## Introduction and Vectors

## CHAPTER OUTLINE

1.1 Standards of Length, Mass, and Time
1.2 Dimensional Analysis
1.3 Conversion of Units
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1.5 Significant Figures
1.6 Coordinate Systems
1.7 Vectors and Scalars
1.8 Some Properties of Vectors
1.9 Components of a Vector and Unit Vectors
1.10 Modeling, Alternative Representations, and Problem-Solving Strategy

* An asterisk indicates an item new to this edition.


## ANSWERS TO OBJECTIVE QUESTIONS

*OQ1.1 The answer is yes for (a), (c), and (e). You cannot add or subtract a number of apples and a number of jokes. The answer is no for (b) and (d). Consider the gauge of a sausage, $4 \mathrm{~kg} / 2 \mathrm{~m}$, or the volume of a cube, ( 2 m$)^{3}$. Thus we have (a) yes (b) no (c) yes (d) no (e) yes.
*OQ1.2 $41 € \approx 41 €(1 \mathrm{~L} / 1.3 €)(1 \mathrm{qt} / 1 \mathrm{~L})(1 \mathrm{gal} / 4 \mathrm{qt}) \approx(10 / 1.3)$ gal $\approx 8$ gallons, answer (c).
*OQ1.3 In the base unit we have (a) $0.032 \mathrm{~kg}(\mathrm{~b}) 0.015 \mathrm{~kg}$ (c) 0.270 kg (d) 0.041 kg (e) 0.27 kg . Then the ranking is $\mathrm{c}=\mathrm{e}>\mathrm{d}>\mathrm{a}>\mathrm{b}$.
*OQ1.4 Answer (c). The vector has no $y$ component given. It is therefore 0 .
*OQ1.5 The population is about 6 billion $=6 \times 10^{9}$. Assuming about 100 lb per person $=$ about 50 kg per person ( 1 kg has the weight of about 2.2 lb ),
the total mass is about $\left(6 \times 10^{9}\right)(50 \mathrm{~kg})=3 \times 10^{11} \mathrm{~kg}$, answer $(\mathrm{d})$.
*OQ1.6 The number of decimal places in a sum of numbers should be the same as the smallest number of decimal places in the numbers summed.

$$
\begin{aligned}
& 21.4 \mathrm{~s} \\
& 15 \mathrm{~s} \\
& 17.17 \mathrm{~s} \\
& 4.003 \mathrm{~s} \\
& \frac{57.573 \mathrm{~s}}{}=58 \mathrm{~s} \text {, answer (d). }
\end{aligned}
$$

*OQ1.7 The meterstick measurement, (a), and (b) can all be 4.31 cm . The meterstick measurement and (c) can both be 4.24 cm . Only (d) does not overlap. Thus (a), (b), and (c) all agree with the meterstick measurement.
*OQ1.8 Mass is measured in kg ; acceleration is measured in $\mathrm{m} / \mathrm{s}^{2}$. Force $=$ mass $\times$ acceleration, so the units of force are answer (a) $\mathrm{kg} \cdot \mathrm{m} / \mathrm{s}^{2}$.
*OQ1.9 Answer (d). Take the difference of the $x$ coordinates of the ends of the vector, head minus tail: $-4-2=-6 \mathrm{~cm}$.
*OQ1.10 Answer (a). Take the difference of the $y$ coordinates of the ends of the vector, head minus tail: $1-(-2)=3 \mathrm{~cm}$.
*OQ1.11 The answers are (a) yes (b) no (c) no (d) no (e) no (f) yes (g) no. Only force and velocity are vectors. None of the other quantities requires a direction to be described.
*OQ1.12 Answers (a), (b), and (c). The magnitude can range from the sum of the individual magnitudes, $8+6=14$, to the difference of the individual magnitudes, $8-6=2$. Because magnitude is the "length" of a vector, it is always positive.
*OQ1.13 Answer (a). The vector $-2 \overrightarrow{\mathbf{D}}_{1}$ will be twice as long as $\overrightarrow{\mathbf{D}}_{1}$ and in the opposite direction, namely northeast. Adding $\overrightarrow{\mathbf{D}}_{2}$, which is about equally long and southwest, we get a sum that is still longer and due east.
*OQ1.14 Answer (c). A vector in the second quadrant has a negative $x$ component and a positive $y$ component.
*OQ1.15 Answer (e). The magnitude is $\sqrt{10^{2}+10^{2}} \mathrm{~m} / \mathrm{s}$.
*OQ1.16 Answer (c). The signs of the components of a vector are the same as the signs of the points in the quadrant into which it points. If a vector arrow is drawn to scale, the coordinates of the point of the arrow equal the components of the vector. All $x$ and $y$ values in the third quadrant are negative.

## ANSWERS TO CONCEPTUAL QUESTIONS

*CQ1.1 A unit of time should be based on a reproducible standard so it can be used everywhere. The more accuracy required of the standard, the less the standard should change with time. The current, very accurate standard is the period of vibration of light emitted by a cesium atom. Depending on the accuracy required, other standards could be: the period of light emitted by a different atom, the period of the swing of a pendulum at a certain place on Earth, the period of vibration of a sound wave produced by a string of a specific length, density and tension, and the time interval from full Moon to full Moon.
*CQ1.2 (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms.
*CQ1.3 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.
*CQ1.4 Vectors $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ are perpendicular to each other.
*CQ1.5 (a) The book's displacement is zero, as it ends up at the point from which it started. (b) The distance traveled is 6.0 meters.
*CQ1.6 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.
*CQ1.7 The inverse tangent function gives the correct angle, relative to the $+x$ axis, for vectors in the first or fourth quadrant, and it gives an incorrect answer for vectors in the second or third quadrant. If the $x$ and $y$ components are both positive, their ratio $y / x$ is positive and the vector lies in the first quadrant; if the $x$ component is positive and the $y$ component negative, their ratio $y / x$ is negative and the vector lies in the fourth quadrant. If the $x$ and $y$ components are both negative, their ratio $y / x$ is positive but the vector lies in the third quadrant; if the $x$ component is negative and the $y$ component positive, their ratio $y / x$ is negative but the vector lies in the second quadrant.
*CQ1.8 Addition of a vector to a scalar is not defined. Try adding the speed and velocity, $8.0 \mathrm{~m} / \mathrm{s}+(15.0 \mathrm{~m} / \mathrm{s} \hat{\mathbf{i}})$ : Should you consider the sum to be a vector or a scaler? What meaning would it have?

## SOLUTIONS TO END-OF-CHAPTER PROBLEMS

## Section 1.1 Standards of Length, Mass, and Time

P1.1 For either sphere the volume is $V=\frac{4}{3} \pi r^{3}$ and the mass is $m=\rho V=\rho \frac{4}{3} \pi r^{3}$. We divide this equation for the larger sphere by the same equation for the smaller:

$$
\frac{m_{\ell}}{m_{s}}=\frac{\rho 4 \pi r_{\ell}^{3} 3}{\rho 4 \pi r_{s}^{3} 3}=\frac{r_{\ell}^{3}}{r_{s}^{3}}=5
$$

Then $r_{\ell}=r_{s} \sqrt[3]{5}=4.50 \mathrm{~cm}(1.71)=7.69 \mathrm{~cm}$.
P1.2 (a) Modeling the Earth as a sphere, we find its volume as

$$
\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi\left(6.37 \times 10^{6} \mathrm{~m}\right)^{3}=1.08 \times 10^{21} \mathrm{~m}^{3}
$$

Its density is then

$$
\rho=\frac{m}{V}=\frac{5.98 \times 10^{24} \mathrm{~kg}}{1.08 \times 10^{21} \mathrm{~m}^{3}}=5.52 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}
$$

(b) This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2000 to $3000 \mathrm{~kg} / \mathrm{m}^{3}$. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.3 Let $V$ represent the volume of the model, the same in $\rho=\frac{m}{V}$, for both.
Then $\rho_{\text {iron }}=9.35 \mathrm{~kg} / V$ and $\rho_{\text {gold }}=\frac{m_{\text {gold }}}{V}$.
Next, $\frac{\rho_{\text {gold }}}{\rho_{\text {iron }}}=\frac{m_{\text {gold }}}{9.35 \mathrm{~kg}}$ and

$$
m_{\text {gold }}=(9.35 \mathrm{~kg})\left(\frac{19.3 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}{7.87 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}}\right)=22.9 \mathrm{~kg}
$$

P1.4 The volume of a spherical shell can be calculated from

$$
V=V_{o}-V_{i}=\frac{4}{3} \pi\left(r_{2}^{3}-r_{1}^{3}\right)
$$

From the definition of density, $\rho=\frac{m}{V}$, so

$$
m=\rho V=\rho\left(\frac{4}{3} \pi\right)\left(r_{2}^{3}-r_{1}^{3}\right)=\frac{4 \pi \rho\left(r_{2}^{3}-r_{1}^{3}\right)}{3}
$$

## Section 1.2 Dimensional Analysis

P1.5 (a) This is incorrect since the units of $[a x]$ are $\mathrm{m}^{2} / \mathrm{s}^{2}$, while the units of $[v]$ are $\mathrm{m} / \mathrm{s}$.
(b) This is correct since the units of $[y]$ are $m, \cos (k x)$ is dimensionless if $[k]$ is in $\mathrm{m}^{-1}$, and the constant multiplying $\cos (k x)$ is in units of m .

P1.6 Circumference has dimensions L , area has dimensions $\mathrm{L}^{2}$, and volume has dimensions $L^{3}$. Expression (a) has dimensions $L\left(L^{2}\right)^{1 / 2}=L^{2}$, expression (b) has dimensions L, and expression (c) has dimensions $\mathrm{L}\left(\mathrm{L}^{2}\right)=\mathrm{L}^{3}$.

> The matches are: (a) and (f), (b) and (d), and (c) and (e).

P1.7 The term $x$ has dimensions of $\mathrm{L}, a$ has dimensions of $\mathrm{LT}^{-2}$, and $t$ has dimensions of T. Therefore, the equation $x=k a^{m} t^{n}$ has dimensions of

$$
\mathrm{L}=\left(\mathrm{LT}^{-2}\right)^{m}(\mathrm{~T})^{n} \text { or } \mathrm{L}^{1} \mathrm{~T}^{0}=\mathrm{L}^{m} \mathrm{~T}^{n-2 m}
$$

The powers of L and T must be the same on each side of the equation. Therefore,

$$
\mathrm{L}^{1}=\mathrm{L}^{m} \text { and } m=1
$$

Likewise, equating terms in T, we see that $n-2 m$ must equal 0 . Thus, $n=2$. The value of $k$, a dimensionless constant,
cannot be obtained by dimensional analysis

## Section 1.3 Conversion of Units

P1.8 It is often useful to remember that the $1600-\mathrm{m}$ race at track and field events is approximately 1 mile in length. To be precise, there are 1609 meters in a mile. Thus, 1 acre is equal in area to

$$
(1 \text { acre })\left(\frac{1 \mathrm{mi}^{2}}{640 \text { acres }}\right)\left(\frac{1609 \mathrm{~m}}{\mathrm{mi}}\right)^{2}=4.05 \times 10^{3} \mathrm{~m}^{2}
$$

P1.9

$$
V=A t \text { so } t=\frac{V}{A}=\frac{3.78 \times 10^{-3} \mathrm{~m}^{3}}{25.0 \mathrm{~m}^{2}}=1.51 \times 10^{-4} \mathrm{~m}(\text { or } 151 \mu \mathrm{~m})
$$

P1.10
Apply the following conversion factors:

$$
\begin{aligned}
& 1 \mathrm{in}=2.54 \mathrm{~cm}, 1 \mathrm{~d}=86400 \mathrm{~s}, 100 \mathrm{~cm}=1 \mathrm{~m}, \text { and } 10^{9} \mathrm{~nm}=1 \mathrm{~m} \\
& \left(\frac{1}{32} \mathrm{in} / \text { day }\right) \frac{(2.54 \mathrm{~cm} / \mathrm{in})\left(10^{-2} \mathrm{~m} / \mathrm{cm}\right)\left(10^{9} \mathrm{~nm} / \mathrm{m}\right)}{86400 \mathrm{~s} / \text { day }}=9.19 \mathrm{~nm} / \mathrm{s}
\end{aligned}
$$

This means the proteins are assembled at a rate of many layers of atoms each second!

P1.11 The weight flow rate is

$$
1200 \frac{\text { ton }}{\mathrm{h}}\left(\frac{2000 \mathrm{lb}}{\text { ton }}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{~s}}\right)=667 \mathrm{lb} / \mathrm{s} .
$$

P1.12 We obtain the number of atoms in the Sun by dividing its mass by the mass of a single hydrogen atom:

$$
N_{\text {atoms }}=\frac{m_{\text {Sun }}}{m_{\text {atom }}}=\frac{1.99 \times 10^{30} \mathrm{~kg}}{1.67 \times 10^{-27} \mathrm{~kg}}=1.19 \times 10^{57} \text { atoms }
$$

P1.13 The masses given are for a $1.00 \mathrm{~m}^{3}$ volume. Density is defined as mass per unit volume, so $\rho_{\mathrm{Al}}=2.70 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$ and $\rho_{\mathrm{Fe}}=7.86 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$.
For the spheres to balance, $m_{\mathrm{Fe}}=m_{\mathrm{A} 1}$ or $\rho_{\mathrm{Fe}} V_{\mathrm{Fe}}=\rho_{\mathrm{A} 1} V_{\mathrm{A} 1}$ :

$$
\begin{aligned}
\rho_{\mathrm{Fe}}\left(\frac{4}{3}\right) \pi r_{\mathrm{Fe}}^{3} & =\rho_{\mathrm{Al}}\left(\frac{4}{3}\right) \pi r_{\mathrm{Al}}^{3} \\
r_{\mathrm{Al}} & =r_{\mathrm{Fe}}\left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{Al}}}\right)^{1 / 3}=(2.00 \mathrm{~cm})\left(\frac{7.86}{2.70}\right)^{1 / 3}=2.86 \mathrm{~cm}
\end{aligned}
$$

P1.14 The mass of each sphere is $m_{\mathrm{Al}}=\rho_{\mathrm{Al}} V_{\mathrm{Al}}=\frac{4 \pi \rho_{\mathrm{Al}} r_{\mathrm{Al}}^{3}}{3}$ and $m_{\mathrm{Fe}}=\rho_{\mathrm{Fe}} V_{\mathrm{Fe}}=\frac{4 \pi \rho_{\mathrm{Fe}} r_{\mathrm{Fe}}^{3}}{3}$. Setting these masses equal,

$$
\begin{aligned}
& \frac{4}{3} \pi \rho_{A L} r_{A l}^{3}=\frac{4}{3} \pi \rho_{F e} r_{F e}^{3} \rightarrow r_{A L}=r_{F e}^{3} \sqrt[3]{\frac{\rho_{F e}}{\rho_{A l}}} \\
& r_{A L}=r_{F e} \sqrt[3]{\frac{7.86}{2.70}}=r_{F e}(1.43)
\end{aligned}
$$

The resulting expression shows that the radius of the aluminum sphere is directly proportional to the radius of the balancing iron sphere. The sphere of lower density has larger radius.

P1.15
(a) rate $=\left(\frac{30.0 \mathrm{gal}}{7.00 \mathrm{~min}}\right)\left(\frac{1 \mathrm{mi}}{60 \mathrm{~s}}\right)=7.14 \times 10^{-2} \frac{\mathrm{gal}}{\mathrm{s}}$
(b) rate $=7.14 \times 10^{-2} \frac{\mathrm{gal}_{\mathrm{s}}}{\mathrm{s}}\left(\frac{231 \mathrm{in}^{3}}{1 \mathrm{gal}}\right)\left(\frac{2.54 \mathrm{~cm}}{1 \mathrm{in}}\right)^{3}\left(\frac{1 \mathrm{~m}}{100 \mathrm{~cm}}\right)^{3}$

$$
=2.70 \times 10^{-4} \frac{\mathrm{~m}^{3}}{\mathrm{~s}}
$$

(c) To find the time to fill a $1.00-\mathrm{m}^{3}$ tank, find the rate time/volume:

$$
\begin{gathered}
2.70 \times 10^{-4} \frac{\mathrm{~m}^{3}}{\mathrm{~s}}=\left(\frac{2.70 \times 10^{-4} \mathrm{~m}^{3}}{1 \mathrm{~s}}\right) \\
\text { or }\left(\frac{2.70 \times 10^{-4} \mathrm{~m}^{3}}{1 \mathrm{~s}}\right)^{-1}=\left(\frac{1 \mathrm{~s}}{2.70 \times 10^{-4} \mathrm{~m}^{3}}\right)=3.70 \times 10^{3} \frac{\mathrm{~s}}{\mathrm{~m}^{3}}
\end{gathered}
$$

$$
\text { and so: } 3.70 \times 10^{3} \mathrm{~s}\left(\frac{1 \mathrm{~h}}{3600 \mathrm{~s}}\right)=1.03 \mathrm{~h}
$$

P1.16
(a) $\quad d_{\text {nucleus,scale }}=d_{\text {nucleus,real }}\left(\frac{d_{\text {atom,scale }}}{d_{\text {atom,real }}}\right)=\left(2.40 \times 10^{-15} \mathrm{~m}\right)\left(\frac{300 \mathrm{ft}}{1.06 \times 10^{-10} \mathrm{~m}}\right)$ $=6.79 \times 10^{-3} \mathrm{ft}$, or

$$
d_{\text {nucleus,scale }}=\left(6.79 \times 10^{-3} \mathrm{ft}\right)(304.8 \mathrm{~mm} / 1 \mathrm{ft})=2.07 \mathrm{~mm}
$$

(b) $\frac{V_{\text {atom }}}{V_{\text {nucleus }}}=\frac{4 \pi r_{\text {atom }}^{3} / 3}{4 \pi r_{\text {nucleus }}^{3} / 3}=\left(\frac{r_{\text {atom }}}{r_{\text {nucleus }}}\right)^{3}=\left(\frac{d_{\text {atom }}}{d_{\text {nucleus }}}\right)^{3}=\left(\frac{1.06 \times 10^{-10} \mathrm{~m}}{2.40 \times 10^{-15} \mathrm{~m}}\right)^{3}$

$$
=8.62 \times 10^{13} \text { times as large }
$$

## 8 Introduction and Vectors

## Section 1.4 Order-of-Magnitude Calculations

P1.17 Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m , and each ping-pong ball as a sphere of diameter 0.038 m . The volume of the room is $4 \times 4 \times 3=48 \mathrm{~m}^{3}$, while the volume of one ball is

$$
\frac{4 \pi}{3}\left(\frac{0.038 \mathrm{~m}}{2}\right)^{3}=2.87 \times 10^{-5} \mathrm{~m}^{3}
$$

Therefore, one can fit about $\frac{48}{2.87 \times 10^{-5}} \sim 10^{6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6} \pi \sqrt{2}=0.74$, so that at least $26 \%$ of the space will be empty.
Therefore, the above estimate reduces to $1.67 \times 10^{6} \times 0.740 \sim 10^{6}$.
*P1.18 (a) We estimate the mass of the water in the bathtub. Assume the tub measures 1.3 m by 0.5 m by 0.3 m . One-half of its volume is then

$$
V=(0.5)(1.3)(0.5)(0.3)=0.10 \mathrm{~m}^{3} .
$$

The mass of this volume of water is

$$
m_{\text {water }}=\rho_{\text {water }} V=\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.10 \mathrm{~m}^{3}\right)=100 \mathrm{~kg} \sim 10^{2} \mathrm{~kg}
$$

(b) Pennies are now mostly zinc, but consider copper pennies filling $50 \%$ of the volume of the tub. The mass of copper required is

$$
m_{\text {copper }}=\rho_{\text {copper }} V=\left(8920 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(0.10 \mathrm{~m}^{3}\right)=892 \mathrm{~kg} \sim 10^{3} \mathrm{~kg}
$$

P1.19 Assume: Total population $=10^{7}$; one out of every 100 people has a piano; one tuner can serve about 1000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$
\text { \# tuners } \sim\left(\frac{1 \text { tuner }}{1000 \text { pianos }}\right)\left(\frac{1 \text { piano }}{100 \text { people }}\right)\left(10^{7} \text { people }\right)=100 \text { tuners }
$$

P1.20 A reasonable guess for the diameter of a tire might be 2.5 ft , with a circumference of about 8 ft . Thus, the tire would make

$$
(50000 \mathrm{mi})(5280 \mathrm{ft} / \mathrm{mi})(1 \mathrm{rev} / 8 \mathrm{ft})=3 \times 10^{7} \mathrm{rev} \sim 10^{7} \mathrm{rev}
$$

## Section 1.5 Significant Figures

P1.21 We work to nine significant digits:

$$
\begin{aligned}
1 \mathrm{yr} & =1 \mathrm{yr}\left(\frac{365.242199 \mathrm{~d}}{1 \mathrm{yr}}\right)\left(\frac{24 \mathrm{~h}}{1 \mathrm{~d}}\right)\left(\frac{60 \mathrm{~min}}{1 \mathrm{~h}}\right)\left(\frac{60 \mathrm{~s}}{1 \mathrm{~min}}\right) \\
& =31556926.0 \mathrm{~s}
\end{aligned}
$$

P1.22 (a) $756+37.2+0.83+2=796.03 \rightarrow 796$, since the number with the fewest decimal places is 2 .
(b) $\quad(0.0032)(2$ s.f. $) \times(356.3)(4$ s.f. $)=1.14016=(2$ s.f. $) 1.1$
(c) $5.620(4$ s.f. $) \times \pi(>4$ s.f. $)=17.656=(4$ s.f. $) 17.66$

P1.23
(a) 3
(b)
4 (c)
3 (d)
2

P1.24

$$
r=(6.50 \pm 0.20) \mathrm{cm}=(6.50 \pm 0.20) \times 10^{-2} \mathrm{~m}
$$

$m=(1.85+0.02) \mathrm{kg}$
$\rho=\frac{m}{\left(\frac{4}{3}\right) \pi r^{3}}$
also, $\frac{\delta \rho}{\rho}=\frac{\delta m}{m}+\frac{3 \delta r}{r}$
In other words, the percentages of uncertainty are cumulative.
Therefore,

$$
\begin{aligned}
& \frac{\delta \rho}{\rho}=\frac{0.02}{1.85}+\frac{3(0.20)}{6.50}=0.103 \\
& \rho=\frac{1.85}{\left(\frac{4}{3}\right) \pi\left(6.5 \times 10^{-2} \mathrm{~m}\right)^{3}}=1.61 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}
\end{aligned}
$$

then $\delta \rho=0.103 \rho=0.166 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$
and $\rho \pm \delta \rho=(1.61 \pm 0.17) \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}=(1.6 \pm 0.2) \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$.
P1.25 The volume of concrete needed is the sum of the four sides of sidewalk, or

$$
V=2 V_{1}+2 V_{2}=2\left(V_{1}+V_{2}\right)
$$

The figure on the right gives the dimensions needed to determine the volume of each portion of


ANS. FIG. P1.25 sidewalk:

$$
\begin{aligned}
& V_{1}=(17.0 \mathrm{~m}+1.0 \mathrm{~m}+1.0 \mathrm{~m})(1.0 \mathrm{~m})(0.09 \mathrm{~m})=1.70 \mathrm{~m}^{3} \\
& V_{2}=(10.0 \mathrm{~m})(1.0 \mathrm{~m})(0.090 \mathrm{~m})=0.900 \mathrm{~m}^{3} \\
& V=2\left(1.70 \mathrm{~m}^{3}+0.900 \mathrm{~m}^{3}\right)=5.2 \mathrm{~m}^{3}
\end{aligned}
$$

The uncertainty in the volume is the sum of the uncertainties in each dimension:

$$
\begin{aligned}
& \frac{\delta \ell_{1}}{\ell_{1}}=\frac{0.12 \mathrm{~m}}{19.0 \mathrm{~m}}=0.0063 \\
& \begin{array}{l}
\frac{\delta w_{1}}{w_{1}}=\frac{0.01 \mathrm{~m}}{1.0 \mathrm{~m}}=0.010 \\
\frac{\delta t_{1}}{t_{1}}=\frac{0.1 \mathrm{~cm}}{9.0 \mathrm{~cm}}=0.011
\end{array} \quad \begin{array}{l} 
\\
\end{array}
\end{aligned}
$$

P1.26 Using substitution is to solve simultaneous equations. We substitute $p$ $=3 q$ into each of the other two equations to eliminate $p$ :

$$
\left\{\begin{array}{l}
3 q r=q s \\
\frac{1}{2} 3 q r^{2}+\frac{1}{2} q s^{2}=\frac{1}{2} q t^{2}
\end{array}\right.
$$

These simplify to $\left\{\begin{array}{l}3 r=s \\ 3 r^{2}+s^{2}=t^{2}\end{array}\right.$, assuming $q \neq 0$.
We substitute the upper relation into the lower equation to eliminate $s$ :

$$
3 r^{2}+(3 r)^{2}=t^{2} \rightarrow 12 r^{2}=t^{2} \rightarrow \frac{t^{2}}{r^{2}}=12
$$

We now have the ratio of $t$ to $r: \frac{t}{r}= \pm \sqrt{12}= \pm 3.46$
P1.27 We draw the radius to the initial point and the radius to the final point. The angle $\theta$ between these two radii has its sides perpendicular, right side to right side and left side to left side, to the $35^{\circ}$ angle between the original and final tangential directions of travel. A most useful theorem from geometry then identifies these angles as equal: $\theta=35^{\circ}$. The whole circumference of a $360^{\circ}$ circle of the same radius is


ANS. FIG. P1.27
$2 \pi R$. By proportion, then

$$
\frac{2 \pi R}{360^{\circ}}=\frac{840 \mathrm{~m}}{35^{\circ}}
$$

$$
R=\frac{360^{\circ}}{2 \pi} \frac{840 \mathrm{~m}}{35^{\circ}}=\frac{840 \mathrm{~m}}{0.611}=1.38 \times 10^{3} \mathrm{~m}
$$

We could equally well say that the measure of the angle in radians is

$$
\theta=35^{\circ}=35^{\circ}\left(\frac{2 \pi \text { radians }}{360^{\circ}}\right)=0.611 \mathrm{rad}=\frac{840 \mathrm{~m}}{R}
$$

Solving yields $R=1.38 \mathrm{~km}$.
P1.28 For those who are not familiar with solving equations numerically, we provide a detailed solution. It goes beyond proving that the suggested answer works.

The equation $2 x^{4}-3 x^{3}+5 x-70=0$ is quartic, so we do not attempt to solve it with algebra. To find how many real solutions the equation has and to estimate them, we graph the expression:


ANS. FIG. P1.28

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=2 x^{4}-3 x^{3}+5 x-70$ | 158 | -24 | -70 | -70 | -66 | -52 | 26 | 270 |

We see that the equation $y=0$ has two roots, one around $x=-2.2$ and the other near $x=+2.7$. To home in on the first of these solutions we compute in sequence:
When $x=-2.2, y=-2.20$. The root must be between $x=-2.2$ and $x=-3$.
When $x=-2.3, y=11.0$. The root is between $x=-2.2$ and $x=-2.3$.
When $x=-2.23, y=1.58$. The root is between $x=-2.20$ and $x=-2.23$. When $x=-2.22, y=0.301$. The root is between $x=-2.20$ and -2.22 . When $x=-2.215, y=-0.331$. The root is between $x=-2.215$ and -2.22 . We could next try $x=-2.218$, but we already know to three-digit precision that the root is $x=-2.22$.
P1.29 We require

$$
\sin \theta=-3 \cos \theta, \text { or } \frac{\sin \theta}{\cos \theta}=\tan \theta=-3
$$

For $\tan ^{-1}(-3)=\arctan (-3)$, your calculator may return $-71.6^{\circ}$, but this angle is not between $0^{\circ}$ and $360^{\circ}$ as the problem


ANS. FIG. P1. 29 requires. The tangent function is negative in the second quadrant (between $90^{\circ}$ and $180^{\circ}$ ) and in the fourth quadrant (from $270^{\circ}$ to $360^{\circ}$ ). The solutions to the equation are then

$$
360^{\circ}-71.6^{\circ}=288^{\circ} \text { and } 180^{\circ}-71.6=108^{\circ}
$$

## Section 1.6 Coordinate Systems

P1.30 (a) The distance between the points is given by:

$$
\begin{aligned}
d & =\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& =\sqrt{(2.00-[-3.00])^{2}+(-4.00-3.00)^{2}} \\
d & =\sqrt{25.0+49.0}=8.60 \mathrm{~m}
\end{aligned}
$$

(b) To find the polar coordinates of each point, we measure the radial distance to that point and the angle it makes with the $+x$ axis:

$$
\begin{aligned}
& r_{1}=\sqrt{(2.00)^{2}+(-4.00)^{2}}=\sqrt{20.0}=4.47 \mathrm{~m} \\
& \theta_{1}=\tan ^{-1}\left(-\frac{4.00}{2.00}\right)=-63.4^{\circ} \\
& r_{2}=\sqrt{(-3.00)^{2}+(3.00)^{2}}=\sqrt{18.0}=4.24 \mathrm{~m} \\
& \theta_{2}=135^{\circ} \text { measured from the }+x \text { axis. }
\end{aligned}
$$

P1.31 $x=r \cos \theta=(5.50 \mathrm{~m}) \cos 240^{\circ}=(5.50 \mathrm{~m})(-0.5)=-2.75 \mathrm{~m}$

$$
y=r \sin \theta=(5.50 \mathrm{~m}) \sin 240^{\circ}=(5.50 \mathrm{~m})(-0.866)=-4.76 \mathrm{~m}
$$

P1.32 We have $r=\sqrt{x^{2}+y^{2}}$ and $\theta=\tan ^{-1}\left(\frac{y}{x}\right)$.
(a) The radius for this new point is

$$
\sqrt{(-x)^{2}+y^{2}}=\sqrt{x^{2}+y^{2}}=r
$$

and its angle is

$$
\tan ^{-1}\left(\frac{y}{-x}\right)=180^{\circ}-\theta
$$

(b) $\sqrt{(-2 x)^{2}+(-2 y)^{2}}=2 r$. This point is in the third quadrant if $(x, y)$ is in the first quadrant or in the fourth quadrant if $(x, y)$ is in the second quadrant. It is at an angle of $180^{\circ}+\theta$.
(c) $\sqrt{(3 x)^{2}+(-3 y)^{2}}=3 r$. This point is in the fourth quadrant if $(x, y)$ is in the first quadrant or in the third quadrant if $(x, y)$ is in the second quadrant. It is at an angle of $-\theta$ or $360-\theta$.

P1.33 The $x$ distance out to the fly is 2.00 m and the $y$ distance up to the fly is
1.00 m .
(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$
\text { distance }=\sqrt{x^{2}+y^{2}}=\sqrt{(2.00 \mathrm{~m})^{2}+(1.00 \mathrm{~m})^{2}}=\sqrt{5.00 \mathrm{~m}^{2}}=2.24 \mathrm{~m}
$$

(b) $\theta=\tan ^{-1}\left(\frac{1}{2}\right)=26.6^{\circ} ; \overrightarrow{\mathbf{r}}=2.24 \mathrm{~m}, 26.6^{\circ}$

## Section $1.7 \quad$ Vectors and Scalars

## Section $1.8 \quad$ Some Properties of Vectors

P1.34 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m )
(a) $\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}=5.2 \mathrm{~m}$ at $60^{\circ}$
(b) $\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}=3.0 \mathrm{~m}$ at $330^{\circ}$
(c) $\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}}=3.0 \mathrm{~m}$ at $150^{\circ}$
(d) $\overrightarrow{\mathbf{A}}-2 \mathbf{B}=5.2 \mathrm{~m}$ at $300^{\circ}$


ANS. FIG. P1.34

P1.35
From the figure, we note that the length of the skater's path along the $\operatorname{arc} \mathrm{OA}$ is greater than the length of the displacement arrow $\overrightarrow{\mathbf{O A}}$.


ANS. FIG. P1.35

P1.36 Ans. Fig. P1.36 shows the graphical addition of the vector from the base camp to lake $A$ to the vector connecting lakes $A$ and $B$, with a scale of 1 unit $=20 \mathrm{~km}$. The distance from lake $B$ to base camp is then the negative of this resultant vector, or $-\overrightarrow{\mathbf{R}}=310 \mathrm{~km}$ at $57^{\circ} \mathrm{S}$ of W .


ANS. FIG. P1.36
P1.37 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring $d$ and $\theta$ on the drawing ANS. FIG. P1.37 and applying the scale factor used in making the drawing. The results should be $d=420 \mathrm{ft}$ and $\theta=-3^{\circ}$.

## Section 1.9 Components of a Vector and Unit Vectors

P1.38 (a) See figure to the right.
(b) $\overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}=2.00 \hat{\mathbf{i}}+6.00 \hat{\mathbf{j}}+3.00 \hat{\mathbf{i}}-2.00 \hat{\mathbf{j}}$

$$
=5.00 \hat{\mathbf{i}}+4.00 \hat{\mathbf{j}}
$$

$$
\overrightarrow{\mathbf{D}}=\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}=2.00 \hat{\mathbf{i}}+6.00 \hat{\mathbf{j}}-3.00 \hat{\mathbf{i}}+2.00 \hat{\mathbf{j}}
$$

$$
=-1.00 \hat{\mathbf{i}}+8.00 \hat{\mathbf{j}}
$$



ANS. FIG. P1.38

$$
\overrightarrow{\mathbf{D}}=\sqrt{(-1.00)^{2}+(8.00)^{2}} \text { at } \tan ^{-1}\left(\frac{8.00}{-1.00}\right)
$$

$$
\overrightarrow{\mathbf{D}}=8.06 \text { at }\left(180^{\circ}-82.9^{\circ}\right)=8.06 \text { at } 97.2^{\circ}
$$

P1.39 (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$
6.00 a-8.00 b+26.0=0
$$

and

$$
-8.00 a+3.00 b+19.0=0
$$

Substituting $a=1.33 b-4.33$ into the second equation, we find

$$
-8(1.33 b-4.33)+3 b+19=0 \rightarrow 7.67 b=53.67 \rightarrow b=7.00
$$

and so $a=1.33(7)-4.33=5.00$.
Thus $a=5.00, b=7.00$. Therefore, $5.00 \overrightarrow{\mathbf{A}}+7.00 \overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}=0$.
(b) In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation, as each component gives us one equation.

P1.40 The superhero follows a straight-line path at $30.0^{\circ}$ below the horizontal. If his displacement is 100 m , then the coordinates of the superhero are:

$$
\begin{aligned}
& x=(100 \mathrm{~m}) \cos \left(-30.0^{\circ}\right)=86.6 \mathrm{~m} \\
& y=(100 \mathrm{~m}) \sin \left(-30.0^{\circ}\right)=-50.0 \mathrm{~m}
\end{aligned}
$$



ANS. FIG. P1.40

P1.41
$A_{x}=-25.0$
$A_{y}=40.0$
$A=\sqrt{A_{x}^{2}+A_{y}^{2}}=\sqrt{(-25.0)^{2}+(40.0)^{2}}=47.2$ units.
We observe that

$$
\tan \phi=\frac{\left|A_{y}\right|}{\left|A_{x}\right|}
$$



ANS. FIG. P1.41

So

$$
\phi=\tan ^{-1}\left(\frac{A_{y}}{\left|A_{x}\right|}\right)=\tan ^{-1}\left(\frac{40.0}{25.0}\right)=\tan ^{-1}(1.60)=58.0^{\circ}
$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from $180^{\circ}$ :

$$
\theta=180^{\circ}-58^{\circ}=122^{\circ}
$$

P1.42

$$
\begin{aligned}
\overrightarrow{\mathbf{B}} & =B_{x} \hat{\mathbf{i}}+B_{y} \hat{\mathbf{j}}+B_{z} \hat{\mathbf{k}}=4.00 \hat{\mathbf{i}}+6.00 \hat{\mathbf{j}}+3.00 \hat{\mathbf{k}} \\
|\overrightarrow{\mathbf{B}}| & =\sqrt{4.00^{2}+6.00^{2}+3.00^{2}}=7.81 \\
\alpha & =\cos ^{-1}\left(\frac{4.00}{7.81}\right)=59.2^{\circ} \text { is the angle with the } x \text { axis } \\
\beta & =\cos ^{-1}\left(\frac{6.00}{7.81}\right)=39.8^{\circ} \text { is the angle with the } y \text { axis } \\
\gamma & =\cos ^{-1}\left(\frac{3.00}{7.81}\right)=67.4^{\circ} \text { is the angle with the } z \text { axis }
\end{aligned}
$$

P1.43
(a) $\overrightarrow{\mathbf{A}}=8.00 \hat{\mathbf{i}}+12.0 \hat{\mathbf{j}}-4.00 \hat{\mathbf{k}}$
(b) $\overrightarrow{\mathbf{B}}=\frac{\overrightarrow{\mathbf{A}}}{4}=2.00 \hat{\mathbf{i}}+3.00 \hat{\mathbf{j}}-1.00 \hat{\mathbf{k}}$
(c) $\overrightarrow{\mathbf{C}}=-3 \overrightarrow{\mathbf{A}}=-24.0 \hat{\mathbf{i}}-36.0 \hat{\mathbf{j}}+12.0 \hat{\mathbf{k}}$

P1.44
(a) $R_{x}=40.0 \cos 45.0^{\circ}+30.0 \cos 45.0^{\circ}=49.5$
$R_{y}=40.0 \sin 45.0^{\circ}-30.0 \sin 45.0^{\circ}+20.0=27.1$

$$
\overrightarrow{\mathbf{R}}=49.5 \hat{\mathbf{i}}+27.1 \hat{\mathbf{j}}
$$

(b) $|\overrightarrow{\mathbf{R}}|=\sqrt{(49.5)^{2}+(27.1)^{2}}=56.4$

$$
\theta=\tan ^{-1}\left(\frac{27.1}{49.5}\right)=28.7^{\circ}
$$



ANS. FIG. P1.44
P1.45 We have $\overrightarrow{\mathbf{B}}=\overrightarrow{\mathbf{R}}-\overrightarrow{\mathbf{A}}$ :

$$
\begin{aligned}
& A_{x}=150 \cos 120^{\circ}=-75.0 \mathrm{~cm} \\
& A_{y}=150 \sin 120^{\circ}=130 \mathrm{~cm} \\
& R_{x}=140 \cos 35.0^{\circ}=115 \mathrm{~cm} \\
& R_{y}=140 \sin 35.0^{\circ}=80.3 \mathrm{~cm}
\end{aligned}
$$

ANS. FIG. P1.45


Therefore,

$$
\begin{aligned}
& \overrightarrow{\mathbf{B}}=[115-(-75) \hat{\mathbf{i}}+[80.3-130] \hat{\mathbf{j}}=(190 \hat{\mathbf{i}}-49.7 \hat{\mathbf{j}}) \mathrm{cm} \\
& |\overrightarrow{\mathbf{B}}|=\sqrt{190^{2}+49.7^{2}}=196 \mathrm{~cm} \\
& \theta=\tan ^{-1}\left(-\frac{49.7}{190}\right)=-14.7^{\circ}
\end{aligned}
$$

P1.46
$\overrightarrow{\mathbf{A}}=-8.70 \hat{\mathbf{i}}+15.0 \hat{\mathbf{j}}$ and $\overrightarrow{\mathbf{B}}=13.2 \hat{\mathbf{i}}-6.60 \hat{\mathbf{j}}$
$\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}+3 \overrightarrow{\mathbf{C}}=0$ :
$3 \overrightarrow{\mathbf{C}}=\overrightarrow{\mathbf{B}}-\overrightarrow{\mathbf{A}}=21.9 \hat{\mathbf{i}}-21.6 \hat{\mathbf{j}}$
$\overrightarrow{\mathbf{C}}=7.30 \hat{\mathbf{i}}-7.20 \hat{\mathbf{j}}$ or $\quad C_{x}=7.30 \mathrm{~cm} ; C_{y}=-7.20 \mathrm{~cm}$
P1.47
(a) $(\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}})=(3 \hat{\mathbf{i}}-2 \hat{\mathbf{j}})+(-\hat{\mathbf{i}}-4 \hat{\mathbf{j}})=2 \hat{\mathbf{i}}-6 \hat{\mathbf{j}}$
(b) $\quad(\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}})=(3 \hat{\mathbf{i}}-2 \hat{\mathbf{j}})-(-\hat{\mathbf{i}}-4 \hat{\mathbf{j}})=4 \hat{\mathbf{i}}+2 \hat{\mathbf{j}}$
(c) $|\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}|=\sqrt{2^{2}+6^{2}}=6.32$
(d) $|\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}|=\sqrt{4^{2}+2^{2}}=4.47$
(e) $\quad \theta_{|\mathrm{A}+\mathrm{B}|}=\tan ^{-1}\left(-\frac{6}{2}\right)=-71.6^{\circ}=288^{\circ}$

$$
\theta_{|\mathrm{A}-\mathrm{B}|}=\tan ^{-1}\left(\frac{2}{4}\right)=26.6^{\circ}
$$

P1.48 We use the numbers given in Problem 1.34:

$$
\begin{aligned}
& \overrightarrow{\mathbf{A}}=3.00 \mathrm{~m}, \theta_{A}=30.0^{\circ} \\
& A_{x}=A \cos \theta_{A}=3.00 \cos 30.0^{\circ}=2.60 \mathrm{~m} \\
& A_{y}=A \sin \theta_{A}=3.00 \sin 30.0^{\circ}=1.50 \mathrm{~m}
\end{aligned}
$$

So $\overrightarrow{\mathbf{A}}=A_{x} \hat{\mathbf{i}}+A_{y} \hat{\mathbf{j}}=(2.60 \hat{\mathbf{i}}+1.50 \hat{\mathbf{j}}) \mathrm{m}$

$$
\begin{aligned}
& \overrightarrow{\mathbf{B}}=3.00 \mathrm{~m}, \theta_{B}=90.0^{\circ} \\
& B_{x}=0, B_{y}=3.00 \mathrm{~m} \rightarrow \overrightarrow{\mathbf{B}}=3.00 \hat{\mathbf{j}} \mathrm{~m} \\
& \overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}=(2.60 \hat{\mathbf{i}}+1.50 \hat{\mathbf{j}})+3.00 \hat{\mathbf{j}}=(2.60 \hat{\mathbf{i}}+4.50 \hat{\mathbf{j}}) \mathrm{m}
\end{aligned}
$$

P1.49 Let the positive $x$ direction be eastward, the positive $y$ direction be vertically upward, and the positive $z$ direction be southward. The total displacement is then

$$
\begin{aligned}
\overrightarrow{\mathbf{d}} & =(4.80 \hat{\mathbf{i}}+4.80 \hat{\mathbf{j}}) \mathrm{cm}+(3.70 \hat{\mathbf{j}}-3.70 \hat{\mathbf{k}}) \mathrm{cm} \\
& =(4.80 \hat{\mathbf{i}}+8.50 \hat{\mathbf{j}}-3.70 \hat{\mathbf{k}}) \mathrm{cm}
\end{aligned}
$$

(a) The magnitude is $d=\sqrt{(4.80)^{2}+(8.50)^{2}+(-3.70)^{2}} \mathrm{~cm}=10.4 \mathrm{~cm}$.
(b) Its angle with the $y$ axis follows from

$$
\cos \theta=\frac{8.50}{10.4}, \text { giving } \theta=35.5^{\circ}
$$

P1.50
(a) $\overrightarrow{\mathbf{D}}=\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}=6 \hat{\mathbf{i}}-2 \hat{\mathbf{j}}$

$$
|\overrightarrow{\mathbf{D}}|=\sqrt{6^{2}+2^{2}}=6.32 \mathrm{~m} \text { at } \theta=342^{\circ}
$$

(b) $\overrightarrow{\mathbf{E}}=-\overrightarrow{\mathbf{A}}-\overrightarrow{\mathbf{B}}+\overrightarrow{\mathbf{C}}=-2 \hat{\mathbf{i}}+12 \hat{\mathbf{j}}$

$$
|\overrightarrow{\mathbf{E}}|=\sqrt{2^{2}+12^{2}}=12.2 \mathrm{~m} \text { at } \theta=99.5^{\circ}
$$

P1.51
$\overrightarrow{\mathbf{d}}_{1}=100 \hat{\mathbf{i}}$
$\overrightarrow{\mathbf{d}}_{2}=-300 \hat{\mathbf{i}}$
$\overrightarrow{\mathbf{d}}_{3}=-150 \cos \left(30.0^{\circ}\right) \hat{\mathbf{i}}-150 \sin \left(30.0^{\circ}\right) \hat{\mathbf{j}}=-130 \hat{\mathbf{i}}-75.0 \hat{\mathbf{j}}$
$\overrightarrow{\mathbf{d}}_{4}=-200 \cos \left(60.0^{\circ}\right) \hat{\mathbf{i}}+200 \sin \left(60.0^{\circ}\right) \hat{\mathbf{j}}=-100 \hat{\mathbf{i}}+173 \hat{\mathbf{j}}$
$\overrightarrow{\mathbf{R}}=\overrightarrow{\mathbf{d}}_{1}+\overrightarrow{\mathbf{d}}_{2}+\overrightarrow{\mathbf{d}}_{3}+\overrightarrow{\mathbf{d}}_{4}=(-130 \hat{\mathbf{i}}-202 \hat{\mathbf{j}}) \mathrm{m}$
$|\overrightarrow{\mathbf{R}}|=\sqrt{(-130)^{2}+(-202)^{2}}=240 \mathrm{~m}$


ANS. FIG. P1.51

P1.52
(a) $\overrightarrow{\mathbf{E}}=(17.0 \mathrm{~cm}) \cos \left(27.0^{\circ}\right) \hat{\mathbf{i}}$

$$
\begin{aligned}
& \quad+(17.0 \mathrm{~cm}) \sin \left(27.0^{\circ}\right) \hat{\mathbf{j}} \\
& \overrightarrow{\mathbf{E}}=(15.1 \hat{\mathbf{i}}+7.72 \hat{\mathbf{j}}) \mathrm{cm}
\end{aligned}
$$

(b) $\overrightarrow{\mathbf{F}}=(17.0 \mathrm{~cm}) \cos \left(117.0^{\circ}\right) \hat{\mathbf{i}}$

$$
+(17.0 \mathrm{~cm}) \sin \left(117.0^{\circ}\right) \hat{\mathbf{j}}
$$

$$
\overrightarrow{\mathbf{F}}=(-7.72 \hat{\mathbf{i}}+15.1 \hat{\mathbf{j}}) \mathrm{cm} \text { Note that we }
$$



ANS. FIG. P1.52
did not need to explicitly identify the angle
with the positive $x$ axis, but by doing so, we don't have to keep track of minus signs for the components.
(c) $\quad \overrightarrow{\mathbf{G}}=\left[(-17.0 \mathrm{~cm}) \cos \left(243.0^{\circ}\right)\right] \hat{\mathbf{i}}+\left[(-17.0 \mathrm{~cm}) \sin \left(243.0^{\circ}\right)\right] \hat{\mathbf{j}}$

$$
\overrightarrow{\mathbf{G}}=(-7.72 \hat{\mathbf{i}}-15.1 \hat{\mathbf{j}}) \mathrm{cm}
$$

## Section 1.10 Modeling, Alternative Representations, and Problem-Solving Strategy

P1.53 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text {diag }}=\sqrt{L^{2}+L^{2}}$. Thus, since the atoms are separated by a distance $L=0.200 \mathrm{~nm}$, the diagonal planes are separated by
$\frac{1}{2} \sqrt{L^{2}+L^{2}}=0.141 \mathrm{~nm}$.
P1.54 We note that $-\hat{\mathbf{i}}=$ west and $-\hat{\mathbf{j}}=$ south. The given mathematical representation of the trip can be written as 6.30 b west +4.00 b at $40^{\circ}$ south of west +3.00 b at $50^{\circ}$ south of east +5.00 b south.
(a) The figure on the right shows a map of the successive displacements that the bus undergoes.


ANS. FIG. P1.54
(b) The total odometer distance is the sum of the magnitudes of the four displacements:

$$
6.30 \mathrm{~b}+4.00 \mathrm{~b}+3.00 \mathrm{~b}+5.00 \mathrm{~b}=18.3 \mathrm{~b}
$$

(c) $\quad \overrightarrow{\mathbf{R}}=(-6.30-3.06+1.93) \mathrm{b} \hat{\mathbf{i}}+(-2.57-2.30-5.00) \mathrm{b} \hat{\mathbf{j}}$

$$
=-7.44 \mathrm{~b} \hat{\mathbf{i}}-9.87 \mathrm{~b} \hat{\mathbf{j}}
$$

$$
=\sqrt{(7.44 \mathrm{~b})^{2}+(9.87 \mathrm{~b})^{2}} \text { at } \tan ^{-1}\left(\frac{9.87}{7.44}\right) \text { south of west }
$$

$=12.4 \mathrm{~b}$ at $53.0^{\circ}$ south of west
$=12.4 \mathrm{~b}$ at $233^{\circ}$ counterclockwise from east
Figure P1.55 suggests a right triangle where, relative to angle $\theta$, its adjacent side has length $d$ and its opposite side is equal to the width of the river, $y$; thus,

$$
\begin{aligned}
& \tan \theta=\frac{y}{d} \rightarrow y=d \tan \theta \\
& y=(100 \mathrm{~m}) \tan 35.0^{\circ}=70.0 \mathrm{~m}
\end{aligned}
$$

The width of the river is 70.0 m .
P1.56 The volume of the galaxy is

$$
\pi r^{2} t=\pi\left(10^{21} \mathrm{~m}\right)^{2}\left(10^{19} \mathrm{~m}\right) \sim 10^{61} \mathrm{~m}^{3}
$$

If the distance between stars is $4 \times 10^{16}$, then there is one star in a volume on the order of

$$
\left(4 \times 10^{16} \mathrm{~m}\right)^{3} \sim 10^{50} \mathrm{~m}^{3}
$$

The number of stars is about $\frac{10^{61} \mathrm{~m}^{3}}{10^{50} \mathrm{~m}^{3} / \mathrm{star}} \sim 10^{11}$ stars.

One month is

$$
1 \mathrm{mo}=(30 \text { day })(24 \mathrm{~h} / \text { day })(3600 \mathrm{~s} / \mathrm{h})=2.592 \times 10^{6} \mathrm{~s} .
$$

Applying units to the equation,

$$
V=\left(1.50 \mathrm{Mft}^{3} / \mathrm{mo}\right) t+\left(0.00800 \mathrm{Mft}^{3} / \mathrm{mo}^{2}\right) t^{2}
$$

Since $1 \mathrm{Mft}^{3}=10^{6} \mathrm{ft}^{3}$,

$$
V=\left(1.50 \times 10^{6} \mathrm{ft}^{3} / \mathrm{mo}\right) t+\left(0.00800 \times 10^{6} \mathrm{ft}^{3} / \mathrm{mo}^{2}\right) t^{2}
$$

Converting months to seconds,

$$
V=\frac{1.50 \times 10^{6} \mathrm{ft}^{3} / \mathrm{mo}}{2.592 \times 10^{6} \mathrm{~s} / \mathrm{mo}} t+\frac{0.00800 \times 10^{6} \mathrm{ft}^{3} / \mathrm{mo}^{2}}{\left(2.592 \times 10^{6} \mathrm{~s} / \mathrm{mo}\right)^{2}} t^{2}
$$

Dropping units, the equation is

$$
V=0.579 t+\left(1.19 \times 10^{-9}\right) t^{2}
$$

for $V$ in cubic feet and $t$ in seconds.
P1.59 Since

$$
\overrightarrow{\mathbf{A}}+\overrightarrow{\mathbf{B}}=6.00 \hat{\mathbf{j}},
$$

we have

$$
\left(A_{x}+B_{x}\right) \hat{\mathbf{i}}+\left(A_{y}+B_{y}\right) \hat{\mathbf{j}}=0 \hat{\mathbf{i}}+6.00 \hat{\mathbf{j}}
$$



