

NOT FOR SALE

Section 1.2 Functions

- domain, range, function
- independent, dependent
- No. The input element $x = 3$ cannot be assigned to more than exactly one output element.
- To find $g(x+1)$ for $g(x) = 3x - 2$, substitute x with the quantity $x + 1$.

$$g(x+1) = 3(x+1) - 2$$

$$= 3x + 3 - 2$$

$$= 3x + 1$$
- No. The domain of the function $f(x) = \sqrt{1+x}$ is $[-1, \infty)$ which does not include $x = -2$.
- The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.
- Yes. Each domain value is matched with only one range value.
- No. The domain value of -1 is matched with two output values.
- No. The National Football Conference, an element in the domain, is assigned to three elements in the range, the Giants, the Saints, and the Seahawks; The American Football Conference, an element in the domain, is also assigned to three elements in the range, the Patriots, the Ravens, and the Steelers.
- Yes. Each element, or state, in the domain is assigned to exactly one element, or electoral votes, in the range.
- Yes, the table represents y as a function of x . Each domain value is matched with only one range value.
- No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.
- No, the graph does not represent a function. The input values 1, 2, and 3 are each matched with two outputs.
- Yes, the graph represents a function. Each input value is matched with one output value.
- (a) Each element of A is matched with exactly one element of B , so it does represent a function.
 (b) The element 1 in A is matched with two elements, -2 and 1 of B , so it does not represent a function.
 (c) Each element of A is matched with exactly one element of B , so it does represent a function.
- (a) The element c in A is matched with two elements, 2 and 3 of B , so it is not a function.
 (b) Each element of A is matched with exactly one element of B , so it does represent a function.
 (c) This is not a function from A to B (it represents a function from B to A instead).
- Both are functions. For each year there is exactly one and only one average price of a name brand prescription and average price of a generic prescription.
- Since $b(t)$ represents the average price of a name brand prescription, $b(2009) \approx \$151$. Since $g(t)$ represents the average price of a generic prescription, $g(2006) \approx \$31$.
- $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4-x^2}$
 Thus, y is *not* a function of x . For instance, the values $y = 2$ and $y = -2$ both correspond to $x = 0$.
- $x = y^2 + 1$
 $y = \pm\sqrt{x-1}$
 This is *not* a function of x . For example, the values $y = 2$ and $y = -2$ both correspond to $x = 5$.
- $y = \sqrt{x^2 - 1}$
 This is a function of x .
- $y = \sqrt{x+5}$
 This is a function of x .
- $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$
 Thus, y is a function of x .
- $x = -y + 5 \Rightarrow y = -x + 5$
 This is a function of x .
- $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$
 Thus, y is *not* a function of x . For instance, the values $y = \sqrt{3}$ and $y = -\sqrt{3}$ both correspond to $x = 2$.
- $x + y^2 = 3 \Rightarrow y = \pm\sqrt{3-x}$
 Thus, y is *not* a function of x .
- $y = |4 - x|$
 This is a function of x .
- $|y| = 3 - 2x \Rightarrow y = 3 - 2x$ or $y = -(3 - 2x)$
 Thus, y is *not* a function of x .
- $x = -7$ does not represent y as a function of x . All values of y correspond to $x = -7$.
- $y = 8$ is a function of x , a constant function.
- $f(t) = 3t + 1$
 (a) $f(2) = 3(2) + 1 = 7$
 (b) $f(-4) = 3(-4) + 1 = -11$
 (c) $f(t+2) = 3(t+2) + 1 = 3t + 7$

INSTRUCTOR USE ONLY

32. $g(y) = 7 - 3y$

(a) $g(0) = 7 - 3(0) = 7$

(b) $g\left(\frac{7}{3}\right) = 7 - 3\left(\frac{7}{3}\right) = 0$

(c) $g(s + 5) = 7 - 3(s + 5)$
 $= 7 - 3s - 15 = -3s - 8$

33. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x - 4) = (x - 4)^2 - 2(x - 4)$
 $= x^2 - 8x + 16 - 2x + 8$
 $= x^2 - 10x + 24$

34. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3} \cdot \frac{27}{8}\pi = \frac{9\pi}{2}$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{32\pi r^3}{3}$

35. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

36. $f(x) = \sqrt{x+8} + 2$

(a) $f(-4) = \sqrt{-4+8} + 2 = 4$

(b) $f(8) = \sqrt{8+8} + 2 = 6$

(c) $f(x-8) = \sqrt{x-8+8} + 2 = \sqrt{x} + 2$

37. $q(x) = \frac{1}{x^2 - 9}$

(a) $q(-3) = \frac{1}{(-3)^2 - 9} = \frac{1}{9 - 9} = \frac{1}{0}$ undefined

(b) $q(2) = \frac{1}{(2)^2 - 9} = \frac{1}{4 - 9} = -\frac{1}{5}$

(c) $q(y+3) = \frac{1}{(y+3)^2 - 9} = \frac{1}{y^2 + 6y + 9 - 9} = \frac{1}{y^2 + 6y}$

38. $q(t) = \frac{2t^2 + 3}{t^2}$

(a) $q(2) = \frac{2(2)^2 + 3}{(2)^2} = \frac{8 + 3}{4} = \frac{11}{4}$

(b) $q(0) = \frac{2(0)^2 + 3}{(0)^2}$ Division by zero is undefined.

(c) $q(-x) = \frac{2(-x)^2 + 3}{(-x)^2} = \frac{2x^2 + 3}{x^2}$

39. $f(x) = \frac{|x|}{x}$

(a) $f(9) = \frac{|9|}{9} = 1$

(b) $f(-9) = \frac{|-9|}{-9} = -1$

(c) $f(t) = \frac{|t|}{t} = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$

 $f(0)$ is undefined.

40. $f(x) = |x| + 4$

(a) $f(5) = |5| + 4 = 9$

(b) $f(-5) = |-5| + 4 = 9$

(c) $f(t) = |t| + 4$

41. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

42. $f(x) = \begin{cases} 2x + 5, & x \leq 0 \\ 2 - x, & x > 0 \end{cases}$

(a) $f(-2) = 2(-2) + 5 = 1$

(b) $f(0) = 2(0) + 5 = 5$

(c) $f(1) = 2 - 1 = 1$

43. $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

$$44. f(x) = \begin{cases} x^2 - 4, & x \leq 0 \\ 1 - 2x^2, & x > 0 \end{cases}$$

- (a) $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$
 (b) $f(0) = 0^2 - 4 = -4$
 (c) $f(1) = 1 - 2(1^2) = 1 - 2 = -1$

$$45. f(x) = \begin{cases} x + 2, & x < 0 \\ 4, & 0 \leq x < 2 \\ x^2 + 1, & x \geq 2 \end{cases}$$

- (a) $f(-2) = (-2) + 2 = 0$
 (b) $f(0) = 4$
 (c) $f(2) = (2)^2 + 1 = 5$

$$46. f(x) = \begin{cases} 5 - 2x, & x < 0 \\ 5, & 0 \leq x < 1 \\ 4x + 1, & x \geq 1 \end{cases}$$

- (a) $f(-4) = 5 - 2(-4) = 13$
 (b) $f(0) = 5$
 (c) $f(1) = 4(1) + 1 = 5$

$$47. f(x) = (x - 1)^2$$

$$\{(-2, 9), (-1, 4), (0, 1), (1, 0), (2, 1)\}$$

$$48. f(x) = x^2 - 3$$

$$\{(-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1)\}$$

$$49. f(x) = |x| + 2$$

$$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$$

$$50. f(x) = |x + 1|$$

$$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$$

$$51. h(t) = \frac{1}{2}|t + 3|$$

$$h(-5) = \frac{1}{2}|-5 + 3| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$$

$$h(-4) = \frac{1}{2}|-4 + 3| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$$

$$h(-3) = \frac{1}{2}|-3 + 3| = \frac{1}{2}|0| = 0$$

$$h(-2) = \frac{1}{2}|-2 + 3| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$$

$$h(-1) = \frac{1}{2}|-1 + 3| = \frac{1}{2}|2| = \frac{1}{2}(2) = 1$$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

$$52. f(s) = \frac{|s - 2|}{s - 2}$$

$$f(0) = \frac{|0 - 2|}{0 - 2} = \frac{2}{-2} = -1$$

$$f(1) = \frac{|1 - 2|}{1 - 2} = \frac{1}{-1} = -1$$

$$f\left(\frac{3}{2}\right) = \frac{\left|\frac{3}{2} - 2\right|}{\frac{3}{2} - 2} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$$

$$f\left(\frac{5}{2}\right) = \frac{\left|\frac{5}{2} - 2\right|}{\frac{5}{2} - 2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$f(4) = \frac{|4 - 2|}{4 - 2} = \frac{2}{2} = 1$$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	-1	-1	-1	1	1

$$53. f(x) = 15 - 3x = 0 \\ 3x = 15 \\ x = 5$$

$$54. f(x) = 5x + 1 = 0 \\ 5x = -1 \\ x = -\frac{1}{5}$$

55. $f(x) = \frac{9x - 4}{5} = 0$

$$\begin{aligned} 9x - 4 &= 0 \\ 9x &= 4 \\ x &= \frac{4}{9} \end{aligned}$$

56. $f(x) = \frac{2x - 3}{7} = 0$

$$\begin{aligned} 2x - 3 &= 0 \\ 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

57. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers x .

58. $g(x) = 1 - 2x^2$

Because $g(x)$ is a polynomial, the domain is all real numbers x .

59. $h(t) = \frac{4}{t}$

Domain: All real numbers except $t = 0$

60. $s(y) = \frac{3y}{y + 5}$

$$\begin{aligned} y + 5 &\neq 0 \\ y &\neq -5 \end{aligned}$$

The domain is all real numbers $y \neq -5$.

61. $f(x) = \sqrt[3]{x - 4}$

Domain: all real numbers x

62. $f(x) = \sqrt[4]{x^2 + 3x}$

$$x^2 + 3x = x(x + 3) \geq 0$$

Domain: $x \leq -3$ or $x \geq 0$

63. $g(x) = \frac{1}{x} - \frac{3}{x + 2}$

Domain: All real numbers except $x = 0, x = -2$

64. $h(x) = \frac{10}{x^2 - 2x}$

$$\begin{aligned} x^2 - 2x &\neq 0 \\ x(x - 2) &\neq 0 \end{aligned}$$

The domain is all real numbers except $x = 0, x = 2$.

65. $g(y) = \frac{y + 2}{\sqrt{y - 10}}$

$$\begin{aligned} y - 10 &> 0 \\ y &> 10 \end{aligned}$$

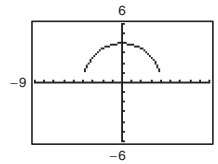
Domain: all $y > 10$

66. $f(x) = \frac{\sqrt{x + 6}}{6 + x}$

$x + 6 \geq 0$ for numerator and $x \neq -6$ for denominator.

Domain: all $x > -6$

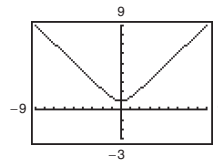
67. $f(x) = \sqrt{16 - x^2}$



Domain: $[-4, 4]$

Range: $[0, 4]$

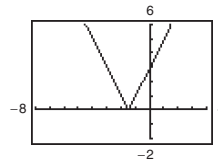
68. $f(x) = \sqrt{x^2 + 1}$



Domain: all real numbers

Range: $1 \leq y$

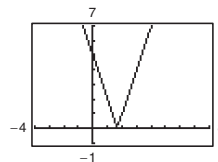
69. $g(x) = |2x + 3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

70. $g(x) = |3x - 5|$



Domain: all real numbers

Range: $y \geq 0$

71. $A = \pi r^2$, $C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi \left(\frac{C}{2\pi} \right)^2 = \frac{C^2}{4\pi}$$

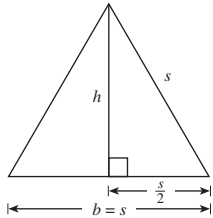
72. $A = \frac{1}{2}bh$, in an equilateral triangle $b = s$ and:

$$s^2 = h^2 + \left(\frac{s}{2} \right)^2$$

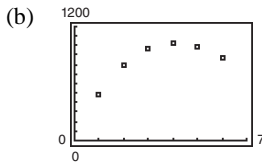
$$h = \sqrt{s^2 - \left(\frac{s}{2} \right)^2}$$

$$h = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \frac{\sqrt{3}s}{2}$$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}s}{2} = \frac{\sqrt{3}s^2}{4}$$



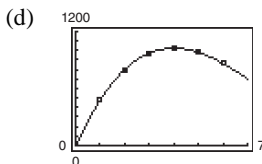
73. (a) From the table, the maximum volume seems to be 1024 cm³, corresponding to $x = 4$.



Yes, V is a function of x .

(c) $V = \text{length} \times \text{width} \times \text{height}$
 $= (24 - 2x)(24 - 2x)x$
 $= x(24 - 2x)^2 = 4x(12 - x)^2$

Domain: $0 < x < 12$



The function is a good fit. Answers will vary.

74. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy$.

Since $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair of points are equal.

$$\frac{1-y}{2-0} = \frac{1-0}{2-x}$$

$$1-y = \frac{2}{2-x}$$

$$y = 1 - \frac{2}{2-x} = \frac{x}{x-2}$$

Therefore, $A = \frac{1}{2}xy = \frac{1}{2}x \left(\frac{x}{x-2} \right) = \frac{x^2}{2x-4}$.

The domain is $x > 2$, since $A > 0$.

75. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

76. (a) $V = (\text{length})(\text{width})(\text{height}) = yx^2$

But, $y + 4x = 108$, or $y = 108 - 4x$.

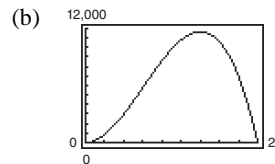
Thus, $V = (108 - 4x)x^2$.

Since $y = 108 - 4x > 0$

$$4x < 108$$

$$x < 27.$$

Domain: $0 < x < 27$



(c) The highest point on the graph occurs at $x = 18$. The dimensions that maximize the volume are $18 \times 18 \times 36$ inches.

77. (a) Total cost = Variable costs + Fixed costs

$$C = 68.75x + 248,000$$

(b) Revenue = Selling price \times Units sold

$$R = 99.99x$$

(c) Since $P = R - C$

$$P = 99.99x - (68.75x + 248,000)$$

$$P = 31.24x - 248,000.$$

78. (a) The independent variable is x and represents the month. The dependent variable is y and represents the monthly revenue.

$$(b) f(x) = \begin{cases} -1.97x + 26.3, & 7 \leq x \leq 12 \\ 0.505x^2 - 1.47x + 6.3, & 1 \leq x \leq 6 \end{cases}$$

Answers will vary.

- (c) $f(5) = 11.575$, and represents the revenue in May: \$11,575.
 (d) $f(11) = 4.63$, and represents the revenue in November: \$4630.
 (e) The values obtained from the model are close approximations to the actual data.
79. (a) The independent variable is t and represents the year. The dependent variable is n and represents the numbers of miles traveled.

t	0	1	2	3	4	5
$n(t)$	3.95	3.96	3.98	3.99	4.00	4.02

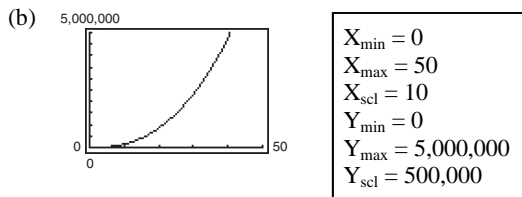
t	6	7	8	9	10	11
$n(t)$	4.03	4.04	4.05	4.07	4.08	4.09

- (c) The model fits the data well.
 (d) Sample answer: No. The function may not accurately model other years.
80. (a) $F(y) = 149.76\sqrt{10}y^{5/2}$

y	5	10	20	30	40
$F(y)$	26,474	149,760	847,170	2,334,527	4,792,320

(Answers will vary.)

F increases very rapidly as y increases.



- (c) From the table, $y \approx 22$ ft (slightly above 20). You could obtain a better approximation by completing the table for values of y between 20 and 30.
 (d) By graphing $F(y)$ together with the horizontal line $y_2 = 1,000,000$, you obtain $y \approx 21.37$ feet.
81. Yes. If $x = 30$, $y = -0.01(30)^2 + 3(30) + 6$
 $y = 6$ feet

Since the child trying to catch the throw is holding the glove at a height of 5 feet, the ball will fly over the glove.

82. (a) $\frac{f(2013) - f(2005)}{2013 - 2005} \approx \525 million/year

This represents the increase in sales per year from 2005 to 2013.

(b)

t	5	6	7	8	9
$S(t)$	217.3	136.9	237.4	518.8	981.1

t	10	11	12	13
$S(t)$	1624.2	2448.2	3453.1	4638.9

The model approximates the data well.

83. $f(x) = 2x$

$$\frac{f(x+c) - f(x)}{c} = \frac{2(x+c) - 2x}{c} = \frac{2c}{c} = 2, c \neq 0$$

84. $g(x) = 3x - 1$

$$g(x+h) = 3(x+h) - 1 = 3x + 3h - 1$$

$$g(x+h) - g(x) = (3x + 3h - 1) - (3x - 1) = 3h$$

$$\frac{g(x+h) - g(x)}{h} = \frac{3h}{h} = 3, h \neq 0$$

85. $f(x) = x^2 - x + 1, f(2) = 3$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - (2+h) + 1 - 3}{h} = \frac{4 + 4h + h^2 - 2 - h + 1 - 3}{h} = \frac{h^2 + 3h}{h} = h + 3, h \neq 0$$

86. $f(x) = x^3 + x$

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$

$$f(x+h) - f(x) = (x^3 + 3x^2h + 3xh^2 + h^3 + x + h) - (x^3 + x) = 3x^2h + 3xh^2 + h^3 + h = h(3x^2 + 3xh + h^2 + 1)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = 3x^2 + 3xh + h^2 + 1, h \neq 0$$

87. False. The range of $f(x)$ is $(-1, \infty)$.

88. True. The first number in each ordered pair corresponds to exactly one second number.

89. $f(x) = \sqrt{x} + 2$

Domain: $[0, \infty)$ or $x \geq 0$

Range: $[2, \infty)$ or $y \geq 2$

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20 Chapter 1 Functions and Their Graphs

90. $f(x) = \sqrt{x+3}$

Domain: $[-3, \infty)$ or $x \geq -3$

Range: $[0, \infty)$ or $y \geq 0$

91. No, f is not the independent variable. Because the value of f depends on the value of x , x is the independent variable and f is the dependent variable.

92. (a) The height h is a function of t because for each value of t there is exactly one corresponding value of h for $0 \leq t \leq 2.6$.

(b) The height after 0.5 second is about 20 feet. The height after 1.25 seconds is about 28 feet.

(c) From the graph, the domain is $0 \leq t \leq 2.6$.

(d) The time t is not a function of h because some values of h correspond to more than one value of t .

93. $12 - \frac{4}{x+2} = \frac{12(x+2) - 4}{x+2} = \frac{12x+20}{x+2}$

94.
$$\begin{aligned} \frac{3}{x^2 + x - 20} + \frac{2x}{x^2 + 4x - 5} &= \frac{3}{(x+5)(x-4)} + \frac{2x}{(x+5)(x-1)} \\ &= \frac{3(x-1)}{(x+5)(x-4)(x-1)} + \frac{2x(x-4)}{(x+5)(x-1)(x-4)} \\ &= \frac{3x-3+2x^2-8x}{(x+5)(x-4)(x-1)} \\ &= \frac{2x^2-5x-3}{(x+5)(x-4)(x-1)} \end{aligned}$$

95.
$$\begin{aligned} \frac{2x^3+11x^2-6x}{5x} \cdot \frac{x+10}{2x^2+5x-3} &= \frac{x(2x^2+11x-6)(x+10)}{5x(2x-1)(x+3)} \\ &= \frac{(2x-1)(x+6)(x+10)}{5(2x-1)(x+3)} \\ &= \frac{(x+6)(x+10)}{5(x+3)}, x \neq 0, \frac{1}{2} \end{aligned}$$

96.
$$\frac{x+7}{2(x-9)} \div \frac{x-7}{2(x-9)} = \frac{x+7}{2(x-9)} \cdot \frac{2(x-9)}{x-7} = \frac{x+7}{x-7}, x \neq 9$$

Section 1.3 Graphs of Functions

1. decreasing

2. even

3. Domain: $1 \leq x \leq 4$ or $[1, 4]$

4. No. If a vertical line intersects the graph more than once, then it does not represent y as a function of x .

5. If $f(2) \geq f(2)$ for all x in $(0, 3)$, then $(2, f(2))$ is a relative maximum of f .

6. Since $f(x) = \lceil x \rceil = n$, where n is an integer and $n \leq x$, the input value of x needs to be greater than or equal to 5 but less than 6 in order to produce an output value of 5. So the interval $[5, 6)$ would yield a function value of 5.

7. Domain: all real numbers, $(-\infty, \infty)$

Range: $(-\infty, 1]$

$f(0) = 1$

8. Domain: all real numbers, $(-\infty, \infty)$

Range: all real numbers, $(-\infty, \infty)$

$f(0) = 2$

9. Domain: $[-4, 4]$

Range: $[0, 4]$

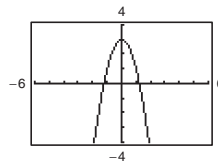
$f(0) = 4$

10. Domain: all real numbers, $(-\infty, \infty)$

Range: $[-3, \infty)$

$f(0) = -3$

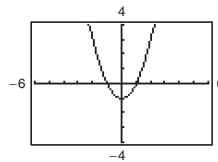
11. $f(x) = -2x^2 + 3$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

12. $f(x) = x^2 - 1$



Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$

INSTRUCTOR USE ONLY