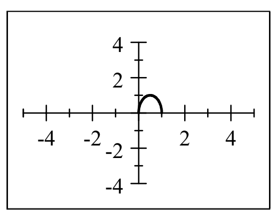
For (b), shift the graph in (a) to the left 3 units; for (c), shift the graph in (a) to the left 3 units and shrink it vertically by a factor of $\frac{1}{2}$; for (d), shift the graph in (a) to the left 3 units, shrink it vertically by a factor of $\frac{1}{2}$, and then shift it downward 3 units. The order in which each operation is applied to the graph in (a) is not important to sketch (c), while it is important in (d).

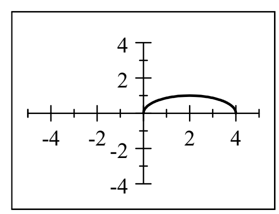
81. **(a)** $y = f(x) = \sqrt{2x - x^2}$

(b) $y = f(2x) = \sqrt{2(2x) - (2x)^2}$
$= \sqrt{4x - 4x^2}$

(c) $y = f\left(\frac{1}{2}x\right) = \sqrt{2\left(\frac{1}{2}x\right) - \left(\frac{1}{2}x\right)^2}$
$= \sqrt{x - \frac{1}{4}x^2}$





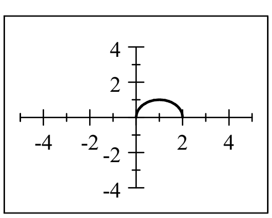


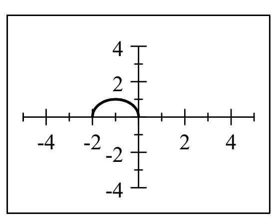
The graph in part (b) is obtained by horizontally shrinking the graph in part (a) by a factor of $\frac{1}{2}$ (so the graph is half as wide). The graph in part (c) is obtained by horizontally stretching the graph in part (a) by a factor of 2 (so the graph is twice as wide).

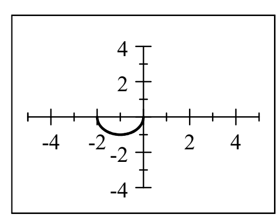
82. **(a)** $y = f(x) = \sqrt{2x - x^2}$

(b) $y = f(-x) = \sqrt{2(-x) - (-x)^2}$
$= \sqrt{-2x - x^2}$

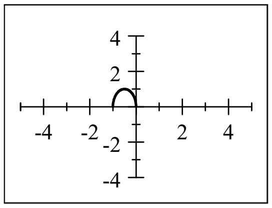
(c) $y = -f(-x) = -\sqrt{2(-x) - (-x)^2}$
$= -\sqrt{-2x = x^2}$



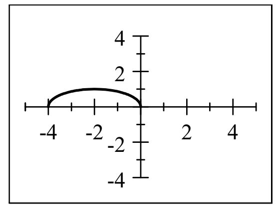




(d) $y = f(-2x) = \sqrt{2(-2x) - (-2x)^2}$
$= -\sqrt{-2x - x^2} = \sqrt{-4x - 4x^2}$

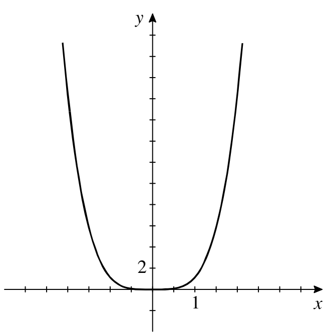


(e) $y = f\left(-\frac{1}{2}x\right) = \sqrt{2\left(-\frac{1}{2}x\right) - \left(-\frac{1}{2}x\right)^2}$
$= \sqrt{-x - \frac{1}{4}x^2}$

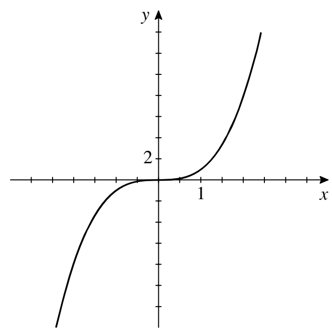


The graph in part (b) is obtained by reflecting the graph in part (a) in the y-axis. The graph in part (c) is obtained by rotating the graph in part (a) through 180° about the origin [or by reflecting the graph in part (a) first in the x-axis and then in the y-axis]. The graph in part (d) is obtained by reflecting the graph in part (a) in the y-axis and then horizontally shrinking the graph by a factor of $\frac{1}{2}$ (so the graph is half as wide). The graph in part (e) is obtained by reflecting the graph in part (a) in the y-axis and then horizontally stretching the graph by a factor of 2 (so the graph is twice as wide).

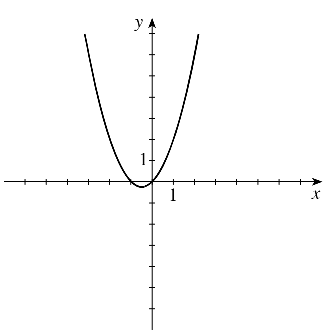
83. $f(x) = x^4$. $f(-x) = (-x)^4 = x^4 = f(x)$. Thus $f(x)$ is even.



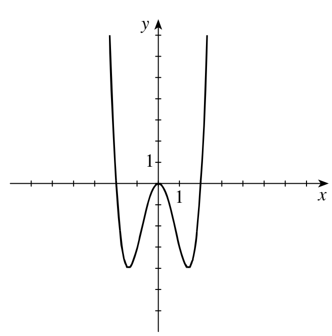
84. $f(x) = x^3$. $f(-x) = (-x)^3 = -x^3 = -f(x)$. Thus $f(x)$ is odd.



85. $f(x) = x^2 + x$. $f(-x) = (-x)^2 + (-x) = x^2 - x$. Thus $f(-x) \neq f(x)$. Also, $f(-x) \neq -f(x)$, so $f(x)$ is neither odd nor even.



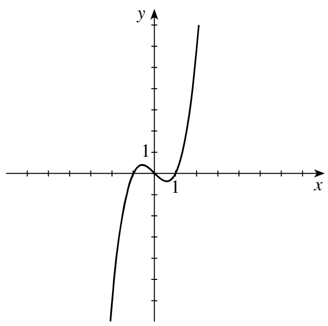
86. $f(x) = x^4 - 4x^2$.
$f(-x) = (-x)^4 - 4(-x)^2 = x^4 - 4x^2 = f(x)$. Thus $f(x)$ is even.



87. $f(x) = x^3 - x$.

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -\left(x^3 - x\right) = -f(x).$$

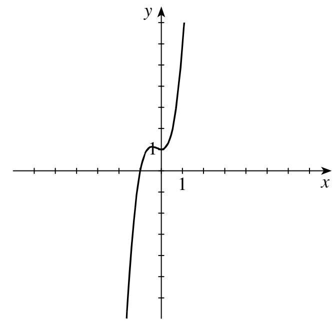
Thus $f(x)$ is odd.



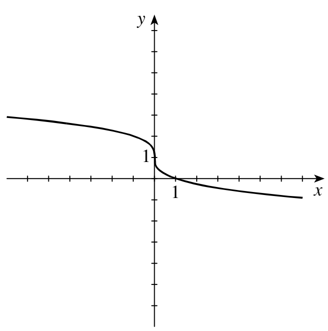
88. $f(x) = 3x^3 + 2x^2 + 1$.

$f(-x) = 3(-x)^3 + 2(-x)^2 + 1 = -3x^3 + 2x^2 + 1$.

Thus $f(-x) \neq f(x)$. Also $f(-x) \neq -f(x)$, so $f(x)$ is neither odd nor even.



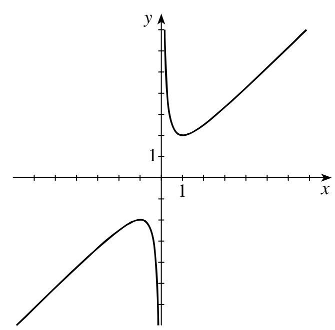
89. $f(x) = 1 - \sqrt[3]{x}$. $f(-x) = 1 - \sqrt[3]{(-x)} = 1 + \sqrt[3]{x}$. Thus $f(-x) \neq f(x)$. Also $f(-x) \neq -f(x)$, so $f(x)$ is neither odd nor even.



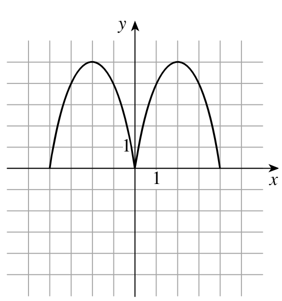
90. $f(x) = x + 1/x$.

$$f(-x) = (-x) + 1/(-x) = -x - 1/x = -(x + 1/x) = -f(x).$$

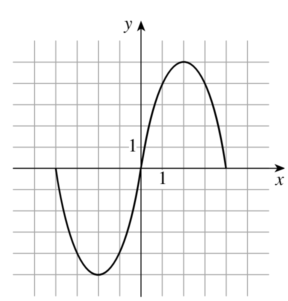
Thus $f(x)$ is odd.



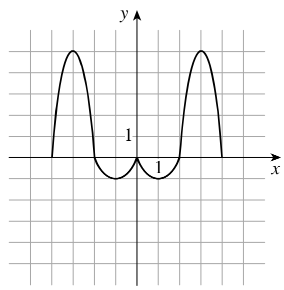
91. **(a)** Even



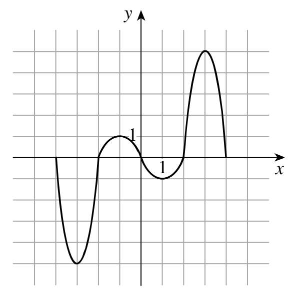
(b) Odd



92. (a) Even

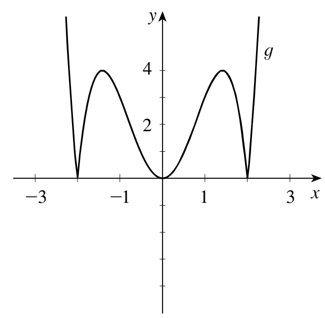


(b) Odd

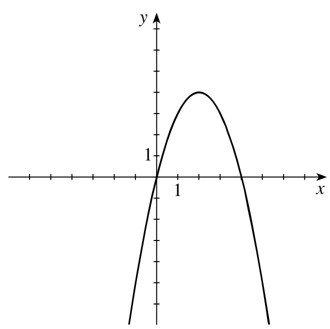


93. Since $f(x) = x^2 - 4 < 0$, for $-2 < x < 2$, the graph of $y = g(x)$ is found by sketching the graph of $y = f(x)$ for $x \le -2$ and $x \ge 2$, then reflecting in the x-axis the part of the graph of $y = f(x)$ for $-2 < x < 2$.

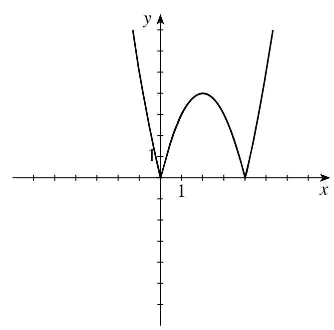
94. $g(x) = \left|x^4 - 4x^2\right|$



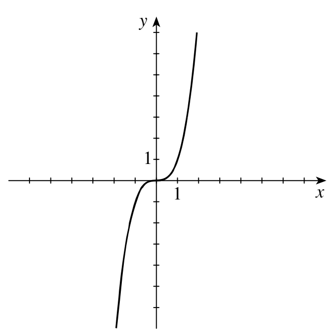
95. (a) $f(x) = 4x - x^2$



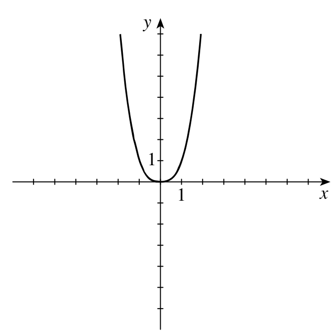
(b) $f(x) = \left|4x - x^2\right|$



96. (a) $f(x) = x^3$

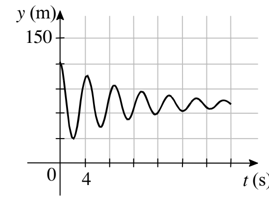


(b) $g(x) = \left|x^3\right|$



97. (a) Luisa drops to a height of 60 m, bounces up and down, then settles at 105 meters.

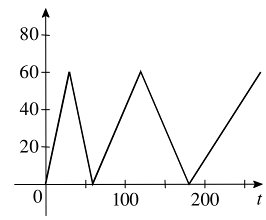
(b)



(c) To obtain the graph of H from that of h, we shift downward 30 meters. Thus, $H(t) = h(t) - 30$.

98. (a) Miyuki swims two and a half laps, slowing down with each successive lap. In the first 30 seconds she swims 50 meters, so her average speed is $\frac{50}{30} \approx 1.67$ m/s.

(b)

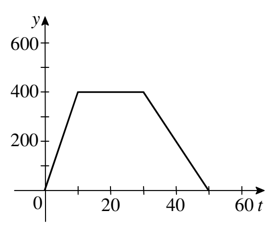


This graph is obtained by stretching the original graph vertically by a factor of 1.2.

(c) Here Miyuki swims 60 meters in 30 seconds, so her average speed is $\frac{60}{30} = 2$ m/s.

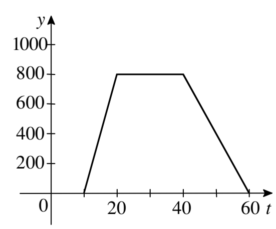
99. (a) The trip to the park corresponds to the first piece of the graph. The class travels 800 meters in 10 minutes, so their average speed is $\frac{800}{10} = 80$ m/min. The second (horizontal) piece of the graph stretches from $t = 10$ to $t = 30$, so the class spends 20 minutes at the park. The park is 800 meters from the school.

(b)



The new graph is obtained by shrinking the original graph vertically by a factor of 0.50. The new average speed is 40 m/min, and the new park is 400 m from the school.

(c)



This graph is obtained by shifting the original graph to the right 10 minutes. The class leaves ten minutes later than it did in the original scenario.

100. To obtain the graph of $g(x) = (x-2)^2 + 5$ from that of $f(x) = (x+2)^2$, we shift to the right 4 units and upward 5 units.

101. To obtain the graph of $g(x)$ from that of $f(x)$, we reflect the graph about the y-axis, then reflect about the x-axis, then shift upward 6 units.

102. f even implies $f(-x) = f(x)$; g even implies $g(-x) = g(x)$; f odd implies $f(-x) = -f(x)$; and g odd implies $g(-x) = -g(x)$

If f and g are both even, then $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$ and $f+g$ is even.

If f and g are both odd, then $(f+g)(-x) = f(-x) + g(-x) = -f(x) - g(x) = -(f+g)(x)$ and $f+g$ is odd.

If f odd and g even, then $(f+g)(-x) = f(-x) + g(-x) = -f(x) + g(x)$, which is neither odd nor even.

103. f even implies $f(-x) = f(x)$; g even implies $g(-x) = g(x)$; f odd implies $f(-x) = -f(x)$; and g odd implies $g(-x) = -g(x)$.

If f and g are both even, then $(fg)(-x) = f(-x) \cdot g(-x) = f(x) \cdot g(x) = (fg)(x)$. Thus fg is even.

If f and g are both odd, then $(fg)(-x) = f(-x) \cdot g(-x) = -f(x) \cdot (-g(x)) = f(x) \cdot g(x) = (fg)(x)$. Thus fg is even

If f if odd and g is even, then $(fg)(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -f(x) \cdot g(x) = -(fg)(x)$. Thus fg is odd.

104. $f(x) = x^n$ is even when n is an even integer and $f(x) = x^n$ is odd when n is an odd integer.
These names were chosen because polynomials with only terms with odd powers are odd functions, and polynomials with only terms with even powers are even functions.

2.7 COMBINING FUNCTIONS

1. From the graphs of f and g in the figure, we find $(f+g)(2) = f(2) + g(2) = 3 + 5 = 8$, $(f-g)(2) = f(2) - g(2) = 3 - 5 = -2$, $(fg)(2) = f(2)g(2) = 3 \cdot 5 = 15$, and $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{5}$.

2. By definition, $f \circ g(x) = f(g(x))$. So, if $g(2) = 5$ and $f(5) = 12$, then $f \circ g(2) = f(g(2)) = f(5) = 12$.

3. If the rule of the function f is "add one" and the rule of the function g is "multiply by 2" then the rule of $f \circ g$ is "*multiply by* 2*, then add one*" and the rule of $g \circ f$ is "*add one, then multiply by* 2."

4. We can express the functions in Exercise 3 algebraically as $f(x) = x + 1$, $g(x) = 2x$, $(f \circ g)(x) = 2x + 1$, and $(g \circ f)(x) = 2(x+1)$.

5. **(a)** The function $(f+g)(x)$ is defined for all values of x that are in the domains of both f and g.
(b) The function $(fg)(x)$ is defined for all values of x that are in the domains of both f and g.
(c) The function $(f/g)(x)$ is defined for all values of x that are in the domains of both f and g, and $g(x)$ is not equal to 0.

6. The composition $(f \circ g)(x)$ is defined for all values of x for which x is in the domain of g and $g(x)$ is in the domain of f.

7. $f(x) = x$ has domain $(-\infty, \infty)$. $g(x) = 2x$ has domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = x + 2x = 3x$, and the domain is $(-\infty, \infty)$. $(f-g)(x) = x - 2x = -x$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = x(2x) = 2x^2$, and the domain is $(-\infty, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{2x} = \dfrac{1}{2}$, and the domain is $(-\infty, 0) \cup (0, \infty)$.

8. $f(x) = x$ has domain $(-\infty, \infty)$. $g(x) = \sqrt{x}$ has domain $[0, \infty)$. The intersection of the domains of f and g is $[0, \infty)$.
$(f+g)(x) = x + \sqrt{x}$, and the domain is $[0, \infty)$. $(f-g)(x) = x - \sqrt{x}$, and the domain is $[0, \infty)$.
$(fg)(x) = x\sqrt{x} = x^{3/2}$, and the domain is $[0, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{\sqrt{x}} = \sqrt{x}$, and the domain is $(0, \infty)$.

9. $f(x) = x^2 + x$ and $g(x) = x^2$ each have domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = 2x^2 + x$, and the domain is $(-\infty, \infty)$. $(f-g)(x) = x$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = x^4 + x^3$, and the domain is $(-\infty, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + x}{x^2} = 1 + \dfrac{1}{x}$, and the domain is $(-\infty, 0) \cup (0, \infty)$.

10. $f(x) = 3 - x^2$ and $g(x) = x^2 - 4$ each have domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = -1$, and the domain is $(-\infty, \infty)$. $(f-g)(x) = -2x^2 + 7$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = \left(3 - x^2\right)\left(x^2 - 4\right) = -x^4 + 7x^2 - 12$, and the domain is $(-\infty, \infty)$. $\left(\dfrac{f}{g}\right)(x) = \dfrac{3 - x^2}{x^2 - 4} = \dfrac{3 - x^2}{(x-2)(x+2)}$, and the domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

11. $f(x) = 5 - x$ and $g(x) = x^2 - 3x$ each have domain $(-\infty, \infty)$. The intersection of the domains of f and g is $(-\infty, \infty)$.
$(f+g)(x) = (5 - x) + \left(x^2 - 3x\right) = x^2 - 4x + 5$, and the domain is $(-\infty, \infty)$.
$(f-g)(x) = (5 - x) - \left(x^2 - 3x\right) = -x^2 + 2x + 5$, and the domain is $(-\infty, \infty)$.
$(fg)(x) = (5 - x)\left(x^2 - 3x\right) = -x^3 + 8x^2 - 15x$, and the domain is $(-\infty, \infty)$.
$\left(\dfrac{f}{g}\right)(x) = \dfrac{5 - x}{x^2 - 3x} = \dfrac{5 - x}{x(x-3)}$, and the domain is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.

12. $f(x)=x^2+2x$ has domain $(-\infty,\infty)$. $g(x)=3x^2-1$ has domain $(-\infty,\infty)$. The intersection of the domains of f and g is $(-\infty,\infty)$.

$(f+g)(x)=x^2+2x+\left(3x^2-1\right)=4x^2+2x-1$, and the domain is $(-\infty,\infty)$.

$(f-g)(x)=x^2+2x-\left(3x^2-1\right)=-2x^2+2x+1$, and the domain is $(-\infty,\infty)$.

$(fg)(x)=\left(x^2+2x\right)\left(3x^2-1\right)=3x^4+6x^3-x^2-2x$, and the domain is $(-\infty,\infty)$.

$\left(\dfrac{f}{g}\right)(x)=\dfrac{x^2+2x}{3x^2-1}$, $3x^2-1\neq 0\Rightarrow x\neq\pm\frac{\sqrt{3}}{3}$, and the domain is $\left\{x\mid x\neq\pm\frac{\sqrt{3}}{3}\right\}$.

13. $f(x)=\sqrt{25-x^2}$, has domain $[-5,5]$. $g(x)=\sqrt{x+3}$, has domain $[-3,\infty)$. The intersection of the domains of f and g is $[-3,5]$.

$(f+g)(x)=\sqrt{25-x^2}+\sqrt{x+3}$, and the domain is $[-3,5]$.

$(f-g)(x)=\sqrt{25-x^2}-\sqrt{x+3}$, and the domain is $[-3,5]$.

$(fg)(x)=\sqrt{\left(25-x^2\right)(x+3)}$, and the domain is $[-3,5]$.

$\left(\dfrac{f}{g}\right)(x)=\sqrt{\dfrac{25-x^2}{x+3}}$, and the domain is $(-3,5]$.

14. $f(x)=\sqrt{16-x^2}$ has domain $[-4,4]$. $g(x)=\sqrt{x^2-1}$ has domain $(-\infty,-1]\cup[1,\infty)$. The intersection of the domains of f and g is $[-4,-1]\cup[1,4]$.

$(f+g)(x)=\sqrt{16-x^2}+\sqrt{x^2-1}$, and the domain is $[-4,-1]\cup[1,4]$.

$(f-g)(x)=\sqrt{16-x^2}-\sqrt{x^2-1}$, and the domain is $[-4,-1]\cup[1,4]$.

$(fg)(x)=\sqrt{\left(16-x^2\right)\left(x^2-1\right)}$, and the domain is $[-4,-1]\cup[1,4]$.

$\left(\dfrac{f}{g}\right)(x)=\sqrt{\dfrac{16-x^2}{x^2-1}}$, and the domain is $[-4,-1)\cup(1,4]$.

15. $f(x)=\dfrac{2}{x}$ has domain $x\neq 0$. $g(x)=\dfrac{4}{x+4}$, has domain $x\neq -4$. The intersection of the domains of f and g is $\{x\mid x\neq 0,-4\}$; in interval notation, this is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

$(f+g)(x)=\dfrac{2}{x}+\dfrac{4}{x+4}=\dfrac{2}{x}+\dfrac{4}{x+4}=\dfrac{2(3x+4)}{x(x+4)}$, and the domain is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

$(f-g)(x)=\dfrac{2}{x}-\dfrac{4}{x+4}=-\dfrac{2(x-4)}{x(x+4)}$, and the domain is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

$(fg)(x)=\dfrac{2}{x}\cdot\dfrac{4}{x+4}=\dfrac{8}{x(x+4)}$, and the domain is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

$\left(\dfrac{f}{g}\right)(x)=\dfrac{\frac{2}{x}}{\frac{4}{x+4}}=\dfrac{x+4}{2x}$, and the domain is $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

16. $f(x) = \dfrac{2}{x+1}$ has domain $x \neq -1$. $g(x) = \dfrac{x}{x+1}$ has domain $x \neq -1$. The intersection of the domains of f and g is $\{x \mid x \neq -1\}$; in interval notation, this is $(-\infty, -1) \cup (-1, \infty)$.

$(f+g)(x) = \dfrac{2}{x+1} + \dfrac{x}{x+1} = \dfrac{x+2}{x+1}$, and the domain is $(-\infty, -1) \cup (-1, \infty)$.

$(f-g)(x) = \dfrac{2}{x+1} - \dfrac{x}{x+1} = \dfrac{2-x}{x+1}$, and the domain is $(-\infty, -1) \cup (-1, \infty)$.

$(fg)(x) = \dfrac{2}{x+1} \cdot \dfrac{x}{x+1} = \dfrac{2x}{(x+1)^2}$, and the domain is $(-\infty, -1) \cup (-1, \infty)$.

$\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{2}{x+1}}{\frac{x}{x+1}} = \dfrac{2}{x}$, so $x \neq 0$ as well. Thus the domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$.

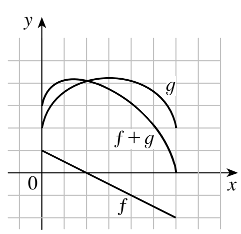
17. $f(x) = \sqrt{x} + \sqrt{3-x}$. The domain of $\sqrt{x}$ is $[0, \infty)$, and the domain of $\sqrt{3-x}$ is $(-\infty, 3]$. Thus, the domain of f is $(-\infty, 3] \cap [0, \infty) = [0, 3]$.

18. $f(x) = \sqrt{x+4} - \dfrac{\sqrt{1-x}}{x}$. The domain of $\sqrt{x+4}$ is $[-4, \infty)$, and the domain of $\dfrac{\sqrt{1-x}}{x}$ is $(-\infty, 0) \cup (0, 1]$. Thus, the domain of f is $[-4, \infty) \cap \{(-\infty, 0) \cup (0, 1]\} = [-4, 0) \cup (0, 1]$.

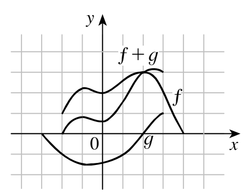
19. $h(x) = (x-3)^{-1/4} = \dfrac{1}{(x-3)^{1/4}}$. Since $1/4$ is an even root and the denominator can not equal 0, $x - 3 > 0 \Leftrightarrow x > 3$. So the domain is $(3, \infty)$.

20. $k(x) = \dfrac{\sqrt{x+3}}{x-1}$. The domain of $\sqrt{x+3}$ is $[-3, \infty)$, and the domain of $\dfrac{1}{x-1}$ is $x \neq 1$. Since $x \neq 1$ is $(-\infty, 1) \cup (1, \infty)$, the domain is $[-3, \infty) \cap \{(-\infty, 1) \cup (1, \infty)\} = [-3, 1) \cup (1, \infty)$.

21.



22.



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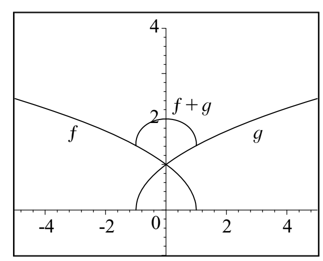

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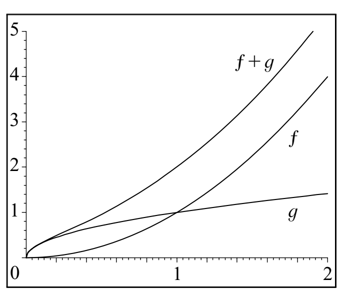

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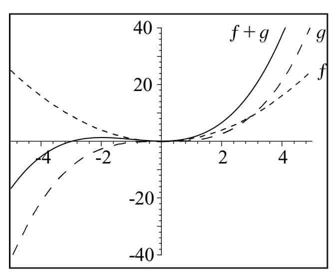
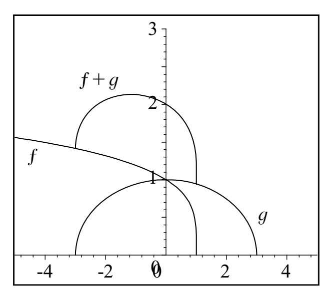


26.



27. $f(x) = 2x - 3$ and $g(x) = 4 - x^2$.

(a) $f(g(0)) = f\left(4 - (0)^2\right) = f(4) = 2(4) - 3 = 5$

(b) $g(f(0)) = g(2(0) - 3) = g(-3) = 4 - (-3)^2 = -5$

28. **(a)** $f(f(2)) = f(2(2) - 3) = f(1) = 2(1) - 3 = -1$

(b) $g(g(3)) = g\left(4 - 3^2\right) = g(-5) = 4 - (-5)^2 = -21$

29. **(a)** $(f \circ g)(-2) = f(g(-2)) = f\left(4 - (-2)^2\right) = f(0) = 2(0) - 3 = -3$

(b) $(g \circ f)(-2) = g(f(-2)) = g(2(-2) - 3) = g(-7) = 4 - (-7)^2 = -45$

30. **(a)** $(f \circ f)(-1) = f(f(-1)) = f(2(-1) - 3) = f(-5) = 2(-5) - 3 = -13$

(b) $(g \circ g)(-1) = g(g(-1)) = g\left(4 - (-1)^2\right) = g(3) = 4 - 3^2 = -5$

31. **(a)** $(f \circ g)(x) = f(g(x)) = f\left(4 - x^2\right) = 2\left(4 - x^2\right) - 3 = 8 - 2x^2 - 3 = 5 - 2x^2$

(b) $(g \circ f)(x) = g(f(x)) = g(2x - 3) = 4 - (2x - 3)^2 = 4 - \left(4x^2 - 12x + 9\right) = -4x^2 + 12x - 5$

32. **(a)** $(f \circ f)(x) = f(f(x)) = f(3x - 5) = 3(3x - 5) - 5 = 9x - 15 - 5 = 9x - 20$

(b) $(g \circ g)(x) = g(g(x)) = g\left(2 - x^2\right) = 2 - \left(2 - x^2\right)^2 = 2 - \left(4 - 4x^2 + x^4\right) = -x^4 + 4x^2 - 2$

33. $f(g(2)) = f(5) = 4$

34. $f(0) = 0$, so $g(f(0)) = g(0) = 3$.

35. $(g \circ f)(4) = g(f(4)) = g(2) = 5$

36. $g(0) = 3$, so $(f \circ g)(0) = f(3) = 0$.

37. $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

38. $f(4) = 2$, so $(f \circ f)(4) = f(2) = -2$.

39. From the table, $g(2) = 5$ and $f(5) = 6$, so $f(g(2)) = 6$.

40. From the table, $f(2) = 3$ and $g(3) = 6$, so $g(f(2)) = 6$.

41. From the table, $f(1) = 2$ and $f(2) = 3$, so $f(f(1)) = 3$.

42. From the table, $g(2) = 5$ and $g(5) = 1$, so $g(g(2)) = 1$.

43. From the table, $g(6) = 4$ and $f(4) = 1$, so $(f \circ g)(6) = 1$.

44. From the table, $f(2) = 3$ and $g(3) = 6$, so $(g \circ f)(2) = 6$.

45. From the table, $f(5) = 6$ and $f(6) = 3$, so $(f \circ f)(5) = 3$.

46. From the table, $g(2) = 5$ and $g(5) = 1$, so $(g \circ g)(5) = 1$.

47. $f(x) = 2x + 3$, has domain $(-\infty, \infty)$; $g(x) = 4x - 1$, has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f(4x - 1) = 2(4x - 1) + 3 = 8x + 1$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g(2x + 3) = 4(2x + 3) - 1 = 8x + 11$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g(4x - 1) = 4(4x - 1) - 1 = 16x - 5$, and the domain is $(-\infty, \infty)$.

48. $f(x) = 6x - 5$ has domain $(-\infty, \infty)$. $g(x) = \dfrac{x}{2}$ has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f\left(\dfrac{x}{2}\right) = 6\left(\dfrac{x}{2}\right) - 5 = 3x - 5$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g(6x - 5) = \dfrac{6x - 5}{2} = 3x - \frac{5}{2}$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f(6x - 5) = 6(6x - 5) - 5 = 36x - 35$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g\left(\dfrac{x}{2}\right) = \dfrac{\frac{x}{2}}{2} = \dfrac{x}{4}$, and the domain is $(-\infty, \infty)$.

49. $f(x) = x^2$, has domain $(-\infty, \infty)$; $g(x) = x + 1$, has domain $(-\infty, \infty)$.
$(f \circ g)(x) = f(x + 1) = (x + 1)^2 = x^2 + 2x + 1$, and the domain is $(-\infty, \infty)$.
$(g \circ f)(x) = g\left(x^2\right) = \left(x^2\right) + 1 = x^2 + 1$, and the domain is $(-\infty, \infty)$.
$(f \circ f)(x) = f\left(x^2\right) = \left(x^2\right)^2 = x^4$, and the domain is $(-\infty, \infty)$.
$(g \circ g)(x) = g(x + 1) = (x + 1) + 1 = x + 2$, and the domain is $(-\infty, \infty)$.

50. $f(x) = x^3 + 2$ has domain $(-\infty, \infty)$. $g(x) = \sqrt[3]{x}$ has domain $(-\infty, \infty)$.

$(f \circ g)(x) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 + 2 = x + 2$, and the domain is $(-\infty, \infty)$.

$(g \circ f)(x) = g\left(x^3 + 2\right) = \sqrt[3]{x^3 + 2}$ and the domain is $(-\infty, \infty)$.

$(f \circ f)(x) = f\left(x^3 + 2\right) = \left(x^3 + 2\right)^3 + 2 = x^9 + 6x^6 + 12x^3 + 8 + 2 = x^9 + 6x^6 + 12x^3 + 10$, and the domain is $(-\infty, \infty)$.

$(g \circ g)(x) = g\left(\sqrt[3]{x}\right) = \sqrt[3]{\sqrt[3]{x}} = \left(x^{1/3}\right)^{1/3} = x^{1/9}$, and the domain is $(-\infty, \infty)$.

51. $f(x) = \dfrac{1}{x}$, has domain $\{x \mid x \neq 0\}$; $g(x) = 2x + 4$, has domain $(-\infty, \infty)$.

$(f \circ g)(x) = f(2x + 4) = \dfrac{1}{2x + 4}$. $(f \circ g)(x)$ is defined for $2x + 4 \neq 0 \Leftrightarrow x \neq -2$. So the domain is $\{x \mid x \neq -2\} = (-\infty, -2) \cup (-2, \infty)$.

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = 2\left(\dfrac{1}{x}\right) + 4 = \dfrac{2}{x} + 4$, the domain is $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

$(f \circ f)(x) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{\left(\dfrac{1}{x}\right)} = x$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$.

$(g \circ g)(x) = g(2x + 4) = 2(2x + 4) + 4 = 4x + 8 + 4 = 4x + 12$, and the domain is $(-\infty, \infty)$.

52. $f(x) = x^2$ has domain $(-\infty, \infty)$. $g(x) = \sqrt{x - 3}$ has domain $[3, \infty)$.

$(f \circ g)(x) = f\left(\sqrt{x - 3}\right) = \left(\sqrt{x - 3}\right)^2 = x - 3$, and the domain is $[3, \infty)$.

$(g \circ f)(x) = g\left(x^2\right) = \sqrt{x^2 - 3}$. For the domain we must have $x^2 \geq 3 \Rightarrow x \leq -\sqrt{3}$ or $x \geq \sqrt{3}$. Thus the domain is $\left(-\infty, -\sqrt{3}\right] \cup \left[\sqrt{3}, \infty\right)$.

$(f \circ f)(x) = f\left(x^2\right) = \left(x^2\right)^2 = x^4$, and the domain is $(-\infty, \infty)$.

$(g \circ g)(x) = g\left(\sqrt{x - 3}\right) = \sqrt{\sqrt{x - 3} - 3}$. For the domain we must have $\sqrt{x - 3} \geq 3 \Rightarrow x - 3 \geq 9 \Rightarrow x \geq 12$, so the domain is $[12, \infty)$.

53. $f(x) = |x|$, has domain $(-\infty, \infty)$; $g(x) = 2x + 3$, has domain $(-\infty, \infty)$

$(f \circ g)(x) = f(2x + 4) = |2x + 3|$, and the domain is $(-\infty, \infty)$.

$(g \circ f)(x) = g(|x|) = 2|x| + 3$, and the domain is $(-\infty, \infty)$.

$(f \circ f)(x) = f(|x|) = ||x|| = |x|$, and the domain is $(-\infty, \infty)$.

$(g \circ g)(x) = g(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9$. Domain is $(-\infty, \infty)$.

54. $f(x) = x - 4$ has domain $(-\infty, \infty)$. $g(x) = |x + 4|$ has domain $(-\infty, \infty)$.

$(f \circ g)(x) = f(|x + 4|) = |x + 4| - 4$, and the domain is $(-\infty, \infty)$.

$(g \circ f)(x) = g(x - 4) = |(x - 4) + 4| = |x|$, and the domain is $(-\infty, \infty)$.

$(f \circ f)(x) = f(x - 4) = (x - 4) - 4 = x - 8$, and the domain is $(-\infty, \infty)$.

$(g \circ g)(x) = g(|x + 4|) = ||x + 4| + 4| = |x + 4| + 4$ ($|x + 4| + 4$ is always positive). The domain is $(-\infty, \infty)$.

55. $f(x)=\dfrac{x}{x+1}$, has domain $\{x \mid x\neq -1\}$; $g(x)=2x-1$, has domain $(-\infty,\infty)$

$(f\circ g)(x)=f(2x-1)=\dfrac{2x-1}{(2x-1)+1}=\dfrac{2x-1}{2x}$, and the domain is $\{x \mid x\neq 0\}=(-\infty,0)\cup(0,\infty)$.

$(g\circ f)(x)=g\left(\dfrac{x}{x+1}\right)=2\left(\dfrac{x}{x+1}\right)-1=\dfrac{2x}{x+1}-1$, and the domain is $\{x \mid x\neq -1\}=(-\infty,-1)\cup(-1,\infty)$

$(f\circ f)(x)=f\left(\dfrac{x}{x+1}\right)=\dfrac{\frac{x}{x+1}}{\frac{x}{x+1}+1}\cdot\dfrac{x+1}{x+1}=\dfrac{x}{x+x+1}=\dfrac{x}{2x+1}$. $(f\circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $x\neq -1$ and $2x+1\neq 0\Rightarrow x\neq -\frac{1}{2}$, which is $(-\infty,-1)\cup\left(-1,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$.

$(g\circ g)(x)=g(2x-1)=2(2x-1)-1=4x-2-1=4x-3$, and the domain is $(-\infty,\infty)$.

56. $f(x)=\dfrac{1}{\sqrt{x}}$ has domain $\{x \mid x>0\}$; $g(x)=x^2-4x$ has domain $(-\infty,\infty)$.

$(f\circ g)(x)=f\left(x^2-4x\right)=\dfrac{1}{\sqrt{x^2-4x}}$. $(f\circ g)(x)$ is defined whenever $0<x^2-4x=x(x-4)$. The product of two numbers is positive either when both numbers are negative or when both numbers are positive. So the domain of $f\circ g$ is $\{x \mid x<0 \text{ and } x<4\}\cup\{x \mid x>0 \text{ and } x>4\}$ which is $(-\infty,0)\cup(4,\infty)$.

$(g\circ f)(x)=g\left(\dfrac{1}{\sqrt{x}}\right)=\left(\dfrac{1}{\sqrt{x}}\right)^2-4\left(\dfrac{1}{\sqrt{x}}\right)=\dfrac{1}{x}-\dfrac{4}{\sqrt{x}}$. $(g\circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined, that is, whenever $x>0$. So the domain of $g\circ f$ is $(0,\infty)$.

$(f\circ f)(x)=f\left(\dfrac{1}{\sqrt{x}}\right)=\dfrac{1}{\sqrt{\frac{1}{\sqrt{x}}}}=x^{1/4}$. $(f\circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined, that is, whenever $x>0$. So the domain of $f\circ f$ is $(0,\infty)$.

$(g\circ g)(x)=g\left(x^2-4x\right)=\left(x^2-4x\right)^2-4\left(x^2-4x\right)=x^4-8x^3+16x^2-4x^2+16x=x^4-8x^3+12x^2+16x$, and the domain is $(-\infty,\infty)$.

57. $f(x)=\dfrac{x}{x+1}$, has domain $\{x \mid x\neq -1\}$; $g(x)=\dfrac{1}{x}$ has domain $\{x \mid x\neq 0\}$.

$(f\circ g)(x)=f\left(\dfrac{1}{x}\right)=\dfrac{\frac{1}{x}}{\frac{1}{x}+1}=\dfrac{1}{x\left(\frac{1}{x}+1\right)}=\dfrac{1}{x+1}$. $(f\circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined, so the domain is $\{x \mid x\neq -1,0\}$.

$(g\circ f)(x)=g\left(\dfrac{x}{x+1}\right)=\dfrac{1}{\frac{x}{x+1}}=\dfrac{x+1}{x}$. $(g\circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined, so the domain is $\{x \mid x\neq -1,0\}$.

$(f\circ f)(x)=f\left(\dfrac{x}{x+1}\right)=\dfrac{\frac{x}{x+1}}{\frac{x}{x+1}+1}=\dfrac{x}{(x+1)\left(\frac{x}{x+1}+1\right)}=\dfrac{x}{2x+1}$. $(f\circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined, so the domain is $\left\{x \mid x\neq -1,-\frac{1}{2}\right\}$.

$(g\circ g)(x)=g\left(\dfrac{1}{x}\right)=\dfrac{1}{\frac{1}{x}}=x$. $(g\circ g)(x)$ is defined whenever both $g(x)$ and $g(g(x))$ are defined, so the domain is $\{x \mid x\neq 0\}$.

58. $f(x) = \dfrac{2}{x}$ has domain $\{x \mid x \neq 0\}$; $g(x) = \dfrac{x}{x+2}$ has domain $\{x \mid x \neq -2\}$.

$(f \circ g)(x) = f\left(\dfrac{x}{x+2}\right) = \dfrac{2}{\dfrac{x}{x+2}} = \dfrac{2x+4}{x}$. $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined; that is, whenever $x \neq 0$ and $x \neq -2$. So the domain is $\{x \mid x \neq 0, -2\}$.

$(g \circ f)(x) = g\left(\dfrac{2}{x}\right) = \dfrac{\dfrac{2}{x}}{\dfrac{2}{x}+2} = \dfrac{2}{2+2x} = \dfrac{1}{1+x}$. $(g \circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined; that is, whenever $x \neq 0$ and $x \neq -1$. So the domain is $\{x \mid x \neq 0, -1\}$.

$(f \circ f)(x) = f\left(\dfrac{2}{x}\right) = \dfrac{2}{\dfrac{2}{x}} = x$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $x \neq 0$. So the domain is $\{x \mid x \neq 0\}$.

$(g \circ g)(x) = g\left(\dfrac{x}{x+2}\right) = \dfrac{\dfrac{x}{x+2}}{\dfrac{x}{x+2}+2} = \dfrac{x}{x+2(x+2)} = \dfrac{x}{3x+4}$. $(g \circ g)(x)$ is defined whenever both $g(x)$ and $g(g(x))$ are defined; that is whenever $x \neq -2$ and $x \neq -\frac{4}{3}$. So the domain is $\left\{x \mid x \neq -2, -\frac{4}{3}\right\}$.

59. $(f \circ g \circ h)(x) = f(g(h(x))) = f(g(x-1)) = f\left(\sqrt{x-1}\right) = \sqrt{x-1} - 1$

60. $(g \circ h)(x) = g\left(x^2+2\right) = \left(x^2+2\right)^3 = x^6 + 6x^4 + 12x^2 + 8$.

$(f \circ g \circ h)(x) = f\left(x^6 + 6x^4 + 12x^2 + 8\right) = \dfrac{1}{x^6 + 6x^4 + 12x^2 + 8}$.

61. $(f \circ g \circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{x}\right)\right) = f\left(\sqrt{x} - 5\right) = \left(\sqrt{x} - 5\right)^4 + 1$

62. $(g \circ h)(x) = g\left(\sqrt[3]{x}\right) = \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}$. $(f \circ g \circ h)(x) = f\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right) = \sqrt{\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}}$.

For Exercises 63–72, many answers are possible.

63. $F(x) = (x-9)^5$. Let $f(x) = x^5$ and $g(x) = x - 9$, then $F(x) = (f \circ g)(x)$.

64. $F(x) = \sqrt{x} + 1$. If $f(x) = x + 1$ and $g(x) = \sqrt{x}$, then $F(x) = (f \circ g)(x)$.

65. $G(x) = \dfrac{x^2}{x^2+4}$. Let $f(x) = \dfrac{x}{x+4}$ and $g(x) = x^2$, then $G(x) = (f \circ g)(x)$.

66. $G(x) = \dfrac{1}{x+3}$. If $f(x) = \dfrac{1}{x}$ and $g(x) = x + 3$, then $G(x) = (f \circ g)(x)$.

67. $H(x) = \left|1 - x^3\right|$. Let $f(x) = |x|$ and $g(x) = 1 - x^3$, then $H(x) = (f \circ g)(x)$.

68. $H(x) = \sqrt{1 + \sqrt{x}}$. If $f(x) = \sqrt{1+x}$ and $g(x) = \sqrt{x}$, then $H(x) = (f \circ g)(x)$.

69. $F(x) = \dfrac{1}{x^2+1}$. Let $f(x) = \dfrac{1}{x}$, $g(x) = x + 1$, and $h(x) = x^2$, then $F(x) = (f \circ g \circ h)(x)$.

70. $F(x) = \sqrt[3]{\sqrt{x} - 1}$. If $g(x) = x - 1$ and $h(x) = \sqrt{x}$, then $(g \circ h)(x) = \sqrt{x} - 1$, and if $f(x) = \sqrt[3]{x}$, then $F(x) = (f \circ g \circ h)(x)$.

71. $G(x) = \left(4 + \sqrt[3]{x}\right)^9$. Let $f(x) = x^9$, $g(x) = 4 + x$, and $h(x) = \sqrt[3]{x}$, then $G(x) = (f \circ g \circ h)(x)$.

72. $G(x) = \dfrac{2}{\left(3 + \sqrt{x}\right)^2}$. If $g(x) = 3 + x$ and $h(x) = \sqrt{x}$, then $(g \circ h)(x) = 3 + \sqrt{x}$, and if $f(x) = \dfrac{2}{x^2}$, then $G(x) = (f \circ g \circ h)(x)$.

73. Yes. If $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$, then $(f \circ g)(x) = f(m_2 x + b_2) = m_1(m_2 x + b_2) + b_1 = m_1 m_2 x + m_1 b_2 + b_1$, which is a linear function, because it is of the form $y = mx + b$. The slope is $m_1 m_2$.

74. $g(x) = 2x + 1$ and $h(x) = 4x^2 + 4x + 7$.

Method 1: Notice that $(2x+1)^2 = 4x^2 + 4x + 1$. We see that adding 6 to this quantity gives $(2x+1)^2 + 6 = 4x^2 + 4x + 1 + 6 = 4x^2 + 4x + 7$, which is $h(x)$. So let $f(x) = x^2 + 6$, and we have $(f \circ g)(x) = (2x+1)^2 + 6 = h(x)$.

Method 2: Since $g(x)$ is linear and $h(x)$ is a second degree polynomial, $f(x)$ must be a second degree polynomial, that is, $f(x) = ax^2 + bx + c$ for some a, b, and c. Thus $f(g(x)) = f(2x+1) = a(2x+1)^2 + b(2x+1) + c \Leftrightarrow 4ax^2 + 4ax + a + 2bx + b + c = 4ax^2 + (4a + 2b)x + (a + b + c) = 4x^2 + 4x + 7$. Comparing this with $f(g(x))$, we have $4a = 4$ (the x^2 coefficients), $4a + 2b = 4$ (the x coefficients), and $a + b + c = 7$ (the constant terms) $\Leftrightarrow a = 1$ and $2a + b = 2$ and $a + b + c = 7 \Leftrightarrow a = 1, b = 0, c = 6$. Thus $f(x) = x^2 + 6$.

$f(x) = 3x + 5$ and $h(x) = 3x^2 + 3x + 2$.

Note since $f(x)$ is linear and $h(x)$ is quadratic, $g(x)$ must also be quadratic. We can then use trial and error to find $g(x)$. Another method is the following: We wish to find g so that $(f \circ g)(x) = h(x)$. Thus $f(g(x)) = 3x^2 + 3x + 2 \Leftrightarrow 3(g(x)) + 5 = 3x^2 + 3x + 2 \Leftrightarrow 3(g(x)) = 3x^2 + 3x - 3 \Leftrightarrow g(x) = x^2 + x - 1$.

75. The price per sticker is $0.15 - 0.000002x$ and the number sold is x, so the revenue is $R(x) = (0.15 - 0.000002x)x = 0.15x - 0.000002x^2$.

76. As found in Exercise 75, the revenue is $R(x) = 0.15x - 0.000002x^2$, and the cost is $0.095x - 0.0000005x^2$, so the profit is $P(x) = 0.15x - 0.000002x^2 - \left(0.095x - 0.0000005x^2\right) = 0.055x - 0.0000015x^2$.

77. **(a)** Because the ripple travels at a speed of 60 cm/s, the distance traveled in t seconds is the radius, so $g(t) = 60t$.

(b) The area of a circle is πr^2, so $f(r) = \pi r^2$.

(c) $f \circ g = \pi(g(t))^2 = \pi(60t)^2 = 3600\pi t^2$ cm^2. This function represents the area of the ripple as a function of time.

78. **(a)** Let $f(t)$ be the radius of the spherical balloon in centimeters. Since the radius is increasing at a rate of 1 cm/s, the radius is $f(t) = t$ after t seconds.

(b) The volume of the balloon can be written as $g(r) = \frac{4}{3}\pi r^3$.

(c) $g \circ f = \frac{4}{3}\pi(t)^3 = \frac{4}{3}\pi t^3$. $g \circ f$ represents the volume as a function of time.

79. Let r be the radius of the spherical balloon in centimeters. Since the radius is increasing at a rate of 2 cm/s, the radius is $r = 2t$ after t seconds. Therefore, the surface area of the balloon can be written as $S = 4\pi r^2 = 4\pi(2t)^2 = 4\pi\left(4t^2\right) = 16\pi t^2$.

80. **(a)** $f(x) = 0.80x$

(b) $g(x) = x - 50$

(c) $(f \circ g)(x) = f(x - 50) = 0.80(x - 50) = 0.80x - 40$. $f \circ g$ represents applying the \$50 coupon, then the 20% discount. $(g \circ f)(x) = g(0.80x) = 0.80x - 50$. $g \circ f$ represents applying the 20% discount, then the \$50 coupon. So applying the 20% discount, then the \$50 coupon gives the lower price.

81. **(a)** $f(x) = 0.90x$

(b) $g(x) = x - 100$

(c) $(f \circ g)(x) = f(x - 100) = 0.90(x - 100) = 0.90x - 90$. $f \circ g$ represents applying the \$100 coupon, then the 10% discount. $(g \circ f)(x) = g(0.90x) = 0.90x - 100$. $g \circ f$ represents applying the 10% discount, then the \$100 coupon. So applying the 10% discount, then the \$100 coupon gives the lower price.

82. Let t be the time since the plane flew over the radar station.

(a) Let s be the distance in kilometers between the plane and the radar station, and let d be the horizontal distance that the plane has flown. Using the Pythagorean theorem, $s = f(d) = \sqrt{1.6 + d^2}$.

(b) Since distance = rate × time, we have $d = g(t) = 560t$.

(c) $s(t) = (f \circ g)(t) = f(560t) = \sqrt{1.6 + (560t)^2} = \sqrt{1.6 + 313{,}600t^2}$.

83. $A(x) = 1.05x$. $(A \circ A)(x) = A(A(x)) = A(1.05x) = 1.05(1.05x) = (1.05)^2 x$.
$(A \circ A \circ A)(x) = A(A \circ A(x)) = A\left((1.05)^2 x\right) = 1.05\left[(1.05)^2 x\right] = (1.05)^3 x$.
$(A \circ A \circ A \circ A)(x) = A(A \circ A \circ A(x)) = A\left((1.05)^3 x\right) = 1.05\left[(1.05)^3 x\right] = (1.05)^4 x$. A represents the amount in the account after 1 year; $A \circ A$ represents the amount in the account after 2 years; $A \circ A \circ A$ represents the amount in the account after 3 years; and $A \circ A \circ A \circ A$ represents the amount in the account after 4 years. We can see that if we compose n copies of A, we get $(1.05)^n x$.

84. If $g(x)$ is even, then $h(-x) = f(g(-x)) = f(g(x)) = h(x)$. So yes, h is always an even function.
If $g(x)$ is odd, then h is not necessarily an odd function. For example, if we let $f(x) = x - 1$ and $g(x) = x^3$, g is an odd function, but $h(x) = (f \circ g)(x) = f\left(x^3\right) = x^3 - 1$ is not an odd function.
If $g(x)$ is odd and f is also odd, then $h(-x) = (f \circ g)(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -(f \circ g)(x) = -h(x)$. So in this case, h is also an odd function.
If $g(x)$ is odd and f is even, then $h(-x) = (f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x) = h(x)$, so in this case, h is an even function.

2.8 ONE-TO-ONE FUNCTIONS AND THEIR INVERSES

1. A function f is one-to-one if different inputs produce *different* outputs. You can tell from the graph that a function is one-to-one by using the *Horizontal Line* Test.

2. (a) For a function to have an inverse, it must be *one-to-one*. $f(x) = x^2$ is not one-to-one, so it does not have an inverse. However $g(x) = x^3$ is one-to-one, so it has an inverse.

(b) The inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$.

3. (a) Proceeding backward through the description of f, we can describe f^{-1} as follows: "Take the third root, subtract 5, then divide by 3."

(b) $f(x) = (3x + 5)^3$ and $f^{-1}(x) = \dfrac{\sqrt[3]{x} - 5}{3}$.

4. Yes, the graph of f is one-to-one, so f has an inverse. Because $f(4) = 1$, $f^{-1}(1) = 4$, and because $f(5) = 3$, $f^{-1}(3) = 5$.

5. If the point $(3, 4)$ is on the graph of f, then the point $(4, 3)$ is on the graph of f^{-1}. [This is another way of saying that $f(3) = 4 \Leftrightarrow f^{-1}(4) = 3$.]

6. (a) False. For instance, if $f(x) = x$, then $f^{-1}(x) = x$, but $\dfrac{1}{f(x)} = \dfrac{1}{x} \neq f^{-1}(x)$.

(b) This is true, by definition.

7. By the Horizontal Line Test, f is not one-to-one.

8. By the Horizontal Line Test, f is one-to-one.

9. By the Horizontal Line Test, f is one-to-one.

10. By the Horizontal Line Test, f is not one-to-one.

11. By the Horizontal Line Test, f is not one-to-one.

12. By the Horizontal Line Test, f is one-to-one.

13. $f(x) = -2x + 4$. If $x_1 \neq x_2$, then $-2x_1 \neq -2x_2$ and $-2x_1 + 4 \neq -2x_2 + 4$. So f is a one-to-one function.

14. $f(x) = 3x - 2$. If $x_1 \neq x_2$, then $3x_1 \neq 3x_2$ and $3x_1 - 2 \neq 3x_2 - 2$. So f is a one-to-one function.

15. $g(x) = \sqrt{x}$. If $x_1 \neq x_2$, then $\sqrt{x_1} \neq \sqrt{x_2}$ because two different numbers cannot have the same square root. Therefore, g is a one-to-one function.

16. $g(x) = |x|$. Because every number and its negative have the same absolute value (for example, $|-1| = 1 = |1|$), g is not a one-to-one function.

17. $h(x) = x^2 - 2x$. Because $h(0) = 0$ and $h(2) = (2) - 2(2) = 0$ we have $h(0) = h(2)$. So f is not a one-to-one function.

18. $h(x) = x^3 + 8$. If $x_1 \neq x_2$, then $x_1^3 \neq x_2^3$ and $x_1^3 + 8 \neq x_2^3 + 8$. So f is a one-to-one function.

19. $f(x) = x^4 + 5$. Every nonzero number and its negative have the same fourth power. For example, $(-1)^4 = 1 = (1)^4$, so $f(-1) = f(1)$. Thus f is not a one-to-one function.

20. $f(x) = x^4 + 5$, $0 \leq x \leq 2$. If $x_1 \neq x_2$, then $x_1^4 \neq x_2^4$ because two different positive numbers cannot have the same fourth power. Thus, $x_1^4 + 5 \neq x_2^4 + 5$. So f is a one-to-one function.

21. $r(t) = t^6 - 3$, $0 \leq t \leq 5$. If $t_1 \neq t_2$, then $t_1^6 \neq t_2^6$ because two different positive numbers cannot have the same sixth power. Thus, $t_1^6 - 3 \neq t_2^6 - 3$. So r is a one-to-one function.

22. $r(t) = t^4 - 1$. Every nonzero number and its negative have the same fourth power. For example, $(-1)^4 = 1 = (1)^4$, so $r(-1) = r(1)$. Thus r is not a one-to-one function.

23. $f(x) = \dfrac{1}{x^2}$. Every nonzero number and its negative have the same square. For example, $\dfrac{1}{(-1)^2} = 1 = \dfrac{1}{(1)^2}$, so $f(-1) = f(1)$. Thus f is not a one-to-one function.

24. $f(x) = \dfrac{1}{x}$. If $x_1 \neq x_2$, then $\dfrac{1}{x_1} \neq \dfrac{1}{x_2}$. So f is a one-to-one function.

25. **(a)** $f(2) = 7$. Since f is one-to-one, $f^{-1}(7) = 2$.
(b) $f^{-1}(3) = -1$. Since f is one-to-one, $f(-1) = 3$.

26. **(a)** $f(5) = 18$. Since f is one-to-one, $f^{-1}(18) = 5$.
(b) $f^{-1}(4) = 2$. Since f is one-to-one, $f(2) = 4$.

27. $f(x) = 5 - 2x$. Since f is one-to-one and $f(1) = 5 - 2(1) = 3$, then $f^{-1}(3) = 1$. (Find 1 by solving the equation $5 - 2x = 3$.)

28. To find $g^{-1}(5)$, we find the x value such that $g(x) = 5$; that is, we solve the equation $g(x) = x^2 + 4x = 5$. Now $x^2 + 4x = 5 \Leftrightarrow x^2 + 4x - 5 = 0 \Leftrightarrow (x-1)(x+5) = 0 \Leftrightarrow x = 1$ or $x = -5$. Since the domain of g is $[-2, \infty)$, $x = 1$ is the only value where $g(x) = 5$. Therefore, $g^{-1}(5) = 1$.

29. **(a)** Because $f(6) = 2$, $f^{-1}(2) = 6$. **(b)** Because $f(2) = 5$, $f^{-1}(5) = 2$. **(c)** Because $f(0) = 6$, $f^{-1}(6) = 0$.

30. **(a)** Because $g(4) = 2$, $g^{-1}(2) = 4$. **(b)** Because $g(7) = 5$, $g^{-1}(5) = 7$. **(c)** Because $g(8) = 6$, $g^{-1}(6) = 8$.

31. From the table, $f(4) = 5$, so $f^{-1}(5) = 4$.

32. From the table, $f(5) = 0$, so $f^{-1}(0) = 5$.

33. $f^{-1}(f(1)) = 1$

34. $f\left(f^{-1}(6)\right) = 6$

35. From the table, $f(6) = 1$, so $f^{-1}(1) = 6$. Also, $f(2) = 6$, so $f^{-1}(6) = 1$. Thus, $f^{-1}\left(f^{-1}(1)\right) = f^{-1}(6) = 1$.

36. From the table, $f(5) = 0$, so $f^{-1}(0) = 5$. Also, $f(4) = 5$, so $f^{-1}(5) = 4$. Thus, $f^{-1}\left(f^{-1}(0)\right) = f^{-1}(5) = 4$.

37. $f(g(x)) = f(x+6) = (x+6) - 6 = x$ for all x.
$g(f(x)) = g(x-6) = (x-6) + 6 = x$ for all x. Thus f and g are inverses of each other.

38. $f(g(x)) = f\left(\dfrac{x}{3}\right) = 3\left(\dfrac{x}{3}\right) = x$ for all x.
$g(f(x)) = g(3x) = \dfrac{3x}{3} = x$ for all x. Thus f and g are inverses of each other.

39. $f(g(x)) = f\left(\dfrac{x-4}{3}\right) = 3\left(\dfrac{x-4}{3}\right) + 4 = x - 4 + 4 = x$ for all x.

$g(f(x)) = g(3x+4) = \dfrac{(3x+4)-4}{3} = x$ for all x. Thus f and g are inverses of each other.

40. $f(g(x)) = f\left(\dfrac{2-x}{5}\right) = 2 - 5\left(\dfrac{2-x}{5}\right) = 2 - (2-x) = x$ for all x.

$g(f(x)) = g(2-5x) = \dfrac{2-(2-5x)}{5} = \dfrac{5x}{5} = x$ for all x. Thus f and g are inverses of each other.

41. $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$ for all $x \neq 0$. Since $f(x) = g(x)$, we also have $g(f(x)) = x$ for all $x \neq 0$. Thus f and g are inverses of each other.

42. $f(g(x)) = f\left(\sqrt[5]{x}\right) = \left(\sqrt[5]{x}\right)^5 = x$ for all x.

$g(f(x)) = g\left(x^5\right) = \sqrt[5]{x^5} = x$ for all x. Thus f and g are inverses of each other.

43. $f(g(x)) = f\left(\sqrt{x+9}\right) = \left(\sqrt{x+9}\right)^2 - 9 = x + 9 - 9 = x$ for all $x \geq -9$.

$g(f(x)) = g\left(x^2 - 9\right) = \sqrt{\left(x^2-9\right)+9} = \sqrt{x^2} = x$ for all $x \geq 0$. Thus f and g are inverses of each other.

44. $f(g(x)) = f\left((x-1)^{1/3}\right) = \left((x-1)^{1/3}\right)^3 + 1 = x - 1 + 1 = x$ for all x.

$g(f(x)) = g\left(x^3+1\right) = \left[(x-1)^{1/3}\right]^3 + 1 = x - 1 + 1 = x$ for all x. Thus f and g are inverses of each other.

45. $f(g(x)) = f\left(\dfrac{1}{x}+1\right) = \dfrac{1}{\left(\dfrac{1}{x}+1\right)-1} = x$ for all $x \neq 0$.

$g(f(x)) = g\left(\dfrac{1}{x-1}\right) = \dfrac{1}{\left(\dfrac{1}{x-1}\right)} + 1 = (x-1) + 1 = x$ for all $x \neq 1$. Thus f and g are inverses of each other.

46. $f(g(x)) = f\left(\sqrt{4-x^2}\right) = \sqrt{4 - \left(\sqrt{4-x^2}\right)^2} = \sqrt{4-4+x^2} = \sqrt{x^2} = x$, for all $0 \leq x \leq 2$. (Note that the last equality is possible since $x \geq 0$.)

$g(f(x)) = g\left(\sqrt{4-x^2}\right) = \sqrt{4 - \left(\sqrt{4-x^2}\right)^2} = \sqrt{4-4+x^2} = \sqrt{x^2} = x$, for all $0 \leq x \leq 2$. (Again, the last equality is possible since $x \geq 0$.) Thus f and g are inverses of each other.

47. $f(g(x)) = f\left(\dfrac{2x+2}{x-1}\right) = \dfrac{\frac{2x+2}{x-1}+2}{\frac{2x+2}{x-1}-2} = \dfrac{2x+2+2(x-1)}{2x+2-2(x-1)} = \dfrac{4x}{4} = x$ for all $x \neq 1$.

$g(f(x)) = g\left(\dfrac{x+2}{x-2}\right) = \dfrac{2\left(\frac{x+2}{x-2}\right)+2}{\frac{x+2}{x-2}-1} = \dfrac{2(x+2)+2(x-2)}{x+2-1(x-2)} = \dfrac{4x}{4} = x$ for all $x \neq 2$. Thus f and g are inverses of each other.

48. $f(g(x)) = f\left(\dfrac{5+4x}{1-3x}\right) = \dfrac{\frac{5+4x}{1-3x}-5}{3\left(\frac{5+4x}{1-3x}\right)+4} = \dfrac{5+4x-5(1-3x)}{3(5+4x)+4(1-3x)} = \dfrac{19x}{19} = x$ for all $x \neq \frac{1}{3}$.

$g(f(x)) = g\left(\dfrac{x-5}{3x+4}\right) = \dfrac{5+4\left(\frac{x-5}{3x+4}\right)}{1-3\left(\frac{x-5}{3x+4}\right)} = \dfrac{5(3x+4)+4(x-5)}{3x+4-3(x-5)} = \dfrac{19x}{19}$ for all $x \neq -\frac{4}{3}$. Thus f and g are inverses of each other.

49. $f(x) = 3x+5$. $y = 3x+5 \Leftrightarrow 3x = y-5 \Leftrightarrow x = \frac{1}{3}(y-5) = \frac{1}{3}y - \frac{5}{3}$. So $f^{-1}(x) = \frac{1}{3}x - \frac{5}{3}$.

50. $f(x) = 7-5x$. $y = 7-5x \Leftrightarrow 5x = 7-y \Leftrightarrow x = \frac{1}{5}(7-y) = -\frac{1}{5}y + \frac{7}{5}$. So $f^{-1}(x) = -\frac{1}{5}x + \frac{7}{5}$.

51. $f(x) = 5 - 4x^3$. $y = 5 - 4x^3 \Leftrightarrow 4x^3 = 5 - y \Leftrightarrow x^3 = \frac{1}{4}(5 - y) \Leftrightarrow x = \sqrt[3]{\frac{1}{4}(5 - y)}$. So $f^{-1}(x) = \sqrt[3]{\frac{1}{4}(5 - x)}$.

52. $f(x) = 3x^3 + 8$. $y = 3x^3 + 8 \Leftrightarrow 3x^3 = y - 8 \Leftrightarrow x^3 = \frac{1}{3}y - \frac{8}{3} \Leftrightarrow x = \sqrt[3]{\frac{1}{3}y - \frac{8}{3}}$. So $f^{-1}(x) = \sqrt[3]{\frac{1}{3}(x - 8)}$.

53. $f(x) = \dfrac{1}{x+2}$. $y = \dfrac{1}{x+2} \Leftrightarrow x + 2 = \dfrac{1}{y} \Leftrightarrow x = \dfrac{1}{y} - 2$. So $f^{-1}(x) = \dfrac{1}{x} - 2$.

54. $f(x) = \dfrac{x-2}{x+2}$. $y = \dfrac{x-2}{x+2} \Leftrightarrow y(x+2) = x - 2 \Leftrightarrow xy + 2y = x - 2 \Leftrightarrow xy - x = -2 - 2y \Leftrightarrow x(y-1) = -2(y+1)$ $\Leftrightarrow x = \dfrac{-2(y+1)}{y-1}$. So $f^{-1}(x) = \dfrac{-2(x+1)}{x-1}$.

55. $f(x) = \dfrac{x}{x+4}$. $y = \dfrac{x}{x+4} \Leftrightarrow y(x+4) = x \Leftrightarrow xy + 4y = x \Leftrightarrow x - xy = 4y \Leftrightarrow x(1-y) = 4y \Leftrightarrow x = \dfrac{4y}{1-y}$. So $f^{-1}(x) = \dfrac{4x}{1-x}$.

56. $f(x) = \dfrac{3x}{x-2}$. $y = \dfrac{3x}{x-2} \Leftrightarrow y(x-2) = 3x \Leftrightarrow xy - 2y = 3x \Leftrightarrow xy - 3x = 2y \Leftrightarrow x(y-3) = 2y \Leftrightarrow x = \dfrac{2y}{y-3}$. So $f^{-1}(x) = \dfrac{2x}{x-3}$.

57. $f(x) = \dfrac{2x+5}{x-7}$. $y = \dfrac{2x+5}{x-7} \Leftrightarrow y(x-7) = 2x + 5 \Leftrightarrow xy - 7y = 2x + 5 \Leftrightarrow xy - 2x = 7y + 5 \Leftrightarrow x(y-2) = 7y + 5$ $\Leftrightarrow x = \dfrac{7y+5}{y-2}$. So $f^{-1}(x) = \dfrac{7x+5}{x-2}$.

58. $f(x) = \dfrac{4x-2}{3x+1}$. $y = \dfrac{4x-2}{3x+1} \Leftrightarrow y(3x+1) = 4x - 2 \Leftrightarrow 3xy + y = 4x - 2 \Leftrightarrow 4x - 3xy = y + 2 \Leftrightarrow x(4-3y) = y + 2$ $\Leftrightarrow x = \dfrac{y+2}{4-3y}$. So $f^{-1}(x) = \dfrac{x+2}{4-3x}$.

59. $f(x) = \dfrac{2x+3}{1-5x}$. $y = \dfrac{2x+3}{1-5x} \Leftrightarrow y(1-5x) = 2x + 3 \Leftrightarrow y - 5xy = 2x + 3 \Leftrightarrow 2x + 5xy = y - 3 \Leftrightarrow x(2+5y) = y - 3$ $\Leftrightarrow x = \dfrac{y-3}{5y+2}$. So $f^{-1}(x) = \dfrac{x-3}{5x+2}$.

60. $f(x) = \dfrac{3-4x}{8x-1}$. $y = \dfrac{3-4x}{8x-1} \Leftrightarrow y(8x-1) = 3 - 4x \Leftrightarrow 8xy - y = 3 - 4x \Leftrightarrow 4x(2y+1) = y + 3 \Leftrightarrow x = \dfrac{y+3}{4(2y+1)}$. So $f^{-1}(x) = \dfrac{x+3}{4(2x+1)}$.

61. $f(x) = 4 - x^2$, $x \ge 0$. $y = 4 - x^2 \Leftrightarrow x^2 = 4 - y \Leftrightarrow x = \sqrt{4-y}$. So $f^{-1}(x) = \sqrt{4-x}$, $x \le 4$. [Note that $x \ge 0 \Rightarrow f(x) \le 4$.]

62. $f(x) = x^2 + x = \left(x^2 + x + \frac{1}{4}\right) - \frac{1}{4} = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$, $x \ge -\dfrac{1}{2}$. $y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} \Leftrightarrow \left(x + \frac{1}{2}\right)^2 = y + \frac{1}{4} \Leftrightarrow$ $x + \frac{1}{2} = \sqrt{y + \frac{1}{4}} \Leftrightarrow x = \sqrt{y + \frac{1}{4}} - \frac{1}{2}$, $y \ge -\frac{1}{4}$. So $f^{-1}(x) = \sqrt{x + \frac{1}{4}} - \frac{1}{2}$, $x \ge -\frac{1}{4}$. (Note that $x \ge -\frac{1}{2}$, so that $x + \frac{1}{2} \ge 0$, and hence $\left(x + \frac{1}{2}\right)^2 = y + \frac{1}{4} \Leftrightarrow x + \frac{1}{2} = \sqrt{y + \frac{1}{4}}$. Also, since $x \ge -\frac{1}{2}$, $y = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4} \ge -\frac{1}{4}$ so that $y + \frac{1}{4} \ge 0$, and hence $\sqrt{y + \frac{1}{4}}$ is defined.)

63. $f(x) = x^6$, $x \ge 0$. $y = x^6 \Leftrightarrow x = \sqrt[6]{y}$ for $x \ge 0$. The range of f is $\{y \mid y \ge 0\}$, so $f^{-1}(x) = \sqrt[6]{x}$, $x \ge 0$.

64. $f(x) = \dfrac{1}{x^2}$, $x > 0$. $y = \dfrac{1}{x^2} \Leftrightarrow x^2 = \dfrac{1}{y} \Leftrightarrow x = \dfrac{1}{\sqrt{y}}$. The range of f is $\{y \mid y > 0\}$, so $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$, $x > 0$.

65. $f(x) = \dfrac{2-x^3}{5}$. $y = \dfrac{2-x^3}{5} \Leftrightarrow 5y = 2 - x^3 \Leftrightarrow x^3 = 2 - 5y \Leftrightarrow x = \sqrt[3]{2-5y}$. Thus, $f^{-1}(x) = \sqrt[3]{2-5x}$.

66. $f(x) = \left(x^5 - 6\right)^7$. $y = \left(x^5 - 6\right)^7 \Leftrightarrow \sqrt[7]{y} = x^5 - 6 \Leftrightarrow x^5 = \sqrt[7]{y} + 6 \Leftrightarrow x = \sqrt[5]{\sqrt[7]{y} + 6}$. Thus, $f^{-1}(x) = \sqrt[5]{\sqrt[7]{x} + 6}$.

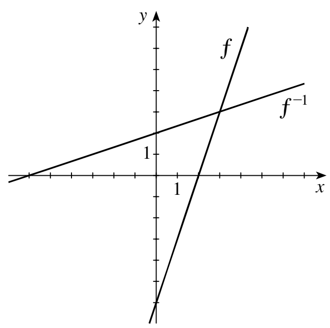
67. $f(x) = \sqrt{5+8x}$. Note that the range of f (and thus the domain of f^{-1}) is $[0, \infty)$. $y = \sqrt{5+8x} \Leftrightarrow y^2 = 5 + 8x \Leftrightarrow$ $8x = y^2 - 5 \Leftrightarrow x = \dfrac{y^2 - 5}{8}$. Thus, $f^{-1}(x) = \dfrac{x^2 - 5}{8}$, $x \geq 0$.

68. $f(x) = 2 + \sqrt{3+x}$. The range of f is $[2, \infty)$. $y = 2 + \sqrt{3+x} \Leftrightarrow y - 2 = \sqrt{3+x} \Leftrightarrow (y-2)^2 = 3 + x \Leftrightarrow$ $x = (y-2)^2 - 3$. Thus, $f^{-1}(x) = (x-2)^2 - 3$, $x \geq 2$.

69. $f(x) = 2 + \sqrt[3]{x}$. $y = 2 + \sqrt[3]{x} \Leftrightarrow y - 2 = \sqrt[3]{x} \Leftrightarrow x = (y-2)^3$. Thus, $f^{-1}(x) = (x-2)^3$.

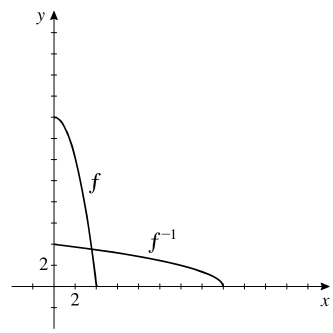
70. $f(x) = \sqrt{4 - x^2}$, $0 \leq x \leq 2$. The range of f is $[0, 2]$. $y = \sqrt{4-x^2} \Leftrightarrow y^2 = 4 - x^2 \Leftrightarrow x^2 = 4 - y^2 \Leftrightarrow x = \sqrt{4-y^2}$. Thus, $f^{-1}(x) = \sqrt{4-x^2}$, $0 \leq x \leq 2$.

71. (a), (b) $f(x) = 3x - 6$



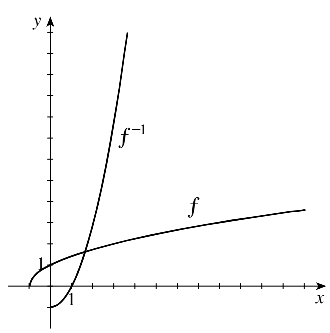
(c) $f(x) = 3x - 6$. $y = 3x - 6 \Leftrightarrow 3x = y + 6 \Leftrightarrow$ $x = \frac{1}{3}(y+6)$. So $f^{-1}(x) = \frac{1}{3}(x+6)$.

72. (a), (b) $f(x) = 16 - x^2$, $x \geq 0$



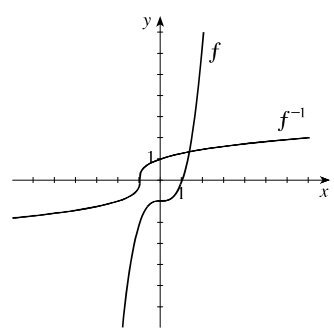
(c) $f(x) = 16 - x^2$, $x \geq 0$. $y = 16 - x^2 \Leftrightarrow$ $x^2 = 16 - y \Leftrightarrow x = \sqrt{16-y}$. So $f^{-1}(x) = \sqrt{16-x}$, $x \leq 16$. (*Note:* $x \geq 0 \Rightarrow$ $f(x) = 16 - x^2 \leq 16$.)

73. (a), (b) $f(x) = \sqrt{x+1}$



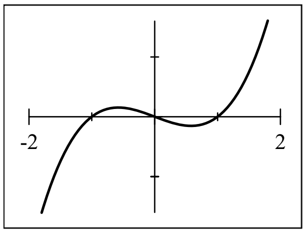
(c) $f(x) = \sqrt{x+1}$, $x \geq -1$. $y = \sqrt{x+1}$, $y \geq 0$ $\Leftrightarrow y^2 = x + 1 \Leftrightarrow x = y^2 - 1$ and $y \geq 0$. So $f^{-1}(x) = x^2 - 1$, $x \geq 0$.

74. (a), (b) $f(x) = x^3 - 1$

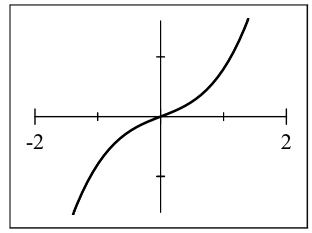


(c) $f(x) = x^3 - 1 \Leftrightarrow y = x^3 - 1 \Leftrightarrow x^3 = y + 1$ $\Leftrightarrow x = \sqrt[3]{y+1}$. So $f^{-1}(x) = \sqrt[3]{x+1}$.

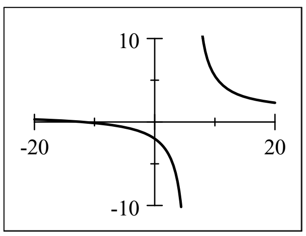
75. $f(x) = x^3 - x$. Using a graphing device and the Horizontal Line Test, we see that f is not a one-to-one function. For example, $f(0) = 0 = f(-1)$.



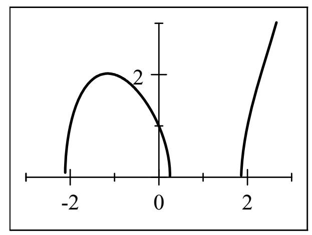
76. $f(x) = x^3 + x$. Using a graphing device and the Horizontal Line Test, we see that f is a one-to-one function.



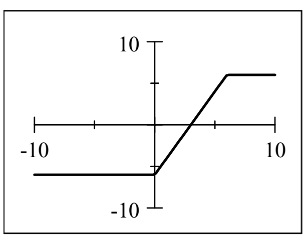
77. $f(x) = \dfrac{x+12}{x-6}$. Using a graphing device and the Horizontal Line Test, we see that f is a one-to-one function.



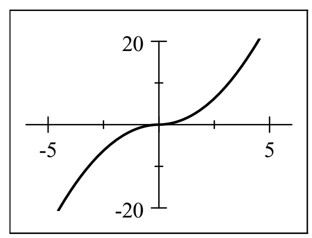
78. $f(x) = \sqrt{x^3 - 4x + 1}$. Using a graphing device and the Horizontal Line Test, we see that f is not a one-to-one function. For example, $f(0) = 1 = f(2)$.



79. $f(x) = |x| - |x-6|$. Using a graphing device and the Horizontal Line Test, we see that f is not a one-to-one function. For example $f(0) = -6 = f(-2)$.

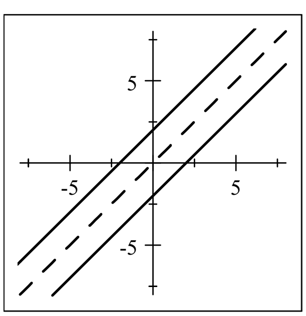


80. $f(x) = x \cdot |x|$. Using a graphing device and the Horizontal Line Test, we see that f is a one-to-one function.



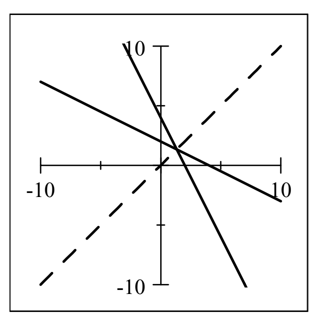
81. **(a)** $y = f(x) = 2 + x \Leftrightarrow x = y - 2$. So $f^{-1}(x) = x - 2$.

(b)



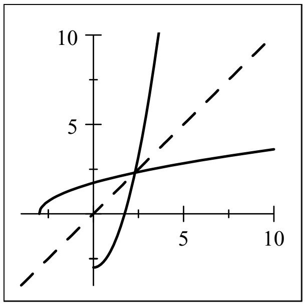
82. **(a)** $y = f(x) = 2 - \frac{1}{2}x \Leftrightarrow \frac{1}{2}x = 2 - y \Leftrightarrow x = 4 - 2y$. So $f^{-1}(x) = 4 - 2x$.

(b)



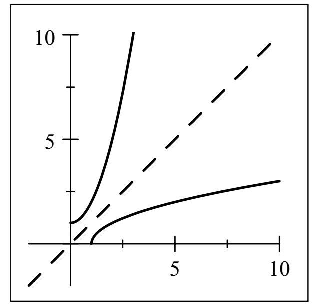
83. **(a)** $y = g(x) = \sqrt{x+3}, y \ge 0 \Leftrightarrow x + 3 = y^2, y \ge 0 \Leftrightarrow x = y^2 - 3, y \ge 0$. So $g^{-1}(x) = x^2 - 3, x \ge 0$.

(b)



84. **(a)** $y = g(x) = x^2 + 1, x \ge 0 \Leftrightarrow x^2 = y - 1, x \ge 0 \Leftrightarrow x = \sqrt{y-1}$. So $g^{-1}(x) = \sqrt{x-1}$.

(b)



85. If we restrict the domain of $f(x)$ to $[0, \infty)$, then $y = 4 - x^2 \Leftrightarrow x^2 = 4 - y \Rightarrow x = \sqrt{4-y}$ (since $x \ge 0$, we take the positive square root). So $f^{-1}(x) = \sqrt{4-x}$.

If we restrict the domain of $f(x)$ to $(-\infty, 0]$, then $y = 4 - x^2 \Leftrightarrow x^2 = 4 - y \Rightarrow x = -\sqrt{4-y}$ (since $x \le 0$, we take the negative square root). So $f^{-1}(x) = -\sqrt{4-x}$.

86. If we restrict the domain of $g(x)$ to $[1, \infty)$, then $y = (x-1)^2 \Rightarrow x - 1 = \sqrt{y}$ (since $x \ge 1$ we take the positive square root) $\Leftrightarrow x = 1 + \sqrt{y}$. So $g^{-1}(x) = 1 + \sqrt{x}$.

If we restrict the domain of $g(x)$ to $(-\infty, 1]$, then $y = (x-1)^2 \Rightarrow x - 1 = -\sqrt{y}$ (since $x \le 1$ we take the negative square root) $\Leftrightarrow x = 1 - \sqrt{y}$. So $g^{-1}(x) = 1 - \sqrt{x}$.

87. If we restrict the domain of $h(x)$ to $[-2, \infty)$, then $y = (x+2)^2 \Rightarrow x + 2 = \sqrt{y}$ (since $x \ge -2$, we take the positive square root) $\Leftrightarrow x = -2 + \sqrt{y}$. So $h^{-1}(x) = -2 + \sqrt{x}$.

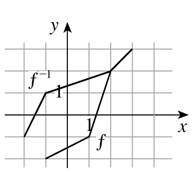
If we restrict the domain of $h(x)$ to $(-\infty, -2]$, then $y = (x+2)^2 \Rightarrow x + 2 = -\sqrt{y}$ (since $x \le -2$, we take the negative square root) $\Leftrightarrow x = -2 - \sqrt{y}$. So $h^{-1}(x) = -2 - \sqrt{x}$.

88. $k(x) = |x - 3| = \begin{cases} -(x-3) & \text{if } x - 3 < 0 \Leftrightarrow x < 3 \\ x - 3 & \text{if } x - 3 \ge 0 \Leftrightarrow x \ge 3 \end{cases}$

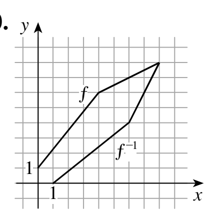
If we restrict the domain of $k(x)$ to $[3, \infty)$, then $y = x - 3 \Leftrightarrow x = 3 + y$. So $k^{-1}(x) = 3 + x$.

If we restrict the domain of $k(x)$ to $(-\infty, 3]$, then $y = -(x-3) \Leftrightarrow y = -x + 3 \Leftrightarrow x = 3 - y$. So $k^{-1}(x) = 3 - x$.

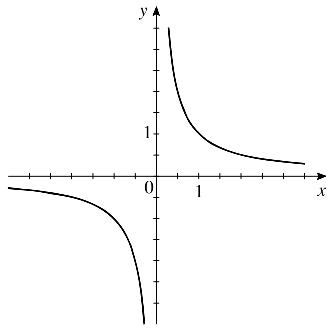
89.



90.



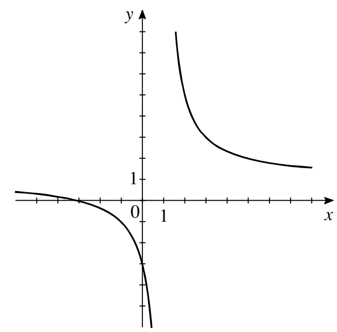
91. (a)



(b) Yes, the graph is unchanged upon reflection about the line $y = x$.

(c) $y = \frac{1}{x} \Leftrightarrow x = \frac{1}{y}$, so $f^{-1}(x) = \frac{1}{x}$.

92. (a)



(b) Yes, the graph is unchanged upon reflection about the line $y = x$.

(c) $y = \frac{x+3}{x-1} \Leftrightarrow y(x-1) = x+3 \Leftrightarrow$

$x(y-1) = y+3 \Leftrightarrow x = \frac{y+3}{y-1}$. Thus,

$f^{-1}(x) = \frac{x+3}{x-1}$.

93. (a) The price of a pizza with no toppings (corresponding to the y-intercept) is \$16, and the cost of each additional topping (the rate of change of cost with respect to number of toppings) is \$1.50. Thus, $f(n) = 16 + 1.5n$.

(b) $p = f(n) = 16 + 1.5n \Leftrightarrow p - 16 = 1.5n \Leftrightarrow n = \frac{2}{3}(p-16)$. Thus, $n = f^{-1}(p) = \frac{2}{3}(p-16)$. This function represents the number of toppings on a pizza that costs x dollars.

(c) $f^{-1}(25) = \frac{2}{3}(25-16) = \frac{2}{3}(9) = 6$. Thus, a \$25 pizza has 6 toppings.

94. (a) $f(x) = 500 + 80x$.

(b) $p = f(x) = 500 + 80x$. $p = 500 + 80x \Leftrightarrow 80x = p - 500 \Leftrightarrow x = \frac{p-500}{80}$. So $x = f^{-1}(p) = \frac{p-500}{80}$. f^{-1} represents the number of hours the investigator spends on a case for x dollars.

(c) $f^{-1}(1220) = \frac{1220-500}{80} = \frac{720}{80} = 9$. If the investigator charges \$1220, he spent 9 hours investigating the case.

95. (a) $V = f(t) = 100\left(1 - \frac{t}{40}\right)^2$, $0 \le t \le 40$. $V = 100\left(1 - \frac{t}{40}\right)^2 \Leftrightarrow \frac{V}{100} = \left(1 - \frac{t}{40}\right)^2 \Rightarrow 1 - \frac{t}{40} = \pm\sqrt{\frac{V}{100}} \Leftrightarrow$ $\frac{t}{40} = 1 \pm \frac{\sqrt{V}}{10} \Leftrightarrow t = 40 \pm 4\sqrt{V}$. Since $t \le 40$, we must have $t = f^{-1}(V) = 40 - 4\sqrt{V}$. f^{-1} represents time that has elapsed since the tank started to leak.

(b) $f^{-1}(15) = 40 - 4\sqrt{15} \approx 24.5$ minutes. In 24.5 minutes the tank has drained to just 15 liters of water.

96. (a) $v = g(r) = 18{,}500\left(0.25 - r^2\right)$. $v = 18{,}500\left(0.25 - r^2\right) \Leftrightarrow v = 4625 - 18{,}500r^2 \Leftrightarrow 18{,}500r^2 = 4625 - v \Leftrightarrow$ $r^2 = \frac{4625 - v}{18{,}500} \Rightarrow r = \pm\sqrt{\frac{4625 - v}{18{,}500}}$. Since r represents a distance, $r \ge 0$, so $g^{-1}(v) = \sqrt{\frac{4625 - v}{18{,}500}}$. $g^{-1}(v)$ represents the radial distance from the center of the vein at which the blood has velocity v.

(b) $g^{-1}(30) = \sqrt{\frac{4625 - 30}{18{,}500}} \approx 0.498$ cm. The velocity is 30 cm/s at a distance of 0.498 cm from the center of the artery or vein.

97. **(a)** $D = f(p) = -3p + 150$. $D = -3p + 150 \Leftrightarrow 3p = 150 - D \Leftrightarrow p = 50 - \frac{1}{3}D$. So $f^{-1}(D) = 50 - \frac{1}{3}D$. $f^{-1}(D)$ represents the price that is associated with demand D.

(b) $f^{-1}(30) = 50 - \frac{1}{3}(30) = 40$. So when the demand is 30 units, the price per unit is \$40.

98. **(a)** $f^{-1}(U) = 1.02396U$.

(b) $U = f(x) = 0.9766x$. $U = 0.9766x \Leftrightarrow x = 1.0240U$. So $f^{-1}(U) = 1.0240U$. $f^{-1}(U)$ represents the value of U US dollars in Canadian dollars.

(c) $f^{-1}(12{,}250) = 1.0240(12{,}250) = 12{,}543.52$. So \$12,250 in US currency is worth \$12,543.52 in Canadian currency.

99. **(a)** $f(x) = \begin{cases} 0.1x, & \text{if } 0 \le x \le 20{,}000 \\ 2000 + 0.2(x - 20{,}000) & \text{if } x > 20{,}000 \end{cases}$

(b) We will find the inverse of each piece of the function f.

$f_1(x) = 0.1x$. $T = 0.1x \Leftrightarrow x = 10T$. So $f_1^{-1}(T) = 10T$.

$f_2(x) = 2000 + 0.2(x - 20{,}000) = 0.2x - 2000$. $T = 0.2x - 2000 \Leftrightarrow 0.2x = T + 2000 \Leftrightarrow x = 5T + 10{,}000$. So $f_2^{-1}(T) = 5T + 10{,}000$.

Since $f(0) = 0$ and $f(20{,}000) = 2000$ we have $f^{-1}(T) = \begin{cases} 10T, & \text{if } 0 \le T \le 2000 \\ 5T + 10{,}000 & \text{if } T > 2000 \end{cases}$ This represents the taxpayer's income.

(c) $f^{-1}(10{,}000) = 5(10{,}000) + 10{,}000 = 60{,}000$. The required income is €60,000.

100. **(a)** $f(x) = 0.85x$.

(b) $g(x) = x - 1000$.

(c) $H(x) = (f \circ g)x = f(x - 1000) = 0.85(x - 1000) = 0.85x - 850$.

(d) $P = H(x) = 0.85x - 850$. $P = 0.85x - 850 \Leftrightarrow 0.85x = P + 850 \Leftrightarrow x = 1.176P + 1000$. So $H^{-1}(P) = 1.176P + 1000$. The function H^{-1} represents the original sticker price for a given discounted price P.

(e) $H^{-1}(13{,}000) = 1.176(13{,}000) + 1000 = 16{,}288$. So the original price of the car is \$16,288 when the discounted price (\$1000 rebate, then 15% off) is \$13,000.

101. $f(x) = mx + b$. Notice that $f(x_1) = f(x_2) \Leftrightarrow mx_1 + b = mx_2 + b \Leftrightarrow mx_1 = mx_2$. We can conclude that $x_1 = x_2$ if and only if $m \neq 0$. Therefore f is one-to-one if and only if $m \neq 0$. If $m \neq 0$, $f(x) = mx + b \Leftrightarrow y = mx + b \Leftrightarrow mx = y - b \Leftrightarrow x = \dfrac{y - b}{m}$. So, $f^{-1}(x) = \dfrac{x - b}{m}$.

102. **(a)** $f(x) = \dfrac{2x + 1}{5}$ is "multiply by 2, add 1, and then divide by 5 ". So the reverse is "multiply by 5, subtract 1, and then divide by 2 " or $f^{-1}(x) = \dfrac{5x - 1}{2}$. Check: $f \circ f^{-1}(x) = f\left(\dfrac{5x - 1}{2}\right) = \dfrac{2\left(\dfrac{5x - 1}{2}\right) + 1}{5} = \dfrac{5x - 1 + 1}{5} = \dfrac{5x}{5} = x$ and $f^{-1} \circ f(x) = f^{-1}\left(\dfrac{2x + 1}{5}\right) = \dfrac{5\left(\dfrac{2x + 1}{5}\right) - 1}{2} = \dfrac{2x + 1 - 1}{2} = \dfrac{2x}{2} = x$.

(b) $f(x) = 3 - \dfrac{1}{x} = \dfrac{-1}{x} + 3$ is "take the negative reciprocal and add 3 ". Since the reverse of "take the negative reciprocal" is "take the negative reciprocal ", $f^{-1}(x)$ is "subtract 3 and take the negative reciprocal ", that is, $f^{-1}(x) = \dfrac{-1}{x-3}$. Check: $f \circ f^{-1}(x) = f\left(\dfrac{-1}{x-3}\right) = 3 - \dfrac{1}{\dfrac{-1}{x-3}} = 3 - \left(1 \cdot \dfrac{x-3}{-1}\right) = 3 + x - 3 = x$ and

$f^{-1} \circ f(x) = f^{-1}\left(3 - \dfrac{1}{x}\right) = \dfrac{-1}{\left(3 - \dfrac{1}{x}\right) - 3} = \dfrac{-1}{-\dfrac{1}{x}} = -1 \cdot \dfrac{x}{-1} = x$.

(c) $f(x) = \sqrt{x^3 + 2}$ is "cube, add 2, and then take the square root". So the reverse is "square, subtract 2, then take the cube root " or $f^{-1}(x) = \sqrt[3]{x^2 - 2}$. Domain for $f(x)$ is $\left[-\sqrt[3]{2}, \infty\right)$; domain for $f^{-1}(x)$ is $[0, \infty)$. Check:

$f \circ f^{-1}(x) = f\left(\sqrt[3]{x^2-2}\right) = \sqrt{\left(\sqrt[3]{x^2-2}\right)^3 + 2} = \sqrt{x^2 - 2 + 2} = \sqrt{x^2} = x$ (on the appropriate domain) and

$f^{-1} \circ f(x) = f^{-1}\left(\sqrt{x^3+2}\right) = \sqrt[3]{\left(\sqrt{x^3+2}\right)^2 - 2} = \sqrt[3]{x^3 + 2 - 2} = \sqrt[3]{x^3} = x$ (on the appropriate domain).

(d) $f(x) = (2x-5)^3$ is "double, subtract 5, and then cube". So the reverse is "take the cube root, add 5, and divide by 2" or $f^{-1}(x) = \dfrac{\sqrt[3]{x}+5}{2}$ Domain for both $f(x)$ and $f^{-1}(x)$ is $(-\infty, \infty)$. Check:

$f \circ f^{-1}(x) = f\left(\dfrac{\sqrt[3]{x}+5}{2}\right) = \left[2\left(\dfrac{\sqrt[3]{x}+5}{2}\right) - 5\right]^3 = \left(\sqrt[3]{x} + 5 - 5\right)^3 = \left(\sqrt[3]{x}\right)^3 = \sqrt[3]{x^3} = x$ and

$f^{-1} \circ f(x) = f^{-1}\left((2x-5)^3\right) = \dfrac{\sqrt[x]{(2x-5)^3} + 5}{2} = \dfrac{(2x-5)+5}{2} = \dfrac{2x}{2} = x$.

In a function like $f(x) = 3x - 2$, the variable occurs only once and it easy to see how to reverse the operations step by step. But in $f(x) = x^3 + 2x + 6$, you apply two different operations to the variable x (cubing and multiplying by 2) and then add 6, so it is not possible to reverse the operations step by step.

103. $f(I(x)) = f(x)$; therefore $f \circ I = f$. $I(f(x)) = f(x)$; therefore $I \circ f = f$.
By definition, $f \circ f^{-1}(x) = x = I(x)$; therefore $f \circ f^{-1} = I$. Similarly, $f^{-1} \circ f(x) = x = I(x)$; therefore $f^{-1} \circ f = I$.

104. (a) We find $g^{-1}(x)$: $y = 2x + 1 \Leftrightarrow 2x = y - 1 \Leftrightarrow x = \frac{1}{2}(y-1)$. So $g^{-1}(x) = \frac{1}{2}(x-1)$. Thus $f(x) = h \circ g^{-1}(x) = h\left(\frac{1}{2}(x-1)\right) = 4\left[\frac{1}{2}(x-1)\right]^2 + 4\left[\frac{1}{2}(x-1)\right] + 7 = x^2 - 2x + 1 + 2x - 2 + 7 = x^2 + 6$.

(b) $f \circ g = h \Leftrightarrow f^{-1} \circ f \circ g = f^{-1} \circ h \Leftrightarrow I \circ g = f^{-1} \circ h \Leftrightarrow g = f^{-1} \circ h$. Note that we compose with f^{-1} on the left on each side of the equation. We find f^{-1}: $y = 3x + 5 \Leftrightarrow 3x = y - 5 \Leftrightarrow x = \frac{1}{3}(y-5)$. So $f^{-1}(x) = \frac{1}{3}(x-5)$. Thus $g(x) = f^{-1} \circ h(x) = f^{-1}\left(3x^2 + 3x + 2\right) = \frac{1}{3}\left[\left(3x^2 + 3x + 2\right) - 5\right] = \frac{1}{3}\left[3x^2 + 3x - 3\right] = x^2 + x - 1$.

CHAPTER 2 REVIEW

1. "Square, then subtract 5" can be represented by the function $f(x) = x^2 - 5$.

2. "Divide by 2, then add 9" can be represented by the function $g(x) = \dfrac{x}{2} + 9$.

3. $f(x) = 3(x + 10)$: "Add 10, then multiply by 3."

4. $f(x) = \sqrt{6x - 10}$: "Multiply by 6, then subtract 10, then take the square root."

5. $g(x) = x^2 - 4x$

x	$g(x)$
-1	5
0	0
1	-3
2	-4
3	-3

6. $h(x) = 3x^2 + 2x - 5$

x	$h(x)$
-2	3
-1	-4
0	-5
1	0
2	11

7. $C(x) = 5000 + 30x - 0.001x^2$

(a) $C(1000) = 5000 + 30(1000) - 0.001(1000)^2 = \$34{,}000$ and $C(10{,}000) = 5000 + 30(10{,}000) - 0.001(10{,}000)^2 = \$205{,}000$.

(b) From part (a), we see that the total cost of printing 1000 copies of the book is \$34,000 and the total cost of printing 10,000 copies is \$205,000.

(c) $C(0) = 5000 + 30(0) - 0.001(0)^2 = \$5000$. This represents the fixed costs associated with getting the print run ready.

(d) The net change in C as x changes from 1000 to 10,000 is $C(10{,}000) - C(1000) = 205{,}000 - 34{,}000 = \$171{,}000$, and the average rate of change is $\dfrac{C(10{,}000) - C(1000)}{10{,}000 - 1000} = \dfrac{171{,}000}{9000} = \$19/\text{copy}$.

8. $E(x) = 400 + 0.03x$

(a) $E(2000) = 400 + 0.03(2000) = \$460$ and $E(15{,}000) = 400 + 0.03(15{,}000) = \$850$.

(b) From part (a), we see that if Reynalda sells \$2000 worth of goods, she makes \$460, and if she sells \$15,000 worth of goods, she makes \$850.

(c) $E(0) = 400 + 0.03(0) = \$400$ is Reynalda's base weekly salary.

(d) The net change in E as x changes from 2000 to 15,000 is $E(15{,}000) - E(2000) = 850 - 460 = \$390$, and the average rate of change is $\dfrac{E(15{,}000) - E(2000)}{15{,}000 - 2000} = \dfrac{390}{13{,}000} = \$0.03$ per dollar.

(e) Because the value of goods sold x is multiplied by 0.03 or 3%, we see that Reynalda earns a percentage of 3% on the goods that she sells.

9. $f(x) = x^2 - 4x + 6$; $f(0) = (0)^2 - 4(0) + 6 = 6$; $f(2) = (2)^2 - 4(2) + 6 = 2$; $f(-2) = (-2)^2 - 4(-2) + 6 = 18$; $f(a) = (a)^2 - 4(a) + 6 = a^2 - 4a + 6$; $f(-a) = (-a)^2 - 4(-a) + 6 = a^2 + 4a + 6$; $f(x+1) = (x+1)^2 - 4(x+1) + 6 = x^2 + 2x + 1 - 4x - 4 + 6 = x^2 - 2x + 3$; $f(2x) = (2x)^2 - 4(2x) + 6 = 4x^2 - 8x + 6$.

10. $f(x) = 4 - \sqrt{3x - 6}$; $f(5) = 4 - \sqrt{15 - 6} = 1$; $f(9) = 4 - \sqrt{27 - 6} = 4 - \sqrt{21}$; $f(a+2) = 4 - \sqrt{3a + 6 - 6} = 4 - \sqrt{3a}$; $f(-x) = 4 - \sqrt{3(-x) - 6} = 4 - \sqrt{-3x - 6}$; $f\left(x^2\right) = 4 - \sqrt{3x^2 - 6}$.

11. By the Vertical Line Test, figures (b) and (c) are graphs of functions. By the Horizontal Line Test, figure (c) is the graph of a one-to-one function.

12. **(a)** $f(-2) = -1$ and $f(2) = 2$.

(b) The net change in f from -2 to 2 is $f(2) - f(-2) = 2 - (-1) = 3$, and the average rate of change is $\dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{3}{4}$.

(c) The domain of f is $[-4, 5]$ and the range of f is $[-4, 4]$.

(d) f is increasing on $(-4, -2)$ and $(-1, 4)$; f is decreasing on $(-2, -1)$ and $(4, 5)$.

(e) f has local maximum values of -1 (at $x = -2$) and 4 (at $x = 4$).

(f) f is not a one-to-one, for example, $f(-2) = -1 = f(0)$. There are many more examples.

13. Domain: We must have $x + 3 \geq 0 \Leftrightarrow x \geq -3$. In interval notation, the domain is $[-3, \infty)$.

Range: For x in the domain of f, we have $x \geq -3 \Leftrightarrow x + 3 \geq 0 \Leftrightarrow \sqrt{x + 3} \geq 0 \Leftrightarrow f(x) \geq 0$. So the range is $[0, \infty)$.

14. $F(t) = t^2 + 2t + 5 = \left(t^2 + 2t + 1\right) + 5 - 1 = (t+1)^2 + 4$. Therefore $F(t) \ge 4$ for all t. Since there are no restrictions on t, the domain of F is $(-\infty, \infty)$, and the range is $[4, \infty)$.

15. $f(x) = 7x + 15$. The domain is all real numbers, $(-\infty, \infty)$.

16. $f(x) = \dfrac{2x+1}{2x-1}$. Then $2x - 1 \ne 0 \Leftrightarrow x \ne \frac{1}{2}$. So the domain of f is $\left\{x \mid x \ne \frac{1}{2}\right\}$.

17. $f(x) = \sqrt{x+4}$. We require $x + 4 \ge 0 \Leftrightarrow x \ge -4$. Thus the domain is $[-4, \infty)$.

18. $f(x) = 3x - \dfrac{2}{\sqrt{x+1}}$. The domain of f is the set of x where $x + 1 > 0 \Leftrightarrow x > -1$. So the domain is $(-1, \infty)$.

19. $f(x) = \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{1}{x+2}$. The denominators cannot equal 0, therefore the domain is $\{x \mid x \ne 0, -1, -2\}$.

20. $g(x) = \dfrac{2x^2 + 5x + 3}{2x^2 - 5x - 3} = \dfrac{2x^2 + 5x + 3}{(2x+1)(x-3)}$. The domain of g is the set of all x where the denominator is not 0. So the domain is $\{x \mid 2x + 1 \ne 0 \text{ and } x - 3 \ne 0\} = \left\{x \mid x \ne -\frac{1}{2} \text{ and } x \ne 3\right\}$.

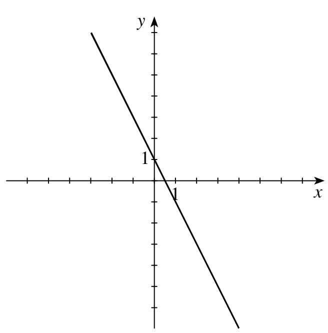
21. $h(x) = \sqrt{4-x} + \sqrt{x^2 - 1}$. We require the expression inside the radicals be nonnegative. So $4 - x \ge 0 \Leftrightarrow 4 \ge x$; also $x^2 - 1 \ge 0 \Leftrightarrow (x-1)(x+1) \ge 0$. We make a table:

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
Sign of $x - 1$	$-$	$-$	$+$
Sign of $x + 1$	$-$	$+$	$+$
Sign of $(x-1)(x+1)$	$+$	$-$	$+$

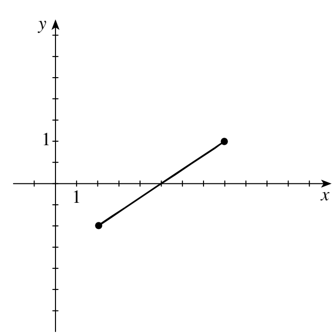
Thus the domain is $(-\infty, 4] \cap \{(-\infty, -1] \cup [1, \infty)\} = (-\infty, -1] \cup [1, 4]$.

22. $f(x) = \dfrac{\sqrt[3]{2x+1}}{\sqrt[3]{2x}+2}$. Since we have an odd root, the domain is the set of all x where the denominator is not 0. Now $\sqrt[3]{2x} + 2 \ne 0 \Leftrightarrow \sqrt[3]{2x} \ne -2 \Leftrightarrow 2x \ne -8 \Leftrightarrow x \ne -4$. Thus the domain of f is $\{x \mid x \ne -4\}$.

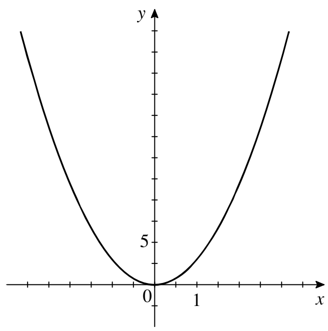
23. $f(x) = 1 - 2x$



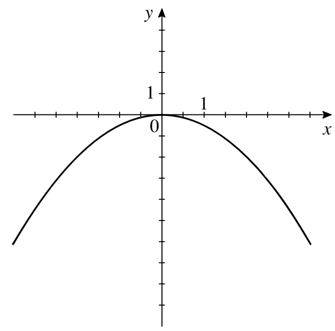
24. $f(x) = \frac{1}{3}(x - 5), 2 \le x \le 8$



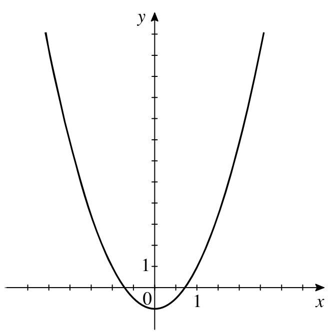
25. $f(x) = 3x^2$



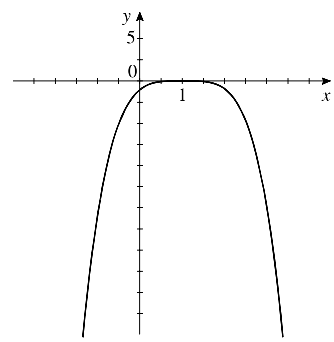
26. $f(x) = -\frac{1}{4}x^2$



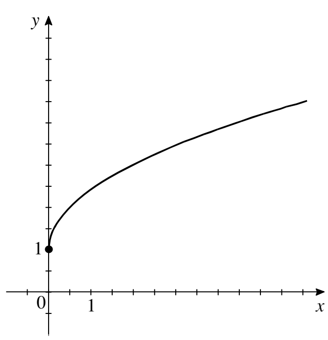
27. $f(x) = 2x^2 - 1$



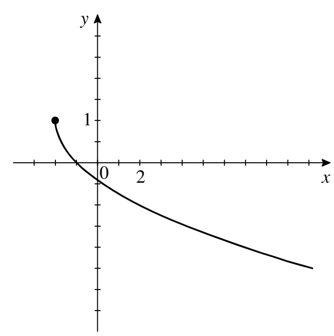
28. $f(x) = -(x-1)^4$



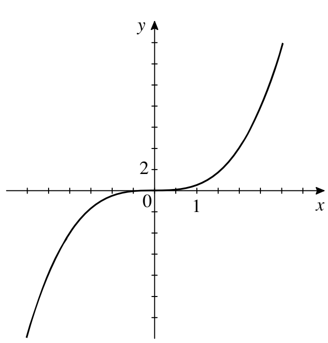
29. $f(x) = 1 + \sqrt{x}$



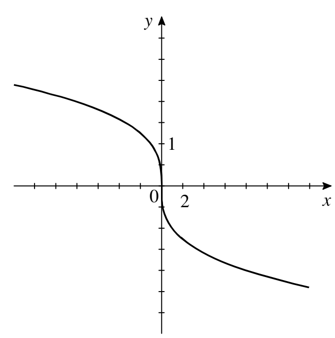
30. $f(x) = 1 - \sqrt{x+2}$



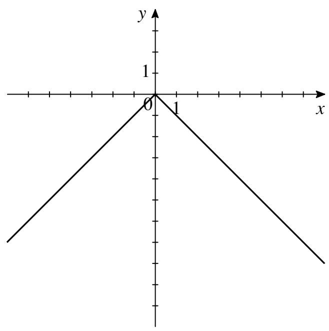
31. $f(x) = \frac{1}{2}x^3$



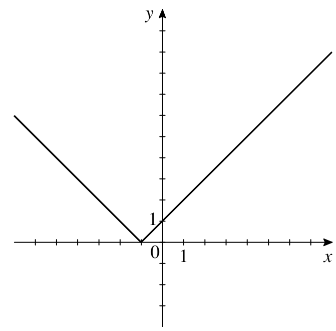
32. $f(x) = \sqrt[3]{-x}$



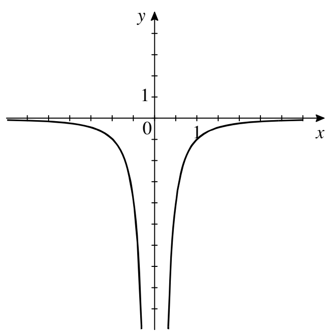
33. $f(x) = -|x|$



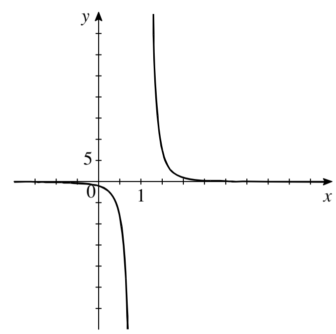
34. $f(x) = |x+1|$



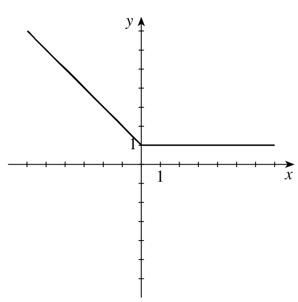
35. $f(x) = -\dfrac{1}{x^2}$



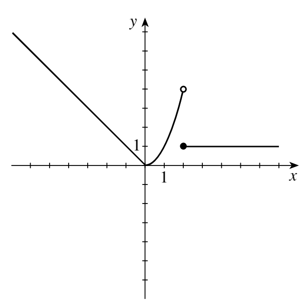
36. $f(x) = \dfrac{1}{(x-1)^3}$



37. $f(x) = \begin{cases} 1-x & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$



38. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$



39. $x + y^2 = 14 \Rightarrow y^2 = 14 - x \Rightarrow y = \pm\sqrt{14-x}$, so the original equation does not define y as a function of x.

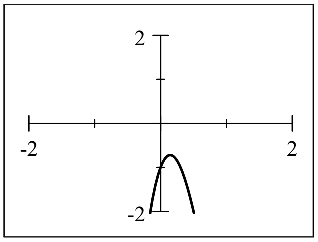
40. $3x - \sqrt{y} = 8 \Rightarrow \sqrt{y} = 3x - 8 \Rightarrow y = (3x-8)^2$, so the original equation defines y as a function of x.

41. $x^3 - y^3 = 27 \Leftrightarrow y^3 = x^3 - 27 \Leftrightarrow y = \left(x^3 - 27\right)^{1/3}$, so the original equation defines y as a function of x (since the cube root function is one-to-one).

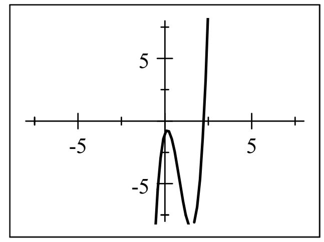
42. $2x = y^4 - 16 \Leftrightarrow y^4 = 2x + 16 \Leftrightarrow y = \pm\sqrt[4]{2x+16}$, so the original equation does not define y as a function of x.

43. $f(x) = 6x^3 - 15x^2 + 4x - 1$

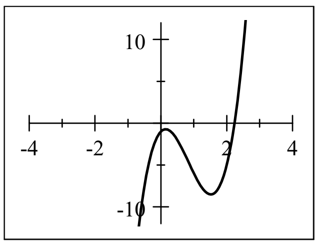
(i) $[-2, 2]$ by $[-2, 2]$



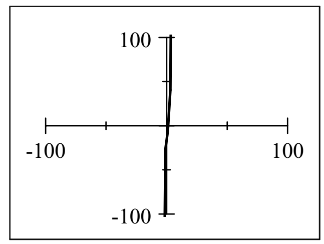
(ii) $[-8, 8]$ by $[-8, 8]$



(iii) $[-4, 4]$ by $[-12, 12]$



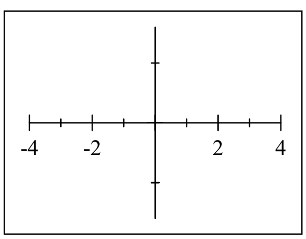
(iv) $[-100, 100]$ by $[-100, 100]$



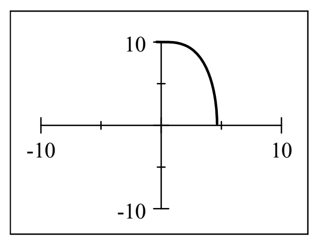
From the graphs, we see that the viewing rectangle in (iii) produces the most appropriate graph.

44. $f(x) = \sqrt{100 - x^3}$

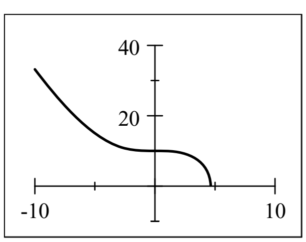
(i) $[-4, 4]$ by $[-4, 4]$



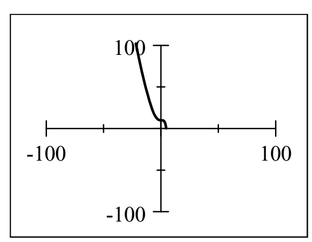
(ii) $[-10, 10]$ by $[-10, 10]$



(iii) $[-10, 10]$ by $[-10, 40]$

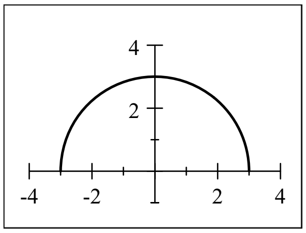


(iv) $[-100, 100]$ by $[-100, 100]$



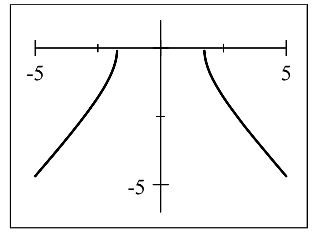
From the graphs, we see that the viewing rectangle in (iii) produces the most appropriate graph of f.

45. **(a)** We graph $f(x) = \sqrt{9 - x^2}$ in the viewing rectangle $[-4, 4]$ by $[-1, 4]$.



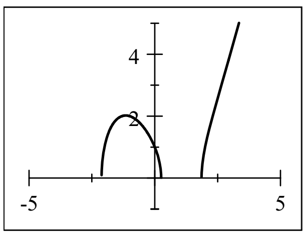
(b) From the graph, the domain of f is $[-3, 3]$ and the range of f is $[0, 3]$.

46. **(a)** We graph $f(x) = -\sqrt{x^2 - 3}$ in the viewing rectangle $[-5, 5]$ by $[-6, 1]$.



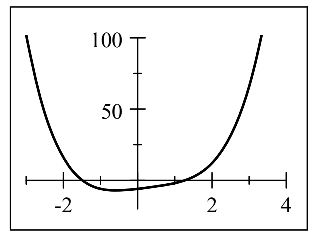
(b) From the graph, the domain of f is $(-\infty, -1.73] \cup [1.73, \infty)$ and the range of f is $(-\infty, 0]$.

47. **(a)** We graph $f(x) = \sqrt{x^3 - 4x + 1}$ in the viewing rectangle $[-5, 5]$ by $[-1, 5]$.



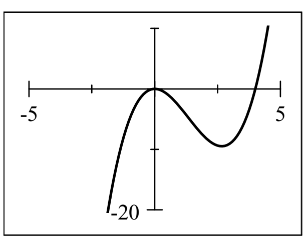
(b) From the graph, the domain of f is approximately $[-2.11, 0.25] \cup [1.86, \infty)$ and the range of f is $[0, \infty)$.

48. **(a)** We graph $f(x) = x^4 - x^3 + x^2 + 3x - 6$ in the viewing rectangle $[-3, 4]$ by $[-20, 100]$.

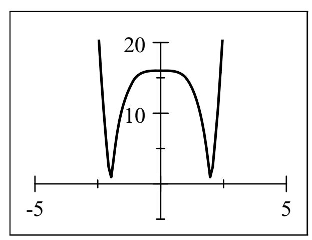


(b) From the graph, the domain of f is $(-\infty, \infty)$ and the range of f is approximately $[-7.10, \infty)$.

49. $f(x) = x^3 - 4x^2$ is graphed in the viewing rectangle $[-5, 5]$ by $[-20, 10]$. $f(x)$ is increasing on $(-\infty, 0)$ and $(2.67, \infty)$. It is decreasing on $(0, 2.67)$.



50. $f(x) = \left|x^4 - 16\right|$ is graphed in the viewing rectangle $[-5, 5]$ by $[-5, 20]$. $f(x)$ is increasing on $(-2, 0)$ and $(2, \infty)$. It is decreasing on $(-\infty, -2)$ and $(0, 2)$.



51. The net change is $f(8) - f(4) = 8 - 12 = -4$ and the average rate of change is $\dfrac{f(8) - f(4)}{8 - 4} = \dfrac{-4}{4} = -1$.

52. The net change is $g(30) - g(10) = 30 - (-5) = 35$ and the average rate of change is $\dfrac{g(30) - g(10)}{30 - 10} = \dfrac{35}{20} = \dfrac{7}{4}$.

53. The net change is $f(2) - f(-1) = 6 - 2 = 4$ and the average rate of change is $\dfrac{f(2) - f(-1)}{2 - (-1)} = \dfrac{4}{3}$.

54. The net change is $f(3) - f(1) = -1 - 5 = -6$ and the average rate of change is $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{-6}{2} = -3$.

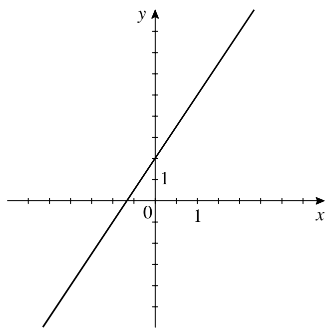
55. The net change is $f(4) - f(1) = \left[4^2 - 2(4)\right] - \left[1^2 - 2(1)\right] = 8 - (-1) = 9$ and the average rate of change is $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{9}{3} = 3$.

56. The net change is $g(a+h) - g(a) = (a+h+1)^2 - (a+1)^2 = 2ah + 2h + h^2$ and the average rate of change is $\dfrac{g(a+h) - g(a)}{a+h-a} = \dfrac{2ah + 2h + h^2}{h} = 2a + 2 + h$.

57. $f(x) = (2+3x)^2 = 9x^2 + 12x + 4$ is not linear. It cannot be expressed in the form $f(x) = ax + b$ with constant a and b.

58. $g(x) = \dfrac{x+3}{5} = \frac{1}{5}x + \frac{3}{5}$ is linear with $a = \frac{1}{5}$ and $b = \frac{3}{5}$.

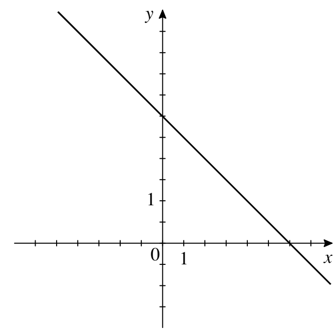
59. **(a)**



(b) The slope of the graph is the value of a in the equation $f(x) = ax + b = 3x + 2$; that is, 3.

(c) The rate of change is the slope of the graph, 3.

60. **(a)**



(b) The slope of the graph is the value of a in the equation $f(x) = ax + b = -\frac{1}{2}x + 3$; that is, $-\frac{1}{2}$.

(c) The rate of change is the slope of the graph, $-\frac{1}{2}$.

61. The linear function with rate of change -2 and initial value 3 has $a = -2$ and $b = 3$, so $f(x) = -2x + 3$.

62. The linear function whose graph has slope $\frac{1}{2}$ and y-intercept -1 has $a = \frac{1}{2}$ and $b = -1$, so $f(x) = \frac{1}{2}x - 1$.

63. Between $x = 0$ and $x = 1$, the rate of change is $\dfrac{f(1) - f(0)}{1 - 0} = \dfrac{5 - 3}{1} = 2$. At $x = 0$, $f(x) = 3$. Thus, an equation is $f(x) = 2x + 3$.

64. Between $x = 0$ and $x = 2$, the rate of change is $\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{5.5 - 6}{2} = -\frac{1}{4}$. At $x = 0$, $f(x) = 6$. Thus, an equation is $f(x) = -\frac{1}{4}x + 6$.

65. The points $(0, 4)$ and $(8, 0)$ lie on the graph, so the rate of change is $\dfrac{0 - 4}{8 - 0} = -\dfrac{1}{2}$. At $x = 0$, $y = 4$. Thus, an equation is $y = -\frac{1}{2}x + 4$.

66. The points $(0, -4)$ and $(2, 0)$ lie on the graph, so the rate of change is $\dfrac{0 - (-4)}{2 - 0} = 2$. At $x = 0$, $y = -4$. Thus, an equation is $y = 2x - 4$.

67. $P(t) = 3000 + 200t + 0.1t^2$

(a) $P(10) = 3000 + 200(10) + 0.1(10)^2 = 5010$ represents the population in its 10th year (that is, in 1995), and $P(20) = 3000 + 200(20) + 0.1(20)^2 = 7040$ represents its population in its 20th year (in 2005).

(b) The average rate of change is $\dfrac{P(20) - P(10)}{20 - 10} = \dfrac{7040 - 5010}{10} = \dfrac{2030}{10} = 203$ people/year. This represents the average yearly change in population between 1995 and 2005.

68. $D(t) = 3500 + 15t^2$

(a) $D(0) = 3500 + 15(0)^2 = \$3500$ represents the amount deposited in 1995 and $D(15) = 3500 + 15(15)^2 = \$6875$ represents the amount deposited in 2010.

(b) Solving the equation $D(t) = 17{,}000$, we get $17{,}000 = 3500 + 15t^2 \Leftrightarrow 15t^2 = 13{,}500 \Leftrightarrow t^2 = \dfrac{13500}{15} = 900 \Leftrightarrow$ $t = 30$, so thirty years after 1995 (that is, in the year 2025) she will deposit \$17,000.

(c) The average rate of change is $\dfrac{D(15) - D(0)}{15 - 0} = \dfrac{6875 - 3500}{15} = \$225/\text{year}$. This represents the average annual increase in contributions between 1995 and 2010.

69. $f(x) = \frac{1}{2}x - 6$

(a) The average rate of change of f between $x = 0$ and $x = 2$ is

$\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{\left[\frac{1}{2}(2) - 6\right] - \left[\frac{1}{2}(0) - 6\right]}{2} = \dfrac{-5 - (-6)}{2} = \dfrac{1}{2}$, and the average rate of change of f between $x = 15$ and $x = 50$ is

$\dfrac{f(50) - f(15)}{50 - 15} = \dfrac{\left[\frac{1}{2}(50) - 6\right] - \left[\frac{1}{2}(15) - 6\right]}{35} = \dfrac{19 - \frac{3}{2}}{35} = \dfrac{1}{2}$.

(b) The rates of change are the same.

(c) Yes, f is a linear function with rate of change $\frac{1}{2}$.

70. $f(x) = 8 - 3x$

(a) The average rate of change of f between $x = 0$ and $x = 2$ is $\dfrac{f(2) - f(0)}{2 - 0} = \dfrac{[8 - 3(2)] - [8 - 3(0)]}{2} = \dfrac{2 - 8}{2} = -3$, and the average rate of change of f between $x = 15$ and $x = 50$ is

$\dfrac{f(50) - f(15)}{50 - 15} = \dfrac{[8 - 3(50)] - [8 - 3(15)]}{35} = \dfrac{-142 - (-37)}{35} = -3$.

(b) The rates of change are the same.

(c) Yes, f is a linear function with rate of change -3.

71. **(a)** $y = f(x) + 8$. Shift the graph of $f(x)$ upward 8 units.

(b) $y = f(x + 8)$. Shift the graph of $f(x)$ to the left 8 units.

(c) $y = 1 + 2f(x)$. Stretch the graph of $f(x)$ vertically by a factor of 2, then shift it upward 1 unit.

(d) $y = f(x - 2) - 2$. Shift the graph of $f(x)$ to the right 2 units, then downward 2 units.

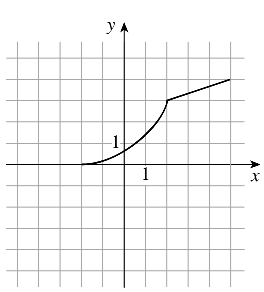
(e) $y = f(-x)$. Reflect the graph of $f(x)$ about the y-axis.

(f) $y = -f(-x)$. Reflect the graph of $f(x)$ first about the y-axis, then reflect about the x-axis.

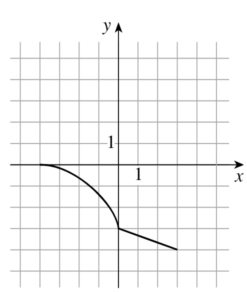
(g) $y = -f(x)$. Reflect the graph of $f(x)$ about the x-axis.

(h) $y = f^{-1}(x)$. Reflect the graph of $f(x)$ about the line $y = x$.

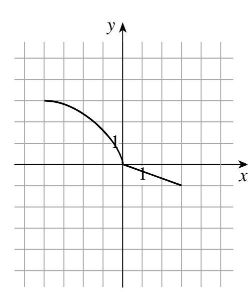
72. **(a)** $y = f(x - 2)$



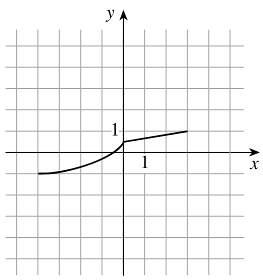
(b) $y = -f(x)$



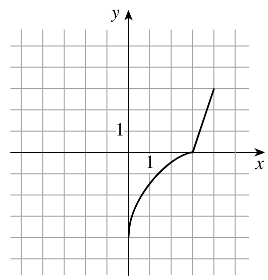
(c) $y = 3 - f(x)$



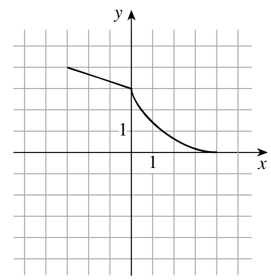
(d) $y = \frac{1}{2} f(x) - 1$



(e) $y = f^{-1}(x)$



(f) $y = f(-x)$



73. **(a)** $f(x) = 2x^5 - 3x^2 + 2$. $f(-x) = 2(-x)^5 - 3(-x)^2 + 2 = -2x^5 - 3x^2 + 2$. Since $f(x) \neq f(-x)$, f is not even. $-f(x) = -2x^5 + 3x^2 - 2$. Since $-f(x) \neq f(-x)$, f is not odd.

(b) $f(x) = x^3 - x^7$. $f(-x) = (-x)^3 - (-x)^7 = -\left(x^3 - x^7\right) = -f(x)$, hence f is odd.

(c) $f(x) = \dfrac{1 - x^2}{1 + x^2}$. $f(-x) = \dfrac{1 - (-x)^2}{1 + (-x)^2} = \dfrac{1 - x^2}{1 + x^2} = f(x)$. Since $f(x) = f(-x)$, f is even.

(d) $f(x) = \dfrac{1}{x + 2}$. $f(-x) = \dfrac{1}{(-x) + 2} = \dfrac{1}{2 - x}$. $-f(x) = -\dfrac{1}{x + 2}$. Since $f(x) \neq f(-x)$, f is not even, and since $f(-x) \neq -f(x)$, f is not odd.

74. **(a)** This function is odd.

(b) This function is neither even nor odd.

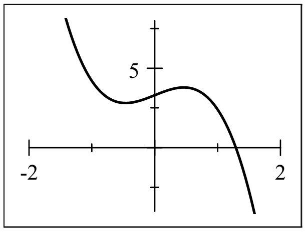
(c) This function is even.

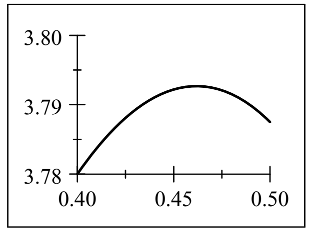
(d) This function is neither even nor odd.

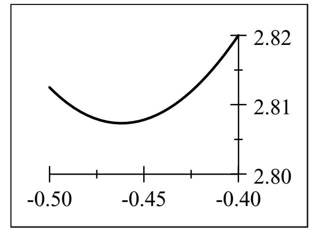
75. $g(x) = 2x^2 + 4x - 5 = 2\left(x^2 + 2x\right) - 5 = 2\left(x^2 + 2x + 1\right) - 5 - 2 = 2(x + 1)^2 - 7$. So the local minimum value -7 when $x = -1$.

76. $f(x) = 1 - x - x^2 = -\left(x^2 + x\right) + 1 = -\left(x^2 + x + \frac{1}{4}\right) + 1 + \frac{1}{4} = -\left(x + \frac{1}{2}\right)^2 + \frac{5}{4}$. So the local maximum value is $\frac{5}{4}$ when $x = -\frac{1}{2}$.

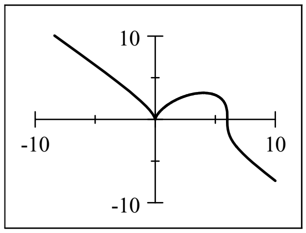
77. $f(x) = 3.3 + 1.6x - 2.5x^3$. In the first viewing rectangle, $[-2, 2]$ by $[-4, 8]$, we see that $f(x)$ has a local maximum and a local minimum. In the next viewing rectangle, $[0.4, 0.5]$ by $[3.78, 3.80]$, we isolate the local maximum value as approximately 3.79 when $x \approx 0.46$. In the last viewing rectangle, $[-0.5, -0.4]$ by $[2.80, 2.82]$, we isolate the local minimum value as 2.81 when $x \approx -0.46$.

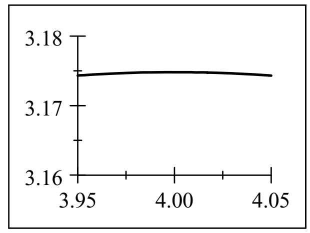






78. $f(x) = x^{2/3}(6-x)^{1/3}$. In the first viewing rectangle, $[-10, 10]$ by $[-10, 10]$, we see that $f(x)$ has a local maximum and a local minimum. The local minimum is 0 at $x = 0$ (and is easily verified). In the next viewing rectangle, $[3.95, 4.05]$ by $[3.16, 3.18]$, we isolate the local maximum value as approximately 3.175 when $x \approx 4.00$.





79. $h(t) = -5t^2 + 15t + 10 = -5\left(t^2 - 3t\right) + 10 = -5\left(t^2 - 3t + \frac{9}{4}\right) + 10 + 11.25$

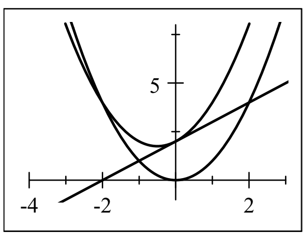
$= -5\left(t^2 - 3t + \frac{9}{4}\right) + 21.25 = -16\left(t - \frac{3}{2}\right)^2 + 21.25$

The stone reaches a maximum height of 21.25 meters.

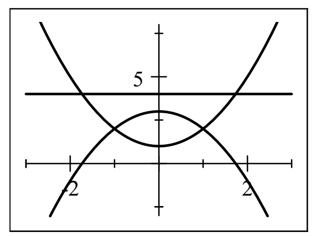
80. $P(x) = -1500 + 12x - 0.0004x^2 = -0.0004\left(x^2 - 30{,}000x\right) - 1500 = -0.0004\left(x^2 - 30{,}000x + 225{,}000{,}000\right) - 1500 + 90{,}000 = -0.0004(x - 15{,}000)^2 + 88{,}500$

The maximum profit occurs when 15,000 units are sold, and the maximum profit is \$88,500.

81. $f(x) = x + 2$, $g(x) = x^2$



82. $f(x) = x^2 + 1$, $g(x) = 3 - x^2$



83. $f(x) = x^2 - 3x + 2$ and $g(x) = 4 - 3x$.

(a) $(f+g)(x) = \left(x^2 - 3x + 2\right) + (4 - 3x) = x^2 - 6x + 6$

(b) $(f-g)(x) = \left(x^2 - 3x + 2\right) - (4 - 3x) = x^2 - 2$

(c) $(fg)(x) = \left(x^2 - 3x + 2\right)(4 - 3x) = 4x^2 - 12x + 8 - 3x^3 + 9x^2 - 6x = -3x^3 + 13x^2 - 18x + 8$

(d) $\left(\frac{f}{g}\right)(x) = \dfrac{x^2 - 3x + 2}{4 - 3x}, x \neq \frac{4}{3}$

(e) $(f \circ g)(x) = f(4 - 3x) = (4 - 3x)^2 - 3(4 - 3x) + 2 = 16 - 24x + 9x^2 - 12 + 9x + 2 = 9x^2 - 15x + 6$

(f) $(g \circ f)(x) = g\left(x^2 - 3x + 2\right) = 4 - 3\left(x^2 - 3x + 2\right) = -3x^2 + 9x - 2$

84. $f(x) = 1 + x^2$ and $g(x) = \sqrt{x - 1}$. (Remember that the proper domains must apply.)

(a) $(f \circ g)(x) = f\left(\sqrt{x-1}\right) = 1 + \left(\sqrt{x-1}\right)^2 = 1 + x - 1 = x$

(b) $(g \circ f)(x) = g\left(1 + x^2\right) = \sqrt{\left(1 + x^2\right) - 1} = \sqrt{x^2} = |x|$

(c) $(f \circ g)(2) = f(g(2)) = f\left(\sqrt{(2) - 1}\right) = f(1) = 1 + (1)^2 = 2$.

(d) $(f \circ f)(2) = f(f(2)) = f\left(1 + (2)^2\right) = f(5) = 1 + (5)^2 = 26$.

(e) $(f \circ g \circ f)(x) = f((g \circ f)(x)) = f(|x|) = 1 + (|x|)^2 = 1 + x^2$. Note that $(g \circ f)(x) = |x|$ by part (b).

(f) $(g \circ f \circ g)(x) = g((f \circ g)(x)) = g(x) = \sqrt{x - 1}$. Note that $(f \circ g)(x) = x$ by part (a).

85. $f(x) = 3x - 1$ and $g(x) = 2x - x^2$.

$(f \circ g)(x) = f\left(2x - x^2\right) = 3\left(2x - x^2\right) - 1 = -3x^2 + 6x - 1$, and the domain is $(-\infty, \infty)$.

$(g \circ f)(x) = g(3x - 1) = 2(3x - 1) - (3x - 1)^2 = 6x - 2 - 9x^2 + 6x - 1 = -9x^2 + 12x - 3$, and the domain is $(-\infty, \infty)$

$(f \circ f)(x) = f(3x - 1) = 3(3x - 1) - 1 = 9x - 4$, and the domain is $(-\infty, \infty)$.

$(g \circ g)(x) = g\left(2x - x^2\right) = 2\left(2x - x^2\right) - \left(2x - x^2\right)^2 = 4x - 2x^2 - 4x^2 + 4x^3 - x^4 = -x^4 + 4x^3 - 6x^2 + 4x$, and domain is $(-\infty, \infty)$.

86. $f(x) = \sqrt{x}$, has domain $\{x \mid x \geq 0\}$. $g(x) = \dfrac{2}{x-4}$, has domain $\{x \mid x \neq 4\}$.

$(f \circ g)(x) = f\left(\dfrac{2}{x-4}\right) = \sqrt{\dfrac{2}{x-4}}$. $(f \circ g)(x)$ is defined whenever both $g(x)$ and $f(g(x))$ are defined; that is, whenever $x \neq 4$ and $\dfrac{2}{x-4} \geq 0$. Now $\dfrac{2}{x-4} \geq 0 \Leftrightarrow x - 4 > 0 \Leftrightarrow x > 4$. So the domain of $f \circ g$ is $(4, \infty)$.

$(g \circ f)(x) = g\left(\sqrt{x}\right) = \dfrac{2}{\sqrt{x} - 4}$. $(g \circ f)(x)$ is defined whenever both $f(x)$ and $g(f(x))$ are defined; that is, whenever $x \geq 0$ and $\sqrt{x} - 4 \neq 0$. Now $\sqrt{x} - 4 \neq 0 \Leftrightarrow x \neq 16$. So the domain of $g \circ f$ is $[0, 16) \cup (16, \infty)$.

$(f \circ f)(x) = f\left(\sqrt{x}\right) = \sqrt{\sqrt{x}} = x^{1/4}$. $(f \circ f)(x)$ is defined whenever both $f(x)$ and $f(f(x))$ are defined; that is, whenever $x \geq 0$. So the domain of $f \circ f$ is $[0, \infty)$.

$(g \circ g)(x) = g\left(\dfrac{2}{x-4}\right) = \dfrac{2}{\dfrac{2}{x-4} - 4} = \dfrac{2(x-4)}{2 - 4(x-4)} = \dfrac{x-4}{9-2x}$. $(g \circ g)(x)$ is defined whenever both $g(x)$ and $g(g(x))$ are defined; that is, whenever $x \neq 4$ and $9 - 2x \neq 0$. Now $9 - 2x \neq 0 \Leftrightarrow 2x \neq 9 \Leftrightarrow x \neq \frac{9}{2}$. So the domain of $g \circ g$ is $\left\{x \mid x \neq \frac{9}{2}, 4\right\}$.

87. $f(x) = \sqrt{1-x}$, $g(x) = 1 - x^2$ and $h(x) = 1 + \sqrt{x}$.

$$\begin{aligned}(f \circ g \circ h)(x) &= f(g(h(x))) = f\left(g\left(1 + \sqrt{x}\right)\right) = f\left(1 - \left(1 + \sqrt{x}\right)^2\right) = f\left(1 - \left(1 + 2\sqrt{x} + x\right)\right) \\ &= f\left(-x - 2\sqrt{x}\right) = \sqrt{1 - \left(-x - 2\sqrt{x}\right)} = \sqrt{1 + 2\sqrt{x} + x} = \sqrt{\left(1 + \sqrt{x}\right)^2} = 1 + \sqrt{x}\end{aligned}$$

88. If $h(x) = \sqrt{x}$ and $g(x) = 1 + x$, then $(g \circ h)(x) = g\left(\sqrt{x}\right) = 1 + \sqrt{x}$. If $f(x) = \dfrac{1}{\sqrt{x}}$, then $(f \circ g \circ h)(x) = f\left(1 + \sqrt{x}\right) = \dfrac{1}{\sqrt{1 + \sqrt{x}}} = T(x)$.

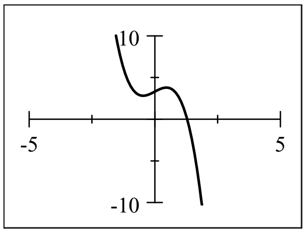
89. $f(x) = 3 + x^3$. If $x_1 \neq x_2$, then $x_1^3 \neq x_2^3$ (unequal numbers have unequal cubes), and therefore $3 + x_1^3 \neq 3 + x_2^3$. Thus f is a one-to-one function.

90. $g(x) = 2 - 2x + x^2 = \left(x^2 - 2x + 1\right) + 1 = (x-1)^2 + 1$. Since $g(0) = 2 = g(2)$, as is true for all pairs of numbers equidistant from 1, g is not a one-to-one function.

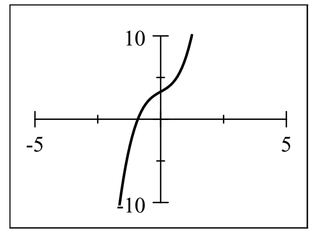
91. $h(x) = \dfrac{1}{x^4}$. Since the fourth powers of a number and its negative are equal, h is not one-to-one. For example, $h(-1) = \dfrac{1}{(-1)^4} = 1$ and $h(1) = \dfrac{1}{(1)^4} = 1$, so $h(-1) = h(1)$.

92. $r(x) = 2 + \sqrt{x+3}$. If $x_1 \neq x_2$, then $x_1 + 3 \neq x_2 + 3$, so $\sqrt{x_1 + 3} \neq \sqrt{x_2 + 3}$ and $2 + \sqrt{x_1 + 3} \neq 2 + \sqrt{x_2 + 3}$. Thus r is one-to-one.

93. $p(x) = 3.3 + 1.6x - 2.5x^3$. Using a graphing device and the Horizontal Line Test, we see that p is not a one-to-one function.



94. $q(x) = 3.3 + 1.6x + 2.5x^3$. Using a graphing device and the Horizontal Line Test, we see that q is a one-to-one function.



95. $f(x) = 3x - 2 \Leftrightarrow y = 3x - 2 \Leftrightarrow 3x = y + 2 \Leftrightarrow x = \frac{1}{3}(y+2)$. So $f^{-1}(x) = \frac{1}{3}(x+2)$.

96. $f(x) = \dfrac{2x+1}{3}$. $y = \dfrac{2x+1}{3} \Leftrightarrow 2x + 1 = 3y \Leftrightarrow 2x = 3y - 1 \Leftrightarrow x = \frac{1}{2}(3y-1)$. So $f^{-1}(x) = \frac{1}{2}(3x-1)$.

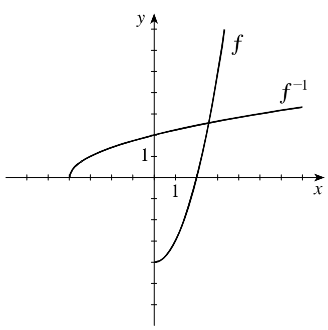
97. $f(x) = (x+1)^3 \Leftrightarrow y = (x+1)^3 \Leftrightarrow x + 1 = \sqrt[3]{y} \Leftrightarrow x = \sqrt[3]{y} - 1$. So $f^{-1}(x) = \sqrt[3]{x} - 1$.

98. $f(x) = 1 + \sqrt[5]{x-2}$. $y = 1 + \sqrt[5]{x-2} \Leftrightarrow y - 1 = \sqrt[5]{x-2} \Leftrightarrow x - 2 = (y-1)^5 \Leftrightarrow x = 2 + (y-1)^5$. So $f^{-1}(x) = 2 + (x-1)^5$.

99. The graph passes the Horizontal Line Test, so f has an inverse. Because $f(1) = 0$, $f^{-1}(0) = 1$, and because $f(3) = 4$, $f^{-1}(4) = 3$.

100. The graph fails the Horizontal Line Test, so f does not have an inverse.

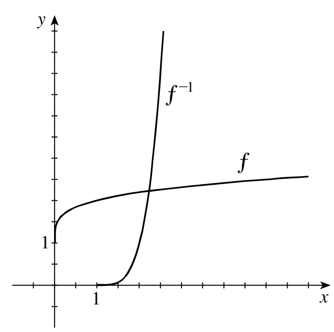
101. (a), (b) $f(x) = x^2 - 4$, $x \geq 0$



(c) $f(x) = x^2 - 4$, $x \geq 0 \Leftrightarrow y = x^2 - 4$, $y \geq -4$ $\Leftrightarrow x^2 = y + 4 \Leftrightarrow x = \sqrt{y+4}$. So $f^{-1}(x) = \sqrt{x+4}$, $x \geq -4$.

102. (a) If $x_1 \neq x_2$, then $\sqrt[4]{x_1} \neq \sqrt[4]{x_2}$, and so $1 + \sqrt[4]{x_1} \neq 1 + \sqrt[4]{x_2}$. Therefore, f is a one-to-one function.

(b), (c)



(d) $f(x) = 1 + \sqrt[4]{x}$. $y = 1 + \sqrt[4]{x} \Leftrightarrow \sqrt[4]{x} = y - 1$ $\Leftrightarrow x = (y-1)^4$. So $f^{-1}(x) = (x-1)^4$, $x \geq 1$. Note that the domain of f is $[0, \infty)$, so $y = 1 + \sqrt[4]{x} \geq 1$. Hence, the domain of f^{-1} is $[1, \infty)$.

CHAPTER 2 TEST

1. By the Vertical Line Test, figures (a) and (b) are graphs of functions. By the Horizontal Line Test, only figure (a) is the graph of a one-to-one function.

2. (a) $f(0) = \dfrac{\sqrt{0}}{0+1} = 0$; $f(2) = \dfrac{\sqrt{2}}{2+1} = \dfrac{\sqrt{2}}{3}$; $f(a+2) = \dfrac{\sqrt{a+2}}{a+2+1} = \dfrac{\sqrt{a+2}}{a+3}$.

(b) $f(x) = \dfrac{\sqrt{x}}{x+1}$. Our restrictions are that the input to the radical is nonnegative and that the denominator must not be 0. Thus, $x \geq 0$ and $x + 1 \neq 0 \Leftrightarrow x \neq -1$. (The second restriction is made irrelevant by the first.) In interval notation, the domain is $[0, \infty)$.

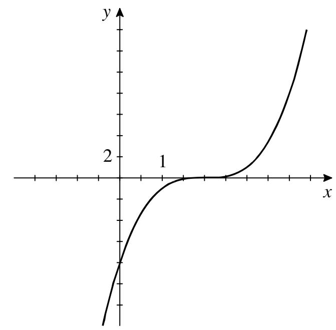
(c) The average rate of change is $\dfrac{f(10) - f(2)}{10 - 2} = \dfrac{\dfrac{\sqrt{10}}{10+1} - \dfrac{\sqrt{2}}{2+1}}{10 - 2} = \dfrac{3\sqrt{10} - 11\sqrt{2}}{264}$.

3. (a) "Subtract 2, then cube the result" can be expressed algebraically as $f(x) = (x-2)^3$.

(b)

x	$f(x)$
-1	-27
0	-8
1	-1
2	0
3	1
4	8

(c)



(d) We know that f has an inverse because it passes the Horizontal Line Test. A verbal description for f^{-1} is, "Take the cube root, then add 2."

(e) $y = (x-2)^3 \Leftrightarrow \sqrt[3]{y} = x - 2 \Leftrightarrow x = \sqrt[3]{y} + 2$. Thus, a formula for f^{-1} is $f^{-1}(x) = \sqrt[3]{x} + 2$.

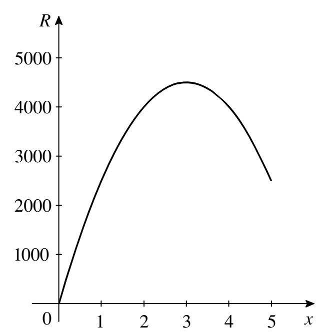
4. (a) f has a local minimum value of -4 at $x = -1$ and local maximum values of -1 at $x = -4$ and 4 at $x = 3$.

(b) f is increasing on $(-\infty, -4)$ and $(-1, 3)$ and decreasing on $(-4, -1)$ and $(3, \infty)$.

5. $R(x) = -500x^2 + 3000x$

(a) $R(2) = -500(2)^2 + 3000(2) = \$4000$ represents their total sales revenue when their price is \$2 per bar and $R(4) = -500(4)^2 + 3000(4) = \$4000$ represents their total sales revenue when their price is \$4 per bar

(b)

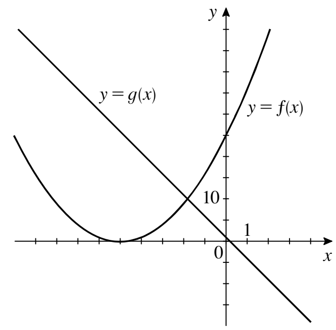


(c) The maximum revenue is \$4500, and it is achieved at a price of $x = \$3$.

6. The net change is $f(2+h) - f(2) = \left[(2+h)^2 - 2(2+h)\right] - \left[2^2 - 2(2)\right] = \left(4 + h^2 + 4h - 4 - 2h\right) - 0 = 2h + h^2$ and the average rate of change is $\dfrac{f(2+h) - f(2)}{2+h-2} = \dfrac{2h+h^2}{h} = 2 + h$.

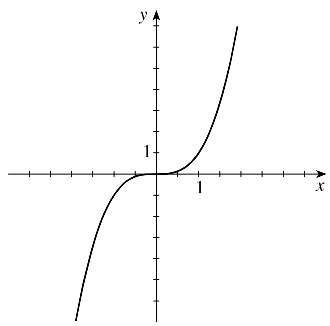
7. **(a)** $f(x) = (x+5)^2 = x^2 + 10x + 25$ is not linear because it cannot be expressed in the form $f(x) = ax + b$ for constants a and b. $g(x) = 1 - 5x$ is linear.

(b)

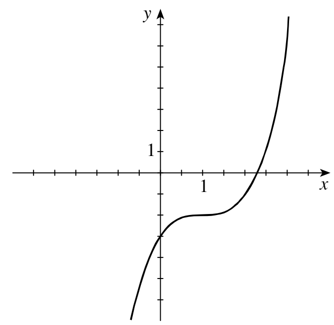


(c) $g(x)$ has rate of change -5.

8. **(a)** $f(x) = x^3$



(b) $g(x) = (x-1)^3 - 2$. To obtain the graph of g, shift the graph of f to the right 1 unit and downward 2 units.

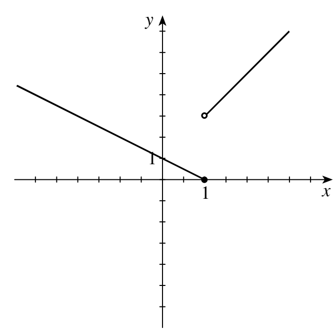


9. **(a)** $y = f(x-3) + 2$. Shift the graph of $f(x)$ to the right 3 units, then shift the graph upward 2 units.

(b) $y = f(-x)$. Reflect the graph of $f(x)$ about the y-axis.

10. **(a)** $f(-2) = 1 - (-2) = 1 + 2 = 3$ (since $-2 \le 1$).
$f(1) = 1 - 1 = 0$ (since $1 \le 1$).

(b)



11. $f(x) = x^2 + x + 1$; $g(x) = x - 3$.

(a) $(f+g)(x) = f(x) + g(x) = \left(x^2 + x + 1\right) + (x-3) = x^2 + 2x - 2$

(b) $(f-g)(x) = f(x) - g(x) = \left(x^2 + x + 1\right) - (x-3) = x^2 + 4$

(c) $(f \circ g)(x) = f(g(x)) = f(x-3) = (x-3)^2 + (x-3) + 1 = x^2 - 6x + 9 + x - 3 + 1 = x^2 - 5x + 7$

(d) $(g \circ f)(x) = g(f(x)) = g\left(x^2 + x + 1\right) = \left(x^2 + x + 1\right) - 3 = x^2 + x - 2$

(e) $f(g(2)) = f(-1) = (-1)^2 + (-1) + 1 = 1$. [We have used the fact that $g(2) = 2 - 3 = -1$.]

(f) $g(f(2)) = g(7) = 7 - 3 = 4$. [We have used the fact that $f(2) = 2^2 + 2 + 1 = 7$.]

(g) $(g \circ g \circ g)(x) = g(g(g(x))) = g(g(x-3)) = g(x-6) = (x-6) - 3 = x - 9$. [We have used the fact that $g(x-3) = (x-3) - 3 = x - 6$.]

12. **(a)** $f(x) = x^3 + 1$ is one-to-one because each real number has a unique cube.

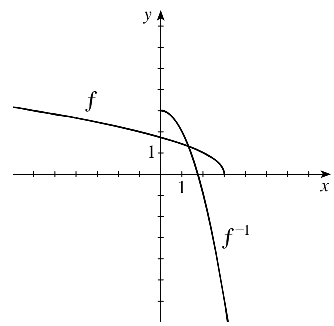
(b) $g(x) = |x + 1|$ is not one-to-one because, for example, $g(-2) = g(0) = 1$.

13. $f(g(x)) = \dfrac{1}{\left(\dfrac{1}{x} + 2\right) - 2} = \dfrac{1}{\dfrac{1}{x}} = x$ for all $x \neq 0$, and $g(f(x)) = \dfrac{1}{\dfrac{1}{x-2}} + 2 = x - 2 + 2 = x$ for all $x \neq -2$. Thus, by the Inverse Function Property, f and g are inverse functions.

14. $f(x) = \dfrac{x-3}{2x+5}$. $y = \dfrac{x-3}{2x+5} \Leftrightarrow (2x+5)y = x - 3 \Leftrightarrow x(2y-1) = -5x - 3 \Leftrightarrow x = -\dfrac{5y+3}{2y-1}$. Thus, $f^{-1}(x) = -\dfrac{5x+3}{2x-1}$.

15. **(a)** $f(x) = \sqrt{3-x}$, $x \leq 3 \Leftrightarrow y = \sqrt{3-x} \Leftrightarrow$ $y^2 = 3 - x \Leftrightarrow x = 3 - y^2$. Thus $f^{-1}(x) = 3 - x^2$, $x \geq 0$.

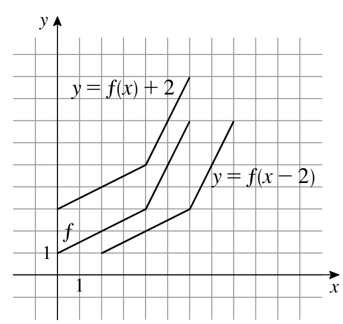
(b) $f(x) = \sqrt{3-x}$, $x \leq 3$ and $f^{-1}(x) = 3 - x^2$, $x \geq 0$



16. The domain of f is $[0, 6]$, and the range of f is $[1, 7]$.

17. The graph passes through the points $(0, 1)$ and $(4, 3)$, so $f(0) = 1$ and $f(4) = 3$.

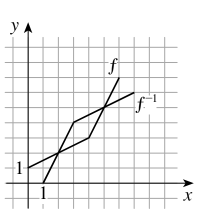
18. The graph of $f(x-2)$ can be obtained by shifting the graph of $f(x)$ to the right 2 units. The graph of $f(x) + 2$ can be obtained by shifting the graph of $f(x)$ upward 2 units.



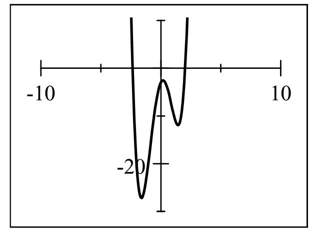
19. The net change of f between $x = 2$ and $x = 6$ is $f(6) - f(2) = 7 - 2 = 5$ and the average rate of change is $\dfrac{f(6) - f(2)}{6 - 2} = \dfrac{5}{4}$.

20. Because $f(0) = 1$, $f^{-1}(1) = 0$. Because $f(4) = 3$, $f^{-1}(3) = 4$.

21.

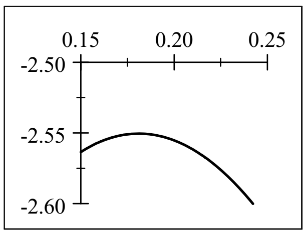


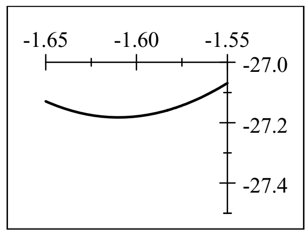
22. (a) $f(x) = 3x^4 - 14x^2 + 5x - 3$. The graph is shown in the viewing rectangle $[-10, 10]$ by $[-30, 10]$.

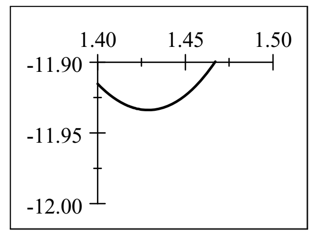


(b) No, by the Horizontal Line Test.

(c) The local maximum is approximately -2.55 when $x \approx 0.18$, as shown in the first viewing rectangle $[0.15, 0.25]$ by $[-2.6, -2.5]$. One local minimum is approximately -27.18 when $x \approx -1.61$, as shown in the second viewing rectangle $[-1.65, -1.55]$ by $[-27.5, -27]$. The other local minimum is approximately -11.93 when $x \approx 1.43$, as shown is the viewing rectangle $[1.4, 1.5]$ by $[-12, -11.9]$.







(d) Using the graph in part (a) and the local minimum, -27.18, found in part (c), we see that the range is $[-27.18, \infty)$.

(e) Using the information from part (c) and the graph in part (a), $f(x)$ is increasing on the intervals $(-1.61, 0.18)$ and $(1.43, \infty)$ and decreasing on the intervals $(-\infty, -1.61)$ and $(0.18, 1.43)$.

FOCUS ON MODELING Modeling with Functions

1. Let w be the width of the building lot. Then the length of the lot is $3w$. So the area of the building lot is $A(w) = 3w^2$, $w > 0$.

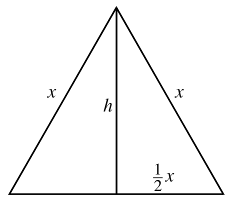
2. Let w be the width of the poster. Then the length of the poster is $w + 10$. So the area of the poster is $A(w) = w(w + 10) = w^2 + 10w$.

3. Let w be the width of the base of the rectangle. Then the height of the rectangle is $\frac{1}{2}w$. Thus the volume of the box is given by the function $V(w) = \frac{1}{2}w^3$, $w > 0$.

4. Let r be the radius of the cylinder. Then the height of the cylinder is $4r$. Since for a cylinder $V = \pi r^2 h$, the volume of the cylinder is given by the function $V(r) = \pi r^2 (4r) = 4\pi r^3$.

5. Let P be the perimeter of the rectangle and y be the length of the other side. Since $P = 2x + 2y$ and the perimeter is 20, we have $2x + 2y = 20 \Leftrightarrow x + y = 10 \Leftrightarrow y = 10 - x$. Since area is $A = xy$, substituting gives $A(x) = x(10 - x) = 10x - x^2$, and since A must be positive, the domain is $0 < x < 10$.

6. Let A be the area and y be the length of the other side. Then $A = xy = 16 \Leftrightarrow y = \dfrac{16}{x}$. Substituting into $P = 2x + 2y$ gives $P = 2x + 2 \cdot \dfrac{16}{x} = 2x + \dfrac{32}{x}$, where $x > 0$.

7. 

Let h be the height of an altitude of the equilateral triangle whose side has length x, as shown in the diagram. Thus the area is given by $A = \frac{1}{2}xh$. By the Pythagorean Theorem, $h^2 + \left(\frac{1}{2}x\right)^2 = x^2 \Leftrightarrow h^2 + \frac{1}{4}x^2 = x^2 \Leftrightarrow h^2 = \frac{3}{4}x^2 \Leftrightarrow h = \frac{\sqrt{3}}{2}x$. Substituting into the area of a triangle, we get

$A(x) = \frac{1}{2}xh = \frac{1}{2}x\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$, $x > 0$.

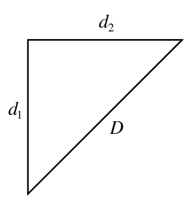
8. Let d represent the length of any side of a cube. Then the surface area is $S = 6d^2$, and the volume is $V = d^3 \Leftrightarrow d = \sqrt[3]{V}$. Substituting for d gives $S(V) = 6\left(\sqrt[3]{V}\right)^2 = 6V^{2/3}$, $V > 0$.

9. We solve for r in the formula for the area of a circle. This gives $A = \pi r^2 \Leftrightarrow r^2 = \dfrac{A}{\pi} \Rightarrow r = \sqrt{\dfrac{A}{\pi}}$, so the model is $r(A) = \sqrt{\dfrac{A}{\pi}}$, $A > 0$.

10. Let r be the radius of a circle. Then the area is $A = \pi r^2$, and the circumference is $C = 2\pi r \Leftrightarrow r = \dfrac{C}{2\pi}$. Substituting for r gives $A(C) = \pi\left(\dfrac{C}{2\pi}\right)^2 = \dfrac{C^2}{4\pi}$, $C > 0$.

11. Let h be the height of the box in meters. The volume of the box is $V = 60$. Then $x^2h = 60 \Leftrightarrow h = \dfrac{60}{x^2}$. The surface area, S, of the box is the sum of the area of the 4 sides and the area of the base and top. Thus $S = 4xh + 2x^2 = 4x\left(\dfrac{60}{x^2}\right) + 2x^2 = \dfrac{240}{x} + 2x^2$, so the model is $S(x) = \dfrac{240}{x} + 2x^2$, $x > 0$.

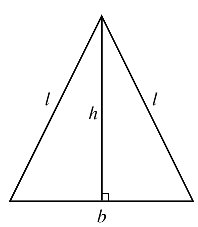
12. By similar triangles, $\dfrac{1.5}{L} = \dfrac{3.6}{L+d} \Leftrightarrow 1.5(L+d) = 3.6L \Leftrightarrow 1.5d = 2.1L \Leftrightarrow L = \dfrac{1.5d}{2.1}$. The model is $L(d) = 0.714d$.

13. 

Let d_1 be the distance traveled south by the first ship and d_2 be the distance traveled east by the second ship. The first ship travels south for t hours at 15 km/h, so $d_1 = 15t$ and, similarly, $d_2 = 20t$. Since the ships are traveling at right angles to each other, we can apply the Pythagorean Theorem to get

$D(t) = \sqrt{d_1^2 + d_2^2} = \sqrt{(15t)^2 + (20t)^2} = \sqrt{225t^2 + 400t^2} = 25t$.

14. Let n be one of the numbers. Then the other number is $60 - n$, so the product is given by the function $P(n) = n(60 - n) = 60n - n^2$.

15. 

Let b be the length of the base, l be the length of the equal sides, and h be the height in centimeters. Since the perimeter is 8, $2l + b = 8 \Leftrightarrow 2l = 8 - b \Leftrightarrow l = \frac{1}{2}(8 - b)$. By the Pythagorean Theorem, $h^2 + \left(\frac{1}{2}b\right)^2 = l^2 \Leftrightarrow h = \sqrt{l^2 - \frac{1}{4}b^2}$. Therefore the area of the triangle is

$$A = \frac{1}{2}\cdot b\cdot h = \frac{1}{2}\cdot b\sqrt{l^2 - \frac{1}{4}b^2} = \frac{b}{2}\sqrt{\frac{1}{4}(8-b)^2 - \frac{1}{4}b^2}$$
$$= \frac{b}{4}\sqrt{64 - 16b + b^2 - b^2} = \frac{b}{4}\sqrt{64 - 16b} = \frac{b}{4}\cdot 4\sqrt{4-b} = b\sqrt{4-b}$$

so the model is $A(b) = b\sqrt{4-b}$, $0 < b < 4$.

16. Let x be the length of the shorter leg of the right triangle. Then the length of the other triangle is $2x$. Since it is a right triangle, the length of the hypotenuse is $\sqrt{x^2+(2x)^2}=\sqrt{5x^2}=\sqrt{5}\,x$ (since $x \geq 0$). Thus the perimeter of the triangle is $P(x)=x+2x+\sqrt{5}\,x=\left(3+\sqrt{5}\right)x$.

17. Let w be the length of the rectangle. By the Pythagorean Theorem, $\left(\frac{1}{2}w\right)^2+h^2=10^2 \Leftrightarrow \dfrac{w^2}{4}+h^2=10^2 \Leftrightarrow$ $w^2=4\left(100-h^2\right) \Leftrightarrow w=2\sqrt{100-h^2}$ (since $w>0$). Therefore, the area of the rectangle is $A=wh=2h\sqrt{100-h^2}$, so the model is $A(h)=2h\sqrt{100-h^2}$, $0<h<10$.

18. Using the formula for the volume of a cone, $V=\frac{1}{3}\pi r^2h$, we substitute $V=100$ and solve for h. Thus $100=\frac{1}{3}\pi r^2h \Leftrightarrow$ $h(r)=\dfrac{300}{\pi r^2}$.

19. **(a)** We complete the table.

First number	Second number	Product
1	18	18
2	17	34
3	16	48
4	15	60
5	14	70
6	13	78
7	12	84
8	11	88
9	10	90
10	9	90
11	8	88

From the table we conclude that the numbers is still increasing, the numbers whose product is a maximum should both be 9.5.

(b) Let x be one number: then $19-x$ is the other number, and so the product, p, is

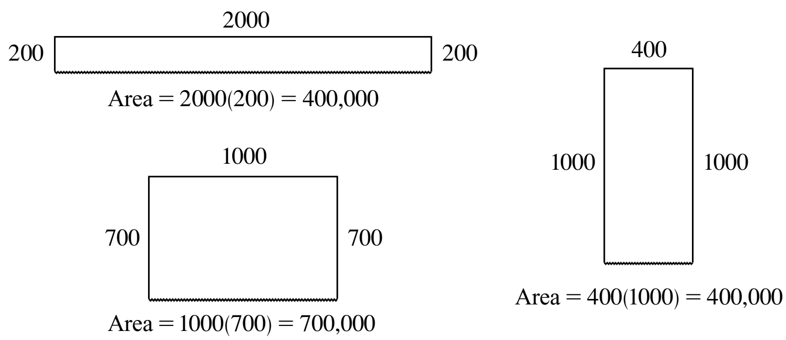
$p(x)=x(19-x)=19x-x^2$.

(c)
$$\begin{aligned} p(x) &= 19x-x^2=-\left(x^2-19x\right) \\ &= -\left[x^2-19x+\left(\tfrac{19}{2}\right)^2\right]+\left(\tfrac{19}{2}\right)^2 \\ &= -(x-9.5)^2+90.25 \end{aligned}$$

So the product is maximized when the numbers are both 9.5.

20. Let the positive numbers be x and y. Since their sum is 100, we have $x+y=100 \Leftrightarrow y=100-x$. We wish to minimize the sum of squares, which is $S=x^2+y^2=x^2+(100-x)^2$. So $S(x)=x^2+(100-x)^2=x^2+10{,}000-200x+x^2=2x^2-200x+10{,}000=2\left(x^2-100x\right)+10{,}000=2\left(x^2-100x+2500\right)+10{,}000-5000=2(x-50)^2+5000$. Thus the minimum sum of squares occurs when $x=50$. Then $y=100-50=50$. Therefore both numbers are 50.

21. (a) Let x be the width of the field (in meters) and l be the length of the field (in meters). Since the farmer has 2400 m of fencing we must have $2x + l = 2400$.



Width	Length	Area
200	2000	400,000
300	1800	540,000
400	1600	640,000
500	1400	700,000
600	1200	720,000
700	1000	700,000
800	800	640,000

It appears that the field of largest area is about 600 m × 1200 m.

(b) Let x be the width of the field (in meters) and l be the length of the field (in meters). Since the farmer has 2400 m of fencing we must have $2x + l = 2400 \Leftrightarrow l = 2400 - 2x$. The area of the fenced-in field is given by $A(x) = l \cdot x = (2400 - 2x)\,x = -2x^2 + 2400x = -2\left(x^2 - 1200x\right)$.

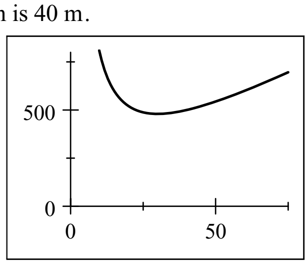
(c) The area is $A(x) = -2\left(x^2 - 1200x + 600^2\right) + 2\left(600^2\right) = -2(x - 600)^2 + 720{,}000$. So the maximum area occurs when $x = 600$ meters and $l = 2400 - 2(600) = 1200$ meters.

22. (a) Let w be the width of the rectangular area (in meters) and l be the length of the field (in meters). Since the farmer has 750 meters of fencing, we must have $5w + 2l = 750 \Leftrightarrow 2l = 750 - 5w \Leftrightarrow l = \frac{5}{2}(150 - w)$. Thus the total area of the four pens is $A(w) = l \cdot w = \frac{5}{2}w(150 - w) = -\frac{5}{2}\left(w^2 - 150w\right)$.

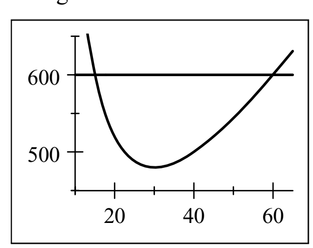
(b) We complete the square to get $A(w) = -\frac{5}{2}\left(w^2 - 150w\right) = -\frac{5}{2}\left(w^2 - 150w + 75^2\right) + \left(\frac{5}{2}\right) \cdot 75^2 = -\frac{5}{2}(w - 75)^2 + 14062.5$. Therefore, the largest possible total area of the four pens is 14,062.5 square meters.

23. (a) Let x be the length of the fence along the road. If the area is 1200, we have $1200 = x \cdot$ width, so the width of the garden is $\dfrac{1200}{x}$. Then the cost of the fence is given by the function $C(x) = 5(x) + 3\left[x + 2 \cdot \dfrac{1200}{x}\right] = 8x + \dfrac{7200}{x}$.

(b) We graph the function $y = C(x)$ in the viewing rectangle $[0, 75] \times [0, 800]$. From this we get the cost is minimized when $x = 30$ m. Then the width is $\frac{1200}{30} = 40$ m. So the length is 30 m and the width is 40 m.



(c) We graph the function $y = C(x)$ and $y = 600$ in the viewing rectangle $[10, 65] \times [450, 650]$. From this we get that the cost is at most \$600 when $15 \le x \le 60$. So the range of lengths he can fence along the road is 15 meters to 60 meters.



24. **(a)** Let x be the length of wire in cm that is bent into a square. So $10 - x$ is the length of wire in cm that is bent into the second square. The width of each square is $\frac{x}{4}$ and $\frac{10-x}{4}$, and the area of each square is $\left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$ and $\left(\frac{10-x}{4}\right)^2 = \frac{100-20x+x^2}{16}$. Thus the sum of the areas is $A(x) = \frac{x^2}{16} + \frac{100-20x+x^2}{16} = \frac{100-20x+2x^2}{16} = \frac{1}{8}x^2 - \frac{5}{4}x + \frac{25}{4}$.

(b) We complete the square. $A(x) = \frac{1}{8}x^2 - \frac{5}{4}x + \frac{25}{4} = \frac{1}{8}\left(x^2 - 10x\right) + \frac{25}{4} = \frac{1}{8}\left(x^2 - 10x + 25\right) + \frac{25}{4} - \frac{25}{8} = \frac{1}{8}(x-5)^2 + \frac{25}{8}$ So the minimum area is $\frac{25}{8}$ cm^2 when each piece is 5 cm long.

25. **(a)** Let h be the height in meters of the straight portion of the window. The circumference of the semicircle is $C = \frac{1}{2}\pi x$. Since the perimeter of the window is 10 meters, we have $x + 2h + \frac{1}{2}\pi x = 10$. Solving for h, we get $2h = 10 - x - \frac{1}{2}\pi x \Leftrightarrow h = 5 - \frac{1}{2}x - \frac{1}{4}\pi x$. The area of the window is $A(x) = xh + \frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 = x\left(5 - \frac{1}{2}x - \frac{1}{4}\pi x\right) + \frac{1}{8}\pi x^2 = 5x - \frac{1}{2}x^2 - \frac{1}{8}\pi x^2$.

(b) $A(x) = 5x - \frac{1}{8}(\pi+4)x^2 = -\frac{1}{8}(\pi+4)\left[x^2 - \frac{40}{\pi+4}x\right]$

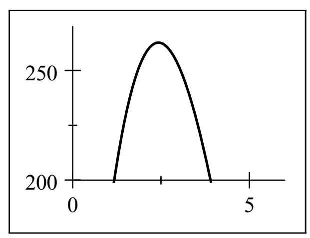
$$= -\frac{1}{8}(\pi+4)\left[x^2 - \frac{40}{\pi+4}x + \left(\frac{20}{\pi+4}\right)^2\right] + \frac{50}{\pi+4} = -\frac{1}{8}(\pi+4)\left(x - \frac{20}{\pi+4}\right)^2 + \frac{50}{\pi+4}$$

The area is maximized when $x = \frac{20}{\pi+4} \approx 2.80$, and hence $h \approx 5 - \frac{1}{2}(2.8) - \frac{1}{4}\pi(2.8) \approx 1.40$.

26. **(a)** The height of the box is x, the width of the box is $12 - 2x$, and the length of the box is $20 - 2x$. Therefore, the volume of the box is

$$\begin{aligned} V(x) &= x(12-2x)(20-2x) \\ &= 4x^3 - 64x^2 + 240x, 0 < x < 6 \end{aligned}$$

(b) We graph the function $y = V(x)$ in the viewing rectangle $[0, 6] \times [200, 270]$.

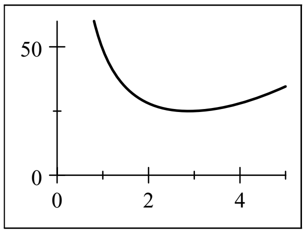


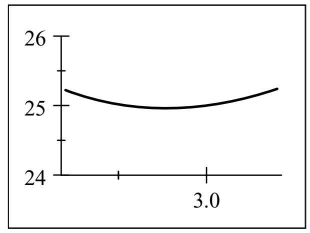
From the calculator we get that the volume of the box is greater than 200 cm^3 for $1.174 \le x \le 3.898$ (accurate to 3 decimal places).

(c) From the graph, the volume of the box with the largest volume is 262.682 cm^3 when $x \approx 2.427$.

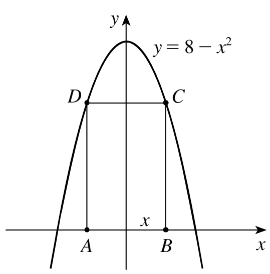
27. **(a)** Let x be the length of one side of the base and let h be the height of the box in meters. Since the volume of the box is $V = x^2h = 12$, we have $x^2h = 12 \Leftrightarrow h = \frac{12}{x^2}$. The surface area, A, of the box is sum of the area of the four sides and the area of the base. Thus the surface area of the box is given by the formula $A(x) = 4xh + x^2 = 4x\left(\frac{12}{x^2}\right) + x^2 = \frac{48}{x} + x^2, x > 0$.

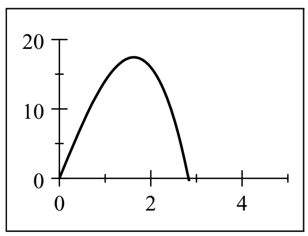
(b) The function $y = A(x)$ is shown in the first viewing rectangle below. In the second viewing rectangle, we isolate the minimum, and we see that the amount of material is minimized when x (the length and width) is 2.88 m. Then the height is $h = \dfrac{12}{x^2} \approx 1.44$ m.

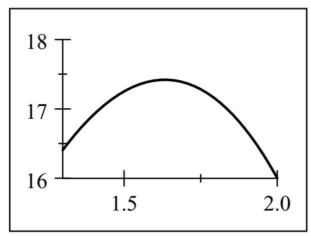




28. Let A, B, C, and D be the vertices of a rectangle with base AB on the x-axis and its other two vertices C and D above the x-axis and lying on the parabola $y = 8 - x^2$. Let C have the coordinates (x, y), $x > 0$. By symmetry, the coordinates of D must be $(-x, y)$. So the width of the rectangle is $2x$, and the length is $y = 8 - x^2$. Thus the area of the rectangle is $A(x) = \text{length} \cdot \text{width} = 2x\left(8 - x^2\right) = 16x - 2x^3$. The graphs of $A(x)$ below show that the area is maximized when $x \approx 1.63$. Hence the maximum area occurs when the width is 3.26 and the length is 5.33.

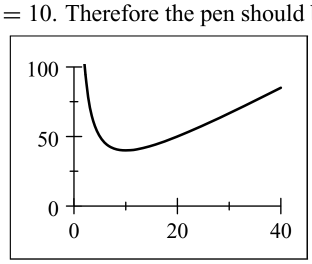


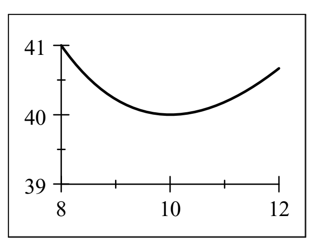




29. (a) Let w be the width of the pen and l be the length in meters. We use the area to establish a relationship between w and l. Since the area is 100 m^2, we have $l \cdot w = 100 \Leftrightarrow l = \dfrac{100}{w}$. So the amount of fencing used is $F = 2l + 2w = 2\left(\dfrac{100}{w}\right) + 2w = \dfrac{200 + 2w^2}{w}$.

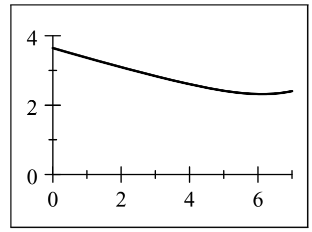
(b) Using a graphing device, we first graph F in the viewing rectangle $[0, 40]$ by $[0, 100]$, and locate the approximate location of the minimum value. In the second viewing rectangle, $[8, 12]$ by $[39, 41]$, we see that the minimum value of F occurs when $w = 10$. Therefore the pen should be a square with side 10 m.





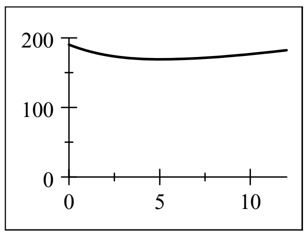
30. (a) Let t_1 represent the time, in hours, spent walking, and let t_2 represent the time spent rowing. Since the distance walked is x and the walking speed is 5 mi/h, the time spent walking is $t_1 = \frac{1}{5}x$. By the Pythagorean Theorem, the distance rowed is $d = \sqrt{2^2 + (7-x)^2} = \sqrt{x^2 - 14x + 53}$, and so the time spent rowing is $t_2 = \frac{1}{2} \cdot \sqrt{x^2 - 14x + 53}$. Thus the total time is $T(x) = \frac{1}{2}\sqrt{x^2 - 14x + 53} + \frac{1}{5}x$.

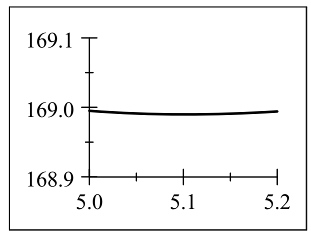
(b) We graph $y = T(x)$. Using the zoom function, we see that T is minimized when $x \approx 6.13$. He should land at a point 6.13 kilometers from point B.



31. (a) Let x be the distance from point B to C, in kilometers. Then the distance from A to C is $\sqrt{x^2 + 25}$, and the energy used in flying from A to C then C to D is $f(x) = 14\sqrt{x^2 + 25} + 10(12 - x)$.

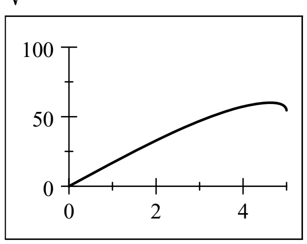
(b) By using a graphing device, the energy expenditure is minimized when the distance from B to C is about 5.1 kilometers.

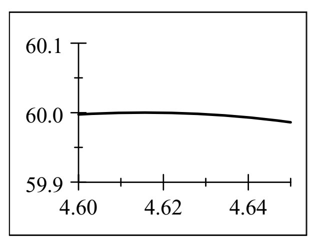




32. (a) Using the Pythagorean Theorem, we have that the height of the upper triangles is $\sqrt{25 - x^2}$ and the height of the lower triangles is $\sqrt{144 - x^2}$. So the area of the each of the upper triangles is $\frac{1}{2}x\sqrt{25 - x^2}$, and the area of the each of the lower triangles is $\frac{1}{2}x\sqrt{144 - x^2}$. Since there are two upper triangles and two lower triangles, we get that the total area is $A(x) = 2 \cdot \left[\frac{1}{2}x\sqrt{25 - x^2}\right] + 2 \cdot \left[\frac{1}{2}x\sqrt{144 - x^2}\right] = x\left(\sqrt{25 - x^2} + \sqrt{144 - x^2}\right)$.

(b) The function $y = A(x) = x\left(\sqrt{25 - x^2} + \sqrt{144 - x^2}\right)$ is shown in the first viewing rectangle below. In the second viewing rectangle, we isolate the maximum, and we see that the area of the kite is maximized when $x \approx 4.615$. So the length of the horizontal crosspiece must be $2 \cdot 4.615 = 9.23$. The length of the vertical crosspiece is $\sqrt{5^2 - (4.615)^2} + \sqrt{12^2 - (4.615)^2} \approx 13.00$.





Name ____ Date ___________

Chapter 2 Functions

2.1 Functions

I. Functions All Around Us

Give a real-life example of a function.

II. Definition of Function

A **function** f is a ____
____.

The symbol $f(x)$ is read ____________________________ and is called the ____________________, or the _______________________________.

The set A is called the _____________________ of the function, and the **range** of f is ______________
____.

The symbol that represents an arbitrary number in the domain of a function f is called ________________
_________________. The symbol that represents a number in the range of f is called ______________
_________________. If we write $y = f(x)$, then _________ is the independent variable and ______ is the dependent variable.

III. Evaluating a Function

To evaluate a function f at a number, ____.

Example 1: If $f(x) = 50 - 2x^2$, then evaluate $f(5)$.

A **piecewise-defined function** is ____
______________________________.

IV. Domain of a Function

The domain of a function is ____. If the function is given by an algebraic expression and the domain is not stated explicitly, then by convention the domain of the function is ____
____.

Example 2: Find the domain of the function $g(x) = \sqrt{x^2 - 16}$.

V. Four Ways to Represent a Function

List and describe the four ways in which a specific function can be described.

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2.2 Graphs of Functions

I. Graphing Functions by Plotting Points

To graph a function *f*, ____

____

____.

If *f* is a function with domain *A*, then the **graph** of *f* is the set of ordered pairs ______________________, plotted in a coordinate plane. In other words, the graph of *f* is the set of all points (x, y) such that __________; that is, the graph of *f* is the graph of the equation ____________________________.

A function *f* of the form $f(x) = mx + b$ is called a ____________________________ because its graph is the graph of the equation $y = mx + b$, which represents a line with slope ______________ and *y*-intercept ________________. The function $f(x) = b$, where b is a given number, is called a __________________ ______________________ because all its values are ____________________________________. Its graph is ____.

II. Graphing Functions with a Graphing Calculator

Describe how to use a graphing calculator to graph the function $f(x) = 5x^3 - 2x + 2$.

III. Graphing Piecewise Defined Functions

Describe how to graph the piecewise-defined function $f(x)=\begin{cases} 3-\frac{1}{5}x, & \text{if } x<0 \\ 2x^2, & \text{if } x\geq 0 \end{cases}$.

The **greatest integer function** is defined by [[x]] = ____.

The greatest integer function is an example of a ________________________________.

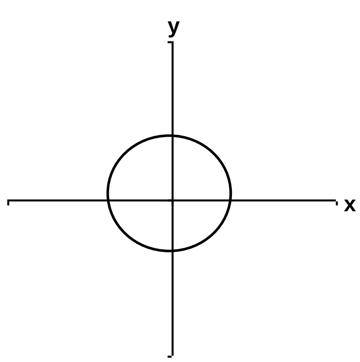
A function is called **continuous** if ____.

IV. The Vertical Line Test: Which Graphs Represent Functions?

The **Vertical Line Test** states that ____

____.

Is the graph below the graph of a function? Explain.

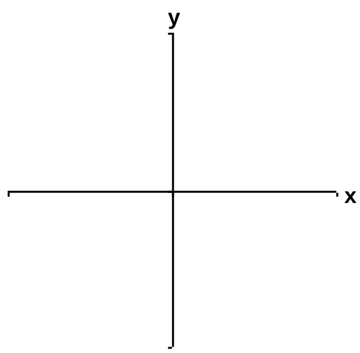


V. Which Equations Represent Functions?

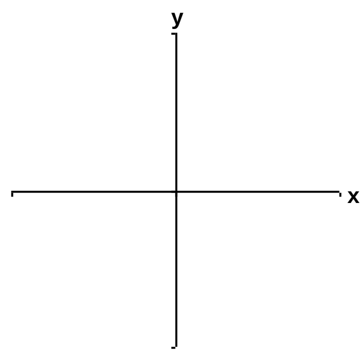
Any equation in the variables x and y defines a relationship between these variables. Does every equation in x and y define y as a function of x? ____________________

Draw an example of the graph of each type of function.

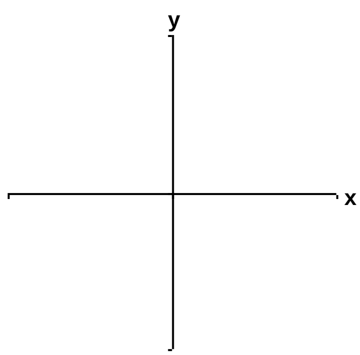
Linear Function



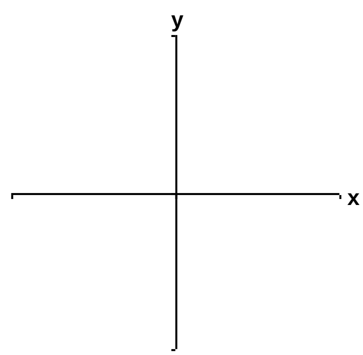
Power Function



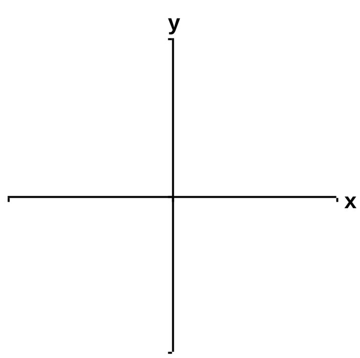
Root Function



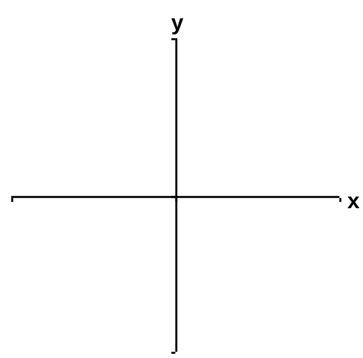
Reciprocal Function



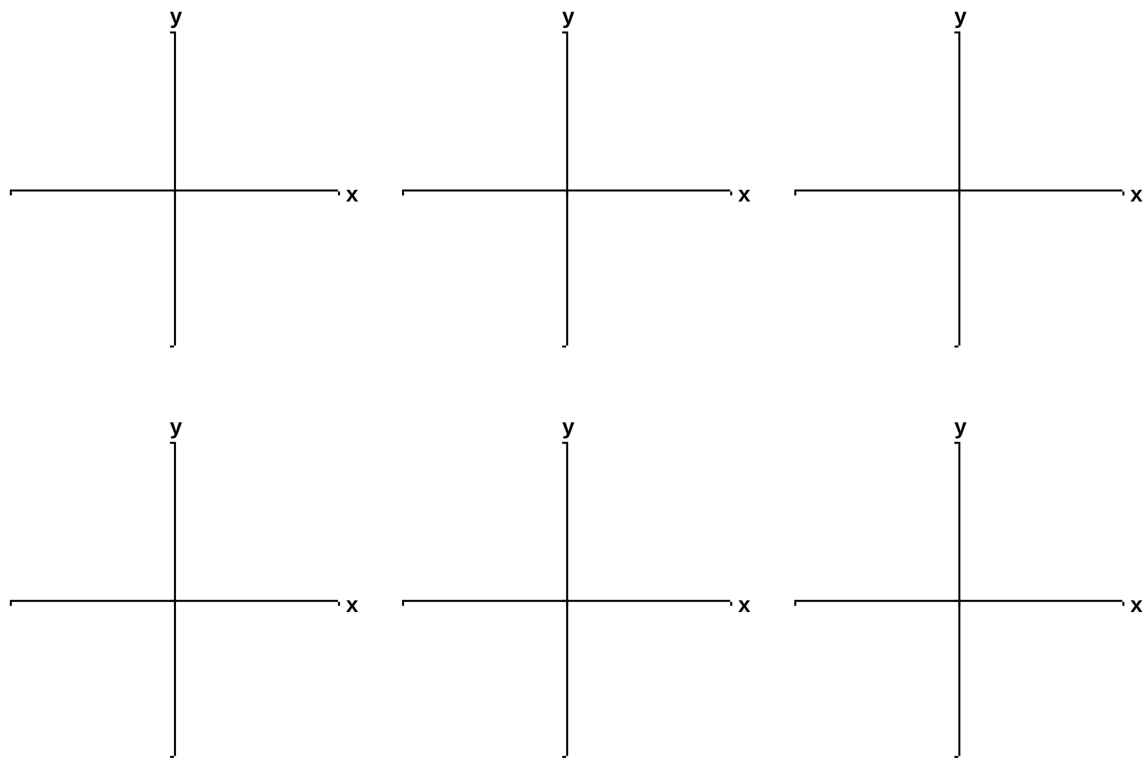
Absolute Value Function



Greatest Integer Function



Additional notes



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2.3 Getting Information from the Graph of a Function

I. Values of a Function; Domain and Range

To analyze the graph of a function, ____

____.

Describe how to use the graph of a function to find the function's domain and range.

II. Comparing Function Values: Solving Equations and Inequalities Graphically

The **solution(s) of the equation** $f(x) = g(x)$ are ____

______________________________. The **solution(s) of the inequality** $f(x) < g(x)$ are __________________

____.

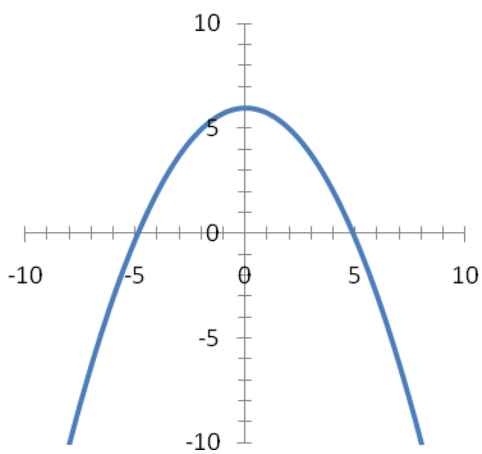
Describe how to solve an equation graphically.

III. Increasing and Decreasing Functions

A function f is said to be *increasing* when ______________________________________ and is said to be decreasing when ____.

According to the definition of increasing and decreasing functions, f is **increasing** on an interval I if ______________________________ whenever ________________________ in I. Similarly, f is **decreasing** on an interval I if ______________________________ whenever ________________________ in I.

Example 1: Use the graph to determine (a) the domain, (b) the range, (c) the intervals on which the function is increasing, and (d) the intervals on which the function is decreasing.

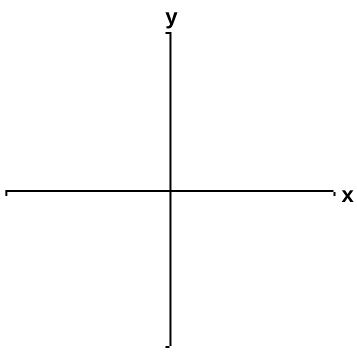


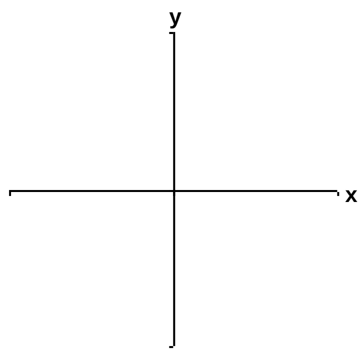
IV. Local Maximum and Minimum Values of a Function

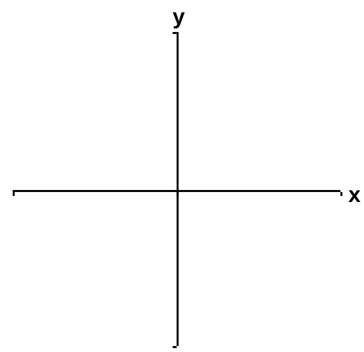
The function value $f(a)$ is a **local maximum value** of f if ____________________ when x is near a. In this case we say that f has ______________________________.

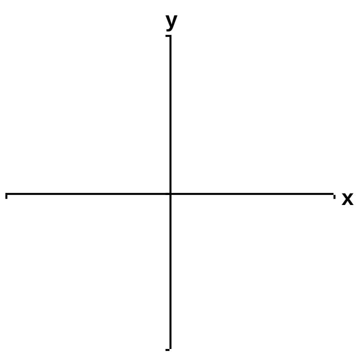
The function value $f(a)$ is a **local minimum value** of f if ____________________ when x is near a. In this case we say that f has ______________________________.

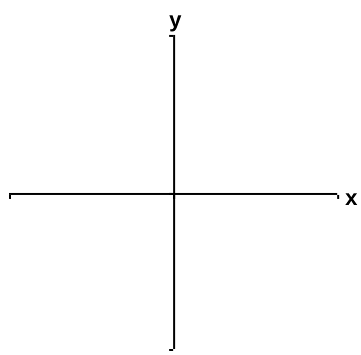
Describe how to use a graphing calculator to find the local maximum and minimum values of a function.

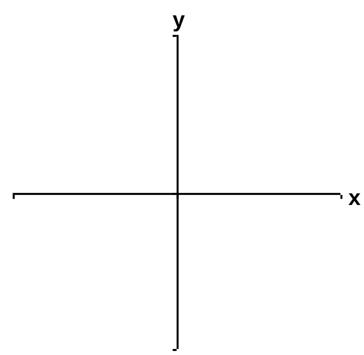












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2.4 Average Rate of Change of a Function

I. Average Rate of Change

The **average rate of change** of the function $y = f(x)$ between $x = a$ and $x = b$ is

____

The average rate of change is the slope of the ________________________ between $x = a$ and $x = b$ on the graph of *f*, that is, the line that passes through _____________________________________.

Example 1: For the function $f(x) = 3x^2 - 2$, find the average rate of change between $x = 2$ and $x = 4$.

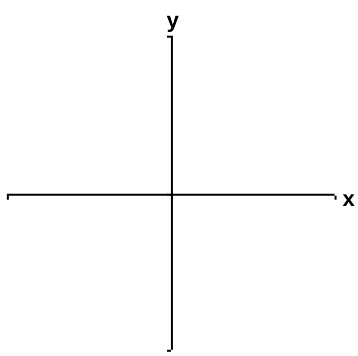
II. Linear Functions Have Constant Rate of Change

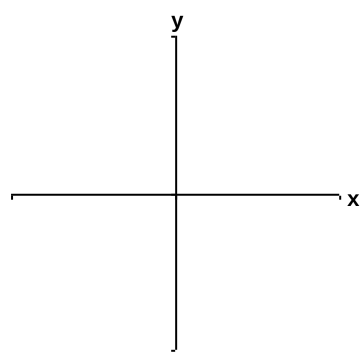
For a linear function $f(x) = mx + b$, the average rate of change between any two points is ________________ _____________________________. If a function *f* has constant average rate of change, then it must be ______ ________________________________.

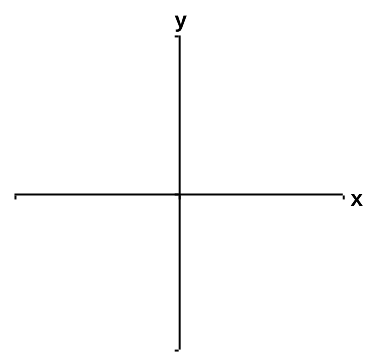
Example 2: For the function $f(x) = 14 - 6x$, find the average rate of change between the following points.

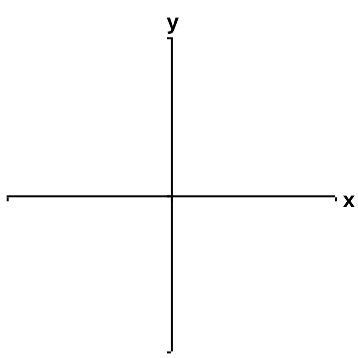
(a) $x = -10$ and $x = -5$
(b) $x = 0$ and $x = 3$
(c) $x = 4$ and $x = 9$

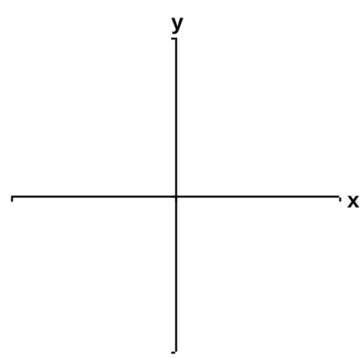
Additional notes

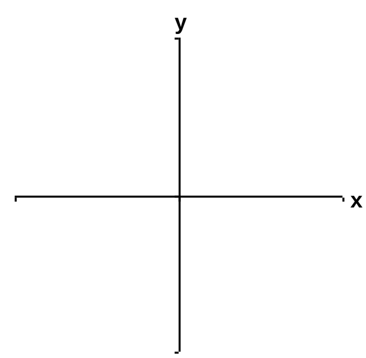












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2.5 Linear Functions and Models

I. Linear Functions

A **linear function** is a function of the form __________________________________. The graph of a linear function is ____.

II. Slope and Rate of Change

For the linear function $f(x) = ax + b$, the slope of the graph of *f* and the rate of change of *f* are both equal to

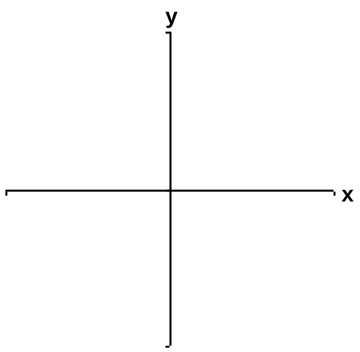
____.

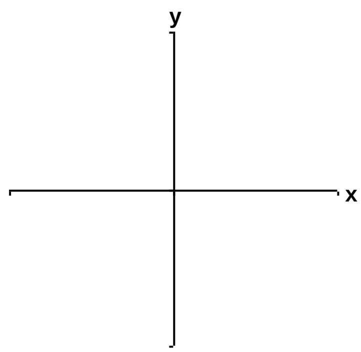
III. Making and Using Linear Models

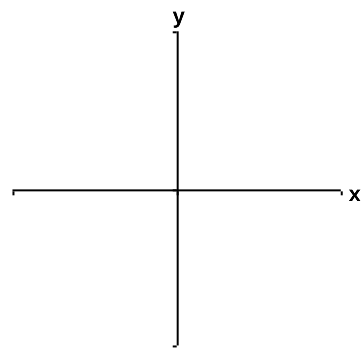
When a linear function is used to model the relationship between two quantities, the slope of the graph of the function is ____.

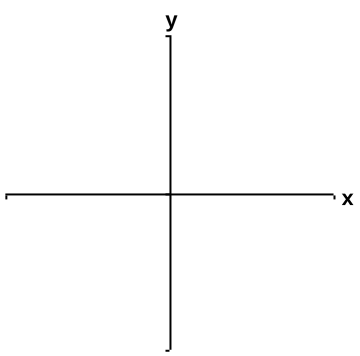
Give an example of a real-world situation that involves a constant rate of change.

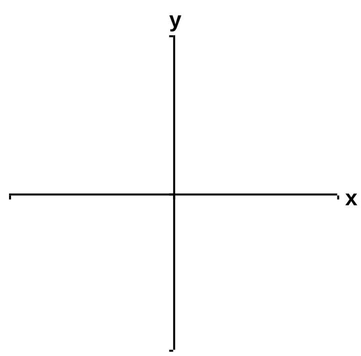
Additional notes

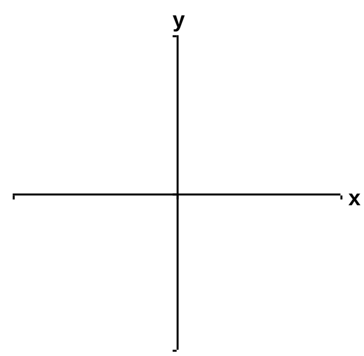












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2.6 Transformations of Functions

I. Vertical Shifting

Adding a constant to a function shifts its graph ________________: upward if the constant is ________________ and downward if it is ________________.

Consider vertical shifts of graphs. Suppose $c > 0$. To graph $y = f(x) + c$, shift ________________ ________________. To graph $y = f(x) - c$, shift ________________ ________________.

II. Horizontal Shifting

Consider horizontal shifts of graphs. Suppose $c > 0$. To graph $y = f(x - c)$, shift ________________ ________________. To graph $y = f(x + c)$, shift ________________ ________________.

III. Reflecting Graphs

To graph $y = -f(x)$, reflect the graph of $y = f(x)$ in the ________________.

To graph $y = f(-x)$, reflect the graph of $y = f(x)$ in the ________________.

IV. Vertical Stretching and Shrinking

Multiplying the *y*-coordinates of the graph of $y = f(x)$ by c has the effect of ________________ ________________.

To graph $y = cf(x)$:

If $c > 1$, ________________________________.

If $0 < c < 1$, ________________________________.

V. Horizontal Stretching and Shrinking

To graph $y = f(cx)$:

If $c > 1$, ________________________________.

If $0 < c < 1$, ________________________________.

VI. Even and Odd Functions

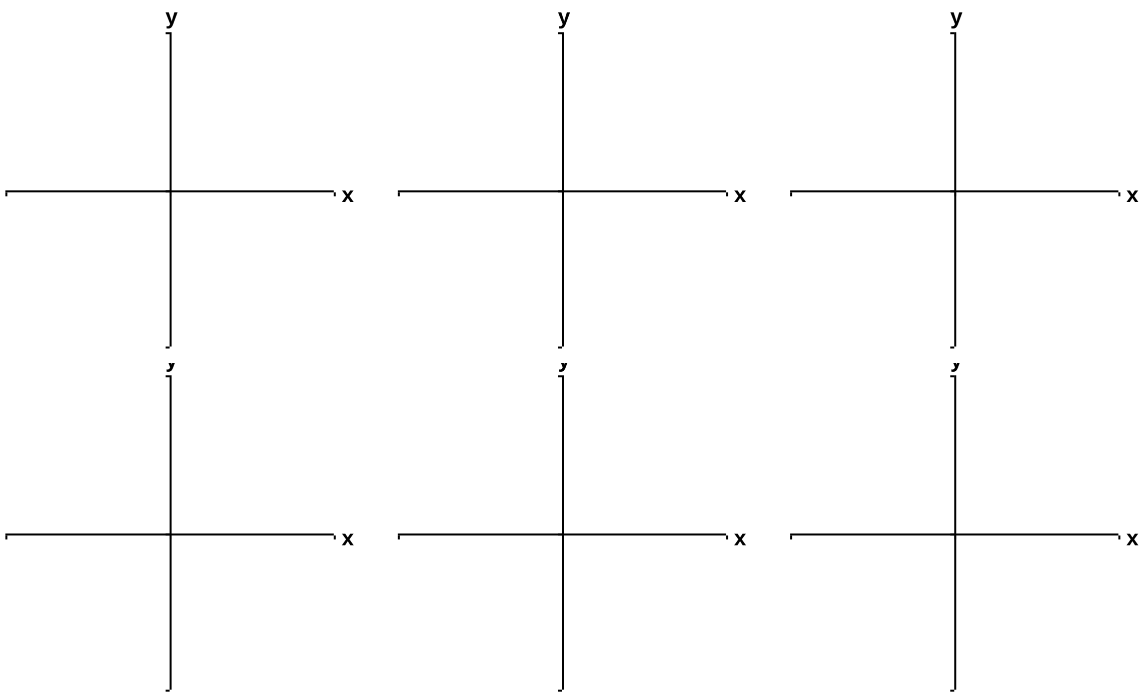
Let *f* be a function.

Then *f* is **even** if ____.

Then *f* is **odd** if ____.

The graph of an even function is symmetric with respect to ____________________.

The graph of an odd function is symmetric with respect to ____________________.



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2.7 Combining Functions

I. Sums, Differences, Products, and Quotients

Given two functions f and g, we define the new function $f+g$ by ______________________________.
The new function $f+g$ is called ____. Its value at x is ______________________________.

Let f and g be functions with domains A and B. Then the functions $f+g$, $f-g$, fg, and f/g are defined as follows.

$(f+g)(x) =$ ______________________________, Domain is ____________________

$(f-g)(x) =$ ______________________________, Domain is ____________________

$(fg)(x) =$ ______________________________, Domain is ____________________

$\left(\dfrac{f}{g}\right)(x) =$ ______________________________, Domain is ______________________________

The graph of the function $f+g$ can be obtained from the graphs of f and g by **graphical addition,** meaning that we ____.

Example 2: Let $f(x) = 3x+1$ and $g(x) = 2x^2 - 1$.

(a) Find the function $g - f$.

(b) Find the function $f + g$.

II. Composition of Functions

Given two functions f and g, the **composite function** $f \circ g$ (also called ______________________________ ______________________) is defined by ____ .

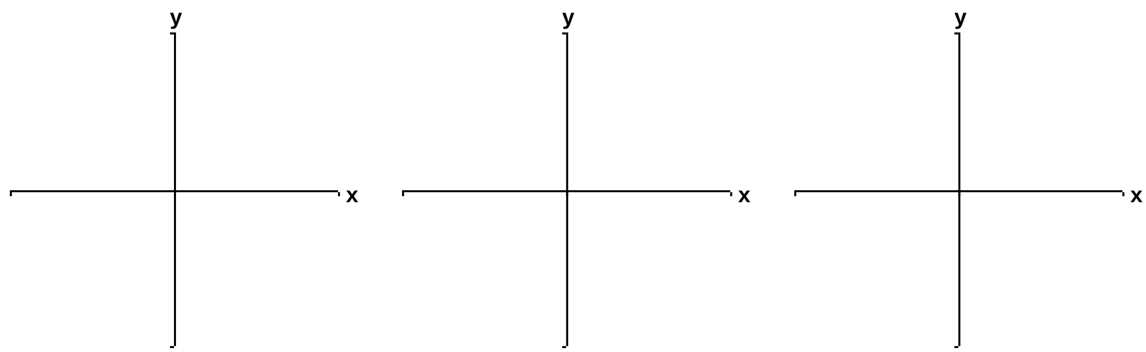
The domain of $f \circ g$ is ____ __________________________________. In other words, $(f \circ g)(x)$ is defined whenever ____________________ ____.

Example 2: Let $f(x) = 3x + 1$ and $g(x) = 2x^2 - 1$.

(a) Find the function $f \circ g$.

(b) Find $(f \circ g)(2)$.

It is possible to take the composition of three or more functions. For instance, the composite function $f \circ g \circ h$ is found by ____ .



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2.8 One-to-One Functions and Their Inverses

I. One-to-One Functions

A function with domain A is called a **one-to-one function** if ____

____ .

An equivalent way of writing the condition for a one-to-one function is this:

____ .

The **Horizontal Line Test** states that ____

____.

Every increasing function and every decreasing function is ______________________________.

II. The Inverse of a Function

Let f be a one-to-one function with domain A and range B. Then its __________________________ f^{-1} has domain B and range A and is defined by $f^{-1}(y) = x \iff f(x) = y$ for any y in B.

Let f be a one-to-one function with domain A and range B. The inverse function f^{-1} satisfies the following cancellation properties:

1)

2)

Conversely, any function f^{-1} satisfying these equations is ______________________________________.

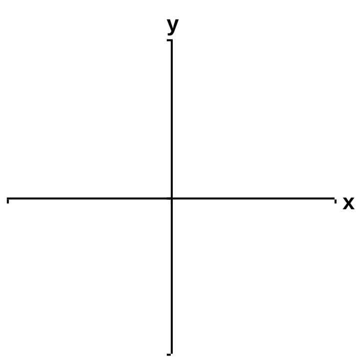
III. Finding the Inverse of a Function

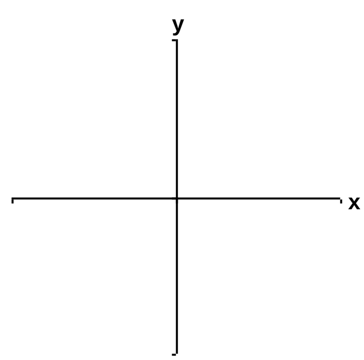
Describe how to find the inverse of a one-to-one function.

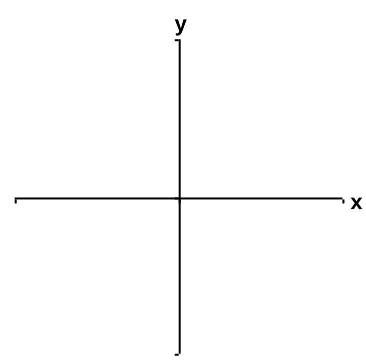
Example 1: Find the inverse of the function $f(x) = 9 - 2x$.

IV. Graphing the Inverse of a Function

The graph of f^{-1} is obtained by ____.







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