Chapter 3: Resistive Network Analysis – Instructor Notes

Chapter 3 presents the principal topics in the analysis of resistive (DC) circuits. The presentation of node voltage and mesh current analysis is supported by several solved examples and drill exercises, with emphasis placed on developing consistent solution methods, and on reinforcing the use of a systematic approach. The aim of this style of presentation, which is perhaps more detailed than usual in a textbook written for a non-majors audience, is to develop good habits early on, with the hope that the orderly approach presented in Chapter 3 may facilitate the discussion of AC and transient analysis in Chapters 4 and 5. *Make The Connection* sidebars (pp. 83-85) introduce analogies between electrical and thermal circuit elements. These analogies are to be encountered again in Chapter 5. A brief discussion of the principle of superposition precedes the discussion of Thèvenin and Norton equivalent circuits. Again, the presentation is rich in examples and drill exercises, because the concept of equivalent circuits will be heavily exploited in the analysis of AC and transient circuits in later chapters. The *Focus on Methodology* boxes (p. 84 – Node Analysis; p. 94 – Mesh Analysis; pp. 111, 115, 119 – Equivalent Circuits) provide the student with a systematic approach to the solution of all basic network analysis problems.

Following a brief discussion of maximum power transfer, the chapter closes with a section on nonlinear circuit elements and load-line analysis. This section can be easily skipped in a survey course, and may be picked up later, in conjunction with Chapter 9, if the instructor wishes to devote some attention to load-line analysis of diode circuits. Finally, **those instructors who are used to introducing the op-amp as a circuit element, will find that sections 8.1 and 8.2 can be covered together with Chapter 3, and that a good complement of homework problems and exercises devoted to the analysis of the op-amp as a circuit element is provided in Chapter 8.** Modularity is a recurrent feature of this book, and we shall draw attention to it throughout these *Instructor Notes*.

The homework problems present a graded variety of circuit problems. Since the aim of this chapter is to teach solution techniques, there are relatively few problems devoted to applications. We should call the instructor's attention to the following end-of-chapter problems: 3.30 on the Wheatstone bridge; 3.33 and 3.34 on fuses; 3.35-3.37 on electrical power distribution systems; 3.76-83 on various nonlinear resistance devices. The 5th Edition of this book includes 19 new problems; some of the 4th Edition problems were removed, increasing the end-of-chapter problem count from 66 to 83.

Learning Objectives for Chapter 3

1. Compute the solution of circuits containing linear resistors and independent and dependent sources using *node analysis*.
2. Compute the solution of circuits containing linear resistors and independent and dependent sources using *mesh analysis*.
3. Apply the *principle of superposition* to linear circuits containing independent sources.
4. Compute *Thévenin and Norton equivalent circuits* for networks containing linear resistors and independent and dependent sources.
5. Use equivalent circuits ideas to compute the *maximum power transfer* between a source and a load.
6. Use the concept of equivalent circuit to determine voltage, current and power for nonlinear loads using *load-line analysis* and analytical methods.

Sections 3.1, 3.2, 3.3, 3.4: Nodal and Mesh Analysis

Focus on Methodology: Node Voltage Analysis Method

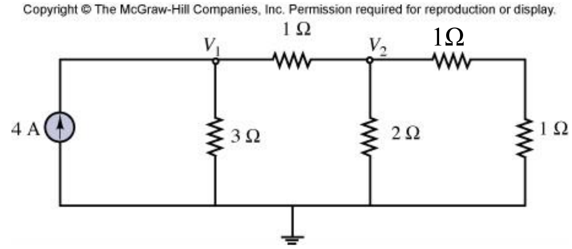
1. Select a reference node(usually ground). This node usually has most elements tied to it. All other nodes will be referenced to this node.
2. Define the remaining $n-1$ node voltages as the independent or dependent variables. Each of the m voltage sources in the circuit will be associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.
4. Solve the linear system of $n-1-m$ unknowns.

Focus on Methodology: Mesh Current Analysis Method

1. Define each mesh current consistently. Unknown mesh currents will be always defined in the clockwise direction; known mesh currents (i.e., when a current source is present) will always be defined in the direction of the current source.
2. In a circuit with n meshes and m current sources, $n-m$ independent equations will result. The unknown mesh currents are the $n-m$ independent variables.
3. Apply KVL to each mesh containing an unknown mesh current, expressing each voltage in terms of one or more mesh currents.
4. Solve the linear system of $n-m$ unknowns.

Problem 3.1

Use node voltage analysis to find the voltages V_1 and V_2 for the circuit of Figure P3.1.



Solution:

Known quantities:

Circuit shown in Figure P3.1

Find:

Voltages v_1 and v_2.

Analysis:

Applying KCL at each of the two nodes, we obtain the following equations:

$$\frac{V_1}{3} + \frac{V_1 - V_2}{1} - 4 = 0$$

$$\frac{V_2}{2} + \frac{V_2}{2} + \frac{V_2 - V_1}{1} = 0$$

Rearranging the equations,

$$\frac{4}{3}V_1 - V_2 = 4$$

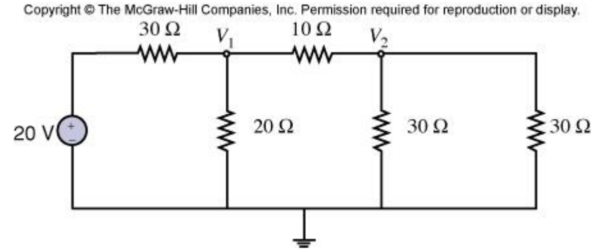
$$-V_1 + 2V_2 = 0$$

Solving the equations,

$$V_1 = 4.8\ V \text{ and } V_2 = 2.4\ V$$

Problem 3.2

Using node voltage analysis, find the voltages V_1 and V_2 for the circuit of Figure P3.2.



Solution:

Known quantities:
Circuit shown in Figure P3.2

Find:
Voltages v_1 and v_2.

Analysis:

Applying KCL at each node, we obtain:

$$\frac{v_1 - 20}{30} + \frac{v_1}{20} + \frac{v_1 - v_2}{10} = 0$$

$$\frac{v_2}{30} + \frac{v_2}{30} + \frac{v_2 - v_1}{10} = 0$$

Rearranging the equations,

$$5.5v_1 - 3v_2 = 20$$

$$-3v_1 + 5v_2 = 0$$

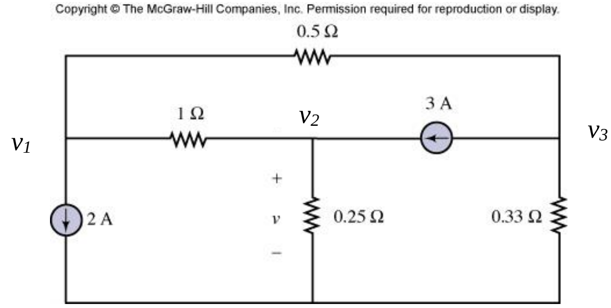
Solving the two equations,

$$v_1 = 5.41 \text{ V} \quad \text{and} \quad v_2 = 3.24 \text{ V}$$

Problem 3.3

Using node voltage analysis in the circuit of Figure P3.3, find the voltage v across the 0.25-ohm resistance.

Solution:



Known quantities:
Circuit shown in Figure P3.3

Find:
Voltages across the resistance, v.

Analysis:
Label the nodes: v_1, v_2, and v_3 as shown.

At node 1:

$$\frac{v_1 - v_2}{1} + \frac{v_1 - v_3}{0.5} = -2$$

At node 2:

$$\frac{v_2 - v_1}{1} + \frac{v_2}{0.25} = 3$$

At node 3:

$$\frac{v_3 - v_1}{0.5} + \frac{v_3}{0.33} = -3$$

Solving for v_2, we find $v_2 = 0.34\text{V}$ and, therefore, $v = 0.34\text{V}$.

Problem 3.4

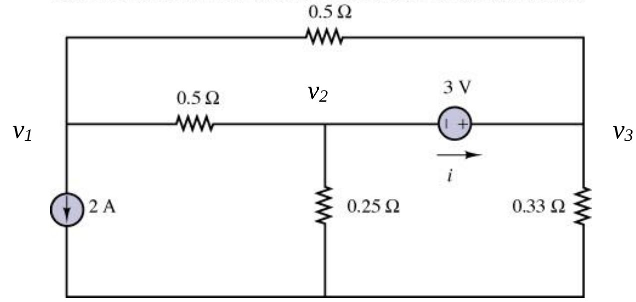
Using node voltage analysis in the circuit of Figure P3.4, find the current *i* through the voltage source.

Solution:

Known quantities:
Circuit shown in Figure P3.4

Find:
Current through the voltage source.

Analysis:
Label the nodes, v_1, v_2, and v_3 as shown.



At node 1:

$$\frac{v_1 - v_2}{0.5} + \frac{v_1 - v_3}{0.5} = -2 \qquad (1)$$

At node 2:

$$\frac{v_2 - v_1}{0.5} + \frac{v_2}{0.25} + i = 0 \qquad (2)$$

At node 3:

$$\frac{v_3 - v_1}{0.5} + \frac{v_3}{0.33} - i = 0 \qquad (3)$$

Further, we know that $v_3 = v_2 + 3$. Now we can eliminate either v_2 or v_3 from the equations, and be left with three equations in three unknowns:

$$\frac{v_1 - v_2}{0.5} + \frac{v_1 - (v_2 + 3)}{0.5} = -2 \qquad (1)$$

$$\frac{v_2 - v_1}{0.5} + \frac{v_2}{0.25} + i = 0 \qquad (2)$$

$$\frac{(v_2 + 3) - v_1}{0.5} + \frac{(v_2 + 3)}{0.33} - i = 0 \qquad (3)$$

Solving the three equations we compute

$$i = 8.310\,\mathrm{A}$$

Problem 3.5

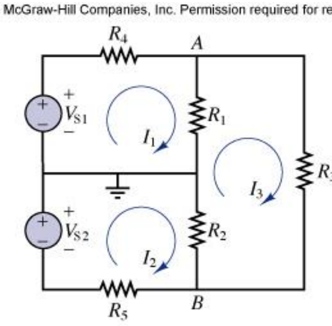
In the circuit shown in Figure P3.5, the mesh currents are $I_1 =$ 5A $I_2 =$ 3A $I_3 =$ 7A
Determine the branch currents through: a. R_1. b. R_2. c. R_3.

Solution:

Known quantities:
Circuit shown in Figure P3.5 with mesh currents: $I_1 = 5$ A, $I_2 = 3$ A, $I_3 =$ 7 A.

Find:
The branch currents through:

a) R_1,
b) R_2,
c) R_3.



Analysis:
a) Assume a direction for the current through R_1 (e.g., from node A toward node B). Then summing currents at node A:

KCL: $-I_1 + I_{R1} + I_3 = 0$

$I_{R1} = I_1 - I_3 = -2$ A

This can also be done by inspection noting that the assumed direction of the current through R_1 and the direction of I_1 are the same.

b) Assume a direction for the current through R_2 (e.g., from node B toward node A). Then summing currents at node B:

KCL: $I_2 + I_{R2} - I_3 = 0$

$I_{R2} = I_3 - I_2 = 4$ A

This can also be done by inspection noting that the assumed direction of the current through R_2 and the direction of I_3 are the same.

c) Only one mesh current flows through R_3. If the current through R_3 is assumed to flow in the same direction, then:

$I_{R3} = I_3 = 7\text{A}$.

Problem 3.6

In the circuit shown in Figure P3.5, the source and node voltages are
VS1 = VS2 = 110 V VA = 103 V VB = −107 V

Solution:

Known quantities:
Circuit shown in Figure P3.5 with source and node voltages: $V_{S1} = V_{S2} = 110\ V$, $V_A = 103\ V$, $V_B = -107\ V$.

Find:
The voltage across each of the five resistors.

Analysis:
Assume a polarity for the voltages across R_1 and R_2 (e.g., from ground to node A, and from node B to ground). R_1 is connected between node A and ground; therefore, the voltage across R_1 is equal to this node voltage. R_2 is connected between node B and ground; therefore, the voltage across R_2 is equal to this voltage.

$V_{R1} = V_A = 103\text{V}, V_{R2} = V_B = -107\text{V}$ (i.e. the voltage drop across R_2 is from ground to node B.)

The two node voltages are with respect to the ground which is given.
Assume a polarity for the voltage across R_3 (e.g., from node B to node A). Then:

KVL: $-V_A - V_{R3} + V_B = 0$

$V_{R3} = V_B - V_A = -210\text{V}$ (i.e. the voltage drop across R_3 is from node A to node B.)

Assume polarities for the voltages across R_4 and R_5 (e.g., from node A to ground , and from ground to node B):

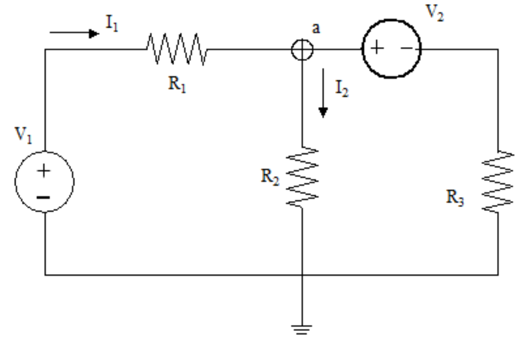
KVL: $-V_{S1} + V_{R4} + V_A = 0$

$V_{R4} = V_{S1} - V_A = 7$ V

KVL: $-V_{S2} - V_B - V_{R5} = 0$

$V_{R5} = -V_{S2} - V_B = -3$ V (i.e. the voltage drop is from node B toward ground

Problem 3.7



Use nodal analysis in the circuit of Figure P3.7 to find the Va. Let R1 = 12Ω, R2 = 6Ω, R3 = 10Ω, V1 = 4V, V2 = 1V.

Solution:

Known quantities:

Circuit shown in Figure P3.7 with known source voltages and resistances, $R_1 = 12\Omega, R_2 = 6\,\Omega, R_3 = 10\,\Omega$.

Find:

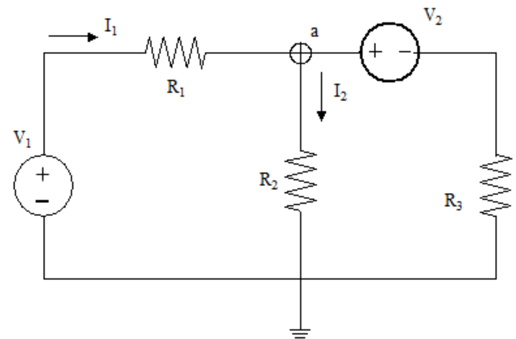
The voltage Va.

Analysis:

Using the node voltage analysis:

$$\frac{V_1 - V_a}{R_1} - \frac{V_a}{R_2} + \frac{V_2 - V_a}{R_3} = 0 \Rightarrow V_a = \frac{26}{21} V = 1.24\text{V}$$

Problem 3.8



Use mesh analysis in the circuit of Figure P3.7 to find the Va. Let R1 = 12Ω, R2 = 6Ω, R3 = 10Ω, V1 = 4V, V2 = 1V.

Solution:

I_1 & I_2 as shown in this diagram are branch currents, not mesh currents. Use i_A & i_B (to avoid confusion with I_1 & I_2) as the mesh currents for the left hand pane and the right hand pane, respectively. Assume that both mesh currents are clockwise.

Known quantities:

Circuit shown in Figure P3.7 with known source voltages and resistances, $R_1 = 12\Omega, R_2 = 6\,\Omega, R_3 = 10\,\Omega$.

Find:

The voltage Va.

Analysis:

Using the mesh current analysis:

$$-V_1 + R_1 i_A + R_2(i_A - i_B) = 0 \quad \textbf{Mesh 1}$$

$$R_2(i_B - i_A) + V_2 + R_3 i_B = 0 \quad \textbf{Mesh 2}$$

Collect coefficients:

$$18 * i_A - 6 * i_B = 4 \textbf{ Mesh 1}$$

$$6 * i_A - 16 * i_B = 1 \textbf{ Mesh 2}$$

Solve set of equations:

$$i_A = \frac{29}{126} A\text{=230mA}$$

$$i_B = \frac{1}{42} A\text{=23.8mA}$$

Find Va:

$$\boldsymbol{V_a = (i_A - i_B) * R_2 = \frac{26}{21}}\textbf{=1.24V}$$

Problem 3.9

Use nodal analysis in the circuit of Figure P3.9 to find V1, V2, and V3. Let R1 = 10Ω, R2 = 8Ω, R3 = 10Ω, R4 = 5, is = 2A, Vs = 1V.

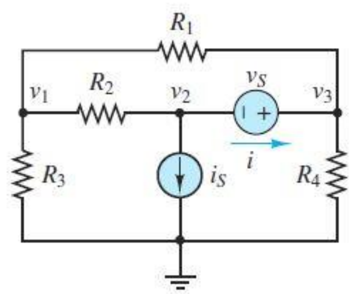


Figure P3.9

Solution:

Known quantities:

Circuit shown in Figure P3.9 with known source voltages and resistances.

Find:

The voltages V1, V2, V3.

Analysis:

Use KCL at all the nodes:

$$\frac{V_3 - V_1}{R_1} - \frac{V_1 - 0}{R_3} - \frac{V_1 - V_2}{R_2} = 0 \; \boldsymbol{Node\ 1}$$

$$\frac{V_1 - V_2}{R_2} - i_s - i = 0 \; \boldsymbol{Node\ 2}$$

$$i - \frac{V_3 - V_1}{R_1} - \frac{V_3}{R_4} = 0 \; \boldsymbol{Node\ 3}$$

$$V_3 - V_2 = 1 \; \boldsymbol{Known\, Voltage}$$

Enter coefficients into matrix:

$$\begin{array}{ccccc} V_1 & V_2 & V_3 & i & X \\ -0.325 & 1/8 & 1/10 & 0 & 0 \\ 1/8 & -1/8 & 0 & -1 & 2 \\ 1/10 & 0 & -0.3 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{array}$$

Solve:

$$\boldsymbol{V_1 = -5.43V \;\; V_2 = -8.29V \;\; V_3 = -7.29V}$$

Problem 3.10

Use nodal analysis in the circuit of Figure P3.10 to find the voltages at nodes A, B, and C. Let $V_1 = 12V$, $V_2 = 10V$, $R_1 = 2\Omega$, $R_2 = 8\Omega$, $R_3 = 12\Omega$, $R_4 = 8\Omega$.

Solution:

Known quantities:

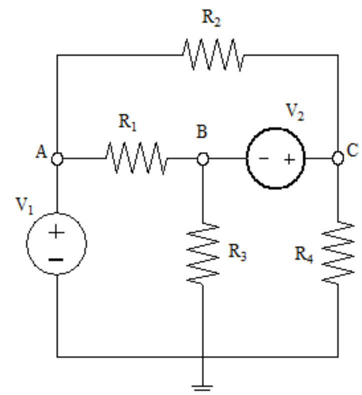
The current source value, the voltage source value and the resistance values for the circuit shown in Figure P3.10.

Find:

The three node voltages indicated in Figure P3.10 using node voltage analysis.

Analysis:

Using the node voltage analysis:



Designate the current through V_2 as i

$$v_A = V_1$$

$$\frac{v_B - v_A}{R_1} + \frac{v_B}{R_3} + i = 0$$

$$\frac{v_C - v_A}{R_3} - i + \frac{v_C}{R_4} = 0$$

$$v_C = v_B + V_2$$

Substituting the know quantities:

$$v_A = 12$$

$$\frac{v_B - 12}{2} + \frac{v_B}{12} + i = 0$$

$$\frac{v_C - 12}{12} - i + \frac{v_C}{8} = 0$$

$$v_C = v_B + 10$$

or

$$v_A = 12\text{V}$$

$$v_B = 6.2\text{V}$$

$$v_C = 16.2\text{V}$$

$$i = 2.38\text{A}$$

Problem 3.11

Use nodal analysis in the circuit of Figure P3.9 to find Va and Vb. Let R1 = 10Ω, R2 = 4Ω, R3 = 6Ω, R4 = 6 Ω, I1 = 2A, V1 = 2V, V2 = 4V.

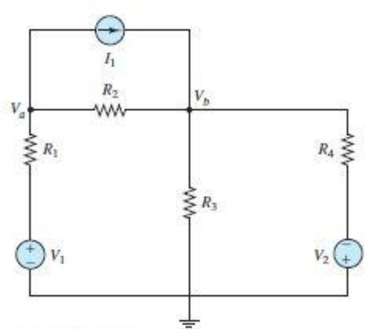


Figure P3.11

Solution:

Known quantities:

Circuit shown in Figure P3.11 with known source voltages and resistances.

Find:

The voltages Va, Vb

Analysis:

Use KCL at all the nodes:

$$\frac{V_1 - V_a}{R_1} - I_1 - \frac{V_a - V_b}{R_2} = 0 \; \boldsymbol{Node\ 1}$$
$$\frac{V_a - V_b}{R_2} + I_1 - \frac{V_b}{R_3} - \frac{V_b - -V_2}{R_4} = 0 \; \boldsymbol{Node\ 2}$$

Substitute known values and collect coefficients:

$$-0.35 * V_a + 0.25 * V_b = 1.8 \; \textbf{Node 1}$$
$$0.25 * V_a - \frac{7}{12} * V_b = -1.33 \; \textbf{Node 2}$$

Solve:

$$\boldsymbol{V_a = -5.06V \;\; V_b = 0.117V}$$

Problem 3.12

Find the power delivered to the load resistor R_0 for the circuit of Figure P3.12, using node voltage analysis, given that $R_1 = 2\Omega$, $R_V = R_2 = R_0 = 4\Omega$, $V_S = 4$ V , and $I_S = 0.5$A.

Solution:

Known quantities:

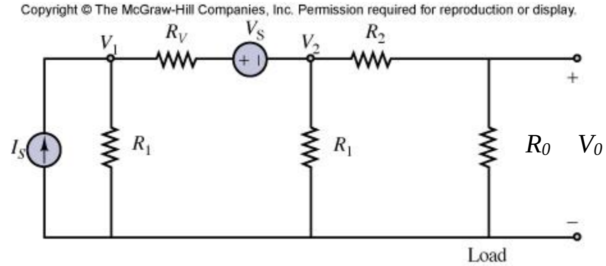
Circuit shown in Figure P3.12.

Find:

Power delivered to the load resistance.

Analysis:

Add node V_3 between the voltage source and its resistance, R_V. Also, add the current i that flows through the voltage source. Choose the negative terminal of V_0 as the ground (reference) node.



KCL at node 1:

$$-0.5 + \frac{V_1}{R_1} + \frac{V_1 - V_3}{R_V} = 0 \qquad \text{(Eq. 1)}$$

KCL at node 2:

$$-i + \frac{V_2}{R_1} + \frac{V_2 - V_0}{R_2} = 0 \qquad \text{(Eq. 2)}$$

KCL at node 3:

$$\frac{V_3 - V_1}{R_V} + i = 0 \qquad \text{(Eq. 3)}$$

KCL at node 0:

$$\frac{V_0 - V_2}{R_2} + \frac{V_0}{R_0} = 0 \qquad \text{(Eq. 4)}$$

And finally:

$V_2 = V_3 + 4$ (Eq. 5)

Solving yields:

$$V_0 = 526.32 \text{ mV}$$

$$P_L = \frac{V_0^2}{R_0} = 69.25 \text{ mW}$$

Problem 3.13

(a) For the circuit of Figure P3.13, write the node equations necessary to find voltages V_1, V_2, and V_3. Note that G = 1/R = conductance. From the results, note the interesting form that the matrices [G] and [I] have taken in the equation [G] [V] = [I] where

$$[G] = \begin{bmatrix} g_{11} & g_{12} & g_{13} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & \cdots & g_{2n} \\ g_{31} & & \ddots & & \\ \vdots & & & \ddots & \\ g_{n1} & g_{n2} & \cdots & \cdots & g_{nn} \end{bmatrix} \quad \text{and } [I] = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ \vdots \\ I_n \end{bmatrix}$$

(b) Write the matrix form of the node voltage equations again, using the following formulas:

g_{ii} =Σ conductances connected to node i

g_{ij} = −Σ conductances shared by nodes i and j

I_i =Σ all source currents into node i

Solution:

Known quantities:

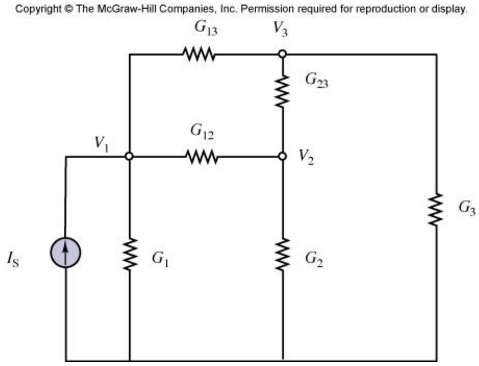
Circuit shown in Figure P3.13.

Find:

a) Voltages

b) Write down the equations in matrix form.

Analysis:



a) Using conductances, apply KCL at node 1:

$$(G_1 + G_{12} + G_{13})V_1 - G_{12}V_2 - G_{13}V_3 = I_s$$

Then apply KCL at node 2:

$$-G_{12}V_1 + (G_2 + G_{12} + G_{23})V_2 - G_{23}V_3 = 0$$

and at node 3:

$$-G_{13}V_1 - G_{23}V_2 + (G_3 + G_{13} + G_{23})V_3 = 0$$

Rewriting in the form

$$[G][V]=[I]$$

we have

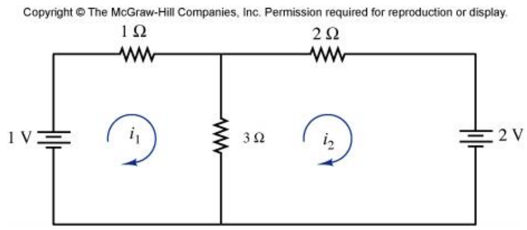
$$\begin{bmatrix} G_1+G_{12}+G_{13} & -G_{12} & -G_{13} \\ -G_{12} & G_2+G_{12}+G_{23} & -G_{23} \\ -G_{13} & -G_{23} & G_3+G_{13}+G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_S \\ 0 \\ 0 \end{bmatrix}$$

b) The result is identical to that obtained in part a).

Problem 3.14

Using mesh analysis, find the currents i1 and i2 for the circuit of Figure P3.14.

Solution:



Circuit shown in Figure P3.14.

Find:

Current i_1 and i_2.

Analysis:

For mesh #1:

$$-1+1i_1+3(i_1-i_2)=0$$

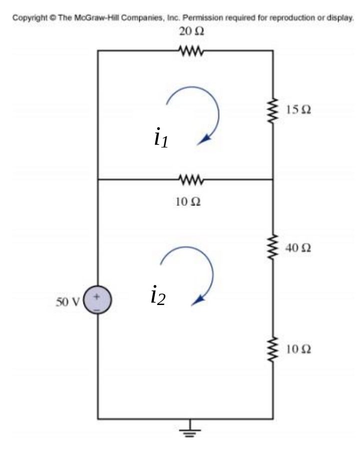
For mesh #2:

$$3(i_2-i_1)+2i_2+2=0$$

Solving,

$$i_1=-0.091\text{A}$$
$$i_2=-0.455\text{A}$$

Problem 3.15



Using mesh analysis, find the currents i_1 and i_2 and the voltage across the top 10-ohm resistor in the circuit of Figure P3.15.

Solution:

Circuit shown in Figure P3.15.

Find:

Current i_1 and i_2 and voltage across the resistance 10Ω.

Analysis:

Mesh #1 $\quad 20i_1+15i_1+10(i_1-i_2)=0$

Mesh #2 $\quad -50+10(i_2-i_1)+40i_2+10i_2=0$

Therefore,

$I_1=0.1923$ A and $I_2=0.865$ A,

$v_{10\Omega}=10(i_2 - i_1)=6.73$ V

Problem 3.16

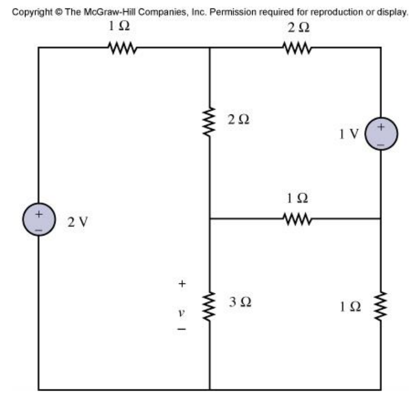
Using mesh analysis, find the voltage, v, across the 3-**ohm** resistor in the circuit of Figure P3.16.

Solution:

Circuit shown in Figure P3.16.

Find:

Voltage across the 3Ω resistance.



Analysis:

Meshes 1, 2 and 3 are clockwise from the left and are oriented clockwise.

For mesh #1:

$$-2 + 1i_1 + 2(i_1 - i_2) + 3(i_1 - i_3) = 0$$

For mesh #2:

$$2(i_2 - i_1) + 2i_2 + 1 + 1(i_2 - i_3) = 0$$

For mesh #3:

$$3(i_3 - i_1) + 1(i_3 - i_2) + 1i_3 = 0$$

Solving,

$$i_1 = 0.5224 \text{ A}$$
$$\text{i}_2 = 0.0746 \text{ A}$$
$$i_3 = 0.3284 \text{ A}$$

and $\quad v = 3(i_1 - i_3) = 3(0.194) = 0.582 \text{ V}$

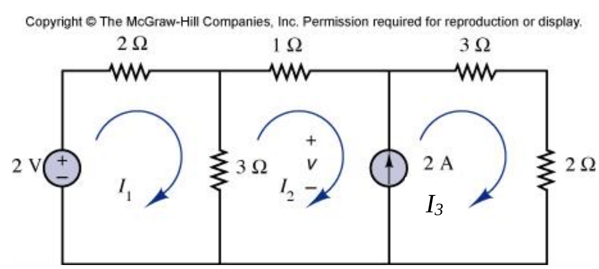
Problem 3.17

Using mesh analysis, find the currents I_1, I_2, and I_3 in the circuit of Figure P3.17 (assume polarity according to I_2).

Solution:

Mesh #1 (on the left-hand side)

$2 - 2I_1 - 3(I_1 - I_2) = 0$



If we treat mesh #2 (middle) and mesh #3 (on the right-hand side) as a single loop containing the four resistors (but not the current source), we can write

$-1I_2 - 3I_3 - 2I_3 - 3(I_2 - I_1) = 0$

From the current source:

$I_3 - I_2 = 2$

Solving the system of equations:

I_1 = - 0.333 A I_2 = - 1.222 A I_3 = 0.778 A

Problem 3.18

Using mesh analysis, find the voltage, V , across the current source in Figure P3.18.

Solution:

Circuit shown in Figure P3.18.

Find:

Voltage across the current source.

Analysis:

The analysis for the mesh currents is exactly the same as Problem 3.17.

Solving,

$i_3 = 0.778\text{A}$

The voltage across the current source is exactly the same as the voltage across the series combination of the 3Ω and 2Ω resistor

or $v = i_3(3+2) = 3.89\text{V}$

Problem 3.19

3.19 a. For the circuit of Figure P3.19, write the mesh equations in matrix form. Notice the form of the $[R]$ and $[V]$ matrices in the $[R][I] = [V]$, where

$$[R] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & \cdots & r_{2n} \\ r_{31} & & \ddots & & \\ \vdots & & & \ddots & \\ r_{n1} & r_{n2} & \cdots & \cdots & r_{nn} \end{bmatrix} \quad \text{and } [V] = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ \vdots \\ V_n \end{bmatrix}$$

b. Write the matrix form of the mesh equations again by using the following formulas:

$r_{ii} = \sum$ resistances around loop i

$r_{ij} = -\sum$ resistances *shared* by loops i and j

$V_i = \sum$ source voltages around loop i

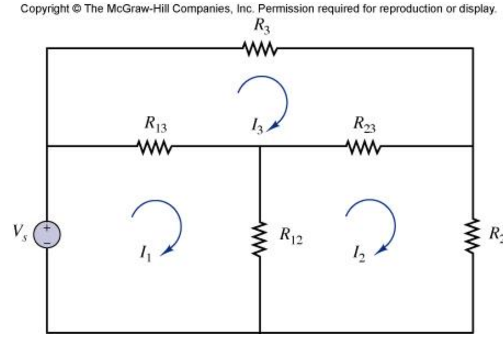
Solution:

Circuit shown in Figure P3.19.

Find:

Mesh equation in matrix form.

Analysis:



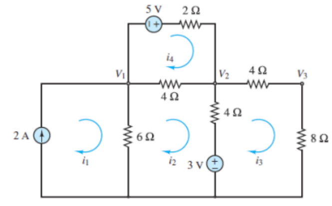
a) $$\begin{bmatrix} R_{12}+R_{13} & -R_{12} & -R_{13} \\ -R_{12} & R_{12}+R_2+R_{23} & -R_{23} \\ -R_{13} & -R_{23} & R_3+R_{13}+R_{23} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \\ 0 \end{bmatrix}$$

b) same result as a).

Problem 3.20

For the circuit of Figure P3.20, use mesh analysis to find 4 equations in the 4 mesh currents. Collect coefficients and solve for the mesh currents.

Solution:



Circuit shown in Figure P3.20

.Figure P3.20

Find:

Foura mesh equations and solve for currents.

Analysis:

The mesh equations are:

$i_1 = 2$ Eqn.1

$6(i_2 - i_1) + 4(i_2 - i_4) + 4(i_2 - i_3) + 3 = 0$ Eqn.2

$-3 + 4(i_3 - i_2) + 4i_3 + 8i_3 = 0$ Eqn.3

$-5 + 2i_4 + 4(i_4 - i_2) = 0$ Eqn.4

Solving the equations:

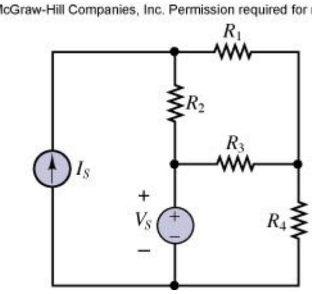
$$i_1 = 2\text{A}$$

$$i_2 = 1.2661\,\text{A}$$

$$i_3 = 0.5040\ \text{A}$$

$$i_4 = 1.6774\ \text{A}$$

Problem 3.21



In the circuit in Figure P3.21, assume the source voltage and source current and all resistances are known.
a. Write the node equations required to determine the node voltages.
b. Write the matrix solution for each node voltage in terms of the known parameters.

Solution:

Known quantities:

Circuit of Figure P3.21 with voltage source, V_S , current source, I_S, and all resistances.

Find:

a. The node equations required to determine the node voltages.
b. The matrix solution for each node voltage in terms of the known parameters.

Analysis:

a) Specify the nodes (e.g., A on the upper left corner of the circuit in Figure P3.10, and B on the right corner). Choose one node as the reference or ground node. If possible, ground one of the sources in the circuit. Note that this is possible here. When using KCL, assume all unknown current flow out of the node. The direction of the current supplied by the current source is specified and must flow into node A.

KCL:

$$-I_S + \frac{V_a - V_S}{R_2} + \frac{V_a - V_b}{R_1} = 0$$

$$V_a\left(\frac{1}{R_2} + \frac{1}{R_1}\right) + V_b\left(-\frac{1}{R_1}\right) = I_S + \frac{V_S}{R_2}$$

KCL:

$$\frac{V_b - V_a}{R_1} + \frac{V_b - V_S}{R_3} + \frac{V_b - 0}{R_4} = 0$$

$$V_a\left(-\frac{1}{R_1}\right) + V_b\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{V_S}{R_3}$$

b) Matrix solution:

$$V_a = \frac{\begin{vmatrix} I_S + \frac{V_S}{R_2} & -\frac{1}{R_1} \\ \frac{V_S}{R_3} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}} = \frac{\left(I_S + \frac{V_S}{R_2}\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \left(\frac{V_S}{R_3}\right)\left(-\frac{1}{R_1}\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \left(-\frac{1}{R_1}\right)\left(-\frac{1}{R_1}\right)}$$

$$V_b = \frac{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & I_S + \frac{V_S}{R_2} \\ -\frac{1}{R_1} & \frac{V_S}{R_3} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}} = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{V_S}{R_3}\right) - \left(-\frac{1}{R_1}\right)\left(I_S + \frac{V_S}{R_2}\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \left(-\frac{1}{R_1}\right)\left(-\frac{1}{R_1}\right)}$$

Notes:
1. The denominators are the same for both solutions.
2. The main diagonal of a matrix is the one that goes to the right and down.
3. The denominator matrix is the "conductance" matrix and has certain properties:

a) The elements on the main diagonal [i(row) = j(column)] include all the conductance connected to node i=j.
b) The off-diagonal elements are all negative.
c) The off-diagonal elements are all symmetric, i.e., the i j-th element = j i-th element. This is true only because there are no controlled (dependent) sources in this circuit.
d) The off-diagonal elements include all the conductance connected between node i [row] and node j [column].

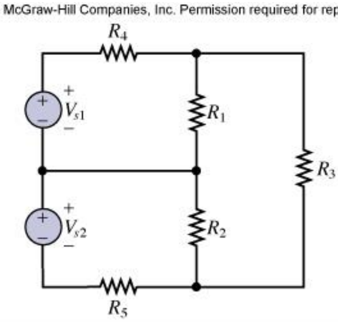
Problem 3.22

For the circuit of Figure P3.22 determine:
a. The most efficient way to solve for the voltage across R_3. Prove your case.
b. The voltage across R_3.
V_{S1} = V_{S2} = 110 V
R_1 = 500mohm R_2 = 167mohm
R_3 = 700mohm
R_4 = 200mohm R_5 = 333mohm

Solution:

Known quantities:

Circuit shown in Figure P3.22



$$V_{S1} = V_{S2} = 110 \text{ V}$$
$$R_1 = 500 \text{ m}\Omega \quad R_2 = 167 \text{ m}\Omega$$
$$R_3 = 700 \text{ m}\Omega$$
$$R_4 = 200 \text{ m}\Omega \quad R_5 = 333 \text{ m}\Omega$$

Find:

a. The most efficient way to solve for the voltage across R_3. Prove your case.

b. The voltage across R_3.

Analysis:

a) There are 3 meshes and 3 mesh currents requiring the solution of 3 simultaneous equations. Only one of these mesh currents is required to determine, using Ohm's Law, the voltage across R_3.
If the terminal (or node) between the two voltage sources is made the ground (or reference) node, then three node voltages are known (the ground or reference voltage and the two source voltages). This leaves only two unknown node voltages (the voltages across R_1, V_{R1}, and across R_2, V_{R2}). Both these voltages are required to determine, using KVL, the voltage across R_3, V_{R3}.
A difficult choice. Choose node analysis due to the smaller number of unknowns. Specify the nodes. Choose one node as the ground node. In KCL, assume unknown currents flow out.
b)

$$KCL: \quad \frac{V_{R1} - V_{S1}}{R_4} + \frac{V_{R1} - 0}{R_1} + \frac{V_{R1} - V_{R2}}{R_3} = 0 \qquad KCL: \quad \frac{V_{R2} - (-V_{S2})}{R_5} + \frac{V_{R2} - 0}{R_2} + \frac{V_{R2} - V_{R1}}{R_3} = 0$$

$$V_{R1}\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right) + V_{R2}\left(-\frac{1}{R_3}\right) = \frac{V_{S1}}{R_4} \qquad V_{R1}\left(-\frac{1}{R_3}\right) + V_{R2}\left(\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3}\right) = -\frac{V_{S2}}{R_5}$$

$$\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{500 \cdot 10^{-3}} + \frac{1}{700 \cdot 10^{-3}} + \frac{1}{200 \cdot 10^{-3}} = 8.43 \ \Omega^{-1}$$

$$\frac{1}{R_5} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{333 \cdot 10^{-3}} + \frac{1}{167 \cdot 10^{-3}} + \frac{1}{700 \cdot 10^{-3}} = 10.42 \ \Omega^{-1}$$

$$\frac{1}{R_3} = \frac{1}{700 \cdot 10^{-3}} = 1.43 \ \Omega^{-1}$$

$$\frac{V_{S1}}{R_4} = \frac{110}{200 \cdot 10^{-3}} = 550 \text{ A} \qquad \frac{V_{S2}}{R_5} = \frac{110}{333 \cdot 10^{-3}} = 330 \text{ A}$$

$$V_{R1} = \frac{\begin{vmatrix} 550 & -1.43 \\ -330 & 10.42 \end{vmatrix}}{\begin{vmatrix} 8.43 & -1.43 \\ -1.43 & 10.42 \end{vmatrix}} = \frac{(5731) - (472)}{(87.84) - (2.04)} = 61.29\text{V}$$

$$V_{R1} = \frac{\begin{vmatrix} 8.43 & 550 \\ -1.43 & -330 \end{vmatrix}}{85.80} = \frac{(-2782) - (-786)}{85.80} = -23.26\text{V}$$

$$KVL: \quad -V_{R1} + V_{R3} + V_{R2} = 0 V_{R3} = V_{R1} - V_{R2} = 84.55\text{V}$$

Problem 3.23

Figure P3.23 represents a temperature measurement system, where temperature T is linearly related to the voltage source V_{S2} by a transduction constant k. Use nodal analysis to determine the

temperature.
V_{S2} = kT k = 10 V/°C
V_{S1} = 24V R_S = R_1 = 12 kohm
R_2 = 3 kohm R_3 = 10 kohm
R_4 = 24 kohm V_{ab} = −2.524 V
In practice, V_{ab} is used as the measure of temperature, which is introduced to the circuit through a temperature sensor modeled by the voltage source V_{S2} in series with R_S .

Solution:

Known quantities:

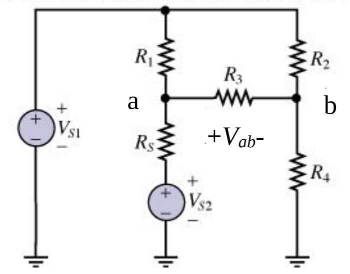
Circuit shown in Figure P3.23

$V_{S2} = \text{kT}\text{k} = 10\text{V/°C}$

$V_{S1} = 24\text{V} R_S = R_1 = 12\text{k}\Omega$

$R_2 = 3\text{k}\Omega R_3 = 10\text{k}\Omega$

$R_4 = 24\text{k}\Omega V_{ab} = -2.524\text{V}$



The voltage across R_3, which is given, indicates the temperature.

Find:

The temperature, T.

Analysis:

Specify nodes (a between R_1 and R_3, b between R_3 and R_2) and polarities of voltages (V_a from ground to a, V_b from ground to b, and V_{ab} from b to a). When using KCL, assume unknown currents flow out.

$$KVL: \quad -V_a + V_{ab} + V_b = 0$$
$$V_b = V_a - V_{ab}$$

Now write KCL at node b, substitute for V_b, solve for V_a:

$$KCL: \quad \frac{V_b - V_{S1}}{R_2} + \frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} = 0$$

$$-\frac{V_a}{R_3} - \frac{V_{S1}}{R_2} + (V_a - V_{ab})\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = 0$$

$$V_a = \frac{\frac{V_{S1}}{R_2} + V_{ab}\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)}{\frac{1}{R_2} + \frac{1}{R_4}} = \frac{\frac{24}{3\cdot 10^3} + (-2.524)\left(\frac{1}{3\cdot 10^3} + \frac{1}{10\cdot 10^3} + \frac{1}{24\cdot 10^3}\right)}{\frac{1}{3\cdot 10^3} + \frac{1}{24\cdot 10^3}} = 18.14\text{V}$$

Now write KCL at node a and solve for V_{S2} then T:

$$KCL: \quad \frac{V_a - V_{S1}}{R_1} + \frac{V_a - V_{S2}}{R_S} + \frac{V_{ab}}{R_3} = 0$$

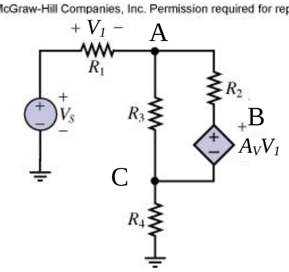
$$V_{S2} = V_a + \frac{R_S}{R_1}\left(V_a - V_{S1}\right) + \frac{R_S}{R_3}V_{ab} =$$

$$= 18.14 + \frac{12 \cdot 10^3}{12 \cdot 10^3}\left(18.14 - 24\right) + \frac{12 \cdot 10^3}{10 \cdot 10^3}\left(-2.524\right) = 9.25\text{V}$$

$$\text{T} = \frac{V_{S2}}{\text{k}} = \frac{9.25}{10} = 0.925°\text{C}$$

Problem 3.24

Use nodal analysis on the circuit in Figure P3.24 to determine the voltage across R_4. Note that one source is a dependent (controlled) voltage source! Let
V_S = 5V; A_V = 70; R_1 = 2.2 kohm; R_2 = 1.8 kohm;
R_3 = 6.8 kohm; R_4 = 220ohm.



Solution:

Known quantities:

Circuit shown in Figure P3.24

$$V_S = 5\ \text{V} \qquad A_V = 70 \qquad R_1 = 2.2\ \text{k}\Omega$$

$$R_2 = 1.8\ \text{k}\Omega \qquad R_3 = 6.8\ \text{k}\Omega \qquad R_4 = 220\ \Omega$$

Find:

The voltage across R_4 using node voltage analysis.

Analysis:

Node analysis is not a method of choice because the dependent source is [1] a voltage source and [2] a floating source. Both factors cause difficulties in a node analysis. A ground is specified. There are three unknown node voltages (labeled A, B, & C in the figure above), one of which is the voltage across R_4. The dependent source will introduce two additional unknowns, the current through the dependent source, I_{DS} and the controlling voltage (across R_1) that is not a node voltage. Therefore 5 equations are required:

$$[1]\text{KCL: } \frac{V_A - V_S}{R_1} + \frac{V_A - V_C}{R_3} + \frac{V_A - V_B}{R_2} = 0$$

$$[2]\text{KCL: } \frac{V_B - V_A}{R_2} - I_{DS} = 0$$

$$[3]\text{KCL: } \frac{V_C - V_A}{R_3} + I_{DS} + \frac{V_C}{R_4} = 0$$

$$[4]\text{KVL: } -V_S + V_1 + V_A = 0$$

$$[5]\text{KVL: } -V_C - A_V V_1 + V_B = 0$$

Solving these five equations simultaneously we find::

$$V_C = V_4 = 8.8\text{mV}$$

We also find:

$$V_A = 4.91\text{V}$$

$$V_B = 6.14\text{V}$$

$$V_1 = 87.6\text{mV, and}$$

$$I_{DS} = 681\mu\text{A}$$

Notes:

1. This solution was not difficult in terms of theory. It would be terribly long and arithmetically cumbersome to solve by hand. However, with modern calculators and the use of computer tools such as MatLab, the solution is straight forward.

Problem 3.25

Use mesh analysis to find the mesh currents in Figure P3.25. Let $R_1 =$ 10ohm, $R_2 =$ 5ohm, $V_1 =$ 2 V, $V_2 =$ 1V, $I_S =$ 2A.

Solution:

Known quantities:

The values of the resistors and of the voltage and current sources (see Figure P3.25).

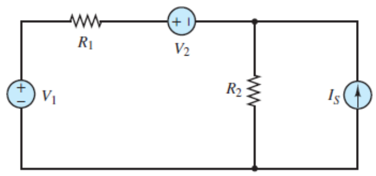


Figure P3.25

Find:

Find the mesh currents in the circuit of Figure P3.25 using mesh current analysis.

Analysis:

Let I_1 be the left hand mesh and I_2 be the right hand mesh. Both mesh currents are clockwise in direction. Using the mesh current analysis:

$$\begin{cases} V_1 - R_1 I_1 - V_2 - R_2 (I_1 - I_2) = 0 \\ I_2 = -I_S \end{cases} \Rightarrow \begin{cases} I_1 = -0.6\text{A} \\ I_2 = 2\text{A} \end{cases}$$

Problem 3.26

Use mesh analysis to find the mesh currents in Figure P3.26. Let $R_1 =$ 6ohm, $R_2 =$ 3ohm, $R_3 =$ 3ohm $V_1 =$ 4V, $V_2 =$ 1V, $V_3 =$ 2V.

Solution:

Known quantities:

The values of the resistors, of the voltage source in the circuit of Figure P3.26.

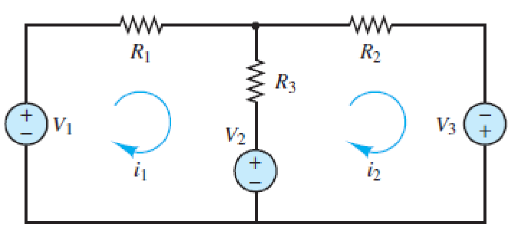


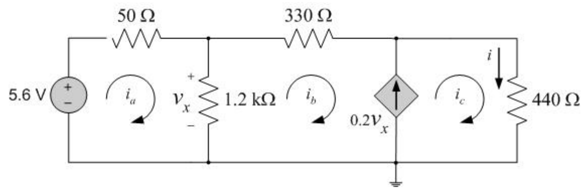
Figure P3.26

Find:

Find the mesh currents in the circuit of Figure P3.26 using mesh current analysis.

Analysis:
$$\begin{cases} V_1 - R_1 i_1 - R_3 (i_1 - i_2) - V_2 = 0 \\ V_2 - R_3 (i_2 - i_1) - R_2 I_2 + V_3 = 0 \end{cases} \Rightarrow \begin{cases} I_1 = 467\text{mA} \\ I_2 = 733\text{mA} \end{cases}$$

Problem 3.27



Use mesh analysis to find the current *i* in Figure P3.27. Let V_S = 5.6 V; R_1 = 50ohm; R_2 = 1.2 kohm; R_3 = 330ohm; g_m = 0.2 S; R_4 = 440ohm.

Solution:

Known quantities:
The values of the resistors and of the voltage source in the circuit of Figure P3.27.

Find:
The current i through the resistance R_4 mesh current analysis.

Analysis:
For mesh (a):

$$50i_a + 1200(i_a - i_b) = 5.6$$

Combining meshes (b) and (c) around the current source:

$$1200(i_b - i_a) + 330i_b + 440i_c = 0$$

For the current source:

$$i_c - i_b = 0.2v_x \text{ and}$$
$$v_x = 1200(i_a - i_b)$$

Solving,

$i_a = 136 \text{ mA}$, $i_b = 137 \text{ mA}$, $i_c = -106 \text{ mA}$, and $v_x = -1.2\text{V}$.

Therefore,
$i = i_c = -106 \text{ mA}$.

Problem 3.28

Use mesh analysis to find the V_4 in Figure P3.28.
Let R_2 = 6ohm, R_3 = 3ohm, R_4 = 3ohm, R_5 = 3ohm, V_S = 4V, I_S = 2 A.

Solution:

Known quantities:
The values of the resistors, of the voltage source and of the current source in the circuit of Figure P3.28.

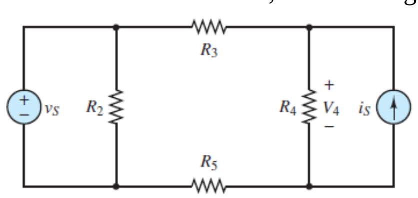


Figure P3.28

Find:
The voltage V_4 using mesh current analysis.

Analysis:
Let I_1 be the left hand mesh, I_2 be the middle mesh, and I_3 be the right hand mesh. All mesh currents are clockwise in direction.

Using the mesh current analysis:

$$\begin{cases} v_S - R_2(I_1 - I_2) = 0 \\ R_2(I_2 - I_1) + R_3 I_2 + R_4(I_2 - I_3) + R_5 I_2 = 0 \\ I_3 = -i_S \end{cases}$$

$$\begin{cases} 4 - 6(I_1 - I_2) = 0 \\ 6(I_2 - I_1) + 3I_2 + 3(I_2 - I_3) + 3I_2 = 0 \\ I_3 = -2 \end{cases} \Rightarrow \begin{cases} I_1 = 444\text{mA} \\ I_2 = -222\text{mA} \\ I_3 = -2\text{A} \end{cases}$$

$$V_4 = 3(I_2 - I_3) = 5.334\text{V}$$

Problem 3.29

Use mesh analysis in the circuit of Figure P3.29 to find the mesh currents. Let *R1* = 8Ω, *R2* = 3Ω, *R3* = 5Ω, *R4*=2 Ω, *R5*=4 Ω, *R6*=3 Ω
V1 = 4V, *V2* = 2V, *V3*=1V, *V4*=2V, *V5*=3V, *V6*=6V.

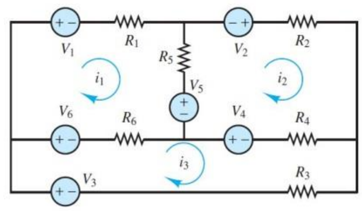


Figure P3.29

Solution:

Known quantities:

Circuit shown in Figure P3.29 with known source voltages and resistances.

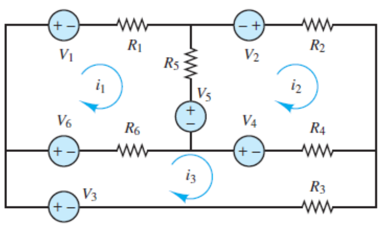


Figure P3.29

Find:

The mesh currents.

Analysis:

Using the mesh current analysis:

$$-V_1 - i_1 * R_1 - (i_1 - i_2) * R_5 - V_5 - (i_1 - i_3) * R_6 + V_6 = 0 \textbf{ Mesh 1}$$

$$-(i_2 - i_3) * R_4 + V_4 + V_5 - (i_2 - i_1) * R_5 + V_2 - i_2 * R_2 = 0 \textbf{ Mesh 2}$$

$$V_3 - V_6 - (i_3 - i_1) * R_6 - V_4 - (i_3 - i_2) * R_4 - i_3 * R_3 = 0 \textbf{ Mesh 3}$$

Collect coefficients:

$$-15 * i_1 + 4 * i_2 + 3 * i_3 = 5 \textbf{ Mesh 1}$$

$$4 * i_1 - 9 * i_2 + 2 * i_3 = -7 \textbf{ Mesh 2}$$

$$3 * i_1 + 2 * i_2 - 10 * i_3 = \mp 3 \textbf{ Mesh 3}$$

Solve set of equations:

$$\boldsymbol{i_1 = -213\text{mA}}$$
$$\boldsymbol{i_2 = 630\text{mA}}$$
$$\boldsymbol{i_3 = -238\text{mA}}$$

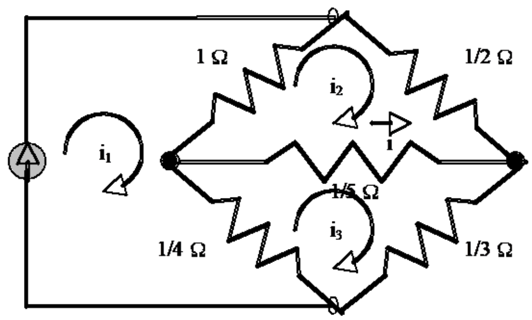
Problem 3.30

Use mesh analysis to find the current i in Figure P3.30.

Solution:

Known quantities:
The values of the resistors in the circuit of Figure P3.30.

Find:
The current in the circuit of Figure P3.30 using mesh current analysis.



Analysis:
For mesh #1, it is obvious that: $i_1 = i_s = 2$

For mesh #2:
$$1(i_2 - i_1) + \tfrac{1}{2} i_2 + \tfrac{1}{5}(i_2 - i_3) = 0$$

For mesh #3: $\tfrac{1}{4}(i_3 - i_1) + \tfrac{1}{5}(i_3 - i_2) + \tfrac{1}{3} i_3 = 0$

Solving, $i_2 = 1.304\text{A}$
$i_3 = 1.087\text{A}$

Then, $i = i_3 - i_2$ or $i = -217\,\text{mA}$

Problem 3.31

Use mesh analysis to find the voltage gain $G_v = v_2/v_s$ in Figure P3.31.

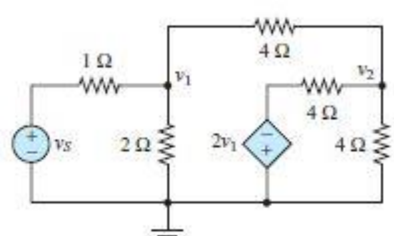


Figure P3.31

Solution:

Known quantities:
The values of the resistors of the circuit in Figure P3.31.

Find:
The voltage gain, in the circuit of Figure P3.31 using mesh current analysis.

Analysis:

Let I_1 be the left hand mesh, I_2 be the middle mesh, and I_3 be the right hand mesh. All mesh currents are clockwise in direction.

Note that

For mesh #1:

$$v_S - 1I_1 - 2(I_1 - I_2) = 0$$

For mesh #2:

$$-2(I_2 - I_1) - 4I_2 - 4(I_2 - I_3) + 2v_1 = 0$$

For mesh #3:

$$-2v_1 - 4(I_3 - I_2) - 4I_3 = 0$$

and finally,

$$v_1 = 2(I_1 - I_2)$$

Solving,

$$I_1 = 0.455v_S$$
$$I_2 = 0.182v_S$$
$$I_3 = -0.045v_S$$

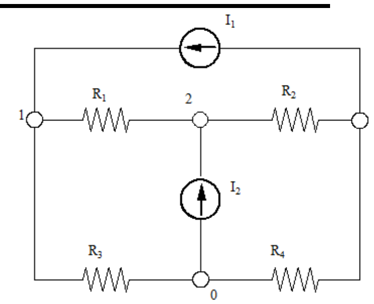
but

$$v_2 = 4I_3 = 4(-0.045v_S)$$

or

$$G_V = \frac{v_2}{v_S} = -0.18$$

Problem 3.32



Use nodal analysis to find node voltages *V1*, *V2* and *V3* in Figure P3.32. Let *R1* = 10ohm, *R2* = 6ohm,

R3 = 7ohm,*R4* = 4ohm, *I1* = 2 V, *I2* = 1V.

Solution:V

Known quantities:

Circuit in Figure P3.32 and the values of the current sources and the values of the four resistors:

Find:

The voltages from V_1, V_2, V_3 to ground using nodal analysis.

Analysis:

Using the nodal analysis and assuming all unknown currents are exiting the node as positive currents:

$$\begin{cases} -I_1 + \dfrac{V_1}{R_3} + \dfrac{V_1 - V_2}{R_1} = 0 \\ \dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - V_3}{R_2} - I_2 = 0 \\ \dfrac{V_3 - V_2}{R_2} + \dfrac{V_3}{R_4} + I_1 = 0 \end{cases}$$

Solving:

$V_1 = 10.89\text{V}$

$V_2 = 6.44\text{V}$

$V_3 = -2.22\text{V}$

Problem 3.33

Use mesh analysis to find the currents through every branch in Figure P3.33. Let $R_1 =$ 10ohm, $R_2 =$ 5ohm, $R_3 =$ 4ohm, $R_4 =$ 1ohm, $V_1 =$ 5V, $V_2 =$ 2V.

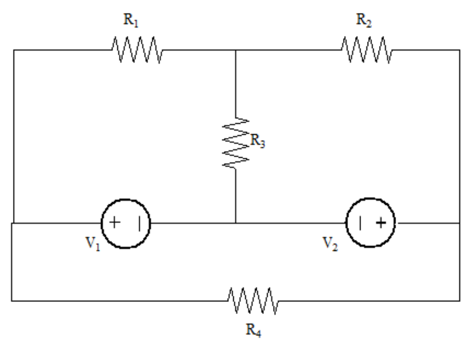
Solution:

Known quantities:

Circuit in Figure P3.33 with the values of the voltage sources and the values of the four resistors.

Find:

The currents through every branch in the circuit.



Analysis:

Assign the mesh currents I_1 to the upper left mesh, I_2 to the upper right mesh, and I_3 to the lower mesh. All mesh currents are assumed to be clockwise.

Using mesh current analysis:

$$\begin{cases} V_1 - R_1 I_1 - R_3(I_1 - I_2) = 0 \\ -V_2 - R_3(I_2 - I_1) - R_2 I_2 = 0 \\ -V_1 + V_2 - R_4 I_3 = 0 \end{cases} \Rightarrow \begin{cases} I_1 = 0.336\text{A} \\ I_2 = -0.073\text{A} \\ I_3 = -3A \end{cases}$$

$$\begin{cases} i_{R1} = I_1 = 0.336\text{A} \\ i_{R2} = -I_2 = 0.073\text{A} \\ i_{R3} = I_1 - I_2 = 0.409\text{A} \\ i_{R4} = -I_3 = 3\text{A} \\ i_{V1} = I_1 - I_3 = 3.336\text{A} \\ i_{V2} = I_3 - I_2 = -2.927\text{A} \end{cases}$$

Problem 3.34

Use nodal analysis to find the current through R_4 in Figure P3.34. Let $R_1 = 10$ohm, $R_2 = 6$ohm, $R_3 = 4$ohm, $R_4 = 3$ohm, $R_5 = 2$ohm, $R_6 = 2$ohm, $I_1 = 2$ A, $I_2 = 3$A, $I_3 = 5$A.

Solution:

Known quantities:

a

Circuit in Figure P3.34 with the values of the current sources and the values of the 6 resistors.

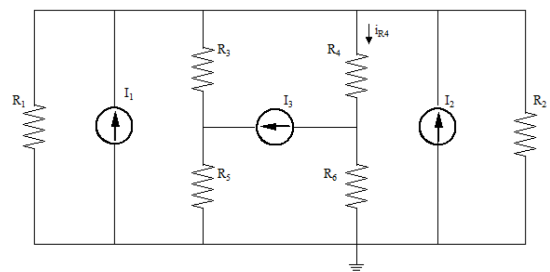
b c

Find:

Find the current through R4.

Analysis:

a, b, c are the nodes in figure, as shown.



Using the voltage node method:

$$\begin{cases} \dfrac{V_a}{R_1} + \dfrac{V_a}{R_2} + \dfrac{V_a - V_b}{R_3} + \dfrac{V_a - V_c}{R_4} = I_1 + I_2 \\ \dfrac{V_b - V_a}{R_3} + \dfrac{V_b}{R_5} = I_3 \\ \dfrac{V_c - V_a}{R_4} + \dfrac{V_c}{R_6} + I_3 = 0 \end{cases}$$

Solving:

$$\begin{cases} V_a = 7.37\text{V} \\ V_b = 9.12\text{V} \\ V_c = -3.05\text{V} \end{cases}$$

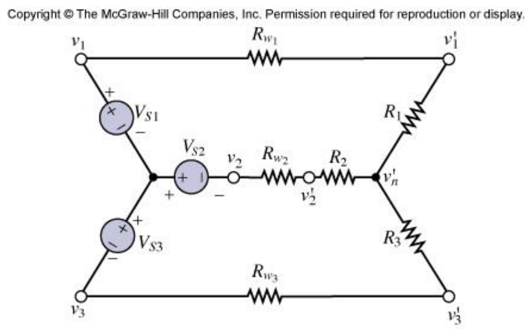
Then

$$i_{R4} = \frac{V_a - V_c}{R_4} = 3.47\text{A}$$

Problem 3.35

The circuit shown in Figure P3.35 is a simplified DC version of an AC three-phase wye-wye (Y-Y) electrical distribution system commonly used to supply industrial loads, particularly rotating machines.
V_{S1} = V_{S2} = V_{S3} = 170 V
R_{w1} = R_{w2} = R_{w3} = 0.7ohm
R_1 = 1.9ohm R_2 = 2.3ohm
R_3 = 11ohm
a. Determine the number of non-reference nodes.
b. Determine the number of unknown node voltages.
c. Compute v_1, v_2, v_3, and v'_n.
Notice that once v'_n is known the other unknown node voltages can be computed directly by voltage division.

Solution:



Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170$ V, and the values of the 6 resistors in the circuit of Figure P3.35:

$R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega$

$R_1 = 1.9\Omega \qquad R_2 = 2.3\ \Omega \qquad R_3 = 11\ \Omega$

Find:

a. The number of non-reference nodes.
b. Number of unknown node voltages.
c. Compute v'_1, v'_2, v'_3, and v_n'.

Analysis:

a) The reference node is the node common to the three sources. There are seven (7) non-reference nodes: v_1, v_1', v_2, v_2', v_3, v_3', and v_n'.

b) The only unknown node voltages are v_1', v_2', v_3', & v_n'. v_1=V_{S1}=170V, v_2=-V_{S2}=-170V, v_3=-V_{S3}=-170V.

A node analysis is the method of choice! Specify polarity of voltages and direction of currents.

$$\frac{v_1' - 170}{0.7} + \frac{v_1' - v_n'}{1.9} = 0 \qquad v_1' = \frac{170(1.9) + v_n'(0.7)}{1.9 + 0.7}$$

$$\frac{v_2' + 170}{0.7} + \frac{v_2' - v_n'}{2.3} = 0 \text{ Notice that this reduces to } v_2' = \frac{-170(2.3) + v_n'(0.7)}{2.3 + 0.7}$$

$$\frac{v_3' + 170}{0.7} + \frac{v_3' - v_n'}{11} = 0 \qquad v_3' = \frac{-170(11) + v_n'(0.7)}{11 + 0.7}$$

The fourth nodal equation is:

$$\frac{v_n' - v_1'}{1.9} + \frac{v_n' - v_2'}{2.3} + \frac{v_n' - v_3'}{11} = 0$$

Solving yields:

$v_1' = 122.28\text{V}$

$v_2' = -132.02\text{V}$

$v_3' = -160.26\text{V}$ and

$v_n' = -7.234\text{V}$

Notice that

$$v_1' = \frac{170(1.9) + v_n'(0.7)}{1.9 + 0.7} = 122.28\text{V}$$

$$v_2' = \frac{-170(2.3) + v_n'(0.7)}{2.3 + 0.7} = -132.02\text{V}$$

$$v_3' = \frac{-170(11) + v_n'(0.7)}{11 + 0.7} = -160.26\text{V}$$

Problem 3.36

The circuit shown in Figure P3.35 is a simplified DC version of an AC three-phase wye-wye (Y-Y) electrical distribution system commonly used to supply industrial loads, particularly rotating machines.

$V_{S1} = V_{S2} = V_{S3} =$ 170 V

$R_{w1} = R_{w2} = R_{w3} =$ 0.7ohm

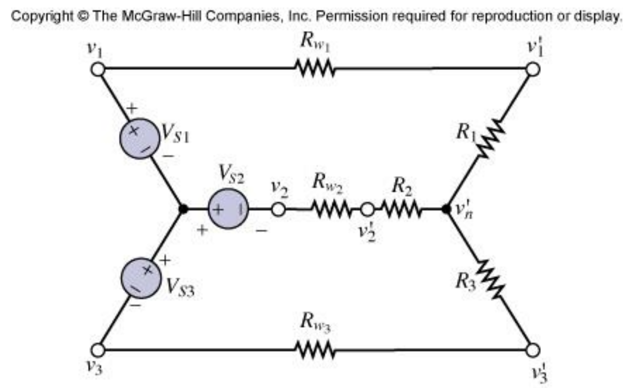
$R_1 =$ 1.9ohm $R_2 =$ 2.3ohm

$R_3 =$ 11ohm

a. Determine the number of meshes.

b. Compute the mesh currents.

c. Use the mesh currents to determine v'_n.



Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170$ V and the values of the 6 resistors in the circuit of Figure P3.35:

$R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega$

$R_1 = 1.9\ \Omega$ $\quad R_2 = 2.3\ \Omega$ $\quad R_3 = 11\ \Omega$

Find:

The number of meshes, the mesh currents, and V'n.

Analysis:

There are two meshes. Call the upper mesh I_1 and the lower mesh I_2. Both meshes are clockwise.

KVL:

$$V_{S1} - R_{w1}I_1 - R_1I_1 - R_2(I_1 - I_2) - R_{w2}(I_1 - I_2) + V_{S2} = 0$$
$$V_{S3} - V_{S2} - R_{w2}(I_2 - I_1) - R_2(I_2 - I_1) - R_3I_2 - R_{w3}I_2 = 0$$

Solving:

$$V_{S1} - R_{w1}I_1 - R_1I_1 - R_2(I_1 - I_2) - R_{w2}(I_1 - I_2) + V_{S2} = 0$$
$$V_{S3} - V_{S2} - R_{w2}(I_2 - I_1) - R_2(I_2 - I_1) - R_3I_2 - R_{w3}I_2 = 0$$

Solve system of two equations:

$$I_1 = 68.17A \quad I_2 = 13.91A$$

But, looking at the lower branch

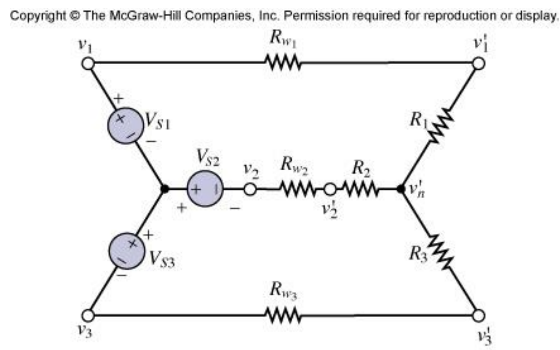
$$\frac{v_n' + 170}{11 + 0.7} = I_2$$

or

$$v_n' = -170 + 11.7(13.91) = -7.25\text{V}$$

Problem 3.37

The circuit shown in Figure P3.35 is a simplified DC version of a typical three-wire, three-phase AC wye-wye (Y-Y) distribution system. Use the Principle of Superposition to determine v_n'. Assume:
$V_{S1} = V_{S2} = V_{S3} =$ 170 V
$R_{w1} = R_{w2} = R_{w3} =$ 0.7ohm
$R_1 =$ 1.9ohm $R_2 =$ 2.3ohm
$R_3 =$ 11ohm



Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170$ V, and the values of the 6 resistors in the circuit of Figure P3.35:

$R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega$

$R_1 = 1.9\ \Omega \qquad R_2 = 2.3\ \Omega \qquad R_3 = 11\ \Omega$

Find:

The voltage at v_n'.

Analysis:

If we know the current through *R1* then we can calculate the voltage drop from v_1 to v_n':

Use superposition on *VS1*(*VS2* and *VS3* become a short circuit):

Find *Req*: $R_{eq} = (R_1 + R_{w1}) + ((R_2 + R_{w2})||(R_3 + R_{w3})) = 4.99\ \Omega$

Current through *R1* because of *VS1* = total current: $i_{1(VS1)} = \frac{170V}{4.99\ \Omega} = 34.07$A

Use superposition on *VS2*(*VS1* and *VS3* become a short circuit):

Find *Req*: $R_{eq} = (R_2 + R_{w2}) + ((R_1 + R_{w1})||(R_3 + R_{w3})) = 5.13\ \Omega$

Total current: $i_T = \frac{170V}{5.13\ \Omega} = 33.14$A

Current through *R1* because of *VS2*: use current division: $i_{1(VS2)} = \frac{R_3 + R_{W3}}{R_1 + R_{W1} + R_3 + R_{W3}} * I_T = 27.12$A

Use superposition on *VS3*(*VS1* and *VS2* become a short circuit):

Find *Req*: $R_{eq} = (R_3 + R_{w3}) + ((R_1 + R_{w1})||(R_2 + R_{w2})) = 13.09\ \Omega$

Total current: $i_T = \frac{170V}{13.09\ \Omega} = 12.99$A

Current through *R1* because of *VS3*: use current division: $i_{1(VS3)} = \frac{R_2 + R_{W2}}{R_1 + R_{W1} + R_2 + R_{W2}} * I_T = 6.96$A

Add up i_1's to get total current:

$$i_1 = 68.15\text{A}$$

Find V'n using Ohm's Law:

$$V_{s1} - V'_n = i_1 * (R_1 + R_{1w})$$
$$\boldsymbol{V'_n = \sim -7.19\text{V}}$$

Problem 3.38

The circuit shown in Figure P3.35 is a simplified DC version of a typical three-wire, three-phase AC wye-wye (Y-Y) distribution system. Use the Principle of source transformations to determine v_n'. Assume:

$V_{S1} = V_{S2} = V_{S3} = 170$ V

$R_{w1} = R_{w2} = R_{w3} = 0.7$ohm

$R_1 = 1.9$ohm $R_2 = 2.3$ohm

$R_3 = 11$ohm

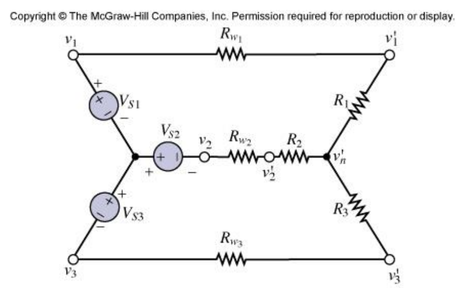
Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = V_{S2} = V_{S3} = 170$ V, and the values of the 6 resistors in the circuit of Figure P3.35:

$R_{W1} = R_{W2} = R_{W3} = 0.7\ \Omega$

$R_1 = 1.9\ \Omega \qquad R_2 = 2.3\ \Omega \qquad R_3 = 11\ \Omega$



Find:

v'_n using source transformations.

Analysis:

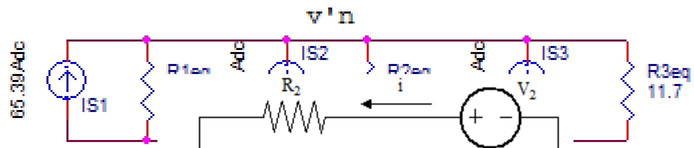
Combine resistors in series i.e R_{w1} and R_1.$VS1$ is a thevenin source with R_{1eq}. The same goes for $Vs2$ and R_{2eq} and $Vs3$ and R_{3eq}. Convert all Thévenin sources to Norton sources and the two nodes of the new circuit are v_n' and ground. Combine parallel resistors and current sources to find R_{eq} and I_S.

First find currents:

$i_{S1} = \frac{V_{S1}}{2.6\Omega} = 65.39\text{A}$ from ground toward v_n'.

$i_{S2} = \frac{V_{S2}}{3\Omega} = 56.67\text{A}$ from v_n' toward ground.

$i_{S3} = \frac{V_{S3}}{11.7\Omega} = 14.53\text{A}$ from v_n' toward ground.



Find Total current:

$I_S = i_{S1} - i_{S2} - i_{S3} = -5.81\text{A}$

Find Equivalent Resistance:

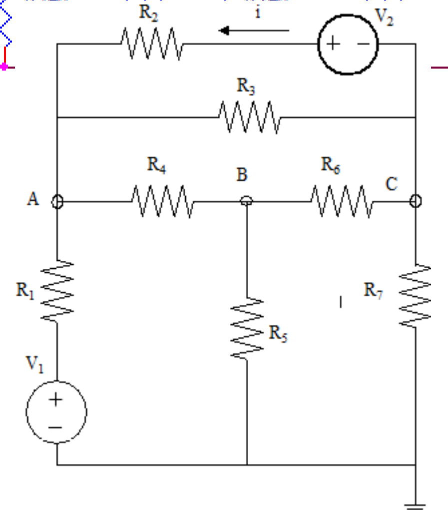
$R_{eq} = (R_{1eq}||R_{2eq}||R_{3eq}) = 1.24\Omega$

Solve for v_n':

$$\boldsymbol{v'_n = I_S * R_{eq} = -7.20\text{V}}$$

Problem 3.39

Use nodal analysis in the circuit of Figure P3.39 to find the three indicated node voltages and the current i.



Assume: R_1 = 10ohm, R_2 = 20ohm,
R_3 = 20ohm, R_4 = 10ohm, R_5 = 10ohm,
R_6 = 10ohm, R_7 = 5ohm, V_1 = 20 V, V_2 = 20 V.

Solution:

Known quantities:

The circuit of Figure P3.39.The value of $V_1, V_2, R_1, R_2, R_3, R_4, R_5, R_6, R_7$.

Find:

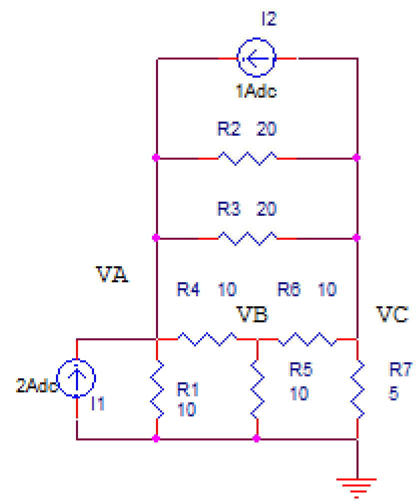
V_a, V_b, V_c using the node voltage method, and the current i.

Analysis:

First of all, we transform voltage sources in current sources: $i_1 = \frac{V_1}{R_1} = 2A$

$$i_2 = \frac{V_2}{R_2} = 1A$$

Using the node voltage method:



$$\begin{cases} \frac{V_A}{R_1} + \frac{V_A - V_B}{R_4} + \frac{V_A - V_C}{R_3} + \frac{V_A - V_C}{R_2} = I_1 + I_2 \\ \frac{V_B - V_A}{R_4} + \frac{V_B}{R_5} + \frac{V_B - V_C}{R_6} = 0 \\ \frac{V_C}{R_7} + \frac{V_C - V_B}{R_6} + \frac{V_C - V_A}{R_3} + \frac{V_C - V_A}{R_2} + I_2 = 0 \end{cases}$$

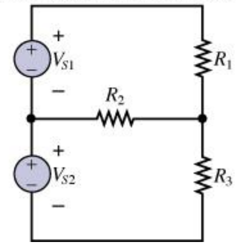
$\Rightarrow$

$$\begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 4 \end{pmatrix} \cdot \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 30 \\ 0 \\ -10 \end{pmatrix}$$

Solving the linear system:

$$\begin{cases} V_A = \frac{145}{12} \text{V} \\ V_B = \frac{55}{12} \text{V} \\ V_C = \frac{5}{3} \text{V} \end{cases}$$

Using KVL:
$$V_A + iR_2 - V_2 = V_C$$
$$i = \frac{V_C - V_A + 20}{20} = \frac{23}{48} \text{A}$$



Section 3.5: Superposition

Problem 3.40

With reference to Figure P3.40, determine the current *i* through R_1 due only to the source V_{S2}.
V_{S1} = 110 V V_{S2} = 90 V R_1 = 560ohm R_2 = 3.5 kohm R_3 = 810ohm

Solution:

Known quantities:

The values of the voltage sources, $V_{S1} = 110\ \text{V}$, $V_{S2} = 90\ \text{V}$ and the values of the 3 resistors in the circuit of Figure P3.40: $R_1 = 560\ \Omega$ $R_2 = 3.5\ k\Omega$ $R_3 = 810\ \Omega$

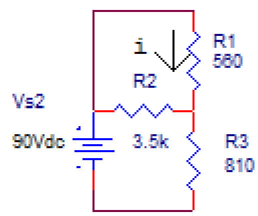
Find:

The current through R_1 due only to the source V_{S2}.

Analysis:

Replace V_{S1} with a short circuit. Use mesh current analysis.

Assign I_1 as the mesh current for the upper mesh and I_2 as the mesh current for the lower mesh. Both mesh currents are in a clockwise direction.



$$R_1 I_1 + R_2 (I_1 - I_2) = 0$$
$$R_2 (I_2 - I_1) + R_3 I_2 = V_{S2}$$

Solving:

$$I_1 = i = 60.02\text{mA}$$
$$I_2 = 69.62\text{mA}$$