Chapter 2

2.1 d. $\gamma = \dfrac{(87.5)(9.81)}{(1000)(0.05)} = \mathbf{17.17\ kN/m^3}$

c. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{17.17}{1+0.15} = \mathbf{14.93\ kN/m^3}$

a. Eq. (2.12): $\gamma = \dfrac{G_s\gamma_w}{1+e}$. $14.93 = \dfrac{(2.68)(9.81)}{1+e}$; $e = \mathbf{0.76}$

b. Eq. (2.6): $n = \dfrac{e}{1+e} = \dfrac{0.76}{1+0.76} = \mathbf{0.43}$

e. Eq. (2.14): $S = \dfrac{V_w}{V_v} = \dfrac{wG_s}{e} = \left[\dfrac{(0.15)(2.68)}{0.76}\right](100) = \mathbf{53\%}$

2.2 a. From Eqs. (2.11) and (2.12), it can be seen that,

$$\gamma_d = \frac{\gamma}{1+w} = \frac{20.1}{1+0.22} = \mathbf{16.48\ kN/m^3}$$

b. $\gamma = 16.48\ \text{kN/m}^3 = \dfrac{G_s\gamma_w}{1+e} = \dfrac{G_s(9.81)}{1+e}$

Eq. (2.14): $e = wG_s = (0.22)(G_s)$. So,

$$16.48 = \frac{9.81G_s}{1+0.22G_s};\quad G_s = \mathbf{2.67}$$

2.3 a. $\gamma = \dfrac{G_s\gamma_w(1+w)}{1+e}$. $119.5 = \dfrac{(2.65)(62.4)(1+0.12)}{1+e}$; $e = \mathbf{0.55}$

b. $n = \dfrac{0.55}{1+0.55} = \mathbf{0.355}$

c. $S = \dfrac{wG_s}{e} = \dfrac{(0.12)(2.65)}{0.55} \times 100 = \mathbf{57.8\%}$

d. $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{119.5}{1+0.12} = \mathbf{106.7\ lb/ft^3}$

2.4 a. $G_s = \dfrac{e}{w}$. $\gamma_d = \dfrac{\left(\dfrac{e}{w}\right)(\gamma_w)}{1+e}$. $85.43 = \dfrac{\left(\dfrac{e}{0.36}\right)(62.4)}{1+e}$; $e = \mathbf{0.97}$

b. $n = \dfrac{e}{1+e} = \dfrac{0.97}{1+0.97} = \mathbf{0.49}$

c. $G_s = \dfrac{e}{w} = \dfrac{0.97}{0.36} = \mathbf{2.69}$

d. $\gamma_{\text{sat}} = \dfrac{(G_s+e)\gamma_w}{1+e} = \dfrac{(2.69+0.97)(62.4)}{1+0.97} = \mathbf{115.9\ lb/ft^3}$

2.5 From Eqs. (2.11) and (2.12): $\gamma_d = \dfrac{116.64}{1+0.08} = 108\ \text{lb/ft}^3$

Eq. (2.12): $\gamma_d = \dfrac{G_s\gamma_w}{1+e}$; $108 = \dfrac{(2.65)(62.4)}{1+e}$; $e = 0.53$

Eq. (2.23): $D_r = 0.82 = \dfrac{e_{\max} - e}{e_{\max} - e_{\min}} = \dfrac{e_{\max} - 0.53}{e_{\max} - 0.44}$; $e_{\max} = \mathbf{0.94}$

$$\gamma_{d\,(\min)} = \frac{G_s\gamma_w}{1+e_{\max}} = \frac{(2.65)(62.4)}{1+0.94} = \mathbf{85.2\ lb/ft^3}$$

2.6 Refer to Table 2.7 for classification.

Soil A: **A-7-6(9)** (*Note*: PI is greater than LL − 30.)

$$\begin{aligned}\text{GI} &= (F_{200} - 35)[0.2 + 0.005(\text{LL} - 40)] + 0.01(F_{200} - 15)(\text{PI} - 10)\\ &= (65 - 35)[0.2 + 0.005(42 - 40)] + 0.01(65 - 15)(16 - 10)\\ &= 9.3 \approx \mathbf{9}\end{aligned}$$

Soil B: **A-6(5)**

$$\begin{aligned}\text{GI} &= (55 - 35)[0.2 + 0.005(38 - 40)] + 0.01(55 - 15)(13 - 10)\\ &= 5.4 \approx \mathbf{5}\end{aligned}$$

Soil C: **A-3-(0)**

Soil D: **A-4(5)**

$$\begin{aligned}\text{GI} &= (64 - 35)[0.2 + 0.005(35 - 40)] + 0.01(64 - 15)(9 - 10)\\ &= 4.585 \approx \mathbf{5}\end{aligned}$$

Soil E: **A-2-6(1)**

$$\text{GI} = 0.01(F_{200} - 15)(\text{PI} - 10) = 0.01(33 - 15)(13 - 10) = 0.54 \approx \mathbf{1}$$

Soil F: **A-7-6(19)** (PI is greater than LL − 30.)

$$\begin{aligned}\text{GI} &= (76 - 35)[0.2 + 0.005(52 - 40)] + 0.01(76 - 15)(24 - 10)\\ &= 19.2 \approx \mathbf{19}\end{aligned}$$

2.7 Soil A: Table 2.8: 65% passing No. 200 sieve.
Fine grained soil; LL = 42; PI = 16
Figure 2.5: **ML**
Figure 2.7: Plus No. 200 > 30%; Plus No. 4 = 0
% sand > % gravel – **sandy silt**

Soil B: Table 2.8: 55% passing No. 200 sieve.
Fine grained soil; LL = 38; PI = 13
Figure 2.5: Plots below A-line – **ML**
Figure 2.7: Plus No. 200 > 30%
% sand > % gravel – **sandy silt**

Soil C: Table 2.8: 8% passing No. 200 sieve.
% sand > % gravel – sandy soil – SP
Figure 2.6: % gravel = 100 – 95 = 5% < 15% – **poorly graded sand**

Soil D: Table 2.8: 64% passing No. 200 sieve
Fine grained soil; LL = 35, PI = 9
Figure 2.5 – **ML**
Figure 2.7: % sand (31%) > % gravel (5%) – **sandy silt**

Soil E: Table 2.8: 33% passing No. 200 sieve; 100% passing No. 4 sieve.
Sandy soil; LL = 38; PI = 13
Figure 2.5: Plots below A-line – **SM**
Figure 2.6: % gravel (0%) < 15% – **silty sand**

Soil F: Table 2.8: 76% passing No. 200 sieve; LL = 52; PI = 24
Figure 2.5: **CH**
Figure 2.7: Plus No. 200 is 100 – 76 = 24%
% gravel > % gravel – **fat clay with sand**

2.8 $\gamma_d = \dfrac{G_s\gamma_w}{1+e}$; $\mathrm{e} = \dfrac{G_s\gamma_w}{\gamma_d} - 1 = \dfrac{(2.68)(62.4)}{117} - 1 = 0.43$

Eq. (2.37): $\dfrac{k_1}{k_2} = \dfrac{\left(\dfrac{e_1^3}{1+e_1}\right)}{\left(\dfrac{e_2^3}{1+e_2}\right)}$; $\dfrac{0.22}{k_2} = \dfrac{\left(\dfrac{0.63^3}{1+0.63}\right)}{\left(\dfrac{0.43^3}{1+0.43}\right)}$; k_2 = **0.08 cm/s**

2.9 From Eq. (2.41):

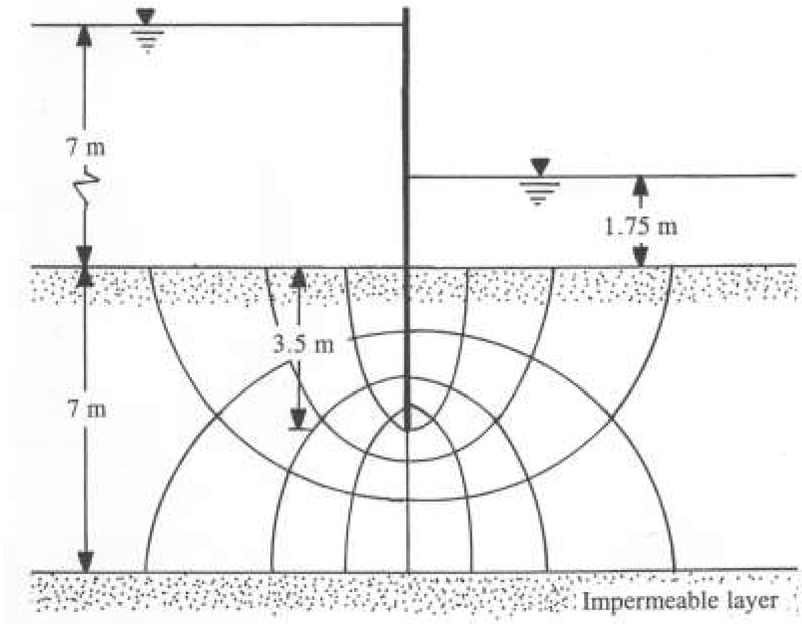
$$\frac{k_1}{k_2} = \left(\frac{1+e_2}{1+e_1}\right)\left(\frac{e_1}{e_2}\right)^n; \quad \text{or } \frac{0.2\times10^{-6}}{0.91\times10^{-6}} = \left(\frac{2.9}{2.2}\right)\left(\frac{1.2}{1.9}\right)^n; \quad 0.1667 = (0.6316)^n$$

$$n = \frac{\log(0.1667)}{\log(0.6316)} = \frac{-0.778}{=0.1996} = 3.898$$

$$C = \frac{k_1(1+e_1)}{e_1^n} = \frac{(0.2\times10^{-6})(2.2)}{1.2^{3.998}} = 0.216\times10^{-6}$$

$$k_3 = C\left(\frac{e^n}{1+e}\right) = \left(\frac{0.9^{3.998}}{1.9}\right)(0.216\times10^{-6}) = \mathbf{0.075\times10^{-6}\ cm/s}$$

2.10 The flow net is shown.



$k = 6.5 \times 10^{-4}$ cm/s; $h_{\max} = H_1 - H_2 = 7 - 1.75 = 5.25$ m. So,

$$q = \left(\frac{6.5 \times 10^{-4}}{10^2}\right)\left[\frac{(5.25)(4)}{8}\right] = \mathbf{17.06 \times 10^{-6}\ m^3/m/s}$$

2.11 a. $k = 2.4622\left[D_{10}^2 \dfrac{e^3}{(1+e)}\right]^{0.7825} = 2.4622\left[(0.2)^2\left(\dfrac{0.6^3}{1+0.6}\right)\right]^{0.7825} = \mathbf{0.041\ cm/s}$

b. $k = 35\left(\dfrac{e^3}{1+e}\right)C_u^{0.6}(D_{10})^{2.32} = (35)\left(\dfrac{0.6^3}{1+0.6}\right)\left(\dfrac{0.4}{0.2}\right)^{0.6}(0.2)^{2.32} = \mathbf{0.171\ cm/s}$

2.12 $\gamma_{\text{dry(sand)}} = \dfrac{G_s\gamma_w}{1+e} = \dfrac{(2.66)(9.81)}{1+0.55} = 16.84\ \text{kN/m}^3$

$$\gamma_{\text{sat(sand)}} = \frac{G_s\gamma_w + e\gamma_w}{1+e} = \frac{(9.81)(2.66+0.48)}{1+0.48} = 20.81\ \text{kN/m}^3$$

$$\gamma_{\text{sat(clay)}} = \frac{G_s\gamma_w(1+w)}{1+wG_s} = \frac{(2.74)(9.81)(1+0.3478)}{1+(0.3478)(2.74)} = 18.55\ \text{kN/m}^3$$

At *A*: σ = **0**; u = **0**; σ' = **0**

At *B*: $\sigma = (16.84)(3) = \mathbf{50.52\ kN/m^2}$
$u = \mathbf{0}$
$\sigma' = \mathbf{50.52\ kN/m^2}$

At *C*: $\sigma = \sigma_B + (20.81)(1.5) = 50.52 + 31.22 = \mathbf{81.74\ kN/m^2}$
$u = (9.81)(1.5) = \mathbf{14.72\ kN/m^2}$
$\sigma' = 81.74 - 14.72 = \mathbf{67.02\ kN/m^2}$

At *D*: $\sigma = \sigma_C + (18.55)(5) = 81.74 + 92.75 = \mathbf{174.49\ kN/m^2}$
$u = (9.81)(6.5) = \mathbf{63.77\ kN/m^2}$
$\sigma' = 174.49 - 63.77 = \mathbf{110.72\ kN/m^2}$

2.13 Eq. (2.54): $C_c = 0.009(\text{LL} - 10) = 0.009(42 - 10) = 0.288$

Eq. (2.65):

$$S_c = \frac{C_cH_c}{1+e_o}\log\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o} = \frac{(0.288)(3.7\times 1000\ \text{mm})}{1+0.82}\log\left(\frac{155}{110}\right) = \mathbf{87.2\ mm}$$

2.14 Eq. (2.69):

$$S_c = \frac{C_sH_c}{1+e_o}\log\left(\frac{\sigma'_c}{\sigma_o}\right) + \frac{C_cH_c}{1+e_o}\log\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}$$

$$= \frac{\left(\frac{0.288}{5}\right)(3700\ \text{mm})}{1+0.75}\log\left(\frac{128}{110}\right) + \frac{(0.288)(3700)}{1+0.82}\log\left(\frac{155}{128}\right) = \mathbf{56.69\ mm}$$

2.15 a. Eq. (2.53): $C_c = \dfrac{e_1 - e_2}{\log\left(\dfrac{\sigma_2'}{\sigma_1'}\right)} = \dfrac{0.91 - 0.792}{\log\left(\dfrac{300}{150}\right)} = 0.392$

From Eq. (2.65): $S_c = \dfrac{C_c H_c}{1+e_o} \log \dfrac{\sigma_o' + \Delta\sigma'}{\sigma_o'}$

Using the results of Problem 2.12,

$$\sigma_o' = (3)(16.84) + 1.5(20.81 - 9.81) + \frac{5}{2}(18.55 - 9.81) = 88.87 \text{ kN/m}^2$$

$$e_o = wG_s = (0.3478)(2.74) = 0.953$$

$$S_c = \frac{(0.392)(5000 \text{ mm})}{1 + 0.953} \log\left(\frac{88.87 + 50}{88.87}\right) = \mathbf{194.54\ mm}$$

b. Eq. (2.73): $T_v = \dfrac{C_v t}{H^2}$. For $U = 50\%$, $T_v = 0.197$ (Table 2.11). So,

$$0.197 = \frac{9.36 \times 10^{-4} t}{(500 \text{ cm})^2}; \quad t = 5262 \times 10^4 \text{ sec} = \mathbf{609\ days}$$

2.16 a. Eq. (2.53): $C_c = \dfrac{e_1 - e_2}{\log\left(\dfrac{\sigma_2'}{\sigma_1'}\right)} = \dfrac{0.82 - 0.64}{\log\left(\dfrac{360}{120}\right)} = \mathbf{0.377}$

b. $C_c = \dfrac{e_1 - e_2}{\log\left(\dfrac{\sigma_2'}{\sigma_1'}\right)}$; $0.377 = \dfrac{0.82 - e_2}{\log\left(\dfrac{200}{120}\right)}$; $e_2 = \mathbf{0.736}$

2.17 Eq. (2.73): $T_v = \dfrac{C_v t}{H^2}$. For 60% consolidation, $T_v = 0.286$ (Table 2.11).

Lab time: $t = 8\tfrac{1}{6}\text{ min} = \dfrac{49}{6}\text{ min}$

$$0.286 = \frac{C_v\left(\dfrac{49}{6}\right)}{(1.5)^2}; \quad C_v = 0.0788\text{ in.}^2/\text{min}$$

Field: $U = 50\%$; $T_v = 0.197$

$$0.197 = \frac{(0.0788)t}{\left(\dfrac{10\times 12}{2}\right)^2}; \quad t = 9000\text{ min} = \mathbf{6.25\ days}$$

2.18 $U = \dfrac{30}{60} = 0.5$

$$T_{v(1)} = \frac{C_{v(1)}t}{H_1^2} = \frac{(2)(t)}{\left(\dfrac{2\times 1000}{2}\right)^2} = 2\times 10^{-6}t$$

$$T_{v(2)} = \frac{C_{v(2)}t}{H_2^2} = \frac{(2)(t)}{\left(\dfrac{1\times 1000}{2}\right)^2} = 8\times 10^{-6}t$$

So, $T_{v(1)} = 0.25T_{v(2)}$. The following table can be prepared for trial and error procedure.

$T_{v(1)}$	$T_{v(2)}$	U_1 (Figure 2.22)	U_2 (Figure 2.22)	$\dfrac{U_1H_1+U_2H_2}{H_1+H_2}=U$
0.05	0.2	0.26	0.51	0.34
0.10	0.4	0.36	0.70	0.473
0.125	0.5	0.40	0.76	0.52
0.1125	0.45	0.385	0.73	0.50

So, $T_{v(1)} = 0.1125 = 2\times 10^{-6}t$; $t = 56{,}250$ min = **39.06 days**

2.19 Eq. (2.84): $T_c = \dfrac{C_v t_c}{H^2}$. t_c = 60 days = 60 × 24 × 60 × 60 sec; $H = \dfrac{2}{2}$ m = 1000 mm.

$$T_c = \frac{(8\times10^{-3})(60\times24\times60\times60)}{(1000)^2} = 0.0415$$

After 30 days: $T_v = \dfrac{C_v t}{H^2} = \dfrac{(8\times10^{-3})(30\times24\times60\times60)}{(1000)^2} = 0.0207$

From Figure 2.24 for $T_v = 0.0207$ and $T_c = 0.0415$, $U = 5\%$. So

$S_c =$ (0.05)(120) = **6 mm**

After 100 days: $T_v = \dfrac{C_v t}{H^2} = \dfrac{(8\times10^{-3})(100\times24\times60\times60)}{(1000)^2} = 0.069$

From Figure 2.24 for $T_v = 0.069$ and $T_c = 0.0415$, $U \approx 23\%$. So

S_c = (0.23)(120) = **27.6 mm**

2.20 $\phi' = \tan^{-1}\left(\dfrac{S}{N}\right)$

Normal force, N (lb)	Shear force, S (lb)	$\phi' = \tan^{-1}\left(\dfrac{S}{N}\right)$ (deg)
50	43.5	41.02
110	95.5	40.96
150	132.0	41.35

From the graph, $\phi' \approx$ **41°**

2.21 Normally consolidated clay; $c' = 0$.

$$\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right);\quad 30 + 96 = 30\tan^2\left(45 + \frac{\phi'}{2}\right);\quad \phi' = \mathbf{38°}$$

2.22 $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right)$; $20 + 40 = 20\tan^2\left(45 + \frac{\phi'}{2}\right)$; $\phi' = \mathbf{30°}$

2.23 $c' = 0$. Eq. (2.91): $\sigma_1' = \sigma_3' \tan^2\left(45 + \frac{\phi'}{2}\right) = 140\tan^2\left(45 + \frac{28}{2}\right) = \mathbf{387.8\ kN/m^2}$

2.24 Eq. (2.91): $\sigma_1' = \sigma_3'\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right)$

$$368 = 140\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right) \qquad \text{(a)}$$

$$701 = 280\tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right) \qquad \text{(b)}$$

Solving Eqs. (a) and (b), $\phi' = \mathbf{24°}$; $c' = \mathbf{12\ kN/m^2}$

2.25 $\phi = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right) = \sin^{-1}\left(\frac{32 - 13}{32 + 13}\right) = \mathbf{25°}$

$$\phi' = \sin^{-1}\left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'}\right)$$

$\sigma_3' = 32 - 5.5 = 26.5\ \text{lb/in.}^2$; $\sigma_3' = 13 - 5.5 = 7.5\ \text{lb/in.}^2$

$$\phi' = \sin^{-1}\left(\frac{26.5 - 7.5}{26.5 + 7.5}\right) = \mathbf{34°}$$

Normally consolidated clay; $c = \mathbf{0}$ and $c' = \mathbf{0}$

2.26 $\sigma_1 = \sigma_3 \tan^2\left(45 + \frac{\phi}{2}\right)$. $\sigma_1 = 150\tan^2\left(45 + \frac{20}{2}\right) = 305.9\ \text{kN/m}^2$

$$\frac{\sigma_1'}{\sigma_3'} = \tan^2\left(45 + \frac{\phi'}{2}\right); \quad \frac{305.9 - u}{150 - u} = \tan^2\left(45 + \frac{28}{2}\right); \quad u = \mathbf{61.9\ kN/m^2}$$

2.27 a. $\phi' = 26° + 10D_r + 0.4C_u + 1.6\log(D_{50})$

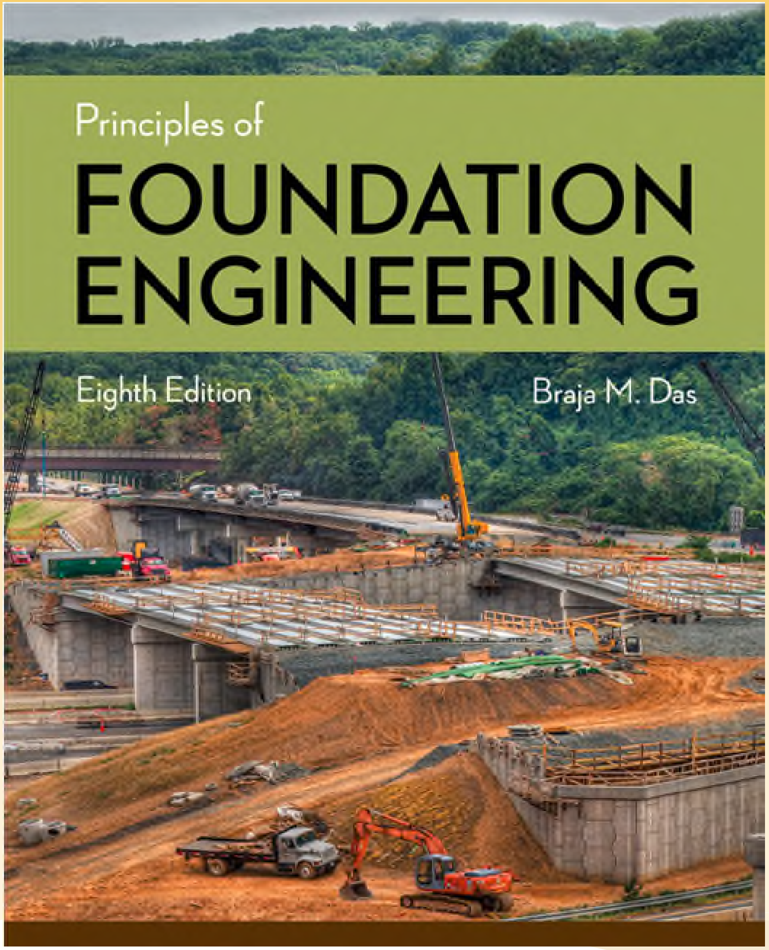
$$= 26° + (10)(0.53) + (0.4)(2.1) + (1.6)[\log(0.13)] = \mathbf{30.7}°$$

b. $\phi' = \dfrac{1}{ae + b}$

$$a = 2.101 + 0.097\left(\frac{D_{85}}{D_{15}}\right) = 2.101 + 0.097\left(\frac{0.21}{0.09}\right) = 2.327$$

$$b = 0.845 - 0.398a = 0.845 - (0.398)(2.327) = -0.081$$

$$\phi' = \frac{1}{(2.327)(0.68) - 0.081} = \mathbf{33.67°}$$



Chapter 2

Geotechnical Properties of Soil

Introduction

Designing foundations for structures such as buildings, bridges, and dams generally requires a knowledge of the following:

1. The load that will be transmitted by the superstructure to the foundation system

2. The requirements of the local building code

3. The behavior and stress-related deformability of soils supporting the foundation system

4. The geological conditions of the soil under consideration

Introduction – Soil Testing

Geotechnical properties of soil such as grain-size distribution, plasticity, compressibility, and shear strength can be assessed by laboratory testing.

In situ determination of strength and deformation properties of soil are considered because these processes avoid disturbing samples during field exploration.

Not all of the needed parameters can be/are determined, because of economic or other reasons.

Engineer Experience

To assess the soil parameters the engineer must have a good grasp of the basic principles of soil mechanics.

Natural soil deposits are not homogeneous in most cases. Thus the engineer must have a thorough understanding of the geology of the area, such as the origin and nature of soil stratification and ground water conditions.

Foundation engineering is a clever combination of soil mechanics, engineering geology, and proper judgment derived from past experience.

Introduction – Chapter Summary

This chapter serves primarily as a review of the basic geotechnical properties of soils.

Focus includes grain-size distribution, plasticity, soil classification, hydraulic conductivity, effective stress, consolidation, and shear strength parameters.

Grain-Size Distribution

Sizes of the grains vary greatly in any soil mass. To classify a soil properly, you must know its *grain-size distribution.*

Grain-size distribution for coarse-grained soil is determined through sieve analysis.

Grain-size distribution for fine-grained soil is conducted by hydrometer analysis.

Sieve Analysis

Conducted by taking a measured amount of dry, well-pulverized soil and passing it through a stack of progressively finer sieves with a pan at the bottom.

The amount of soil retained on each sieve is measured, and the cumulative percentage of soil passing through each is determined and is referred to as percent finer.

Sieve Analysis

Table 2.1 contains a list of U.S. sieve numbers and the corresponding size of their openings.

Table 2.1 U.S. Standard Sieve Sizes

Sieve No.	Opening (mm)
4	4.750
6	3.350
8	2.360
10	2.000
16	1.180
20	0.850
30	0.600
40	0.425
50	0.300
60	0.250
80	0.180
100	0.150
140	0.106
170	0.088
200	0.075
270	0.053

These sieves are commonly used for the analysis of soil for classification purposes.

Sieve Analysis

The percent finer for each sieve is determined by a sieve analysis and plotted on *semilogarithmic graph paper,* as shown here.

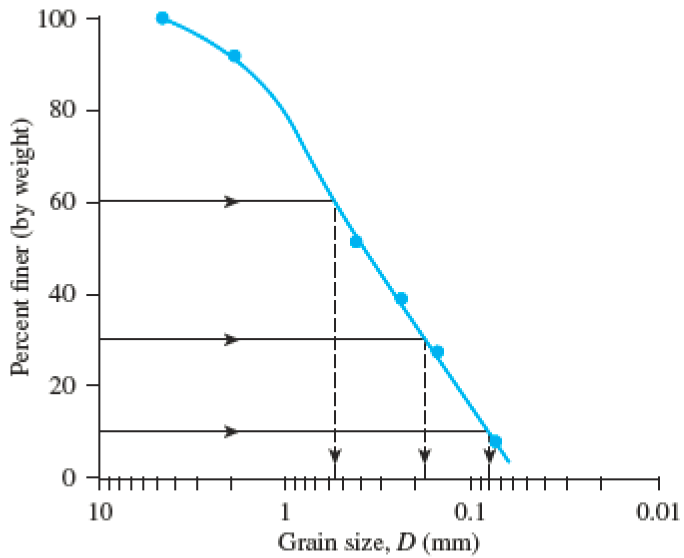


Figure 2.1 Grain-size distribution curve of a coarse-grained soil obtained from sieve analysis

Notice that grain diameter, *D* is plotted on a *logarithmic scale* and the percent finer is plotted an *arithmetic scale.*

Sieve Analysis

Two parameters can be determined from the grain-size distribution curves of coarse-grained soils

1. The *Uniformity coefficient* (C_u),

$$C_u = \frac{D_{60}}{D_{10}}$$

2. The *coefficient of gradation,* or *coefficient of curvature* (C_c)

$$C_c = \frac{D_{30}^2}{(D_{10})(D_{60})}$$

*D*10, *D*30, and *D*60 are the diameters corresponding to percents finer than 10, 30, and 60%, respectively.

Hydrometer Analysis

Based on the principle of sedimentation of soil particles in water.

1. A deflocculating agent is added to dry, pulverized soil.

2. The soil soaks for a minimum of 16 hours.

3. Add distilled water and transfer to 1000 ml cylinder and fill sample with distilled water to 1000 ml mark

4. Place hydrometer in solution to measure the specific gravity of soil and water over 24 hour period.

Hydrometer Analysis

Hydrometers show the amount of soil that is still in suspension at any given time *t.*

The largest diameter of the soil particles still in suspension at time *t* can be determined by Stokes' law

D= diameter of the soil particle

G_s= specific gravity of soil solids

η =dynamic viscosity of water

γ_w=unit weight of water

L=effective length (i.e., length measured from the water surface in the cylinder to the center of gravity of the hydrometer)

t=time

$$D = \sqrt{\frac{18\eta}{(G_s - 1)\gamma_w}}\sqrt{L/t}$$

Hydrometer Analysis

With hydrometer readings taken at various times, the soil *percent finer* than a given diameter *D* can be calculated and a grain-size distribution plot prepared.

The sieve and hydrometer techniques may be combined for a soil having both coarse-grained and fine-grained soil constituents.

Size Limits for Soils

Several organizations have attempted to develop the size limits for *gravel, sand, silt,* and *clay* on the basis of the grain sizes present in soils.

This table presents the size limits recommended by the American Association of State Highway and Transportation Officials (AASHTO) and the Unified Soil Classification systems (Corps of Engineers, Department of the Army, and Bureau of Reclamation)

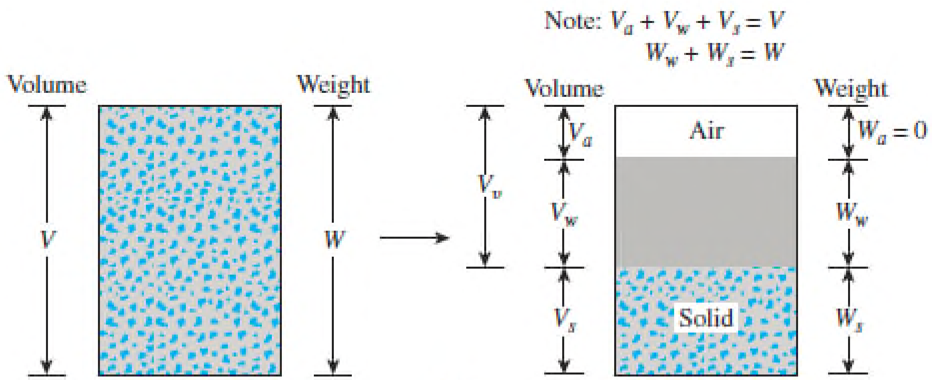
Table 2.2 Soil-Separate Size Limits

Classification system	Grain size (mm)
Unified	Gravel: 75 mm to 4.75 mm Sand: 4.75 mm to 0.075 mm Silt and clay (fines): <0.075 mm
AASHTO	Gravel: 75 mm to 2 mm Sand: 2 mm to 0.05 mm Silt: 0.05 mm to 0.002 mm Clay: <0.002 mm

Weight-Volume Relationships

Soils are three-phase systems consisting of solid soil particles, water, and air.

The phases can be separated



(a)

Based on this separation, the volume relationship can be defined.

Weight Volume Relationships

The *void ratio* (e) is the ratio of the volume of voids to the volume of soil solids in a given soil mass.

$$e = V_v / V_s$$

The *porosity, n,* is the ratio of the volume of voids to the volume of the soil specimen.

$$n = V_v / V$$

V_v = volume of voids

V_s = volume of soil solids

V = total volume

Weight Volume Relationships

The *degree of saturation, S,* is the ratio of the volume of water in the void spaces to the volume of voids.

$$S(\%) = \frac{V_w}{V_v}(100)$$

V_w = volume of water

Weight Volume Relationships

Moisture content, $w(\%) = \frac{W_w}{W_s}(100)$

Moist unit weight, $\gamma = W / V$

Dry unit weight, $\gamma_d = W_s / V$

W_s = weight of the soil solids

W_w = weight of water

W = total weight of the soil specimen $W = W_s + W_w$

Weight Volume Relationships

For moist unit weight of a soil specimen, $\gamma = \frac{w}{v} = \frac{W_s + W_w}{V_s + V_v} = \frac{G_s \gamma_w (1+w)}{1+e}$

For dry unit weight, of soil specimen, $\gamma_d = \frac{w_s}{v} = \frac{W_s}{V_s + V_v} = \frac{G_s \gamma_w}{1+e}$

For saturated unit weight soil, $\gamma_{sat} = \frac{w_s + w_s}{v_s + v_v} = \frac{G_s \gamma_w + e\gamma_w}{1+e}$

G_s= specific gravity of soil solids

γ_w= units weight of water

Weight Volume Relationships

For further relationships

Table 2.3 Various Forms of Relationships for γ, γ_d, and γ_{sat}

Unit-weight relationship	Dry unit weight	Saturated unit weight
$\gamma = \frac{(1+w)G_s\gamma_w}{1+e}$	$\gamma_d = \frac{\gamma}{1+w}$	$\gamma_{sat} = \frac{(G_s+e)\gamma_w}{1+e}$
$\gamma = \frac{(G_s+Se)\gamma_w}{1+e}$	$\gamma_d = \frac{G_s\gamma_w}{1+e}$	$\gamma_{sat} = [(1-n)G_s+n]\gamma_w$
$\gamma = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	$\gamma_d = G_s\gamma_w(1-n)$	$\gamma_{sat} = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$
$\gamma = G_s\gamma_w(1-n)(1+w)$	$\gamma_d = \frac{G_s}{1+\frac{wG_s}{S}}\gamma_w$	$\gamma_{sat} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$
	$\gamma_d = \frac{eS\gamma_w}{(1+e)w}$	$\gamma_{sat} = \gamma_d + n\gamma_w$
	$\gamma_d = \gamma_{sat} - n\gamma_w$	$\gamma_{sat} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$
	$\gamma_d = \gamma_{sat} - \left(\frac{e}{1+e}\right)\gamma_w$	

Weight Volume Relationships

Specific gravities for certain materials can be found in the table below

Table 2.4 Specific Gravities of Some Soils

Type of soil	G_s
Quartz sand	2.64–2.66
Silt	2.67–2.73
Clay	2.70–2.9
Chalk	2.60–2.75
Loess	2.65–2.73
Peat	1.30–1.9

Relative Density

For granular soils the degree of compaction in the field can be measured according to relative density,

$$D_r(\%) = \frac{e_{\max} - e}{e_{\max} - e_{\min}}(100)$$

$e_{\max}$ = void ratio of the soil in the loosest state

$e_{\min}$ = void ratio in the densest state

e = *in situ* void ratio

Relative Density

Clean sand (F_c = 0 to 5%) $e_{max} = 0.072 + 1.53e_{min}$

Sand with fines ($5 < F_c \leq 15\%$) $e_{max} = 0.25 + 1.37e_{min}$

Sands with fines and clay ($15 < P_c \leq 30\%$; F_c = 5 to 20%)

$e_{max} = 0.44 + 1.21e_{min}$

Silty Soils ($30 < F_c \leq 70\%$ P_c = 5 to 20%) $e_{max} = 0.44 + 1.32e_{min}$

F_c = fine fraction for which grain size is smaller that 0.075 mm

P_c = clay-size fraction (<0.005 mm)

Relative Density

The general equation relation for $e_{\min}$ and $e_{\max}$

$$e_{\max} - e_{\min} = 0.23 + \frac{0.06}{D_{50}(mm)}$$

Atterberg Limits

When clayey soil is mixed with an excessive amount of water, it may flow like a *semiliquid.*

If the soil is gradually dried, it will behave like a *plastic, semisolid,* or *solid* material, depending on its moisture content.

Liquid limit (LL): The moisture content, in percent, at which the soil changes from a semiliquid to a plastic state.

Plastic limit (PL): The moisture content, in percent, at which the soil changes from a plastic to a semisolid state.

Shrinkage limit (SL): The moisture content, in percent, at which the soil changes from a semisolid to a solid state.

These limits are referred to as *Atterberg limits.*

Atterberg Limits

The *liquid limit* of a soil is determined by Casagrande's liquid device and is defined as the moisture content at which a groove closure of 12.7 mm (1/2 in.) occurs at 25 blows.

The *plastic limit* is defined as the moisture content at which the soil crumbles when rolled into a thread of 3.18 mm (1/8 in.) in diameter.

The *shrinkage limit* is defined as the moisture content at which the soil does not undergo any further change in volume with loss of moisture.

The difference between the liquid limit and the plastic limit of a soil is defined as the *plasticity index* (PI)

$$PI = LL - PL$$

Liquidity Index

Liquidity Index: The relative consistency of a cohesive soil in its natural state.

$$LI = \frac{w - PL}{LL - PL}$$

w= *in situ* moisture content of soil

The *in situ* moisture content for a sensitive clay may be greater than the liquid limit. In this case $LI > 1$

The soil deposits that are heavily overconsolidated may have a natural moisture content less than the plastic limit. In this case $LI < 0$

Activity

Plasticity of soil is caused by the adsorbed water that surrounds the clay particles. It is expected that the type of clay minerals and their proportional amounts in a soil will affect the liquid and plastic limits.

Plasticity index of a soil increases linearly with the percentage of clay-size fraction (% finer than 2 micrometers by weight) present.

Activity: Slope of the line correlating PI and % finer than 2 micrometers.

Activity is used as an index for identifying swelling potential of clay soils

$$A = \frac{PI}{\%\ \text{of clay-size fraction, by weight}}$$

Soil Classification Systems

Soil classification systems divide soils into groups and subgroups based on common engineering properties such as the *grain-size distribution, liquid limit,* and *plastic limit.*

The two major classification systems presently in use:

(1) the *American Association of State Highway and Transportation Officials* (AASHTO) *System*

(2) the *Unified Soil Classification System* (also ASTM). The AASHTO system is used mainly for the classification of highway subgrades. It is not used in foundation construction.

AASHTO

According to the present form of this system, soils can be classified according to eight major groups, A-1 through A-8, based on their grain-size distribution, liquid limit, and plasticity indices.

Soils listed in groups A-1, A-2, and A-3 are coarse-grained materials.

Soils in groups A-4, A-5, A-6, and A-7 are fine-grained materials.

Peat, muck, and other highly organic soils are classified under A-8. They are identified by visual inspection.

AASHTO

For qualitative evaluation of the desirability of a soil as a highway subgrade material, a number referred to as the *group index* has also been developed.

The higher the value of the group index for a given soil, the weaker will be the soil's performance as a subgrade.

A group index of 20 or more indicates a very poor subgrade material.

The formula for the group index is

$$GI = (F_{200} - 35)[0.2 + 0.005(LL - 40)] + 0.01(F_{200} - 15)(PI - 10)$$

F_{200} = percent passing No.200 sieve, expressed as a whole number

LL = liquid limit

PI = plasticity limit

AASHTO

When calculating group index for a soil of group A-2-6 or A-2-7, use only the partial group-index equation relating to the plasticity index. $GI = 0.01(F_{200} - 15)(PI - 10)$

The group index is rounded to the nearest whole number and written next to the soil group in parentheses.

The group index for soils which fall in groups A-1-a, A-1-b, A-3, A-2-4, and A-2-5 is always zero.

Unified System

The plasticity chart and the table show the procedure for determining the group symbols for various types of soil.

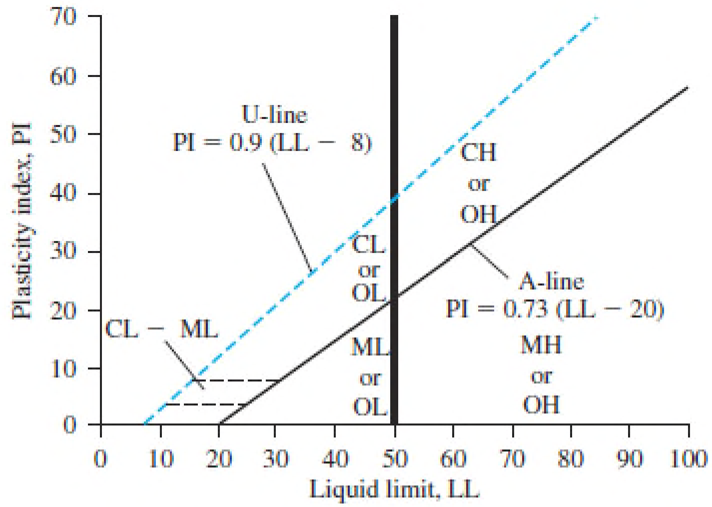


Figure 2.5 Plasticity chart

Unified System

Table 2.8 Unified Soil Classification Chart (after ASTM, 2011) (Based on ASTM D2487-10: Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification)

Criteria for assigning group symbols and group names using laboratory tests[a]				Soil classification: Group symbol	Soil classification: Group name[b]
Coarse-grained soils More than 50% retained on No. 200 sieve	Gravels More than 50% of coarse fraction retained on No. 4 sieve	Clean Gravels Less than 5% fines[c]	$C_u \geq 4$ and $1 \leq C_c \leq 3$[e]	GW	Well-graded gravel[f]
			$C_u < 4$ and/or $1 > C_c > 3$[e]	GP	Poorly graded gravel[f]
		Gravels with Fines More than 12% fines[c]	Fines classify as ML or MH	GM	Silty gravel[f,g,h]
			Fines classify as CL or CH	GC	Clayey gravel[f,g,h]
	Sands 50% or more of coarse fraction passes No. 4 sieve	Clean Sands Less than 5% fines[d]	$C_u \geq 6$ and $1 \leq C_c \leq 3$[e]	SW	Well-graded sand[i]
			$C_u < 6$ and/or $1 > C_c > 3$[e]	SP	Poorly graded sand[i]
		Sand with Fines More than 12% fines[d]	Fines classify as ML or MH	SM	Silty sand[g,h,i]
			Fines classify as CL or CH	SC	Clayey sand[g,h,i]
Fine-grained soils 50% or more passes the No. 200 sieve	Silts and Clays Liquid limit less than 50	Inorganic	PI > 7 and plots on or above "A" line[j]	CL	Lean clay[k,l,m]
			PI < 4 or plots below "A" line[j]	ML	Silt[k,l,m]
		Organic	$\dfrac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$	OL	Organic clay[k,l,m,n]; Organic silt[k,l,m,o]
	Silts and Clays Liquid limit 50 or more	Inorganic	PI plots on or above "A" line	CH	Fat clay[k,l,m]
			PI plots below "A" line	MH	Elastic silt[k,l,m]
		Organic	$\dfrac{\text{Liquid limit—oven dried}}{\text{Liquid limit—not dried}} < 0.75$	OH	Organic clay[k,l,m,p]; Organic silt[k,l,m,q]
Highly organic soils	Primarily organic matter, dark in color, and organic odor			PT	Peat

[a]Based on the material passing the 75-mm. (3-in) sieve.
[b]If field sample contained cobbles or boulders, or both, add "with cobbles or boulders, or both" to group name.
[c]Gravels with 5 to 12% fines require dual symbols: GW-GM well-graded gravel with silt; GW-GC well-graded gravel with clay; GP-GM poorly graded gravel with silt; GP-GC poorly graded gravel with clay.
[d]Sands with 5 to 12% fines require dual symbols: SW-SM well-graded sand with silt; SW-SC well-graded sand with clay; SP-SM poorly graded sand with silt; SP-SC poorly graded sand with clay.

[e]$C_u = D_{60}/D_{10}$ $C_c = \dfrac{(D_{30})^2}{D_{10} \times D_{60}}$

[f]If soil contains ≥ 15% sand, add "with sand" to group name.
[g]If fines classify as CL-ML, use dual symbol GC-GM or SC-SM.
[h]If fines are organic, add "with organic fines" to group name.
[i]If soil contains ≥15% gravel, add "with gravel" to group name.
[j]If Atterberg limits plot in hatched area, soil is a CL-ML, silty clay.
[k]If soil contains 15 to 29% plus No. 200, add "with sand" or "with gravel," whichever is predominant.
[l]If soil contains ≥30% plus No. 200, predominantly sand, add "sandy" to group name.
[m]If soil contains ≥30% plus No. 200, predominantly gravel, add "gravelly" to group name.
[n]PI ≥ 4 and plots on or above "A" line.
[o]PI < 4 or plots below "A" line.
[p]PI plots on or above "A" line.
[q]PI plots below "A" line.

Unified System

When classifying a soil be sure to provide the group name that generally describes the soil, along with the group symbol.

The flowcharts are used for obtaining the group names for coarse-grained soil, inorganic fine-grained soil, and organic fine-grained soil, respectively.

Unified System

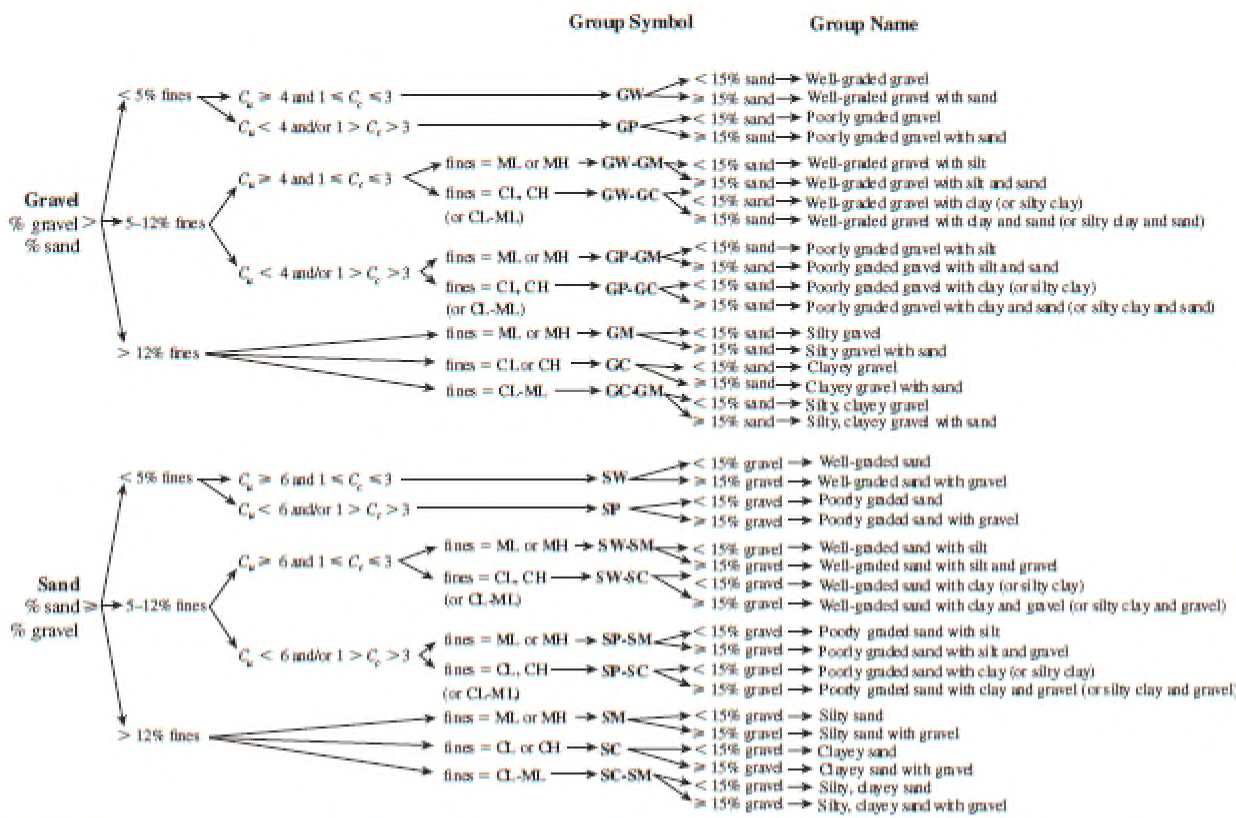


Figure 2.6 Flowchart for classifying coarse-grained soils (more than 50% retained on No. 200 Sieve) (After ASTM, 2011) (Based on ASTM D2487-10: Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification).

Unified System

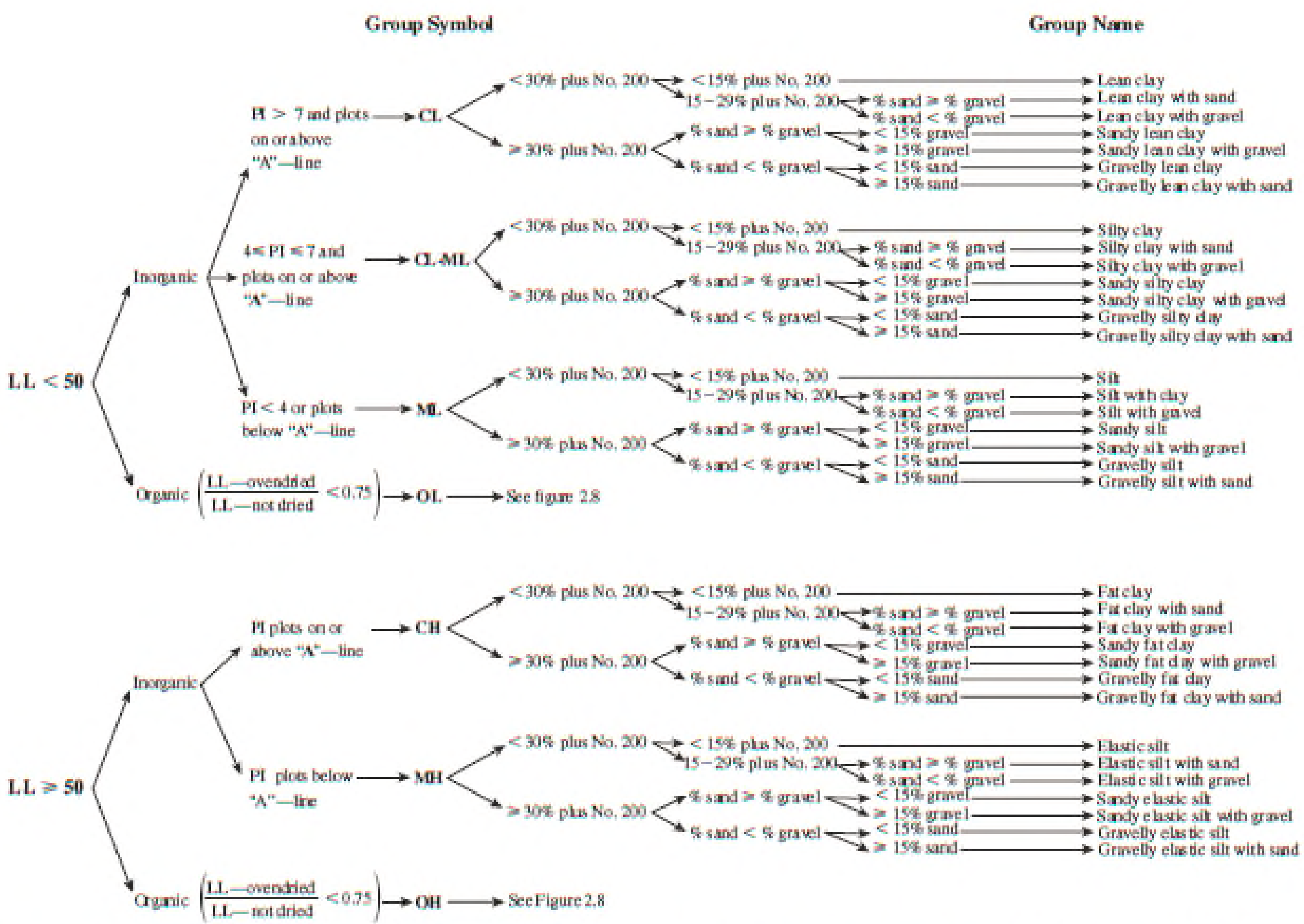


Figure 2.7 Flowchart for classifying fine-grained soil (50% or more passes No. 200 Sieve) (After ASTM, 2011) (Based on ASTM D2487-10: Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification).

Unified System

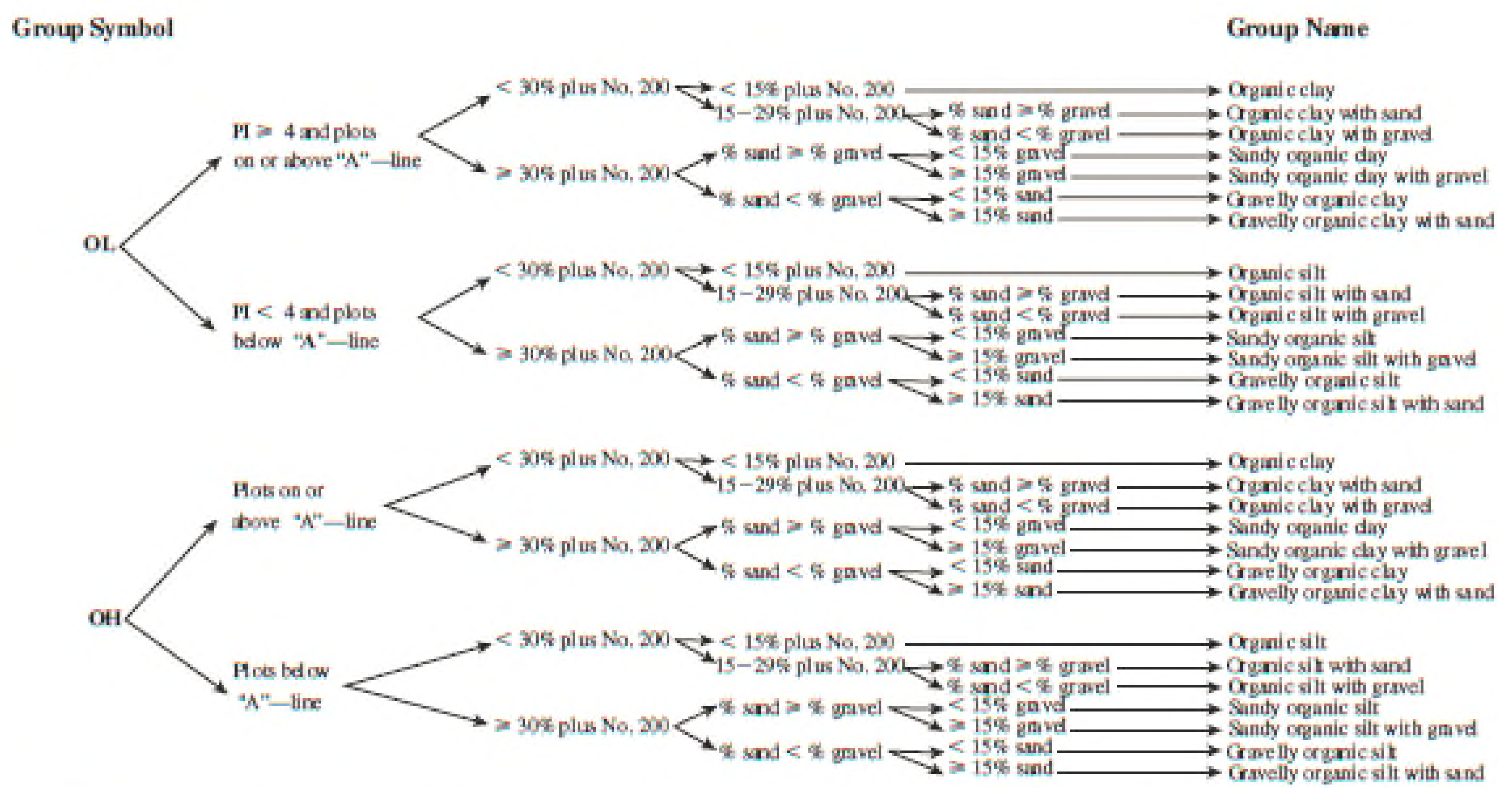


Figure 2.8 Flowchart for classifying organic fine-grained soil (50% or more passes No. 200 Sieve) (After ASTM, 2011) (Based on ASTM D2487-10: Standard Practice for Classification of Soils for Engineering Purposes (Unified Soil Classification).

Unified System

Symbols of the Unified System

G= Gravel

S = Sand

M = Silt

C = Clay

O = Organic silts and clay

Pt = Peat and highly organic soils

H = High plasticity

L = Low plasticity

W = Well graded

P = Poorly graded

Hydraulic Conductivity

Water flows between void spaces or pores between soil grains.

Knowing how much water is flowing through soil per unit of time is important for soil mechanics and foundation engineering.

Water flow knowledge is required for designing earth dams, determining seepage under hydraulic structures, and dewatering foundations before and during construction.

Hydraulic Conductivity

The equation $v = ki$ is used to calculate the velocity of flow of water through a soil.

v =Darcy Velocity (unit: cm/sec)

k =hydraulic conductivity of soil (unit: cm/sec)

i = hydraulic gradient

The hydraulic gradient is defined as $i = \Delta h / L$

Δh = piezometric head difference between the sections at AA and BB

L = distance between the sections at AA and BB

Hydraulic Conductivity

For granular soils, the hydraulic conductivity (k) depends on the void ratio.

Although several equation have been proposed, it is recommended that the equation $k \propto \frac{e^3}{1+e}$ be used to relate k to the void ratio in granular soils.

k = hydraulic conductivity
e = void ratio

The range of hydraulic conductivity for various soils is given in this table.

Table 2.9 Range of the Hydraulic Conductivity for Various Soils

Type of soil	Hydraulic conductivity, *k* (cm/sec)
Medium to coarse gravel	Greater than 10^{-1}
Coarse to fine sand	10^{-1} to 10^{-3}
Fine sand, silty sand	10^{-3} to 10^{-5}
Silt, clayey silt, silty clay	10^{-4} to 10^{-6}
Clays	10^{-7} or less

Hydraulic Conductivity

In determining the hydraulic conductivity of consolidated clays, use the equation

$$k = c\frac{e^n}{1+e}$$

n and c are constants determined experimentally.

Steady-State Seepage

For most cases of seepage under hydraulic structures, the flow path changes direction and is not uniform over the entire area.

One of the ways for determining the rate of seepage is by a graphical construction referred to as the *flow net*.

Steady-State Seepage

The flow at any point A can be determined by the equation

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

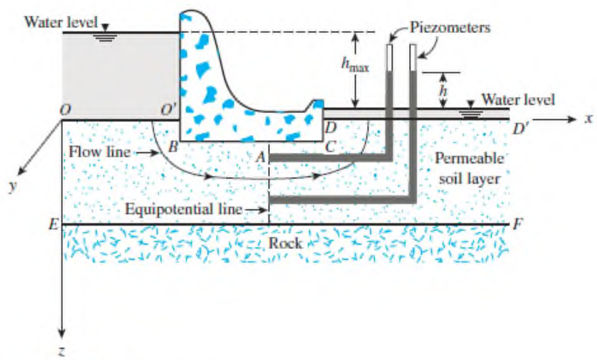


Figure 2.11 Steady-state seepage

k_x, k_y, k_z = hydraulic conductivity of the soil in the x, y, and z directions, respectively

h = hydraulic head at point A (i.e., the head of water that a piezometer placed at *A* would show with the *downstream water level* as *datum,* as shown in Figure 2.11)

Steady-State Seepage

Laplace's equation $$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Laplace's equation is valid for confined flow and represents two orthogonal sets of curves known as *flow lines* and *equipotential lines.*

Steady-State Seepage

A flow net is a combination of numerous equipotential lines and flow lines.

A flow line is a path that a water particle would follow traveling from the upstream side to the downstream side.

An equipotential line is a line along which water, in piezometers, would rise to the same elevation.

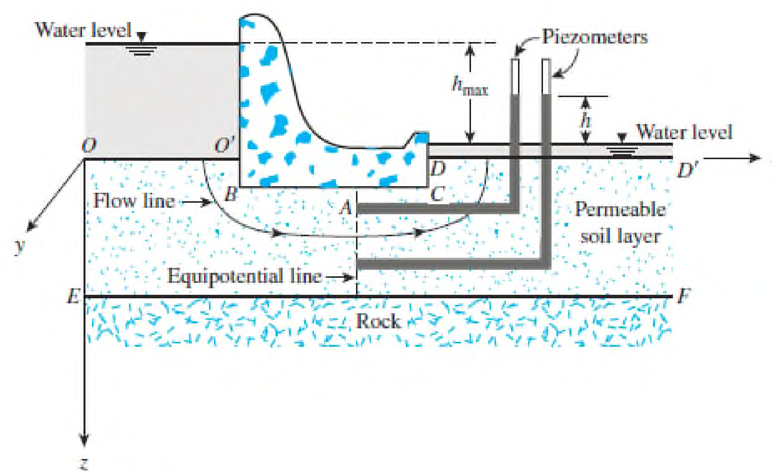


Figure 2.11 Steady-state seepage

Steady-State Seepage

In drawing a flow net, you need to establish the *boundary conditions.*

The ground surfaces on the upstream (*OO*′) and downstream (*DD*′) sides are equipotential lines.

The base of the dam below the ground surface, *O′BCD*, is a flow line. The top of the rock surface, *EF*, is also a flow line.

Steady-State Seepage

Once the boundary conditions are established, a number of flow lines and equipotential lines are drawn by trial and error so that all the flow elements in the net have the same length-to-width ratio (*L*/*B*).

In most cases, *L*/*B* is held to unity, that is, the flow elements are drawn as curvilinear "squares."

All flow lines must intersect all equipotential lines at *right angles*

Steady-State Seepage

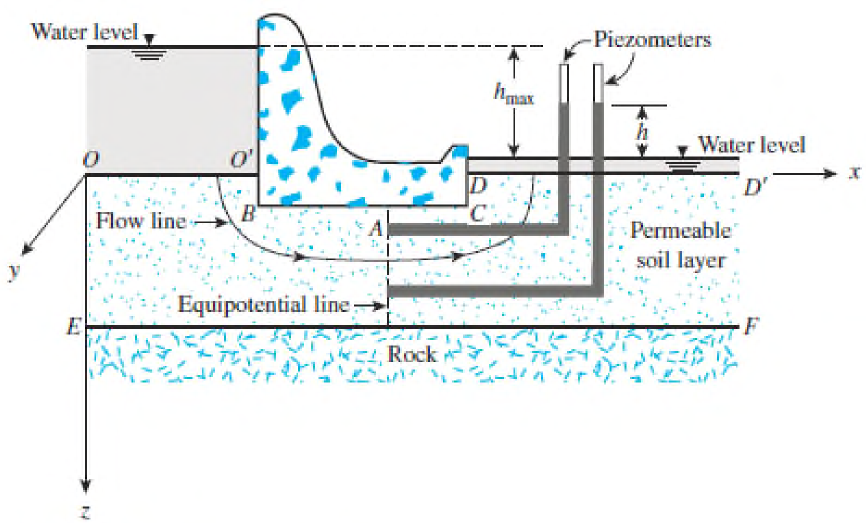


Figure 2.11 Steady-state seepage

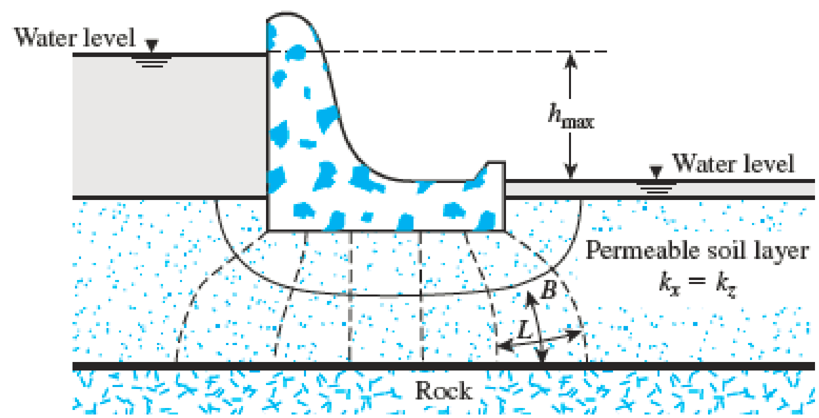


Figure 2.12 Flow net

Steady-State Seepage

Once the flow net is drawn, the seepage (in unit time per unit length of the structure) can be calculated using the equation

$$q = kh_{\max} \frac{N_f}{N_d} n$$

N_f = number of flow channels

N_d = number of drops

n = width-to-length ratio of the flow elements in the flow net (B/L)

$h_{\max}$ = difference in water level between the upstream and downstream sides

Effective Stress

The *total* stress at a given point in a soil mass can be expressed as $\sigma = \sigma' + u$

σ = total stress
σ' = effective stress
u = pore water pressure

Effective Stress

The effective stress σ' is the vertical component of forces at solid-to-solid contact points over a unit cross-sectional area.

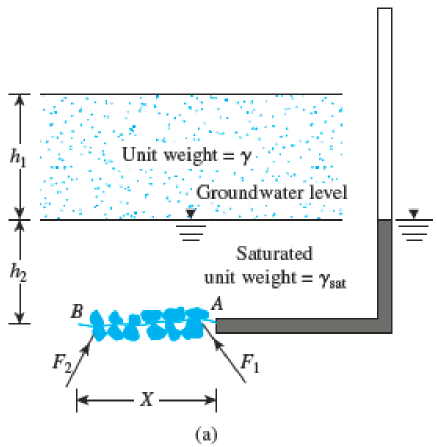


Figure 2.13 Calculation of effective stress

In reference to point A of the figure Effective stress is calculated by the equation

$$\sigma' = \gamma h_1 + \gamma' h_2$$

γ'=effective or submerged unit weight of soil.

Effective Stress

If there is no upward seepage of water in the soil then again referring to point A

$$\sigma = \gamma h_1 + \gamma_{sat} h_2$$

$$u = h_2 \gamma_w$$

γ_w = unit weight of water
γ_{sat} = saturated unit weight of soil

Effective Stress

If upward seepage of water does occur in soil than at point A in the figure

$$\sigma = h_1\gamma_w + h_2\gamma_{sat}$$

$$u = (h_1 + h_2 + h)\gamma_w$$

$$\sigma' = h_2(\gamma' - \frac{h}{h_2}\gamma_w) = h_2(\gamma' - i\gamma_w)$$

i =hydraulic gradient (h/h_2)

Effective Stress

If the hydraulic gradient is very high, then *the effective stress will become zero.*

If there is no contact stress between the soil particles, the soil will break up.

This situation is referred to as the *quick condition,* or *failure by heave.*

For the heave $$i = i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{1 + e}$$

i_{cr} = Critical hydraulic gradient

Consolidation

When stress on a saturated clay layer is increased—for example, by the construction of a foundation—the pore water pressure in the clay will increase.

The hydraulic conductivity of clays is very small and some time will be required for excess pore water pressure to dissipate and for the increase in stress to be transferred to the soil skeleton.

Consolidation

If $\Delta\sigma$ is a surcharge at the ground surface over a very large area, the increase in total stress at any depth of the clay layer will be equal to $\Delta\sigma$.

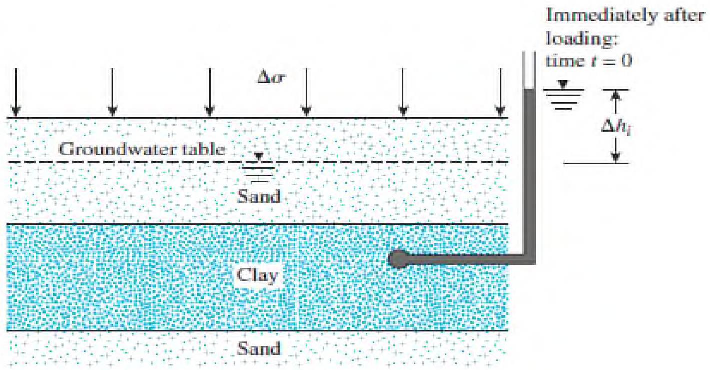


Figure 2.15 Principles of consolidation

At time t = 0 (i.e., immediately after the stress is applied), the excess pore water pressure at any depth

Δu will equal $\Delta\sigma$.

$$\Delta u = \Delta h_i \gamma_w = \Delta\sigma \text{ (at time t=0)}$$

Consolidation

At time $t = \infty$ all the excess pore water pressure in the clay layer should dissipate as a result of drainage into the sand layers.

$\Delta u = 0$ (at time t= ∞)

Then the increase in effective stress in the clay layer is

$$\Delta\sigma' = \Delta\sigma$$

The gradual increase in the effective stress in the clay layer will cause settlement over a period of time and is referred to as *consolidation.*

Consolidation

Laboratory tests on undisturbed saturated clay specimens can be conducted to determine the consolidation settlement caused by various incremental loadings.

Based on the laboratory tests, a graph can be plotted showing the variation of the void ratio *e* at the *end* of consolidation against the corresponding vertical effective stress σ'.

Consolidation

From the *e*–log σ' curve shown in Figure 2.16b, three parameters necessary for calculating settlement in the field can be determined.

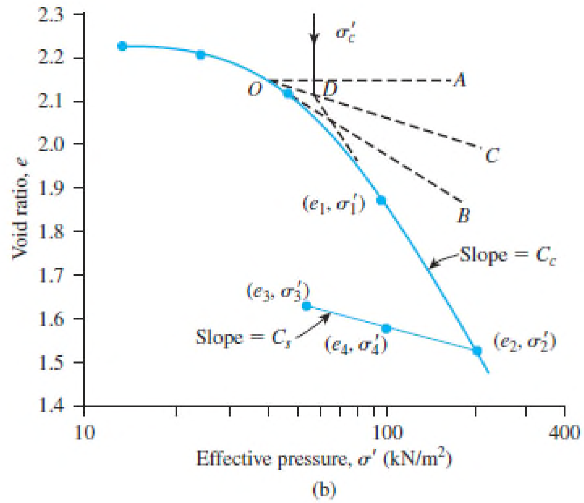


Figure 2.16 (a) Schematic diagram of consolidation test arrangement; (b) *e*–log σ' curve for a soft clay from East St. Louis, Illinois (*Note:* At the end of consolidation, $\sigma = \sigma'$)

The parameters are preconsolidation pressure σ'_c, compression index C_c, and the swelling index C_s.

Preconsolidation Pressure

Preconsolidation pressure, σ', is the *maximum past effective overburden pressure* to which the soil specimen has been subjected.

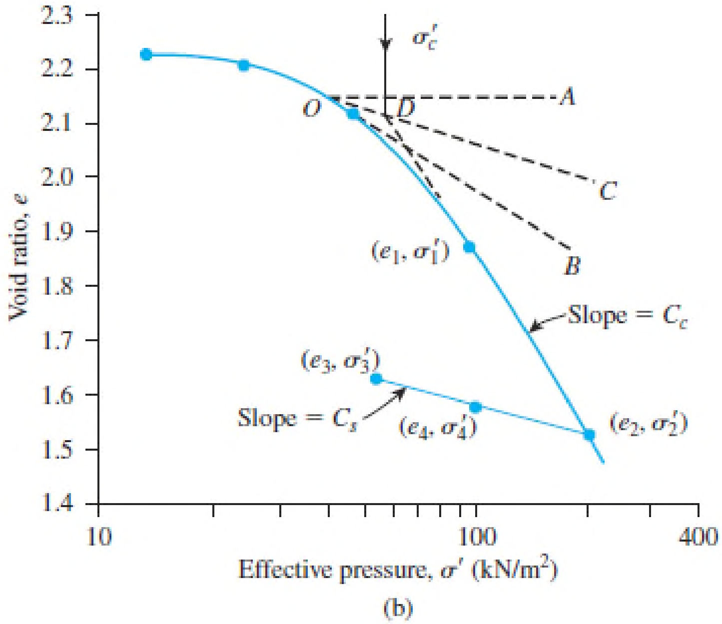


Figure 2.16 (a) Schematic diagram of consolidation test arrangement; (b) e–log σ' curve for a soft clay from East St. Louis, Illinois (*Note:* At the end of consolidation, $\sigma = \sigma'$)

Preconsolidation Pressure

Determining the preconsolidation pressure involves a 5 step process. (Use figure on previous slide for reference)

1. Determine the point *O* on the *e*–log σ'_c curve that has the sharpest curvature (i.e., the smallest radius of curvature).

2. Draw a horizontal line *OA.*

3. Draw a line *OB* that is tangent to the *e*–log σ' curve at *O.*

4. Draw a line *OC* that bisects the angle *AOB.*

5. Produce the straight-line portion of the *e*–log σ' curve backwards to intersect *OC.* This is point *D.* The pressure that corresponds to point *D* is the preconsolidation pressure σ'_c .

Preconsolidation Pressure

Natural soil deposits can be *normally consolidated* or *overconsolidated* (or *preconsolidated*).

If the present effective overburden pressure $\sigma' = \sigma'_0$ is equal to the preconsolidated pressure σ'_c the soil is *normally consolidated.*

If $\sigma'_0 < \sigma'_c$ the soil is *overconsolidated.*

Preconsolidation Pressure

Preconsolidation pressure can be correlated with the liquidity index by the equation

$$\frac{\sigma'_c}{P_a} = 10^{(1.11 - 1.62LI)}$$

P_a = atmospheric pressure

LI = liquidity index

Compression Index

Compression index is the slope of the straight-line portion of the loading curve and is determined by

$$C_c = \frac{e_1 - e_2}{\log \sigma_2' - \log \sigma_1'}$$

e_1 and e_2 are the void ratios at the end of consolidation under effective stress σ_1' and σ_2' respectively.

The *compression index,* as determined from the laboratory curve, will be somewhat different from that encountered in the field.

Compression Index

The swelling index C_s is the slope of the unloading portion of the $e - \log \sigma'$ curve and is determined by

$$C_s = \frac{e_3 - e_4}{\log\left(\frac{\sigma_4'}{\sigma_3'}\right)}$$

The swelling index is also referred to as the *recompression index.*

Calculation of Primary Consolidation Settlement

The one-dimensional primary consolidation settlement (caused by an additional load) of a clay layer having a thickness *Hc* may be calculated by the equation

$$S_c = \frac{\Delta e}{1+e_0} H_c$$

S_c =primary consolidation settlement

Δe=total change of void ratio caused by the additional load application

e_0 =void ratio of the clay before the application of load

Calculation of Primary Consolidation Settlement

For a normally consolidated clay use the equations

$$\Delta e = C_c \log \frac{\sigma_o' + \Delta\sigma'}{\sigma_o'}$$

$$S_c = \frac{C_c H_c}{1+e_0} \log \frac{\sigma_0' + \Delta\sigma'}{\sigma_0'}$$

Calculation of Primary Consolidation Settlement

For overconsolidated clay with $\sigma_0^{'} + \Delta\sigma^{'} \leq \sigma_c^{'}$ use equations

$$\Delta e = C_s \log \frac{\sigma_o^{'} + \Delta\sigma^{'}}{\sigma_o^{'}} \qquad S_c = \frac{C_s H_c}{1+e_0} \log \frac{\sigma_0^{'} + \Delta\sigma^{'}}{\sigma_0^{'}}$$

Calculation of Primary Consolidation Settlement

For overconsolidated clay with $\sigma_0^{'} < \sigma_c^{'} < \sigma_o^{'} + \Delta\sigma^{'}$ use equations

$$S_c = \frac{C_s H_c}{1+e_0} \log \frac{\sigma_c^{'}}{\sigma_0^{'}} + \frac{C_c H_c}{1+e_0} \log \frac{\sigma_0^{'} + \Delta\sigma^{'}}{\sigma_0^{'}}$$

$$\Delta e = \Delta e_1 + \Delta e_2 = C_s \log \frac{\sigma_c^{'}}{\sigma_o^{'}} + C_c \log \frac{\sigma_0^{'} + \Delta\sigma^{'}}{\sigma_c^{'}}$$

Calculation of Primary Consolidation Settlement

Consolidation is the result of the gradual dissipation of the excess pore water pressure from a clay layer.

The dissipation of pore water pressure increases the effective stress, which induces settlement.

To estimate the degree of consolidation of a clay layer at some time *t* after the load is applied, the rate of dissipation of the excess pore water pressure must be determined.

Calculation of Primary Consolidation Settlement

Referring to the figure (see next slide), vertical drainage (drainage in the z direction only) is determined by the equations.

$$\frac{\partial(\Delta u)}{\partial t} = C_v \frac{\partial^2(\Delta u)}{\partial z^2}$$

$$C_v \frac{k}{m_v \gamma_w} = \frac{k}{\dfrac{\Delta e}{\Delta\sigma'(1+e_{av})}\gamma_w}$$

k =hydraulic conductivity of the clay

Δe =total change of void ratio caused by an effective stress increase of $\Delta\sigma'$

e_{av} = average void ratio during consolidation

m_v =volume coefficient of compressibility= $\dfrac{a_v}{1+e_{av}} = \dfrac{\Delta e}{\Delta\sigma'(1+e_{av})}$

Calculation of Primary Consolidation Settlement

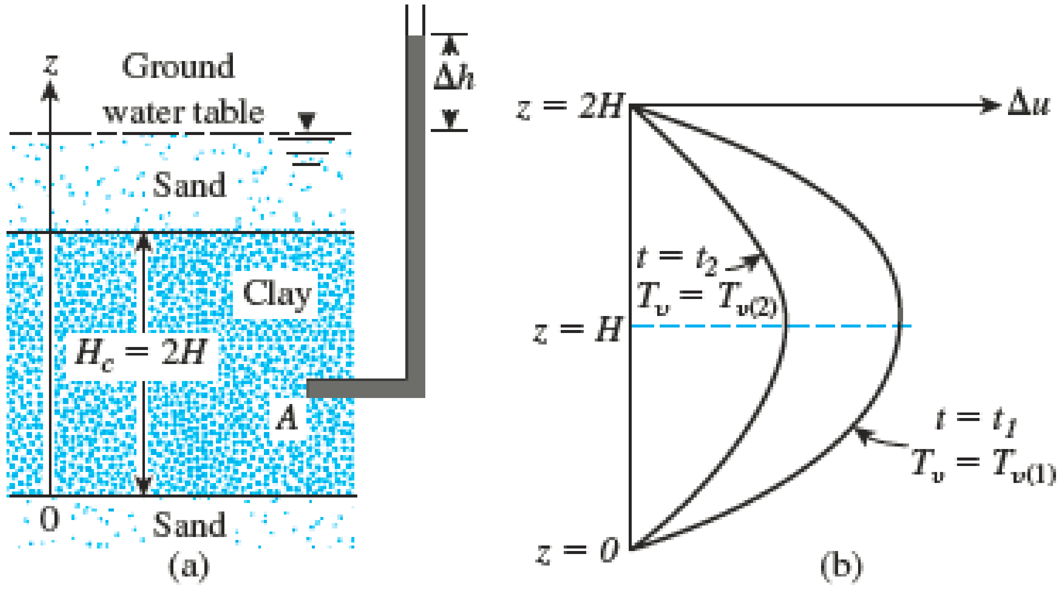


Figure 2.20 (a) Derivation of Eq. (2.72); (b) nature of variation of Δu with time

Calculation of Primary Consolidation Settlement

The *average degree of consolidation* of the clay layer can be defined as

$$U = \frac{S_{c(t)}}{S_{c(\max)}}$$

$S_{c(t)}$ = settlement of a clay layer at time t after the load is applied

$S_{c(\max)}$ = maximum consolidation settlement that the clay will undergo under a given loading

Calculation of Primary Consolidation Settlement

If the initial pore water pressure (Δu_0) distribution is constant with depth the average degree of consolidation can be expressed with relation to time as

$$U = 1 - \sum_{m=0}^{m=\infty} \left(\frac{2}{M^2}\right) e^{-M^2 T_v}$$

Calculation of Primary Consolidation Settlement

The variation of T_v with U have been calculated in this table

Table 2.11 Variation of T_v with U

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774

Calculation of Primary Consolidation Settlement

The variation of T_v with U have been calculated in this table

Table 2.11 Variation of T_v with U (*Continued*)

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.304	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Degree of Consolidation Under Ramp Loading

Previous discussion considered surcharge load per unit area as being applied instantly at t=0. However, surcharge is usually applied over time till max surcharge is met.

In this situation where $T_v \leq T_c$ use equation

$$U = \frac{T_v}{T_c}\{1 - \frac{2}{T_v}\sum_{m=0}^{m=\infty}\frac{1}{M^4}[1 - \exp(M^2 T_v)]\}$$

Degree of Consolidation Under Ramp Loading

If $T_v \geq T_c$ then use equation

$$U = 1 - \frac{2}{T_c} \sum_{m=0}^{m=\infty} \frac{1}{M^4} [\exp(M^2 T_c) - 1] \exp(-M^2 T_c)$$

T_v =nondimensional time factor = $C_v t / H^2$

M = $[(2m+1)\pi]/2$

$$T_c = \frac{C_v t_c}{H^2}$$

Shear Strength

Shear strength of soil is defined in terms of effective stress (Morh-Coulomb failure criterion)

$$s = c' + \sigma' \tan\phi'$$

σ' = effective normal stress on plane of shearing

c' = cohesion, or apparent cohesion (0 for sands and normally consolidated clays and >0 for overconsolidated clays)

ϕ' = effective stress angle of friction

Shear Strength

Shear strength parameters of a soil are determined by two standard laboratory tests:

Direct shear test

Triaxial test

Direct Shear Test

Dry sand can be tested by direct shear tests.

The sand is placed in a shear box that is split into two halves

First a normal load is applied to the specimen.

Then a shear force is applied to the top half of the shear box to cause failure in the sand.

Direct Shear Test

Using the equations $s = R/A$ and $\sigma' = N/A$ give values that are plotted as s against σ' the angle of friction can be determined by the equation

$$\phi' = \tan^{-1}\left(\frac{s}{\sigma'}\right)$$

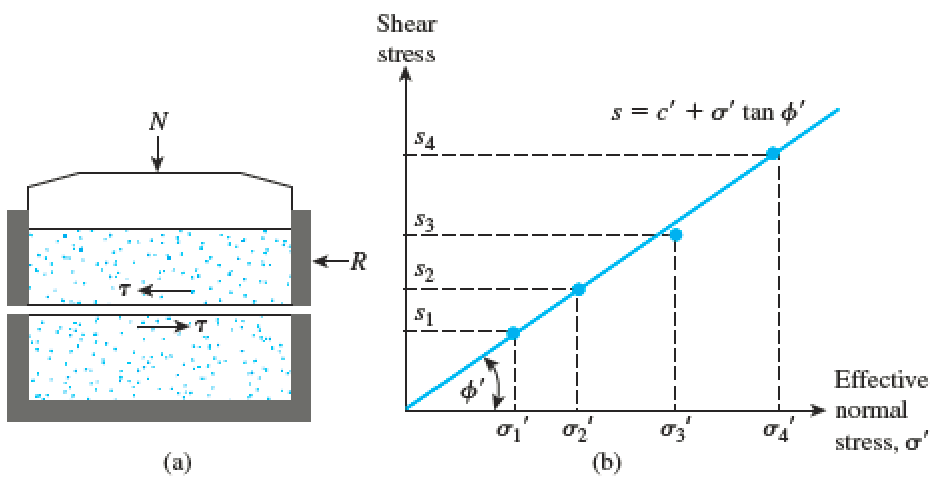


Figure 2.26 Direct shear test in sand: (a) schematic diagram of test equipment; (b) plot of test results to obtain the friction angle ϕ'

Direct Shear Test

Table 2.12 Relationship between Relative Density and Angle of Friction of Cohesionless Soils

State of packing	Relative density (%)	Angle of friction, ϕ' (deg.)
Very loose	<15	<28
Loose	15–35	28–30
Compact	35–65	30–36
Dense	65–85	36–41
Very dense	>85	>41

Direct Shear Test

For granular soils the friction angle can be determined by the equation

$$\phi'(\text{deg}) = \tan^{-1}\left(\frac{1}{ae+b}\right)$$

e = void ratio

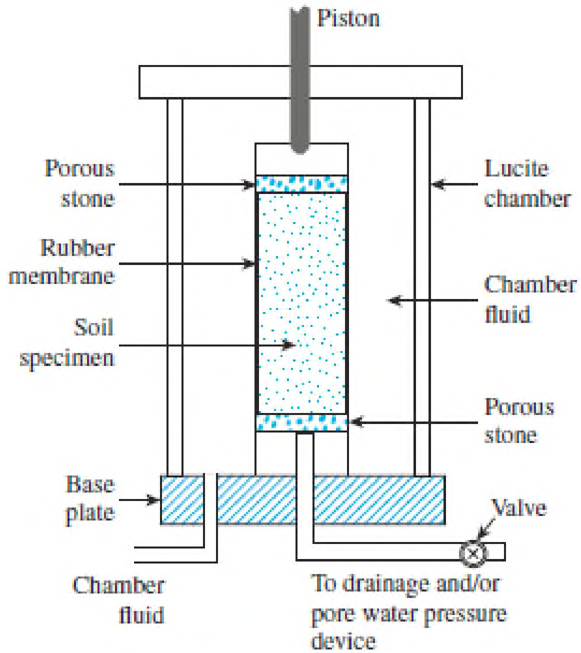
$a = 2.101 + 0.097\left(\frac{D_{85}}{D_{15}}\right)$

$b = 0.845 - 0.398a$

D_{85} and D_{15} = diameters through which respectively 85% and 15% of soil passes

Triaxial Test

Triaxial compression tests can be conducted on sands and clays.



Schematic diagram of triaxial test equipment

(a)

Triaxial Test

Consists of placing a soil specimen confined by a rubber membrane into a lucite chamber then applying an all-around confining pressure to the specimen by means of the chamber fluid (generally, water or glycerin).

An added stress can also be applied to the specimen in the axial direction to cause failure.

Drainage from the specimen can be allowed depending on the conditions being tested.

Triaxial Test

The three main tests are

Consolidated-drained test (CD test)

Consolidated-undrained test (CU test)

Unconsolidated-undrained test (UU test)

Triaxial Test

Consolidated Drained Test

Used on various clay soils

The test will help determine:

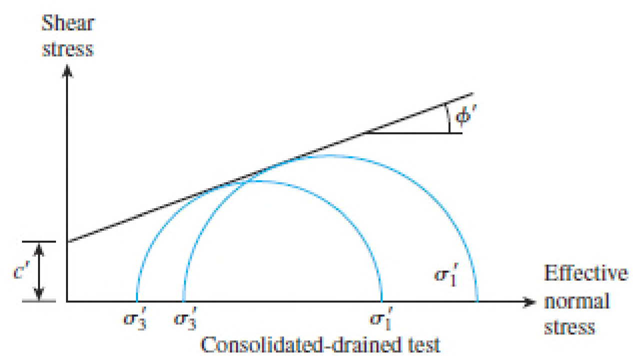
Major principal effective stress= σ_1'

Minor principal effective stress= σ_3'

Triaxial Test

With this test the shear strength can be determined by plotting Mohr's circle as failure and drawing a common tangent to the Mohr's circles. This is the *Mohr-Coulomb Failure Envelope*

Failure = $\sigma_1' = \sigma_3' \tan^2(45 + \frac{\phi'}{2}) + 2c' \tan(45 + \frac{\phi'}{2})$



(b)

Triaxial Test

Consolidated Undrained Test

Used on various soils

The test will help determine:

- Major principal effective stress= σ'_1
- Minor principal effective stress= σ'_3
- Major principal total stress = σ_1
- Minor principal total stress = σ_3

Triaxial Test

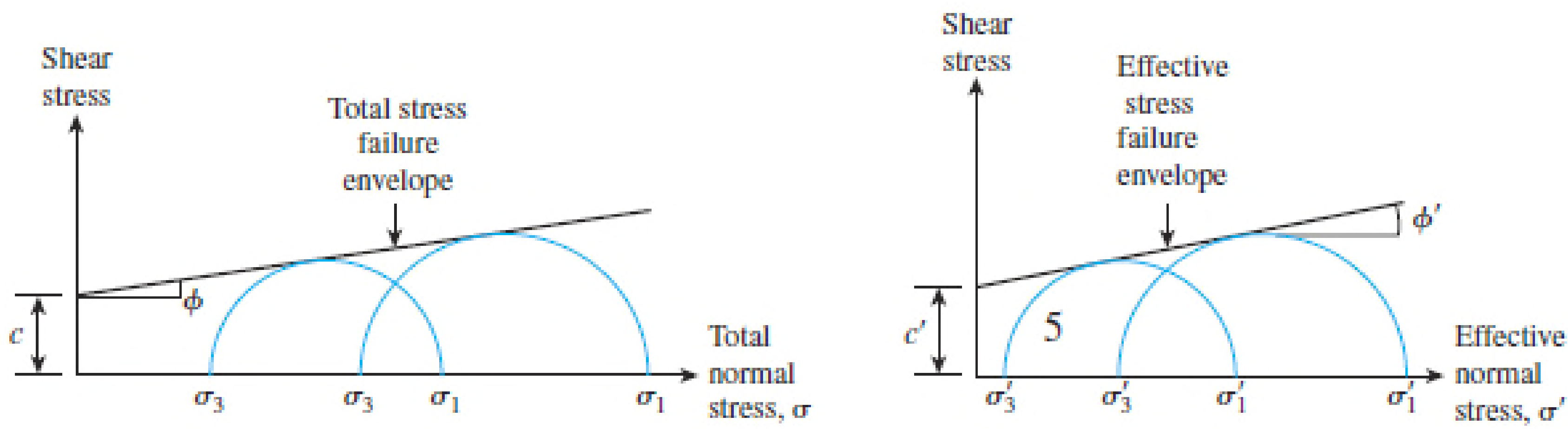
Consolidated-Undrained Tests:

The total stress Mohr's circles at failure can now be plotted. Then a common tangent can be drawn to define the *failure envelope.*

This *total stress failure envelope* is defined by the equation $s = c + \sigma \tan\phi$

c and ϕ are the *consolidated-undrained cohesion* and *angle of friction*, respectively.

Triaxial Test



Consolidated-undrained test

(c)

Triaxial Test

Unconsolidated-Undrained triaxial test

Used on various soils

The test will help determine:

Major principal total stress = σ_1

Minor principal total stress = σ_3

Triaxial Test

The total stress Mohr's circle at failure can now be drawn.

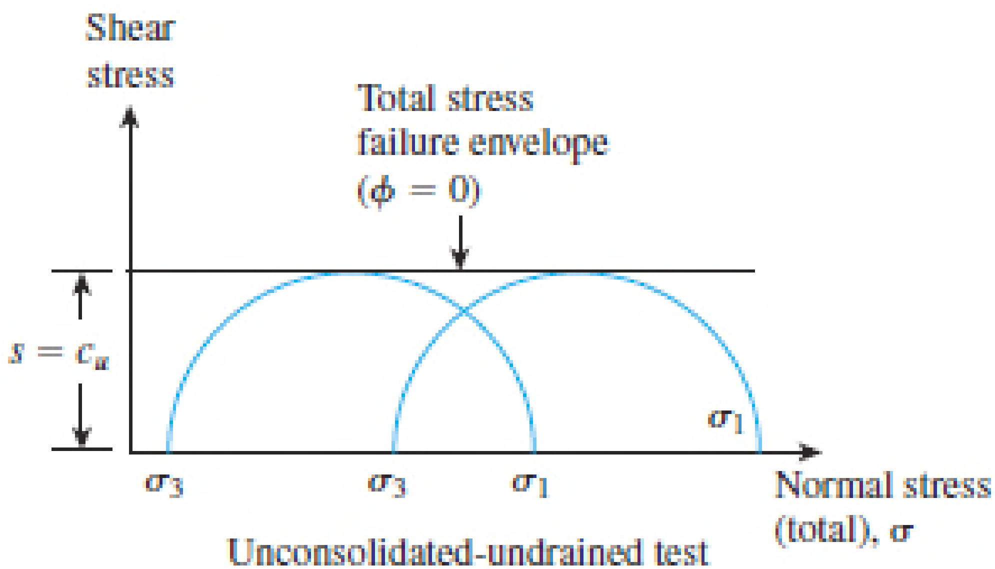
The tangent to these Mohr's circles will be a horizontal line called the $\phi = 0$ condition.

The shear strength for this condition is determined by the equation

$$s = c_u = \frac{\Delta\sigma_f}{2}$$

c_u = undrained cohesion (or undrained shear strength)
$\Delta\sigma_f$ = Failure stress

Triaxial Test



(d)

Unconfined Compression Test

The *unconfined* compression test is a special type of unconsolidated-undrained triaxial test in which the confining pressure $\sigma_3 = 0$.

In this test, an axial stress $\Delta\sigma$ is applied to the specimen to cause failure ($\Delta\sigma = \Delta\sigma_f$)

The axial stress at failure, $\Delta\sigma_f = q_u$, is referred to as the *unconfined compression strength.*

The shear strength of saturated clays under this condition ($\phi = 0$) is determined by the equation

$$s = c_u = \frac{q_u}{2}$$

Unconfined Compression Test

The *unconfined compression test* is a special type of unconsolidated-undrained triaxial test in which the confining pressure $\sigma_3 = 0$.

In this test, an axial stress $\Delta\sigma$ is applied to the specimen to cause failure ($\Delta\sigma = \Delta\sigma_f$)

The axial stress at failure, $\Delta\sigma_f = q_u$ is referred to as the *unconfined compression strength.*

The shear strength of saturated clays under this condition ($\phi = 0$) is determined by the equation

$$s = c_u = \frac{q_u}{2}$$

Unconfined Compression Test

The unconfined compression strength can be used as an indicator of the consistency of clays.

Unconfined compression tests are sometimes conducted on unsaturated soils.

With the void ratio of a soil specimen remaining constant, the unconfined compression strength rapidly decreases with the degree of saturation.

Comments on Friction Angle, ϕ'

In general, the direct shear test yields a higher angle of friction compared with that obtained by the triaxial test.

The failure envelope for a given soil is actually curved.

The Mohr–Coulomb failure criterion is only an approximation. Because of the curved nature of the failure envelope, a soil tested at higher normal stress will yield a lower value of ϕ'.

Sensitivity

For many naturally deposited clay soils, the unconfined compression strength is much less when the soils are tested after remolding without any change in the moisture content.

This is known as *sensitivity* and is the ratio of unconfined compression strength in an undisturbed state to that in a remolded state or

$$S_t = \frac{q_{u(\text{undisturbed})}}{q_{u(\text{remolded})}}$$