INSTRUCTORS' SOLUTIONS MANUAL
FOR DEVORE'S

PROBABILITY
AND
STATISTICS

FOR ENGINEERING AND THE SCIENCES

8TH EDITION

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CONTENTS

CHAPTER 1

Section 1.1

1.

- **a.** *Los Angeles Times, Oberlin Tribune, Gainesville Sun, Washington Post*
- **b.** Duke Energy, Clorox, Seagate, Neiman Marcus
- **c.** Vince Correa, Catherine Miller, Michael Cutler, Ken Lee
- **d.** 2.97, 3.56, 2.20, 2.97

2.

- **a.** 29.1 yd, 28.3 yd, 24.7 yd, 31.0 yd
- **b.** 432 pp, 196 pp, 184 pp, 321 pp
- **c.** 2.1, 4.0, 3.2, 6.3
- **d.** 0.07 g, 1.58 g, 7.1 g, 27.2 g

3.

- **a.** How likely is it that more than half of the sampled computers will need or have needed warranty service? What is the expected number among the 100 that need warranty service? How likely is it that the number needing warranty service will exceed the expected number by more than 10?
- **b.** Suppose that 15 of the 100 sampled needed warranty service. How confident can we be that the proportion of *all* such computers needing warranty service is between .08 and .22? Does the sample provide compelling evidence for concluding that more than 10% of all such computers need warranty service?

4.

a. Concrete populations: all living U.S. Citizens, all mutual funds marketed in the U.S., all books published in 1980
Hypothetical populations: all grade point averages for University of California undergraduates during the next academic year, page lengths for all books published during the next calendar year, batting averages for all major league players during the next baseball season

b. (Concrete) Probability: In a sample of 5 mutual funds, what is the chance that all 5 have rates of return which exceeded 10% last year?
Statistics: If previous year rates-of-return for 5 mutual funds were 9.6, 14.5, 8.3, 9.9 and 10.2, can we conclude that the average rate for all funds was below 10%?
(Hypothetical) Probability: In a sample of 10 books to be published next year, how likely is it that the average number of pages for the 10 is between 200 and 250?
Statistics: If the sample average number of pages for 10 books is 227, can we be highly confident that the average for all books is between 200 and 245?

5.

a. No. All students taking a large statistics course who participate in an SI program of this sort.

b. The advantage to randomly allocating students to the two groups is that the two groups should then be fairly comparable before the study. If the two groups perform differently in the class, we might attribute this to the treatments (SI and control). If it were left to students to choose, stronger or more dedicated students might gravitate toward SI, confounding the results.

c. If all students were put in the treatment group, there would be no firm basis for assessing the effectiveness of SI (nothing to which the SI scores could reasonably be compared).

6. One could take a simple random sample of students from all students in the California State University system and ask each student in the sample to report the distance form their hometown to campus. Alternatively, the sample could be generated by taking a stratified random sample by taking a simple random sample from each of the 23 campuses and again asking each student in the sample to report the distance from their hometown to campus. Certain problems might arise with self reporting of distances, such as recording error or poor recall. This study is enumerative because there exists a finite, identifiable population of objects from which to sample.

7. One could generate a simple random sample of all single-family homes in the city, or a stratified random sample by taking a simple random sample from each of the 10 district neighborhoods. From each of the selected homes, values of all desired variables would be determined. This would be an enumerative study because there exists a finite, identifiable population of objects from which to sample.

8.

a. Number observations equal 2 x 2 x 2 = 8

b. This could be called an analytic study because the data would be collected on an existing process. There is no sampling frame.

9.

a. There could be several explanations for the variability of the measurements. Among them could be measurement error (due to mechanical or technical changes across measurements), recording error, differences in weather conditions at time of measurements, etc.

b. No, because there is no sampling frame.

Section 1.2

10.

a.

```
 5|9
 6|33588
 7|00234677889
 8|127
 9|077          stem: ones
10|7            leaf: tenths
11|368
```

A representative strength for these beams is around 7.8 MPa, but there is a reasonably large amount of variation around that representative value.

(What constitutes large or small variation usually depends on context, but variation is usually considered large when the range of the data – the difference between the largest and smallest value – is comparable to a representative value. Here, the range is 11.8 – 5.9 = 5.9 MPa, which is similar in size to the representative value of 7.8 MPa. So, most researchers would call this a large amount of variation.)

b. The data display is not perfectly symmetric around some middle/representative value. There is some positive skewness in this data.

c. Outliers are data points that appear to be *very* different from the pack. Looking at the stem-and-leaf display in part (a), there appear to be no outliers in this data. (A later section gives a more precise definition of what constitutes an outlier.)

d. From the stem-and-leaf display in part (a), there are 4 values greater than 10. Therefore, the proportion of data values that exceed 10 is 4/27 = .148, or, about 15%.

11.

6L	034	
6H	667899	
7L	00122244	
7H		stem: tens
8L	001111122344	leaf : ones
8H	5557899	
9L	03	
9H	58	

This display brings out the gap in the data—there are no scores in the high 70's.

12. Using the H and L notation suggested in the previous exercise, the stem-and-leaf display would appear as follows:

3L	1	
3H	56678	
4L	000112222234	
4H	5667888	
5L	144	
5H	58	stem: tenths
6L	2	leaf: hundredths
6H	6678	
7L		
7H	5	

The stem-and-leaf display shows that .45 is a good representative value for the data. In addition, the display is not symmetric and appears to be positively skewed. The range of the data is .75 – .31 = .44, which is comparable to the typical value of .45. This constitutes a reasonably large amount of variation in the data. The data value .75 is a possible outlier.

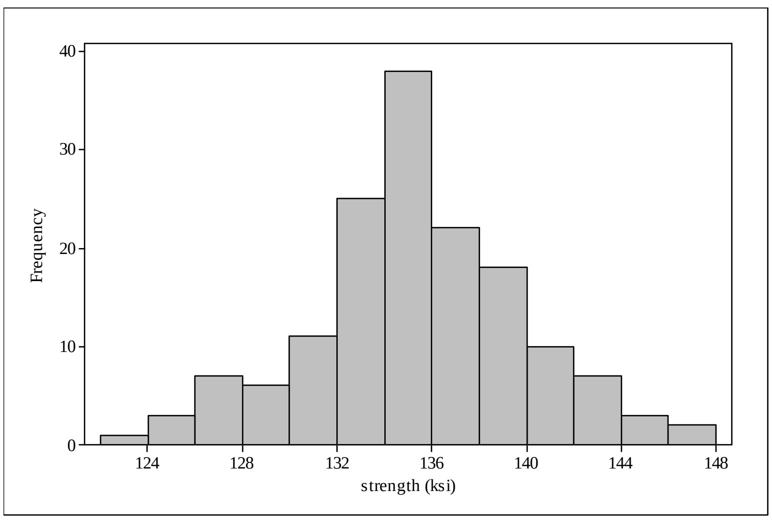
13.

a.

12	2	stem: tens
12	445	leaf: ones
12	6667777	
12	889999	
13	00011111111	
13	2222222222333333333333333	
13	44444444444444444455555555555555555555	
13	6666666666667777777777	
13	888888888888999999	
14	0000001111	
14	2333333	
14	444	
14	77	

The observations are highly concentrated at around 134 or 135, where the display suggests the typical value falls.

b.



The histogram of ultimate strengths is symmetric and unimodal, with the point of symmetry at approximately 135 ksi. There is a moderate amount of variation, and there are no gaps or outliers in the distribution.

14.

a.

2	23	stem: 1.0
3	2344567789	leaf: .10
4	01356889	
5	00001114455666789	
6	0000122223344456667789999	
7	00012233455555668	
8	02233448	
9	012233335666788	
10	2344455688	
11	2335999	
12	37	
13	8	
14	36	
15	0035	
16		
17		
18	9	

b. A representative is around 7.0.

c. The data exhibit a moderate amount of variation (this is subjective).

d. No, the data is skewed to the right, or positively skewed.

e. The value 18.9 appears to be an outlier, being more than two stem units from the previous value.

15.

American		French
	8	1
755543211000	9	00234566
9432	10	2356
6630	11	1369
850	12	223558
8	13	7
	14	
	15	8
2	16	

American movie times are unimodal strongly positively skewed, while French movie times appear to be bimodal. A typical American movie runs about 95 minutes, while French movies are typically either around 95 minutes or around 125 minutes. American movies are generally shorter than French movies and are less variable in length. Finally, both American and French movies occasionally run very long (outliers at 162 minutes and 158 minutes, respectively, in the samples).

16.

a.

Beams		Cylinders
9	5	8
88533	6	16
98877643200	7	012488
721	8	13359
770	9	278
7	10	
863	11	2
	12	6
	13	
	14	1

stem: ones
leaf: tenths

The data appears to be slightly skewed to the right, or positively skewed. The value of 14.1 MPa appears to be an outlier. Three out of the twenty, or 15%, of the observations exceed 10 MPa.

b. The majority of observations are between 5 and 9 MPa for both beams and cylinders, with the modal class being 7.0-7.9 MPa. The observations for cylinders are more variable, or spread out, and the maximum value of the cylinder observations is higher.

c.

```
. .  .  :..  : .: . . .   :         .        .         .
-+---------+---------+---------+---------+---------+-----
6.0       7.5       9.0      10.5      12.0      13.5
                Cylinder strength (MPa)
```

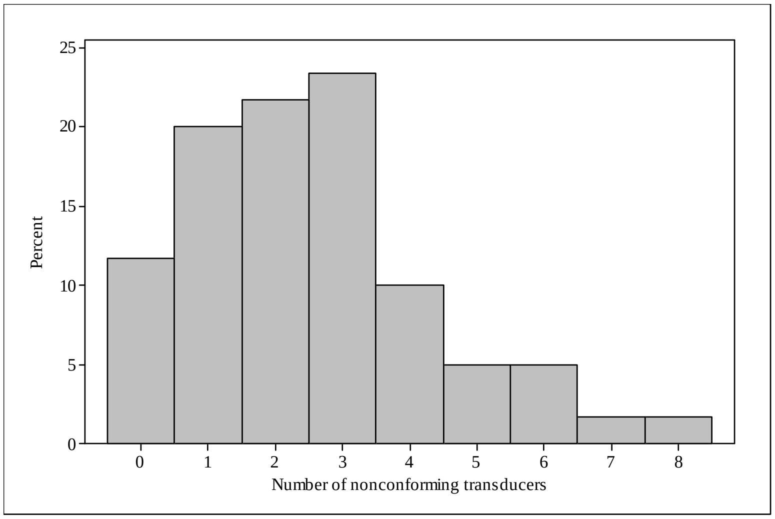
17.

a.

Number Nonconforming	Frequency	Relative Frequency (Freq./60)
0	7	0.117
1	12	0.200
2	13	0.217
3	14	0.233
4	6	0.100
5	3	0.050
6	3	0.050
7	1	0.017
8	1	0.017
		1.001 *rounding error*

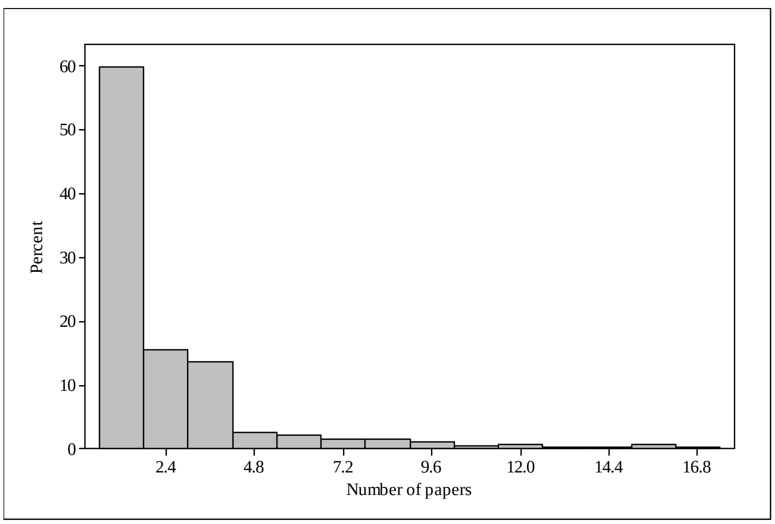
b. The number of batches with at most 5 nonconforming items is 7+12+13+14+6+3 = 55, which is a proportion of 55/60 = .917. The proportion of batches with (strictly) fewer than 5 nonconforming items is 52/60 = .867. Notice that these proportions could also have been computed by using the relative frequencies: e.g., proportion of batches with 5 or fewer nonconforming items = 1– (.05+.017+.017) = .916; proportion of batches with fewer than 5 nonconforming items = 1 – (.05+.05+.017+.017) = .866.

c. The center of the histogram is somewhere around 2 or 3, and there is some positive skewness in the data. The histogram also shows that there is a lot of spread/variation in this data.



18.

a. The most interesting feature of the histogram is the heavy positive skewness of the data.



Note: One way to have Minitab automatically construct a histogram from grouped data such as this is to use Minitab's ability to enter multiple copies of the same number by typing, for example, 784(1) to enter 784 copies of the number 1. The frequency data in this exercise was entered using the following Minitab commands:

```
    MTB > set c1
    DATA> 784(1) 204(2) 127(3) 50(4) 33(5) 28(6) 19(7) 19(8)
6(9) 7(10) 6(11) 7(12) 4(13) 4(14) 5(15) 3(16) 3(17)
    DATA> end
```

b. From the frequency distribution (or from the histogram), the number of authors who published at least 5 papers is 33+28+19+…+5+3+3 = 144, so the proportion who published 5 or more papers is 144/1309 = .11, or 11%. Similarly, by adding frequencies and dividing by n = 1309, the proportion who published 10 or more papers is 39/1309 = .0298, or about 3%. The proportion who published more than 10 papers (i.e., 11 or more) is 32/1309 = .0245, or about 2.5%.

c. No. Strictly speaking, the class described by "≥ 15" has no upper boundary, so it is impossible to draw a rectangle above it having finite area (i.e., frequency).

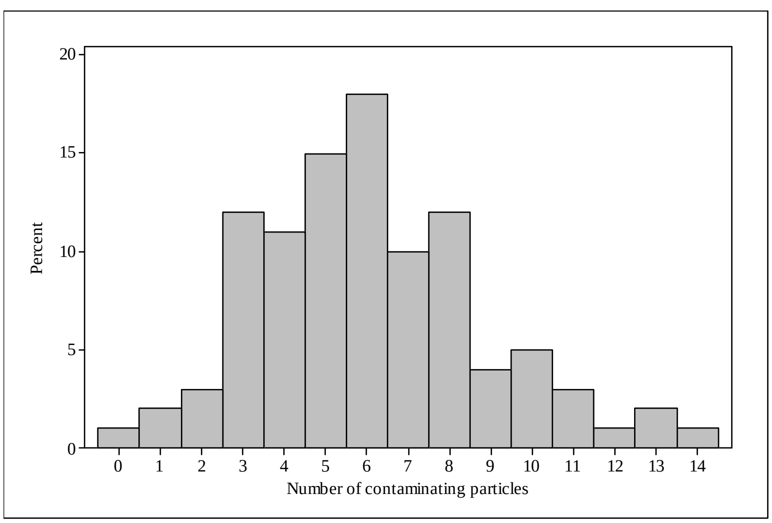
d. The category 15-17 does have a finite width of 3 (15, 16, 17), so the cumulated frequency of 11 can be plotted as a rectangle of height 3.67 over this interval. The rule is to make the area of the bar equal to the class frequency, so area = 11 = (width)(height) = 3(height) yields a height of 3.67. (This is a variant on the *density scale*.)

19.

a. From this frequency distribution, the proportion of wafers that contained at least one particle is (100-1)/100 = .99, or 99%. Note that it is much easier to subtract 1 (which is the number of wafers that contain 0 particles) from 100 than it would be to add all the frequencies for 1, 2, 3,… particles. In a similar fashion, the proportion containing at least 5 particles is (100 - 1-2-3-12-11)/100 = 71/100 = .71, or, 71%.

b. The proportion containing between 5 and 10 particles is (15+18+10+12+4+5)/100 = 64/100 = .64, or 64%. The proportion that contain strictly between 5 and 10 (meaning strictly *more* than 5 and strictly *less* than 10) is (18+10+12+4)/100 = 44/100 = .44, or 44%.

c. The following histogram was constructed using Minitab. The histogram is *almost* symmetric and unimodal; however, the distribution has a few smaller modes and has a very slight positive skew.



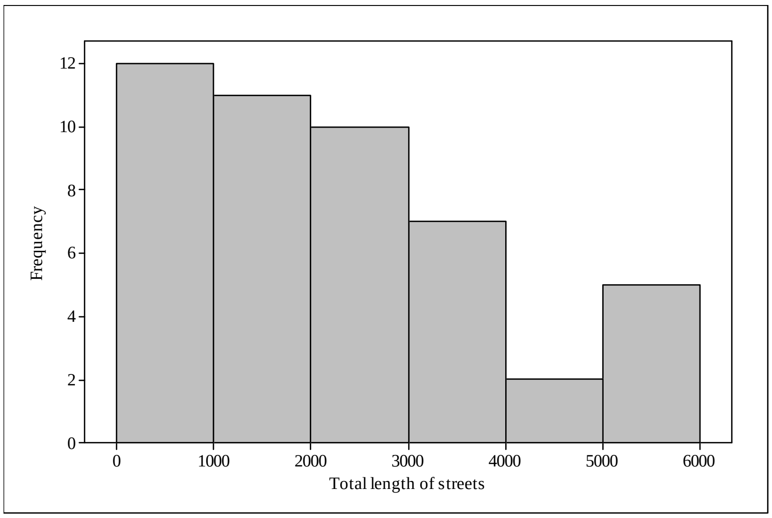
20.

a. The following stem-and-leaf display was constructed:

0	123334555599	
1	00122234688	stem: thousands
2	1112344477	leaf: hundreds
3	0113338	
4	37	
5	23778	

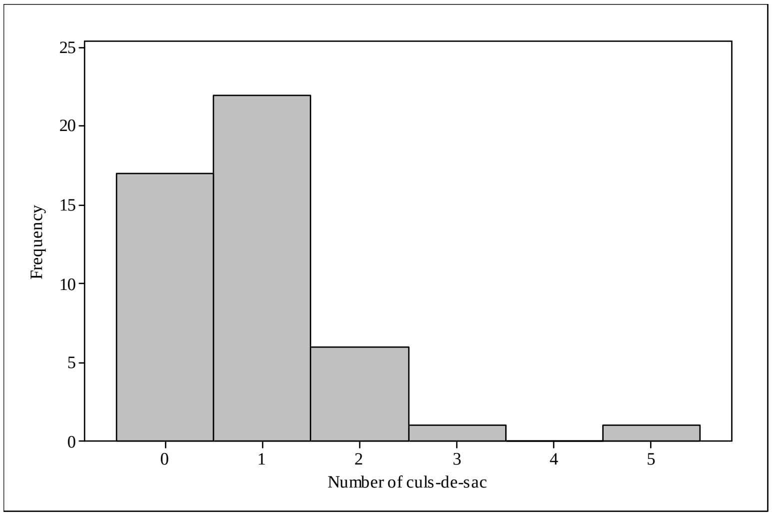
A typical data value is somewhere in the low 2000's. The display is bimodal (the stem at 5 would be considered a mode, the stem at 0 another) and has a positive skew.

b. A histogram of this data, using classes boundaries of 0, 1000, 2000, …, 6000 is shown below. The proportion of subdivisions with total length less than 2000 is (12+11)/47 = .489, or 48.9%. Between 2000 and 4000, the proportion is (10+7)/47 = .362, or 36.2%. The histogram shows the same general shape as depicted by the stem-and-leaf in part (a).

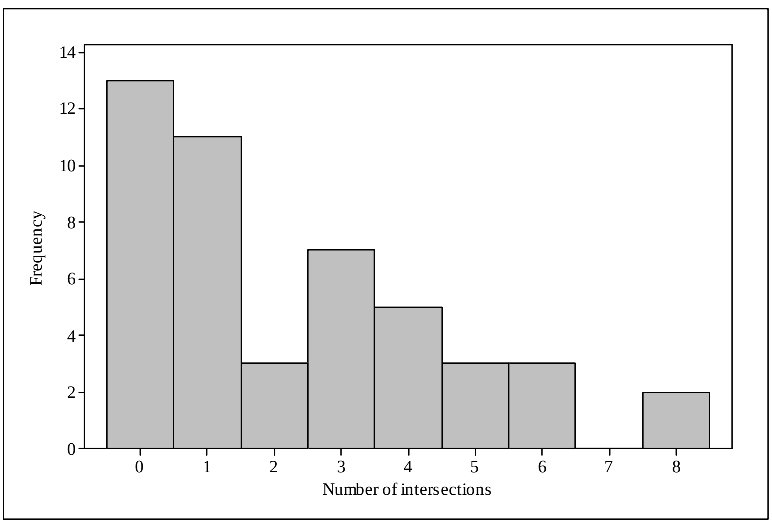


21.

a. A histogram of the y data appears below. From this histogram, the number of subdivisions having no cul-de-sacs (i.e., $y = 0$) is 17/47 = .362, or 36.2%. The proportion having at least one cul-de-sac ($y \geq 1$) is (47 – 17)/47 = 30/47 = .638, or 63.8%. Note that subtracting the number of cul-de-sacs with $y = 0$ from the total, 47, is an easy way to find the number of subdivisions with $y \geq 1$.

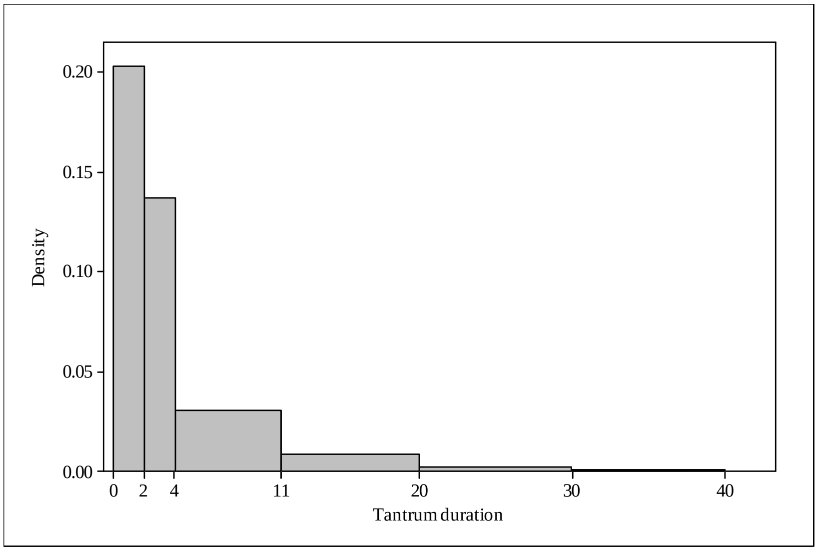


b. A histogram of the z data appears below. From this histogram, the number of subdivisions with at most 5 intersections (i.e., $z \leq 5$) is 42/47 = .894, or 89.4%. The proportion having fewer than 5 intersections (i.e., $z < 5$) is 39/47 = .830, or 83.0%.



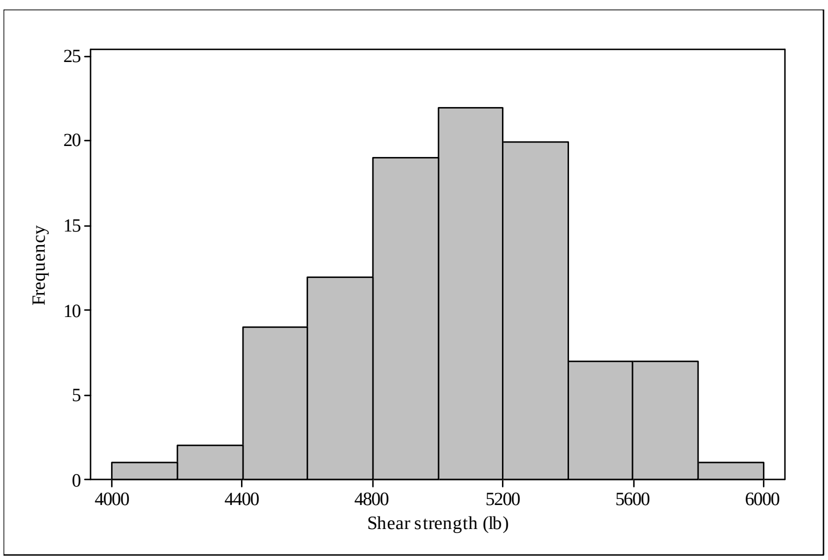
22. A very large percentage of the data values are greater than 0, which indicates that most, but not all, runners do slow down at the end of the race. The histogram is also positively skewed, which means that some runners slow down a *lot* compared to the others. A typical value for this data would be in the neighborhood of 200 seconds. The proportion of the runners who ran the last 5 km faster than they did the first 5 km is very small, about 1% or so.

23. Note: since the class intervals have unequal length, we must use a *density scale*.



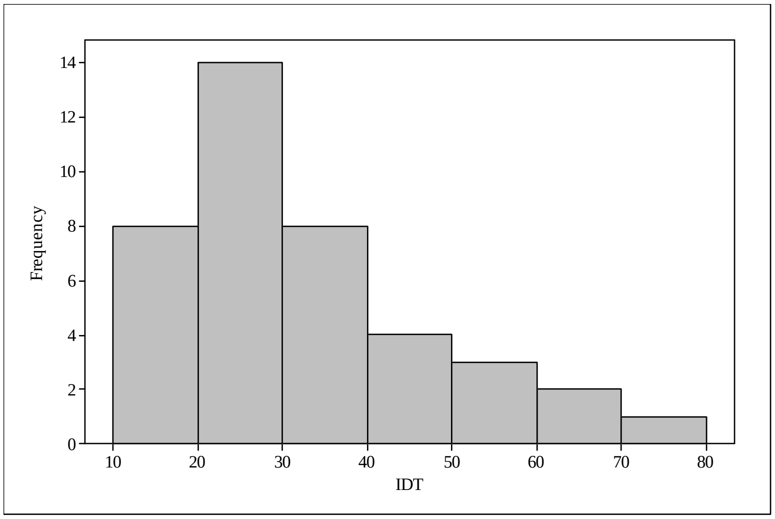
The distribution of tantrum durations is unimodal and heavily positively skewed. Most tantrums last between 0 and 11 minutes, but a few last more than half an hour! With such heavy skewness, it's difficult to give a representative value.

24. The distribution of shear strengths is roughly symmetric and bell-shaped, centered at about 5000 lbs and ranging from about 4000 to 6000 lbs.

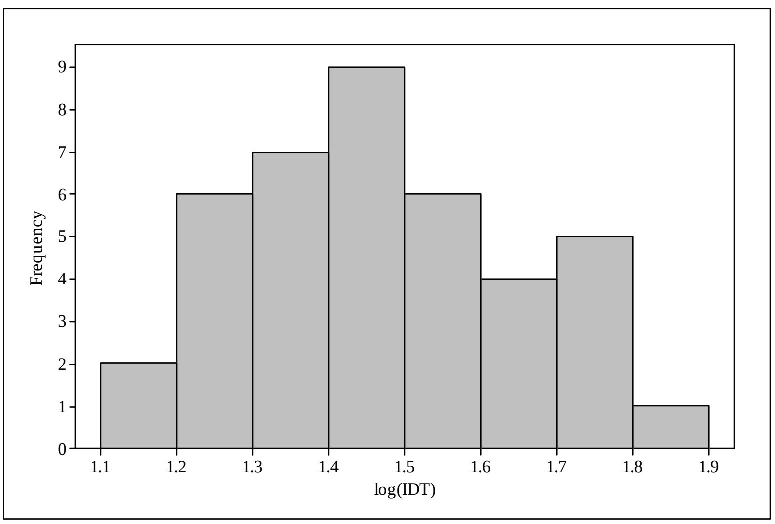


25. The transformation creates a much more symmetric, mound-shaped histogram.

Histogram of original data:



Histogram of transformed data:



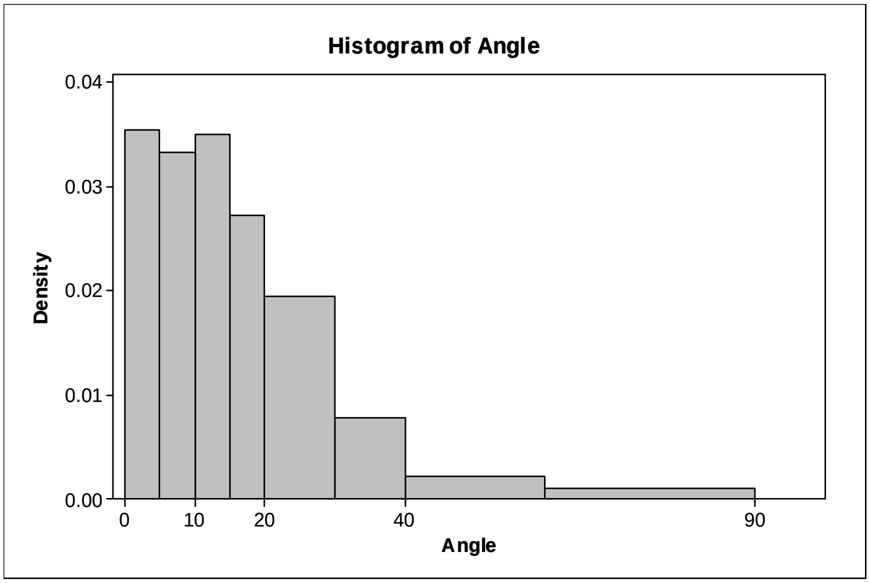
26.

a. Yes: the proportion of sampled angles smaller than 15° is .177 + .166 + .175 = .518.

b. The proportion of sampled angles at least 30° is .078 + .044 + .030 = .152.

c. The proportion of angles between 10° and 25° is roughly .175 + .136 + (.194)/2 = .408.

d. The distribution of misorientation angles is heavily positively skewed. Though angles can range from 0° to 90°, nearly 85% of all angles are less than 30°. Without more precise information, we cannot tell if the data contain outliers.

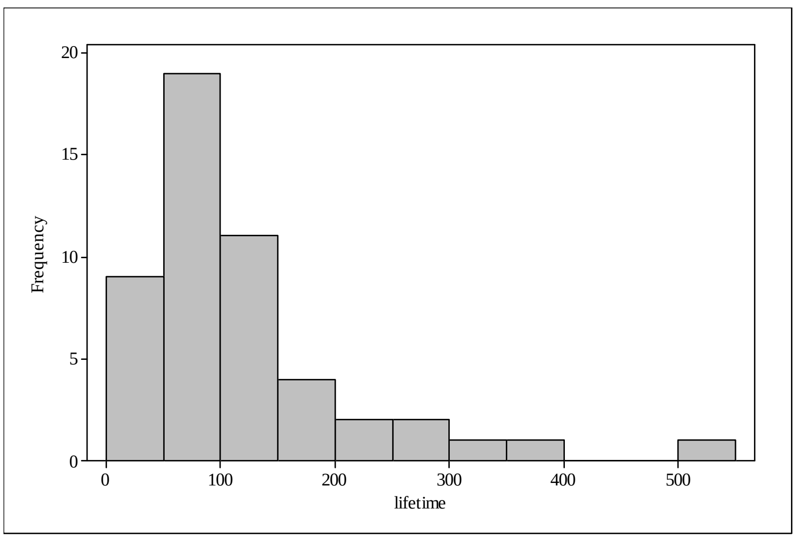


27.

a. The endpoints of the class intervals overlap. For example, the value 50 falls in both of the intervals 0–50 and 50–100.

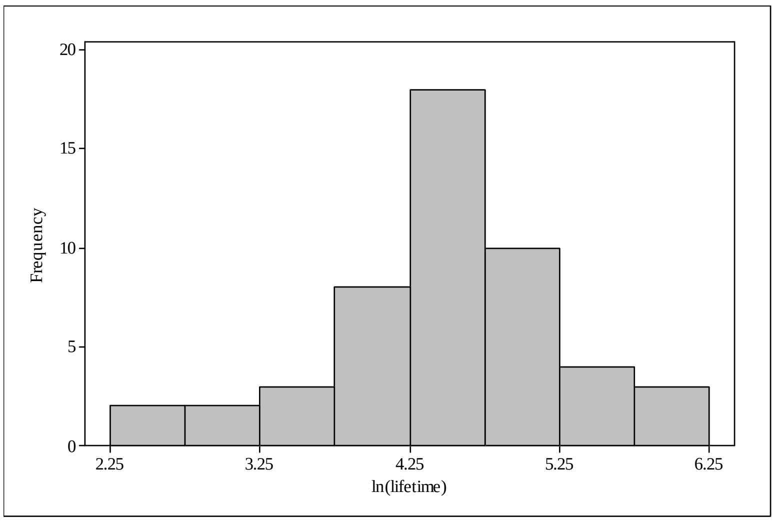
b. The lifetime distribution is positively skewed. A representative value is around 100. There is a great deal of variability in lifetimes and several possible candidates for outliers.

Class Interval	Frequency	Relative Frequency
0–< 50	9	0.18
50–<100	19	0.38
100–<150	11	0.22
150–<200	4	0.08
200–<250	2	0.04
250–<300	2	0.04
300–<350	1	0.02
350–<400	1	0.02
400–<450	0	0.00
450–<500	0	0.00
500–<550	1	0.02
	50	1.00



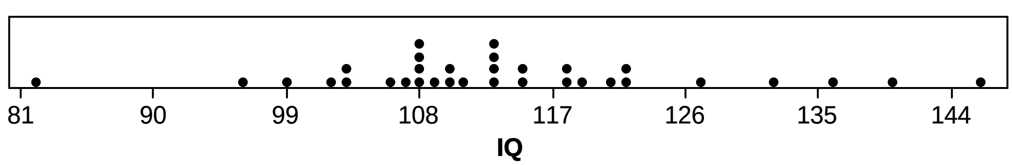
c. There is much more symmetry in the distribution of the transformed values than in the values themselves, and less variability. There are no longer gaps or obvious outliers.

Class Interval	Frequency	Relative Frequency
2.25–<2.75	2	0.04
2.75–<3.25	2	0.04
3.25–<3.75	3	0.06
3.75–<4.25	8	0.16
4.25–<4.75	18	0.36
4.75–<5.25	10	0.20
5.25–<5.75	4	0.08
5.75–<6.25	3	0.06



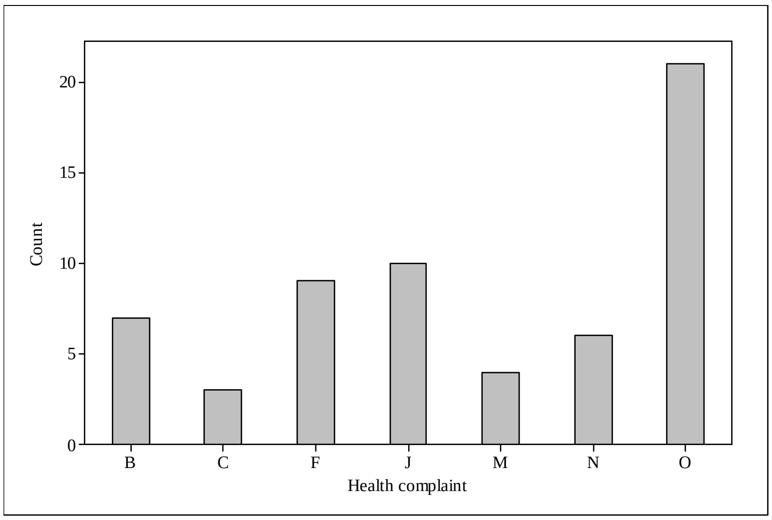
d. The proportion of lifetime observations in this sample that are less than 100 is .18 + .38 = .56, and the proportion that is at least 200 is .04 + .04 + .02 + .02 + .02 = .14.

28. The dot plot below displays the 33 IQ scores. The lowest IQ, 82, certainly stands out as a low outlier. Otherwise, the distribution of IQ scores is slightly positively skewed, with a concentration of IQ scores in the range 106 to 122.

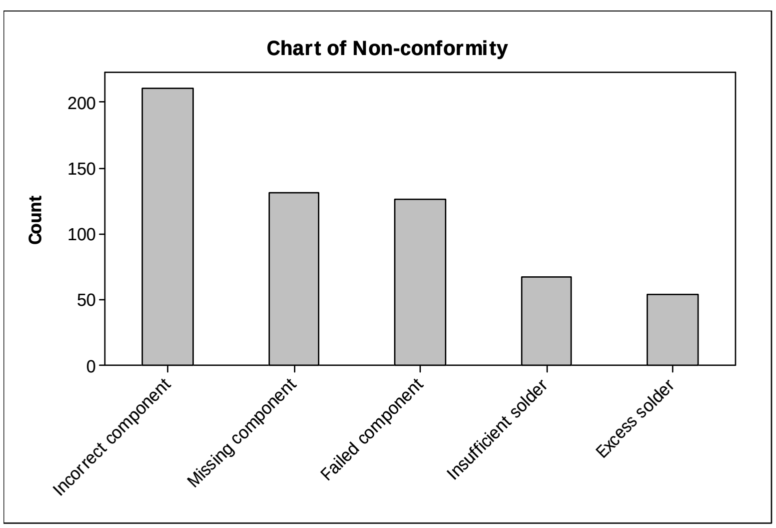


29.

Complaint	Frequency	Relative Frequency
B	7	0.1167
C	3	0.0500
F	9	0.1500
J	10	0.1667
M	4	0.0667
N	6	0.1000
O	21	0.3500
	60	1.0000



30.



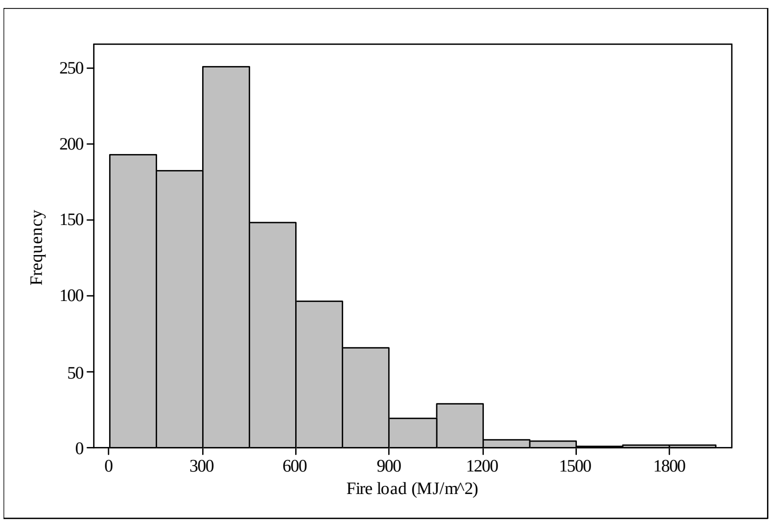
31.

Class	Frequency	Cum. Freq.	Cum. Rel. Freq.
0.0–<4.0	2	2	0.050
4.0–<8.0	14	16	0.400
8.0–<12.0	11	27	0.675
12.0–<16.0	8	35	0.875
16.0–<20.0	4	39	0.975
20.0–<24.0	0	39	0.975
24.0–<28.0	1	40	1.000

32.

a. Cumulative percents must be restored to relative frequencies. Then the histogram may be constructed (see below). The relative frequency distribution is almost unimodal and exhibits a large positive skew. The typical middle value is somewhere between 400 and 450, although the skewness makes it difficult to pinpoint more exactly than this.

Class	Rel. Freq.	Class	Rel. Freq.
0–< 150	.193	900–<1050	.019
150–< 300	.183	1050–<1200	.029
300–< 450	.251	1200–<1350	.005
450–< 600	.148	1350–<1500	.004
600–< 750	.097	1500–<1650	.001
750–< 900	.066	1650–<1800	.002
		1800–<1950	.002



b. The proportion of the fire loads less than 600 is .193 + .183 + .251 + .148 = .775. The proportion of loads that are at least 1200 is .005 + .004 + .001 + .002 + .002 = .014.

c. The proportion of loads between 600 and 1200 is 1 – .775 – .014 = .211.

Section 1.3

33.

a. Using software, $\bar{x}$ = 640.5 ($640,500) and $\tilde{x}$ = 582.5 ($582,500). The average sale price for a home in this sample was $640,500. Half the sales were for less than $582,500, while half were for more than $582,500.

b. Changing that one value lowers the sample mean to 610.5 ($610,500) but has no effect on the sample median.

c. After removing the two largest and two smallest values, $\bar{x}_{tr(20)}$ = 591.2 ($591,200).

d. A 10% trimmed mean from removing just the highest and lowest values is $\bar{x}_{tr(10)}$ = 596.3. To form a 15% trimmed mean, take the average of the 10% and 20% trimmed means to get $\bar{x}_{tr(15)}$ = (591.2 + 596.3)/2 = 593.75 ($593,750).

34.

a. For urban homes, $\bar{x}$ = 21.55 EU/mg; for farm homes, $\bar{x}$ = 8.56 EU/mg. The average endotoxin concentration in urban homes is more than double the average endotoxin concentration in farm homes.

b. For urban homes, $\tilde{x}$ = 17.00 EU/mg; for farm homes, $\tilde{x}$ = 8.90 EU/mg. The median endotoxin concentration in urban homes is nearly double the median endotoxin concentration in farm homes. The mean and median endotoxin concentration for urban homes are so different because the few large values, especially the extreme value of 80.0, raise the mean but not the median.

c. For urban homes, deleting the smallest (x = 4.0) and largest (x = 80.0) values gives a trimmed mean of $\bar{x}_{tr}$ = 153/9 = 17 EU/mg. The corresponding trimming percentage is 100(1/11) ≈ 9.1%. The trimmed mean is less than the mean of the entire sample, since the sample was positively skewed. Coincidentally, the median and trimmed mean are equal.

For farm homes, deleting the smallest (x = 0.3) and largest (x = 21.0) values gives a trimmed mean of $\bar{x}_{tr}$ = 107.1/13 = 8.24 EU/mg. The corresponding trimming percentage is 100(1/15) ≈ 6.7%. The trimmed mean is below, though not far from, the mean and median of the entire sample.

35.

a. From software, the sample mean is $\bar{x}$ = 12.55 and the sample median is $\tilde{x}$ = 12.5. The 12.5% trimmed mean requires that we first trim (.125)(n) = 1 value from the ends of the ordered data set. Then we average the remaining 6 values. The 12.5% trimmed mean $\bar{x}_{tr(12.5)}$ is 74.4/6 = 12.4.

All three measures of center are similar, indicating little skewness to the data set.

b. The smallest value, 8.0, could be increased to any number below 12.0 (a change of less than 4.0) without affecting the value of the sample median.

c. The values obtained in part (a) can be used directly. For example, the sample mean of 12.55 psi could be re-expressed as

$$12.55 \text{ psi} \times \left(\frac{1 \text{ ksi}}{2.2 \text{ psi}} \right) = 5.705 \text{ ksi}$$

36.

a. A stem-and leaf display of this data appears below:

32	55	stem: ones
33	49	leaf: tenths
34		
35	6699	
36	34469	
37	03345	
38	9	
39	2347	
40	23	
41		
42	4	

The display is reasonably symmetric, so the mean and median will be close.

b. The sample mean is $\bar{x}$ = 9638/26 = 370.7 sec, while the sample median is $\tilde{x}$ = (369+370)/2 = 369.50 sec.

c. The largest value (currently 424) could be increased by any amount. Doing so will not change the fact that the middle two observations are 369 and 370, and hence, the median will not change. However, the value x = 424 cannot be changed to a number less than 370 (a change of 424 – 370 = 54) since that will change the middle two values.

d. Expressed in minutes, the mean is (370.7 sec)/(60 sec) = 6.18 min, while the median is 6.16 min.

37. $\bar{x} = 12.01$, $\tilde{x} = 11.35$, $\bar{x}_{tr(10)} = 11.46$. The median or the trimmed mean would be better choices than the mean because of the outlier 21.9.

38.

a. The reported values are (in increasing order) 110, 115, 120, 120, 125, 130, 130, 135, and 140. Thus the median of the reported values is 125.

b. 127.6 is reported as 130, so the median is now 130, a very substantial change. When there is rounding or grouping, the median can be highly sensitive to small change.

39.

a. $\Sigma x_i = 16.475$ so $\bar{x} = \frac{16.475}{16} = 1.0297$; $\tilde{x} = \frac{(1.007 + 1.011)}{2} = 1.009$

b. 1.394 can be decreased until it reaches 1.011 (i.e. by 1.394 – 1.011 = 0.383), the largest of the 2 middle values. If it is decreased by more than 0.383, the median will change.

40. $\tilde{x} = 60.8$, $\bar{x}_{tr(25)} = 59.3083$, $\bar{x}_{tr(10)} = 58.3475$, $\bar{x} = 58.54$. All four measures of center have about the same value.

41.

a. x/n = 7/10 = .7

b. $\bar{x}$ = .70 = the sample proportion of successes

c. To have x/n equal .80 requires x/25 = .80 or x = (.80)(25) = 20. There are 7 successes (S) already, so another 20 – 7 = 13 would be required.

42.

a. $\bar{y} = \frac{\Sigma y_i}{n} = \frac{\Sigma(x_i + c)}{n} = \frac{\Sigma x_i}{n} + \frac{nc}{n} = \bar{x} + c$

$\tilde{y} =$ the median of $(x_1 + c, x_2 + c, ..., x_n + c) =$ median of $(x_1, x_2, ..., x_n) + c = \tilde{x} + c$

b. $\bar{y} = \frac{\Sigma y_i}{n} = \frac{\Sigma(x_i \cdot c)}{n} = \frac{c\Sigma x_i}{n} = c\bar{x}$

$\tilde{y} =$ the median of $(cx_1, cx_2, ..., cx_n) = c \cdot$ the median of $(x_1, x_2, ..., x_n) = c\tilde{x}$

43. The median and certain trimmed means can be calculated, while the mean cannot — the exact values of the “100+” observations are required to calculate the mean. $\tilde{x} = \frac{(57 + 79)}{2} = 68.0$, $\bar{x}_{tr(20)}$ = 66.2, $\bar{x}_{tr(30)}$ = 67.5.

Section 1.4

44.

a. range = 49.3 – 23.5 = 25.8

b.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	x_i^2
29.5	–1.53	2.3409	870.25
49.3	18.27	333.7929	2430.49
30.6	–0.43	0.1849	936.36
28.2	–2.83	8.0089	795.24
28.0	–3.03	9.1809	784.00
26.3	–4.73	22.3729	691.69
33.9	2.87	8.2369	1149.21
29.4	–1.63	2.6569	864.36
23.5	–7.53	56.7009	552.25
31.6	0.57	0.3249	998.56
$\Sigma x_i = 310.3$	$\Sigma(x_i - \bar{x}) = 0$	$\Sigma(x_i - \bar{x})^2 = 443.801$	$\Sigma x_i^2 = 10072.41$

$$\bar{x} = 31.03;\ s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{443.801}{9} = 49.3112$$

c. $s = \sqrt{49.3112} = 7.0222$

d. $s^2 = \dfrac{\Sigma x^2 - (\Sigma x)^2 / n}{n-1} = \dfrac{10072.41 - (310.3)^2 / 10}{9} = 49.3112$

45.

a. $\bar{x}$ = 115.58. The deviations from the mean are 116.4 – 115.58 = .82, 115.9 – 115.58 = .32, 114.6 –115.58 = –.98, 115.2 – 115.58 = –.38, and 115.8 – 115.58 = .22. Notice that the deviations from the mean sum to zero, as they should.

b. s^2 = [(.82)^2 + (.32)^2 + (-.98)^2 + (-.38)^2 + (.22)^2]/(5 – 1) = 1.928/4 = .482, so s = .694.

c. Σx_i^2 = 66795.61, so $s^2 = S_{xx}/(n-1) = \left(\Sigma x_i^2 - (\Sigma x_i)^2 / n\right) / (n-1) =$ (66795.61 –(577.9)^2/5)/4 = 1.928/4 = .482.

d. The new sample values are: 16.4 15.9 14.6 15.2 15.8. While the new mean is 15.58, all the deviations are the same as in part (a), and the variance of the transformed data is identical to that of part (b).

46.

a. From software, $\bar{x}$ = 2887.6 and $\tilde{x}$ = 2888.

b. Subtracting a constant from each observation shifts the data, but does not change its sample variance. For example, by subtracting 2700 from each observation we get the values 81, 200, 313, 156, and 188, which are smaller (fewer digits) and easier to work with. The sum of squares of this transformed data is 204210 and their sum is 938, so the computational formula for the variance gives $s^2 = (204210 - (938)^2/5)/(5 - 1) = 7060.3$.

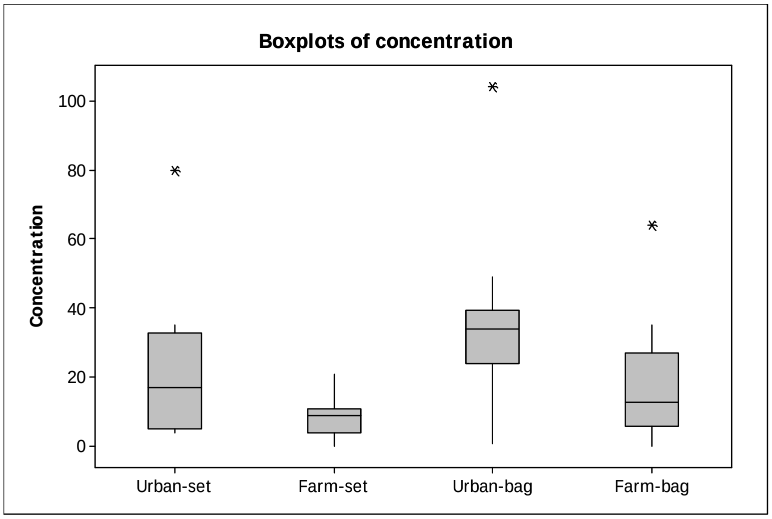
47. From software, $\tilde{x}$ = 109.5 MPa, $\bar{x} = 116.2$ MPa, and s = 25.75 MPa. Half the fracture strength measurements in the sample are below 109.5 MPa, and half are above. On average, we would expect a fracture strength of 116.2 MPa. In general, the size of a typical deviation from the sample mean (116.2) is about 25.75 MPa. Some observations may deviate from 116.2 by more than this and some by less.

48.

a. Using the sums provided for urban homes, $S_{xx} = 10{,}079 - (237.0)^2/11 = 4972.73$, so $s = \sqrt{\dfrac{4972.73}{11-1}}$ = 22.3 EU/mg. Similarly for farm homes, S_{xx} = 518.836 and s = 6.09 EU/mg. The endotoxin concentration in an urban home "typically" deviates from the average of 21.55 by about 22.3 EU/mg. The endotoxin concentration in a farm home "typically" deviates from the average of 8.56 by about 6.09 EU/mg. (These interpretations are very loose, especially since the distributions are not symmetric.) In any case, the variability in endotoxin concentration is far greater in urban homes than in farm homes.

b. The upper and lower fourths of the urban data are 28.0 and 5.5, respectively, for a fourth spread of 22.5 EU/mg. The upper and lower fourths of the farm data are 10.1 and 4, respectively, for a fourth spread of 6.1 EU/mg. Again, we see that the variability in endotoxin concentration is much greater for urban homes than for farm homes.

c. Consider the box plots below. The endotoxin concentration in urban homes generally exceeds that in farm homes, whether measured by settled dust or bag dust. The endotoxin concentration in bag dust generally exceeds that of settled dust, both in urban homes and in farm homes. Settled dust in farm homes shows far less variability than any other scenario.



49.

a. $\Sigma x_i = 2.75 + \cdots + 3.01 = 56.80$, $\Sigma x_i^2 = 2.75^2 + \cdots + 3.01^2 = 197.8040$

b. $s^2 = \dfrac{197.8040 - (56.80)^2/17}{16} = \dfrac{8.0252}{16} = .5016,\quad s = .708$

50. From software or from the sums provided, $\bar{x}$ = 20179/27 = 747.37 and $s = \sqrt{\dfrac{24657511 - (20179)^2/27}{26}} = 606.89$. The maximum award should be $\bar{x} + 2s$ = 747.37 + 2(606.89) = 1961.16, or $1,961,160. This is quite a bit less than the $3.5 million that was awarded originally.

51.

a. From software, $s^2 = 1264.77 \text{ min}^2$ and $s = 35.56$ min. Working by hand, $\Sigma x = 2563$ and $\Sigma x^2 = 368501$, so

$$s^2 = \frac{368501 - (2563)^2/19}{19-1} = 1264.766 \text{ and } s = \sqrt{1264.766} = 35.564$$

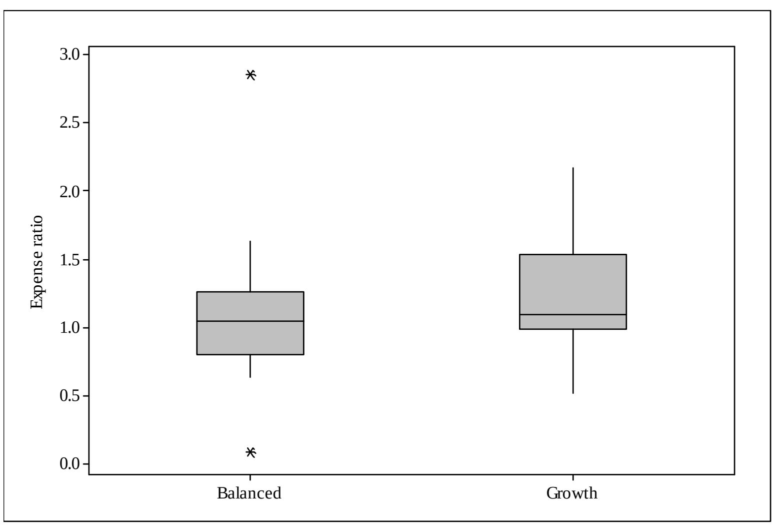
b. If y = time in hours, then $y = cx$ where $c = \frac{1}{60}$. So, $s_y^2 = c^2 s_x^2 = \left(\frac{1}{60}\right)^2 1264.766 = .351\,\text{hr}^2$ and $s_y = cs_x = \left(\frac{1}{60}\right)35.564 = .593\,\text{hr}$.

52. Let d denote the fifth deviation. Then $.3 + .9 + 1.0 + 1.3 + d = 0$ or $3.5 + d = 0$, so $d = -3.5$. One sample for which these are the deviations is $x_1 = 3.8$, $x_2 = 4.4$, $x_3 = 4.5$, $x_4 = 4.8$, $x_5 = 0$. (These were obtained by adding 3.5 to each deviation; adding any other number will produce a different sample with the desired property.)

53.

a. Using software, for the sample of balanced funds we have $\bar{x} = 1.121, \tilde{x} = 1.050, s = 0.536$; for the sample of growth funds we have $\bar{x} = 1.244, \tilde{x} = 1.100, s = 0.448$.

b. The distribution of expense ratios for this sample of balanced funds is fairly symmetric, while the distribution for growth funds is positively skewed. These balanced and growth mutual funds have similar median expense ratios (1.05% and 1.10%, respectively), but expense ratios are generally higher for growth funds. The lone exception is a balanced fund with a 2.86% expense ratio. (There is also one unusually low expense ratio in the sample of balanced funds, at 0.09%.)



54.

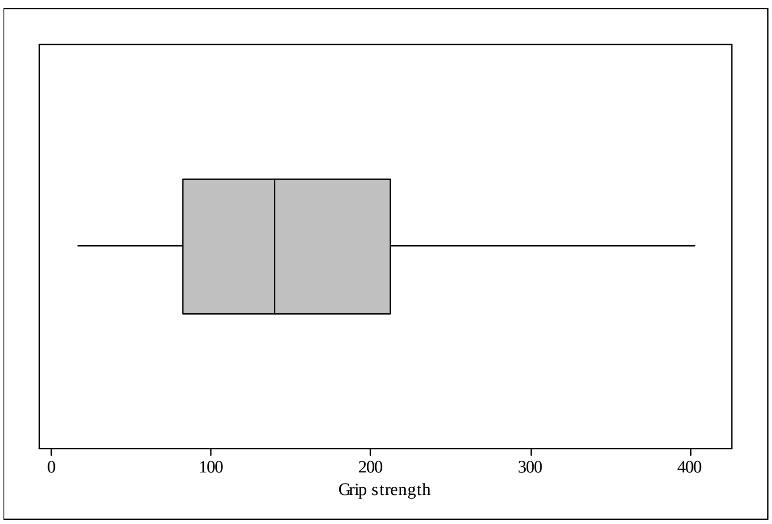
a. Minitab provides the stem-and-leaf display below. Grip strengths for this sample of 42 individuals are positively skewed, and there is one high outlier at 403 N.

```
 6    0  111234
 14   0  55668999
(10)  1  0011223444           Stem = 100s
 18   1  567889               Leaf = 10s
 12   2  01223334
 4    2  59
 2    3  2
 1    3
 1    4  0
```

b. Each half has 21 observations. The lower fourth is the 11th observation, 87 N. The upper fourth is the 32nd observation (11th from the top), 210 N. The fourth spread is the difference: $f_s = 210 - 87 = 123$ N.

c. min = 16; lower fourth = 87; median = 140; upper fourth = 210; max = 403

The boxplot tells a similar story: grip strengths are slightly positively skewed, with a median of 140N and a fourth spread of 123 N.



d. inner fences: 87 – 1.5(123) = –97.5, 210 + 1.5(123) = 394.5
outer fences: 87 – 3(123) = –282, 210 + 3(123) = 579
Grip strength can't be negative, so low outliers are impossible here. A mild high outlier is above 394.5 N and an extreme high outlier is above 579 N. The value 403 N is a mild outlier by this criterion. (Note: some software uses slightly different rules to define outliers — using quartiles and interquartile range — which result in 403 N not being classified as an outlier.)

e. The fourth spread is unaffected unless that observation drops below the current upper fourth, 210. That's a decrease of 403 – 210 = 193 N.

55.

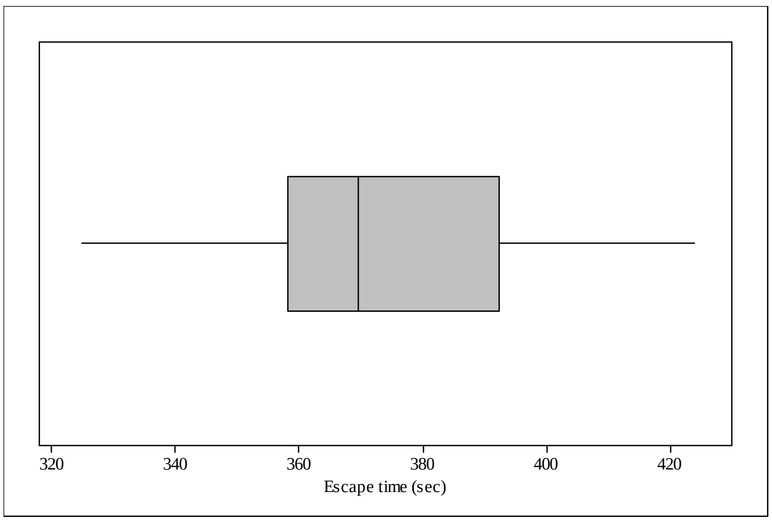
a. Lower half of the data set: 325 325 334 339 356 356 359 359 363 364 364 366 369, whose median, and therefore the lower fourth, is 359 (the 7th observation in the sorted list).

Upper half of the data set: 370 373 373 374 375 389 392 393 394 397 402 403 424, whose median, and therefore the upper fourth is 392.

So, f_s = 392 – 359 = 33.

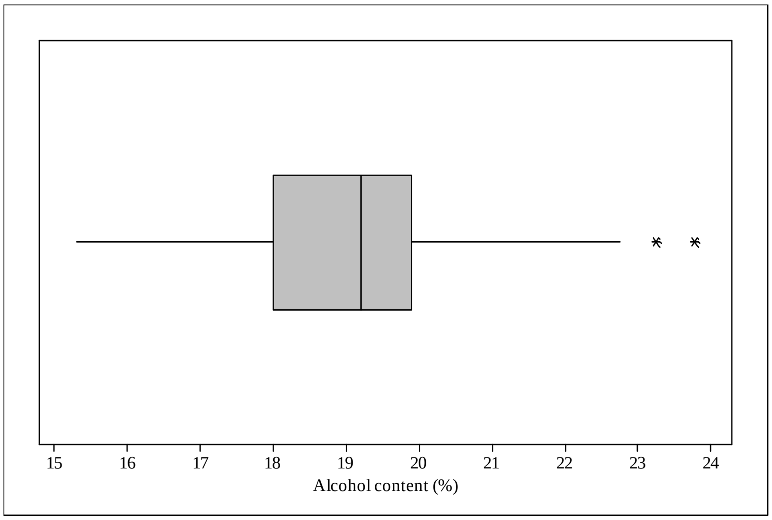
b. inner fences: 359 – 1.5(33) = 309.5, 392 + 1.5(33) = 441.5
To be a mild outlier, an observation must be below 309.5 or above 441.5. There are none in this data set. Clearly, then, there are also no extreme outliers.

c. A boxplot of this data appears below. The distribution of escape times is roughly symmetric with no outliers. Notice the box plot "hides" the fact that the distribution contains two gaps, which can be seen in the stem-and-leaf display.



d. Not until the value x = 424 is lowered below the upper fourth value of 392 would there be any change in the value of the upper fourth (and, thus, of the fourth spread). That is, the value x = 424 could not be decreased by more than 424 – 392 = 32 seconds.

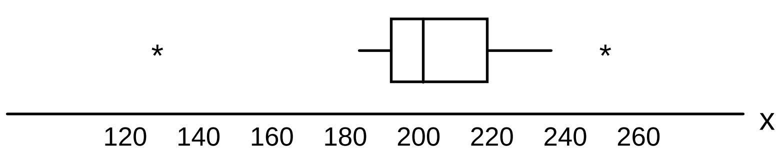
56. The alcohol content distribution of this sample of 35 port wines is roughly symmetric except for two high outliers. The median alcohol content is 19.2% and the fourth spread is 1.42%. [upper fourth = (19.90 + 19.62)/2 = 19.76; lower fourth = (18.00 + 18.68)/2 = 18.34] The two outliers were 23.25% and 23.78%, indicating two port wines with unusually high alcohol content.



57.

a. f_s = 216.8 – 196.0 = 20.8
inner fences: 196 – 1.5(20.8) = 164.6, 216.8 + 1.5(20.8) = 248
outer fences: 196 – 3(20.8) = 133.6, 216.8 + 3(20.8) = 279.2
Of the observations listed, 125.8 is an extreme low outlier and 250.2 is a mild high outlier.

b. A boxplot of this data appears below. There is a bit of positive skew to the data but, except for the two outliers identified in part (a), the variation in the data is relatively small.



58. The most noticeable feature of the comparative boxplots is that machine 2's sample values have considerably more variation than does machine 1's sample values. However, a typical value, as measured by the median, seems to be about the same for the two machines. The only outlier that exists is from machine 1.

59.

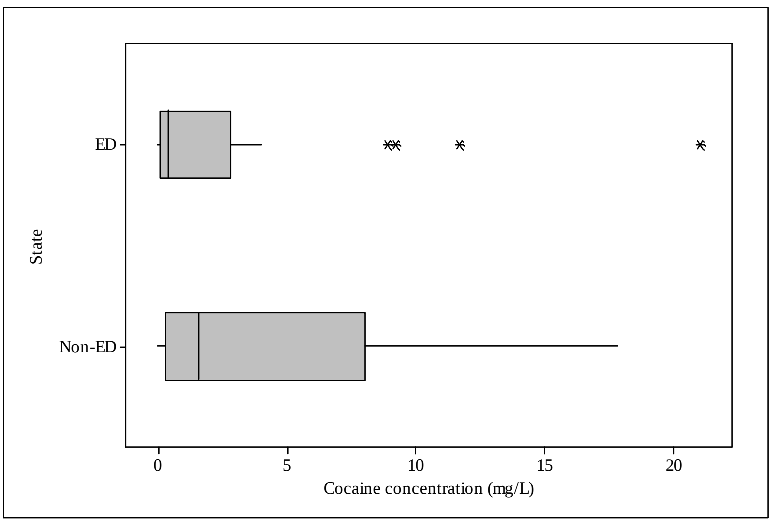
a. If you aren't using software, don't forget to *sort* the data first!
ED: median = .4, lower fourth = (.1 + .1)/2 = .1, upper fourth = (2.7 + 2.8)/2 = 2.75, fourth spread = 2.75 – .1 = 2.65

Non-ED: median = (1.5 + 1.7)/2 = 1.6, lower fourth = .3, upper fourth = 7.9, fourth spread = 7.9 – .3 = 7.6.

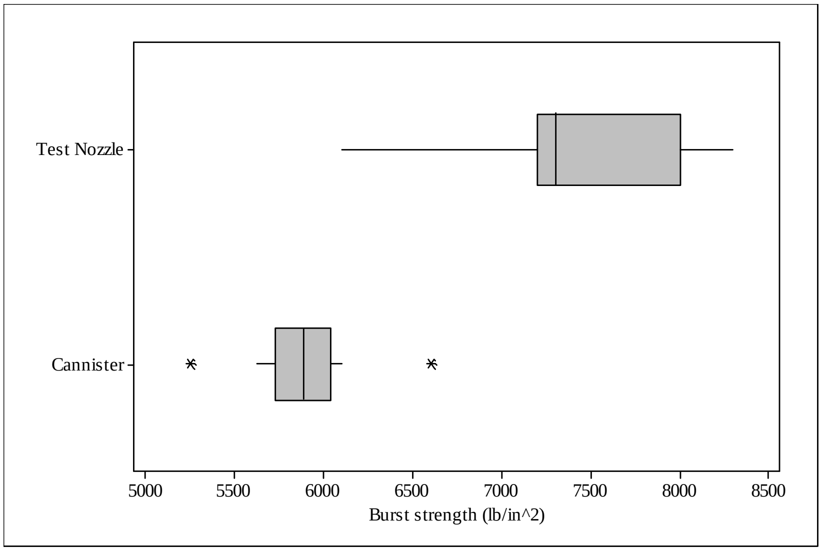
b. *ED*: mild outliers are less than .1 – 1.5(2.65) = –3.875 or greater than 2.75 + 1.5(2.65) = 6.725. Extreme outliers are less than .1 – 3(2.65) = –7.85 or greater than 2.75 + 3(2.65) = 10.7. So, the two largest observations (11.7, 21.0) are extreme outliers and the next two largest values (8.9, 9.2) are mild outliers. There are no outliers at the lower end of the data.

Non-ED: mild outliers are less than .3 – 1.5(7.6) = –11.1 or greater than 7.9 + 1.5(7.6) = 19.3. Note that there are no mild outliers in the data, hence there cannot be any extreme outliers, either.

c. A comparative boxplot appears below. The outliers in the ED data are clearly visible. There is noticeable positive skewness in both samples; the Non-ED sample has more variability then the Ed sample; the typical values of the ED sample tend to be smaller than those for the Non-ED sample.



60. A comparative boxplot (created in Minitab) of this data appears below. The burst strengths for the test nozzle closure welds are quite different from the burst strengths of the production canister nozzle welds. The test welds have much higher burst strengths and the burst strengths are much more variable. The production welds have more consistent burst strength and are consistently lower than the test welds. The production welds data does contain 2 outliers.



61. Outliers occur in the 6a.m. data. The distributions at the other times are fairly symmetric. Variability and the "typical" gasoline-vapor coefficient values increase somewhat until 2p.m., then decrease slightly.

Supplementary Exercises

62. To simplify the math, subtract the mean from each observation; i.e., let $y_i = x_i - \overline{x} = x_i - 76831$. Then y_1 = 76683 – 76831 = –148 and y_4 = 77048 – 76831 = 217; by rescaling, $\overline{y} = \overline{x} - 76831 = 0$, so $y_2 + y_3 = -(y_1 + y_4) = -69$. Also,

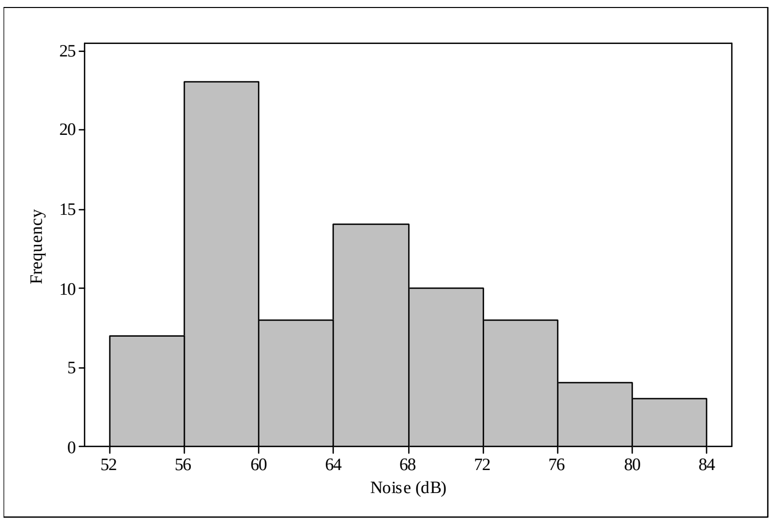
$$180 = s = \sqrt{\frac{\Sigma(x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{\Sigma y_i^2}{3}} \quad \Rightarrow \quad \Sigma y_i^2 = 3(180)^2 = 97200$$

so $y_2^2 + y_3^2 = 97200 - (y_1^2 + y_4^2) = 97200 - ((-148)^2 + (217)^2) = 28207$.

To solve the equations $y_2 + y_3 = -69$ and $y_2^2 + y_3^2 = 28207$, substitute $y_3 = -69 - y_2$ into the second equation and use the quadratic formula to solve. This gives y_2 = 79.14 or –148.14 (one is y_2 and one is y_3).

Finally, x_2 and x_3 are given by y_2 + 76831 and y_3 + 76831, or 79,610 and 76,683.

63. As seen in the histogram below, this noise distribution is bimodal (but close to unimodal) with a positive skew and no outliers. The mean noise level is 64.89 dB and the median noise level is 64.7 dB. The fourth spread of the noise measurements is about 70.4 – 57.8 = 12.6 dB.

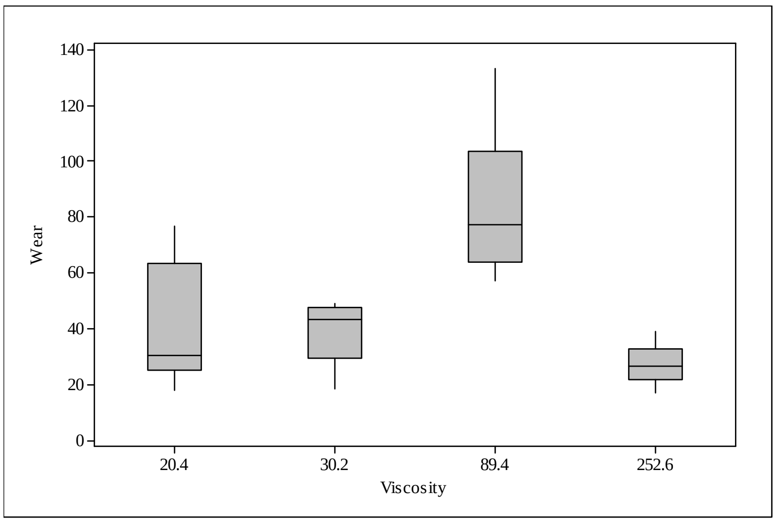


64.

a. The sample coefficient of variation is similar for the three highest oil viscosity levels (29.66, 32.30, 27.86) but is much higher for the lowest viscosity (56.01). At low viscosity, it appears that there is much more variation in volume wear relative to the average or "typical" amount of wear.

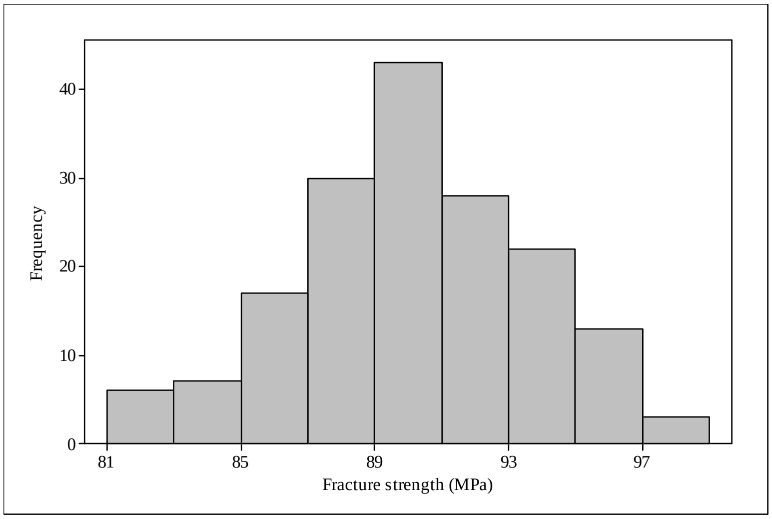
	Wear		
Viscosity	$\bar{x}$	s	cv
20.4	40.17	22.50	56.01
30.2	38.83	11.52	29.66
89.4	84.10	27.20	32.30
252.6	27.10	7.55	27.86

b. Volume wear varies dramatically by viscosity level. At very high viscosity, wear is typically the least and the least variable. Volume wear is actually by far the highest at a "medium" viscosity level and also has the greatest variability at this viscosity level. "Lower" viscosity levels correspond to less wear than a medium level, though there is much greater (relative) variation at a very low viscosity level.



65.

a. The histogram appears below. A representative value for this data would be around 90 MPa. The histogram is reasonably symmetric, unimodal, and somewhat bell-shaped with a fair amount of variability ($s \approx 3$ or 4 MPa).

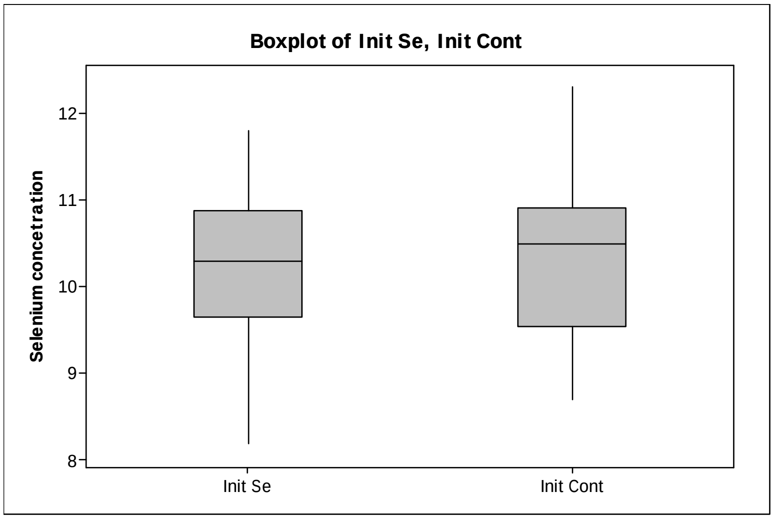


b. The proportion of the observations that are at least 85 is 1 – (6+7)/169 = .9231. The proportion less than 95 is 1 – (13+3)/169 = .9053.

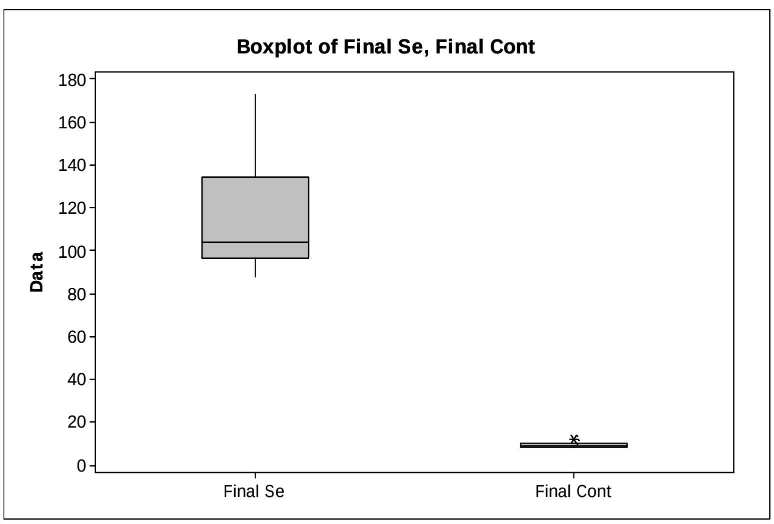
c. 90 is the midpoint of the class 89–<91, which contains 43 observations (a relative frequency of 43/169 = .2544). Therefore about half of this frequency, .1272, should be added to the relative frequencies for the classes to the left of $x = 90$. That is, the approximate proportion of observations that are less than 90 is .0355 + .0414 + .1006 + .1775 + .1272 = .4822.

66.

a. The initial Se concentrations in the treatment and control groups are not that different. The differences in the box plots below are minor. The median initial Se concentrations for the treatment and control groups are 10.3 mg/L and 10.5 mg/L, respectively, each with fourth spread of about 1.25 mg/L. So, the two groups of cows are comparable at the beginning of the study.

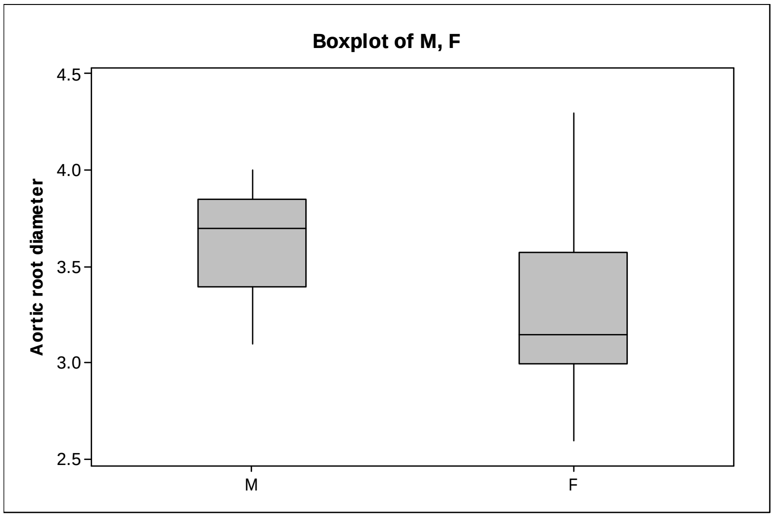


b. The final Se concentrations of the two groups are extremely different, as evidenced by the box plots below. Whereas the median final Se concentration for the control group is 9.3 mg/L (actually slightly lower than the initial concentration), the median Se concentration in the treatment group is now 103.9 mg/L, nearly a 10-fold increase.



67.

a. Aortic root diameters for males have mean 3.64 cm, median 3.70 cm, standard deviation 0.269 cm, and fourth spread 0.40. The corresponding values for females are $\bar{x}$ = 3.28 cm, $\tilde{x}$ = 3.15 cm, s = 0.478 cm, and f_s = 0.50 cm. Aortic root diameters are typically (though not universally) somewhat smaller for females than for males, and females show more variability. The distribution for males is negatively skewed, while the distribution for females is positively skewed (see graphs below).



b. For females (n = 10), the 10% trimmed mean is the average of the middle 8 observations: $\bar{x}_{tr(10)}$ = 3.24 cm. For males (n = 13), the 1/13 trimmed mean is 40.2/11 = 3.6545, and the 2/13 trimmed mean is 32.8/9 = 3.6444. Interpolating, the 10% trimmed mean is $\bar{x}_{tr(10)}$ = 0.7(3.6545) + 0.3(3.6444) = 3.65 cm. (10% is three-tenths of the way from 1/13 to 2/13).

68.

a.
$$\frac{d}{dc}\left\{\sum(x_i-c)^2\right\} = \sum\frac{d}{dc}(x_i-c)^2 = -2\sum(x_i-c) = 0 \Rightarrow \sum(x_i-c) = 0 \Rightarrow$$
$$\sum x_i - \sum c = 0 \Rightarrow \sum x_i - nc = 0 \Rightarrow nc = \sum x_i \Rightarrow c = \frac{\sum x_i}{n} = \bar{x}$$

b. Since $c=\bar{x}$ minimizes $\Sigma(x_i-c)^2$, $\Sigma(x_i-\bar{x})^2 < \Sigma(x_i-\mu)^2$.

69.

a.

$$\bar{y} = \frac{\sum y_i}{n} = \frac{\sum (ax_i + b)}{n} = \frac{a\sum x_i + \sum b}{n} = \frac{a\sum x_i + nb}{n} = a\bar{x} + b$$

$$s_y^2 = \frac{\sum (y_i - \bar{y})^2}{n-1} = \frac{\sum (ax_i + b - (a\bar{x} + b))^2}{n-1} = \frac{\sum (ax_i - a\bar{x})^2}{n-1}$$

$$= \frac{a^2 \sum (x_i - \bar{x})^2}{n-1} = a^2 s_x^2$$

b.

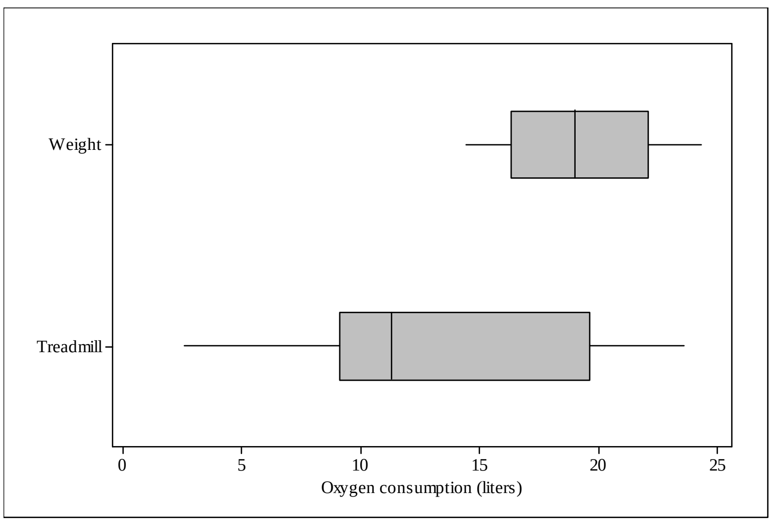
$$x = {}^\circ\text{C}, y = {}^\circ\text{F}$$

$$\bar{y} = \frac{9}{5}(87.3) + 32 = 189.14^\circ\text{F}$$

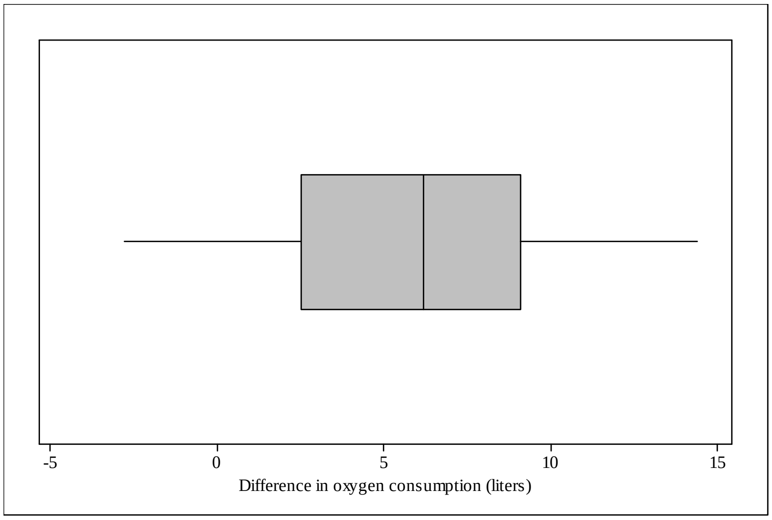
$$s_y = \sqrt{s_y^2} = \sqrt{\left(\frac{9}{5}\right)^2 (1.04)^2} = \sqrt{3.5044} = 1.872^\circ\text{F}$$

70.

a. There is a significant difference in the variability of the two samples. The weight training produced much higher oxygen consumption, on average, than the treadmill exercise, with the median consumptions being approximately 20 and 11 liters, respectively.



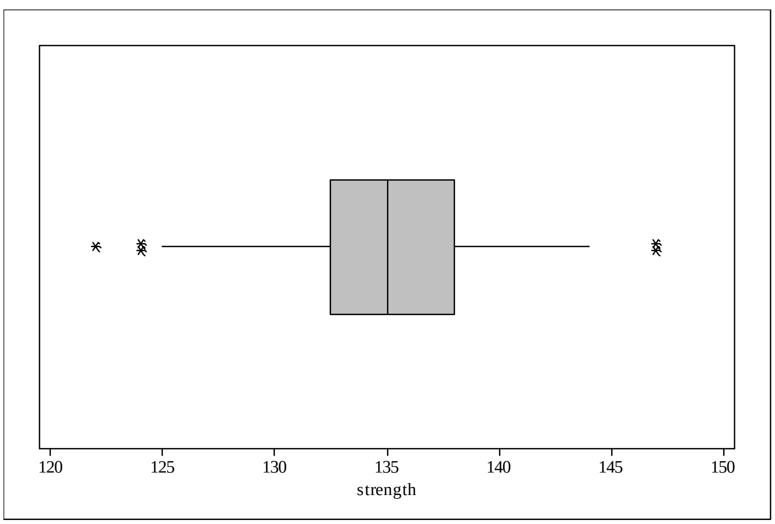
b. The differences in oxygen consumption (weight minus treadmill) for the 15 subjects are 3.3, 9.1, 10.4, 9.1, 6.2, 2.5, 2.2, 8.4, 8.7, 14.4, 2.5, –2.8, –0.4, 5.0, and 11.5. The majority of the differences are positive, which suggests that the weight training produced higher oxygen consumption for most subjects than the treadmill did. The median difference is about 6 liters.



71.

a. The mean, median, and trimmed mean are virtually identical, which suggests symmetry. If there are outliers, they are balanced. The range of values is only 25.5, but half of the values are between 132.95 and 138.25.

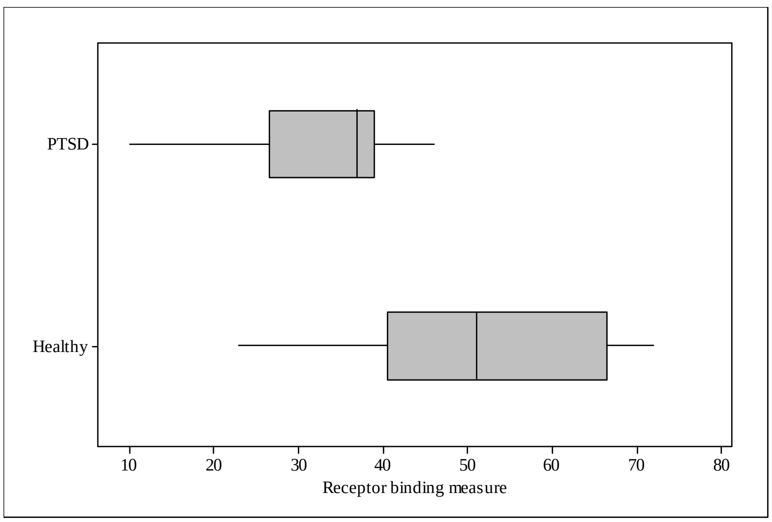
b. See the comments for (a). In addition, using 1.5(Q3 – Q1) as a yardstick, the two largest and three smallest observations are mild outliers.



72. A table of summary statistics, a stem and leaf display, and a comparative boxplot are below. The healthy individuals have higher receptor binding measure, on average, than the individuals with PTSD. There is also more variation in the healthy individuals' values. The distribution of values for the healthy is reasonably symmetric, while the distribution for the PTSD individuals is negatively skewed. The box plot indicates that there are no outliers, and confirms the above comments regarding symmetry and skewness.

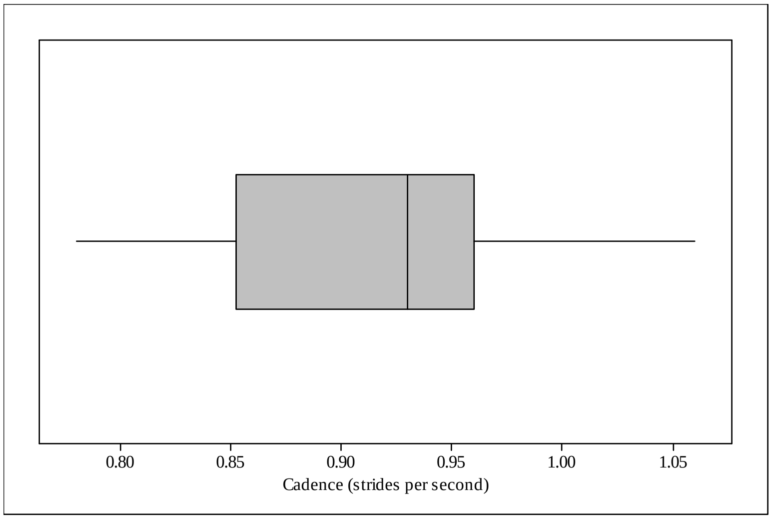
	PTSD	Healthy
Mean	32.92	52.23
Median	37	51
Std Dev	9.93	14.86
Min	10	23
Max	46	72

Healthy		PTSD	
	1	0	stem = tens
3	2	058	leaf = ones
9	3	1578899	
7310	4	26	
81	5		
9763	6		
2	7		



73. From software, $\bar{x}$ = .9255, s = .0809; $\tilde{x}$ = .93, f_s = .1. The cadence observations are slightly skewed (mean = .9255 strides/sec, median = .93 strides/sec) and show a small amount of variability (standard deviation = .0809, fourth spread = .1). There are no apparent outliers in the data.

7	8	stem = tenths
8	11556	leaf = hundredths
9	2233335566	
0	0566	



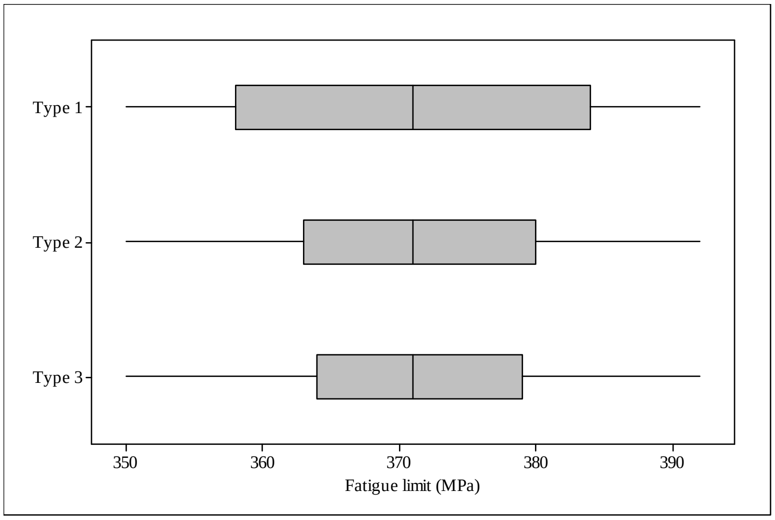
74.

a. The mode is .93. It occurs four times in the data set.

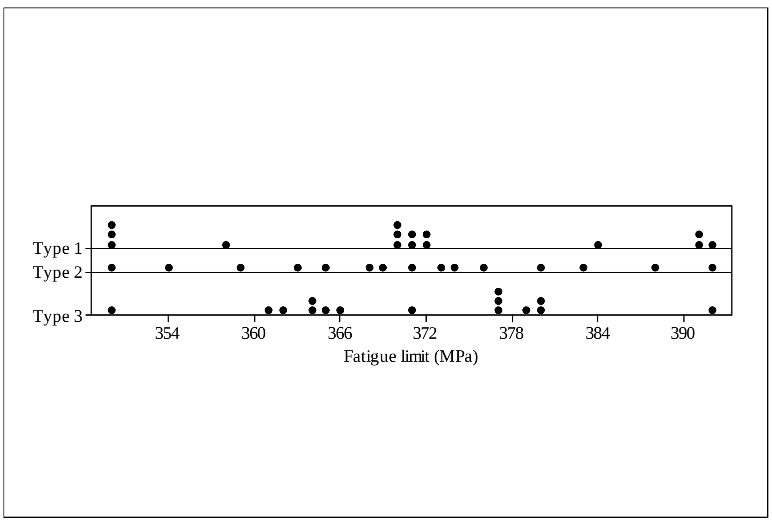
b. The *modal category* is the one in which the most observations occur; i.e., the modal category has the highest frequency. In a survey, the modal category is the most popular answer.

75.

a. The median is the same (371) in each plot and all three data sets are very symmetric. In addition, all three have the same minimum value (350) and same maximum value (392). Moreover, all three data sets have the same lower (364) and upper quartiles (378). So, all three boxplots will be *identical*. (Slight differences in the boxplots below are due to the way Minitab software interpolates to calculate the quartiles.)



b. A comparative dotplot is shown below. These graphs show that there are differences in the variability of the three data sets. They also show differences in the way the values are distributed in the three data sets, especially big differences in the presence of gaps and clusters.



c. The boxplot in (a) is not capable of detecting the differences among the data sets. The primary reason is that boxplots give up some detail in describing data because they use only five summary numbers for comparing data sets.

76. The measures that are sensitive to outliers are: the mean and the midrange. The mean is sensitive because all values are used in computing it. The midrange is sensitive because it uses only the most extreme values in its computation.

The median, the trimmed mean, and the midfourth are not sensitive to outliers.

The median is the most resistant to outliers because it uses only the middle value (or values) in its computation.

The trimmed mean is somewhat resistant to outliers. The larger the trimming percentage, the more resistant the trimmed mean becomes.

The midfourth, which uses the quartiles, is reasonably resistant to outliers because both quartiles are resistant to outliers.

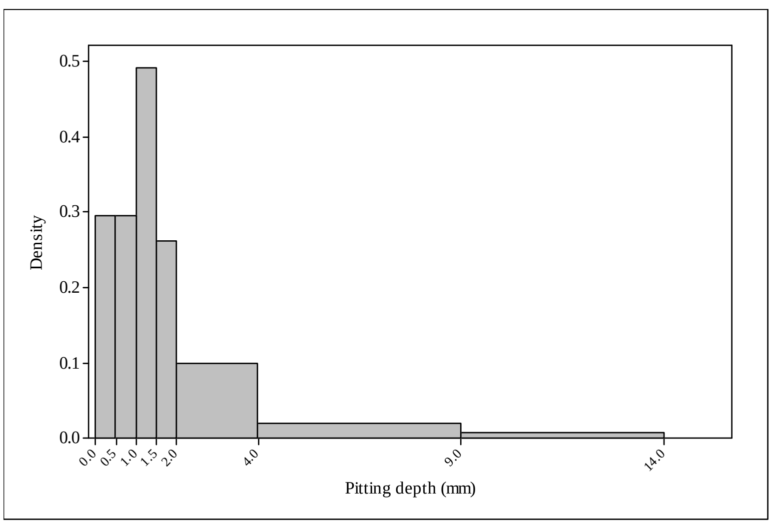
77.

a.

```
0   444444444577888999          leaf = 1.0
1   00011111111124455669999     stem = 0.1
2   1234457
3   11355
4   17
5   3
6
7   67
8   1

HI  10.44, 13.41
```

b. Since the intervals have unequal width, you must use a *density scale*.



c. Representative depths are quite similar for the three types of soils — between 1.5 and 2. Data from the C and CL soils shows much more variability than for the other two types. The boxplots for the first three types show substantial positive skewness both in the middle 50% and overall. The boxplot for the SYCL soil shows negative skewness in the middle 50% and mild positive skewness overall. Finally, there are multiple outliers for the first three types of soils, including extreme outliers.

78.

a. Since the constant $\bar{x}$ is subtracted from each x value to obtain each y value, and addition or subtraction of a constant doesn't affect variability, $s_y^2 = s_x^2$ and $s_y = s_x$.

b. Let $c = 1/s$, where s is the sample standard deviation of the x's (and also, by part (a), of the y's). Then $z_i = cy_i \Rightarrow s_z^2 = c^2 s_y^2 = (1/s)^2 s^2 = 1$ and $s_z = 1$. That is, the "standardized" quantities $z_1, \ldots, z_n$ have a sample variance and standard deviation of 1.

79.

a. $\sum_{i=1}^{n+1} x_i = \sum_{i=1}^{n} x_i + x_{n+1} = n\bar{x}_n + x_{n+1}$, so $\bar{x}_{n+1} = \dfrac{1}{n+1}\sum_{i=1}^{n+1} x_i = \dfrac{n\bar{x}_n + x_{n+1}}{n+1}$.

b. In the second line below, we artificially add and subtract $n\bar{x}_n^2$ to create the term needed for the sample variance:

$$ns_{n+1}^2 = \sum_{i=1}^{n+1}(x_i - \bar{x}_{n+1})^2 = \sum_{i=1}^{n+1} x_i^2 - (n+1)\bar{x}_{n+1}^2$$

$$= \sum_{i=1}^{n} x_i^2 + x_{n+1}^2 - (n+1)\bar{x}_{n+1}^2 = \left[\sum_{i=1}^{n} x_i^2 - n\bar{x}_n^2\right] + n\bar{x}_n^2 + x_{n+1}^2 - (n+1)\bar{x}_{n+1}^2$$

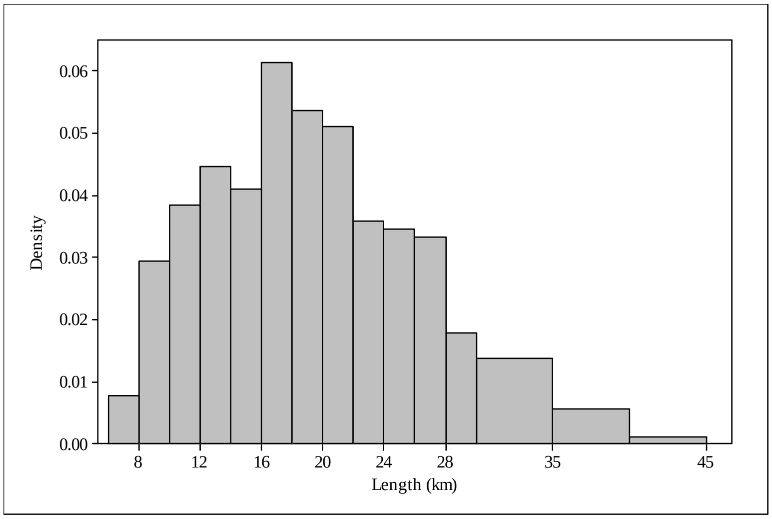
$$= (n-1)s_n^2 + \left\{x_{n+1}^2 + n\bar{x}_n^2 - (n+1)\bar{x}_{n+1}^2\right\}$$

Substitute the expression for $\bar{x}_{n+1}$ from part (a) into the expression in braces, and it simplifies to $\dfrac{n}{n+1}(x_{n+1} - \bar{x}_n)^2$, as desired.

c. First, $\bar{x}_{16} = \dfrac{15(12.58) + 11.8}{16} = \dfrac{200.5}{16} = 12.53$. Then, solving (b) for s_{n+1}^2 gives $s_{n+1}^2 = \dfrac{n-1}{n}s_n^2 + \dfrac{1}{n+1}(x_{n+1} - \bar{x}_n)^2 = \dfrac{14}{15}(.512)^2 + \dfrac{1}{16}(11.8 - 12.58)^2 = .238$. Finally, the standard deviation is $s_{16} = \sqrt{.238} = .532$.

80.

a.



b. There are 391 observations. The proportion of route lengths less than 20 km is (6 + 23 + … + 42)/391 = 216/391 = .552. The proportion of route lengths at least 30 km is (27 + 11 + 2)/391 = 40/391 = .102.

c. First compute (.90)(391 + 1) = 352.8. Thus, the 90th percentile should be about the 352nd ordered value. The 352nd ordered value is the first value in the interval 30–<35. We do not know how the values in that interval are distributed, however, the smallest value (i.e., the 352nd value in the data set) cannot be smaller than 30. So, the 90th percentile is roughly 30.

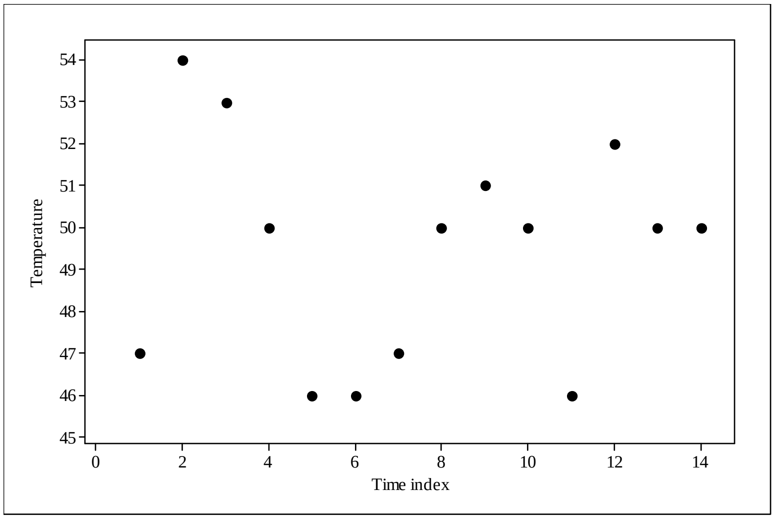
d. First compute (.50)(391 + 1) = 196. Thus the median (50th percentile) should be the 196th ordered value. The 196th observation lies in the interval 18–<20, which includes observations #175 to #216. The 196th observation is about in the middle of these. Thus, we would say the median is roughly 19.

81. Assuming that the histogram is unimodal, then there is evidence of positive skewness in the data since the median lies to the left of the mean (for a symmetric distribution, the mean and median would coincide).

For more evidence of skewness, compare the distances of the 5th and 95th percentiles from the median: median – 5th %ile = 500 – 400 = 100, while 95th %ile – median = 720 – 500 = 220. Thus, the largest 5% of the values (above the 95th percentile) are further from the median than are the lowest 5%. The same skewness is evident when comparing the 10th and 90th percentiles to the median, or comparing the maximum and minimum to the median.

82.

a. There is some evidence of a cyclical pattern.



b. A complete listing of the smoothed values appears below. To illustrate, with α = .1 we have $\bar{x}_2 = .1x_2 + .9\bar{x}_1 = (.1)(54) + (.9)(47) = 47.7$, $\bar{x}_3 = .1x_3 + .9\bar{x}_2$ = (.1)(.53) + (.9)(47.7) = 48.23 ≈ 48.2, etc. It's clear from the values below that α = .1 gives a smoother series.

t	$\bar{x}_t$ for α = .1	$\bar{x}_t$ for α = .5
1	47.0	47.0
2	47.7	50.5
3	48.2	51.8
4	48.4	50.9
5	48.2	48.4
6	48.0	47.2
7	47.9	47.1
8	48.1	48.6
9	48.4	49.8
10	48.5	49.9
11	48.3	47.9
12	48.6	50.0
13	48.8	50.0
14	48.9	50.0

c. As seen below, $\bar{x}_t$ depends on x_t and all previous values. As k increases, the coefficient on x_{t-k} decreases (further back in time implies less weight).

$$\bar{x}_t = \alpha x_t + (1-\alpha)\bar{x}_{t-1} = \alpha x_t + (1-\alpha)[\alpha x_{t-1} + (1-\alpha)\bar{x}_{t-2}]$$
$$= \alpha x_t + \alpha(1-\alpha)x_{t-1} + (1-\alpha)^2[\alpha x_{t-2} + (1-\alpha)\bar{x}_{t-3}] = \cdots$$
$$= \alpha x_t + \alpha(1-\alpha)x_{t-1} + \alpha(1-\alpha)^2 x_{t-2} + \cdots + \alpha(1-\alpha)^{t-2}x_2 + (1-\alpha)^{t-1}x_1$$

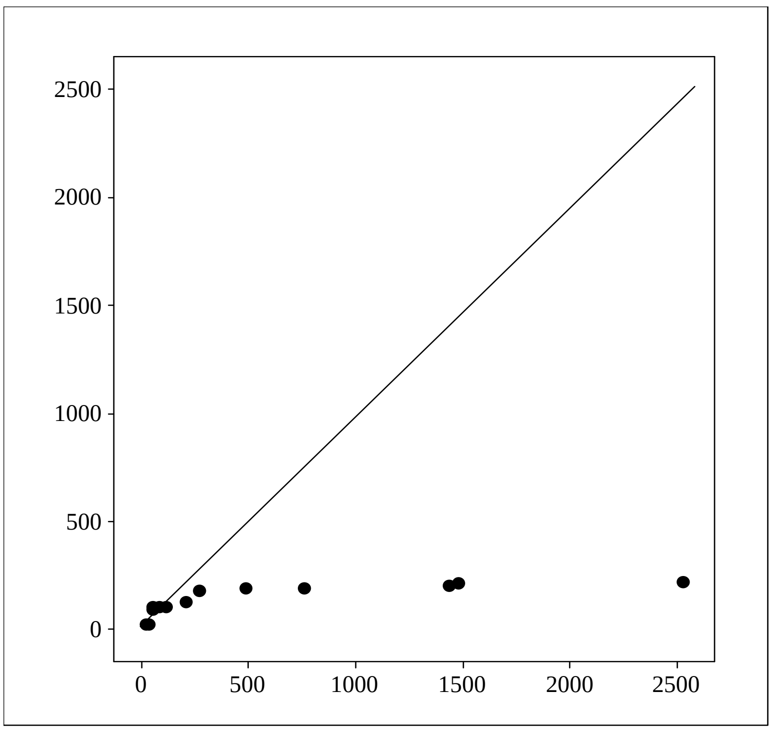
d. For large t, the smoothed series is not very sensitive to the initival value x_1, since the coefficient $(1-\alpha)^{t-1}$ will be very small.

83.

a. When there is perfect symmetry, the smallest observation y_1 and the largest observation y_n will be equidistant from the median, so $y_n - \tilde{x} = \tilde{x} - y_1$. Similarly, the second-smallest and second-largest will be equidistant from the median, so $y_{n-1} - \tilde{x} = \tilde{x} - y_2$, and so on. Thus, the first and second numbers in each pair will be equal, so that each point in the plot will fall exactly on the 45° line.

When the data is positively skewed, y_n will be much further from the median than is y_1, so $y_n - \tilde{x}$ will considerably exceed $\tilde{x} - y_1$ and the point $(y_n - \tilde{x}, \tilde{x} - y_1)$ will fall considerably below the 45° line, as will the other points in the plot.

b. The median of these $n = 26$ observations is 221.6 (the midpoint of the 13th and 14th ordered values). The first point in the plot is (2745.6 – 221.6, 221.6 – 4.1) = (2524.0, 217.5). The others are: (1476.2, 213.9), (1434.4, 204.1), (756.4, 190.2), (481.8, 188.9), (267.5, 181.0), (208.4, 129.2), (112.5, 106.3), (81.2, 103.3), (53.1, 102.6), (53.1, 92.0), (33.4, 23.0), and (20.9, 20.9). The first number in each of the first seven pairs greatly exceeds the second number, so each of those points falls well below the 45° line. A substantial positive skew (stretched upper tail) is indicated.



CHAPTER 2

Section 2.1

1.

a. S = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231}.

b. Event A contains the outcomes where 1 is first in the list:
A = {1324, 1342, 1423, 1432}.

c. Event B contains the outcomes where 2 is first or second:
B = {2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}.

d. The event $A \cup B$ contains the outcomes in A or B or both:
$A \cup B$ = {1324, 1342, 1423, 1432, 2314, 2341, 2413, 2431, 3214, 3241, 4213, 4231}.
$A \cap B = \emptyset$, since 1 and 2 can't both get into the championship game.
$A' = S - A$ = {2314, 2341, 2413, 2431, 3124, 3142, 4123, 4132, 3214, 3241, 4213, 4231}.

2.

a. A = {*RRR*, *LLL*, *SSS*}.

b. B = {*RLS*, *RSL*, *LRS*, *LSR*, *SRL*, *SLR*}.

c. C = {*RRL*, *RRS*, *RLR*, *RSR*, *LRR*, *SRR*}.

d. D = {*RRL*, *RRS*, *RLR*, *RSR*, *LRR*, *SRR*, *LLR*, *LLS*, *LRL*, *LSL*, *RLL*, *SLL*, *SSR*, *SSL*, *SRS*, *SLS*, *RSS*, *LSS*}

e. Event D' contains outcomes where either all cars go the same direction or they all go different directions:
D' = {*RRR*, *LLL*, *SSS*, *RLS*, *RSL*, *LRS*, *LSR*, *SRL*, *SLR*}.
Because event D totally encloses event C (see the lists above), the compound event $C \cup D$ is just event D:
$C \cup D = D$ = {*RRL*, *RRS*, *RLR*, *RSR*, *LRR*, *SRR*, *LLR*, *LLS*, *LRL*, *LSL*, *RLL*, *SLL*, *SSR*, *SSL*, *SRS*, *SLS*, *RSS*, *LSS*}.
Using similar reasoning, we see that the compound event $C \cap D$ is just event C:
$C \cap D = C$ = {*RRL*, *RRS*, *RLR*, *RSR*, *LRR*, *SRR*}.

3.

a. $A = \{SSF, SFS, FSS\}$.

b. $B = \{SSS, SSF, SFS, FSS\}$.

c. For event C to occur, the system must have component 1 working (S in the first position), then at least one of the other two components must work (at least one S in the second and third positions): $C = \{SSS, SSF, SFS\}$.

d. $C' = \{SFF, FSS, FSF, FFS, FFF\}$.
$A \cup C = \{SSS, SSF, SFS, FSS\}$.
$A \cap C = \{SSF, SFS\}$.
$B \cup C = \{SSS, SSF, SFS, FSS\}$. Notice that B contains C, so $B \cup C = B$.
$B \cap C = \{SSS\ SSF, SFS\}$. Since B contains C, $B \cap C = C$.

4.

a. The $2^4 = 16$ possible outcomes have been numbered here for later reference.

	Home Mortgage Number			
Outcome	1	2	3	4
1	*F*	*F*	*F*	*F*
2	*F*	*F*	*F*	*V*
3	*F*	*F*	*V*	*F*
4	*F*	*F*	*V*	*V*
5	*F*	*V*	*F*	*F*
6	*F*	*V*	*F*	*V*
7	*F*	*V*	*V*	*F*
8	*F*	*V*	*V*	*V*
9	*V*	*F*	*F*	*F*
10	*V*	*F*	*F*	*V*
11	*V*	*F*	*V*	*F*
12	*V*	*F*	*V*	*V*
13	*V*	*V*	*F*	*F*
14	*V*	*V*	*F*	*V*
15	*V*	*V*	*V*	*F*
16	*V*	*V*	*V*	*V*

b. Outcome numbers 2, 3, 5, 9 above.

c. Outcome numbers 1, 16 above.

d. Outcome numbers 1, 2, 3, 5, 9 above.

e. In words, the union of (c) and (d) is the event that either all of the mortgages are variable, or that at most one of them is variable-rate: outcomes 1, 2, 3, 5, 9, 16. The intersection of (c) and (d) is the event that all of the mortgages are fixed-rate: outcome 1.

f. The union of (b) and (c) is the event that either exactly three are fixed, or that all four are the same: outcomes 1, 2, 3, 5, 9, 16. The intersection of (b) and (c) is the event that exactly three are fixed and all four are the same type. This cannot happen (the events have no outcomes in common), so the intersection of (b) and (c) is $\varnothing$.

5.

a. The $3^3 = 27$ possible outcomes are numbered below for later reference.

Outcome Number	Outcome	Outcome Number	Outcome
1	111	15	223
2	112	16	231
3	113	17	232
4	121	18	233
5	122	19	311
6	123	20	312
7	131	21	313
8	132	22	321
9	133	23	322
10	211	24	323
11	212	25	331
12	213	26	332
13	221	27	333
14	222		

b. Outcome numbers 1, 14, 27 above.

c. Outcome numbers 6, 8, 12, 16, 20, 22 above.

d. Outcome numbers 1, 3, 7, 9, 19, 21, 25, 27 above.

6.

a. S = {123, 124, 125, 213, 214, 215, 13, 14, 15, 23, 24, 25, 3, 4, 5}.

b. A = {3, 4, 5}.

c. B = {125, 215, 15, 25, 5}.

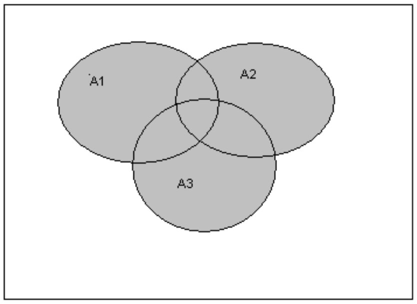
d. C = {23, 24, 25, 3, 4, 5}.

7.

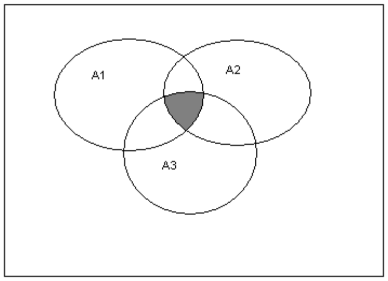
a. *S* = {*BBBAAAA*, *BBABAAA*, *BBAABAA*, *BBAAABA*, *BBAAAAB*, *BABBAAA*, *BABABAA*, *BABAABA*, *BABAAAB*, *BAABBAA*, *BAABABA*, *BAABAAB*, *BAAABBA*, *BAAABAB*, *BAAAABB*, *ABBBAAA*, *ABBABAA*, *ABBAABA*, *ABBAAAB*, *ABABBAA*, *ABABABA*, *ABABAAB*, *ABAABBA*, *ABAABAB*, *ABAAABB*, *AABBBAA*, *AABBABA*, *AABBAAB*, *AABABBA*, *AABABAB*, *AABAABB*, *AAABBBA*, *AAABBAB*, *AAABABB*, *AAAABBB*}.

b. *AAAABBB*, *AAABABB*, *AAABBAB*, *AABAABB*, *AABABAB.*

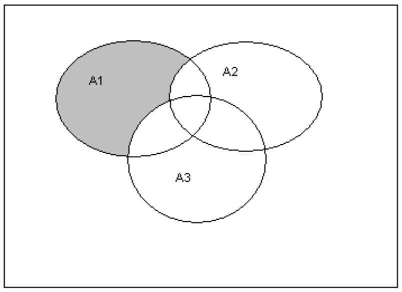
8.



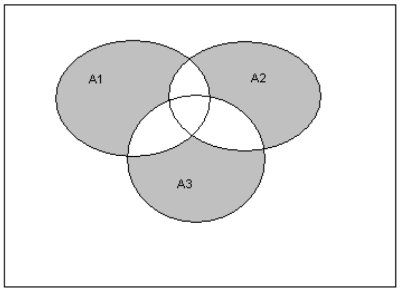
a. $A_1 \cup A_2 \cup A_3$



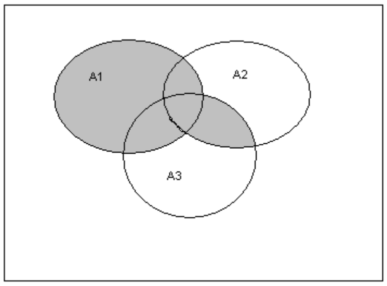
b. $A_1 \cap A_2 \cap A_3$



c. $A_1 \cap A_2' \cap A_3'$



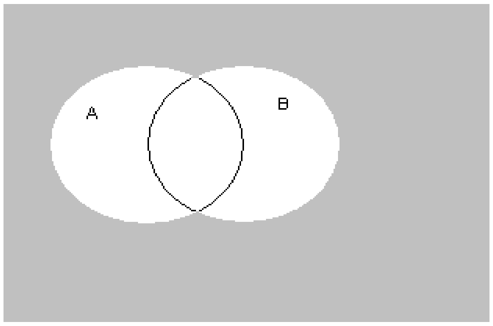
d. $(A_1 \cap A_2' \cap A_3') \cup (A_1' \cap A_2 \cap A_3') \cup (A_1' \cap A_2' \cap A_3)$

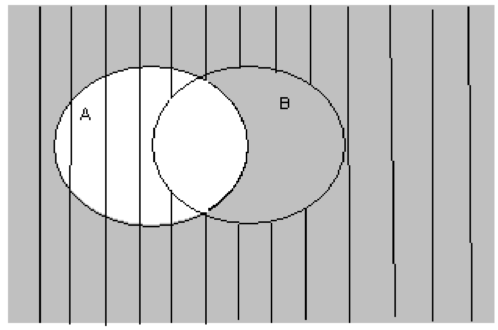


e. $A_1 \cup (A_2 \cap A_3)$

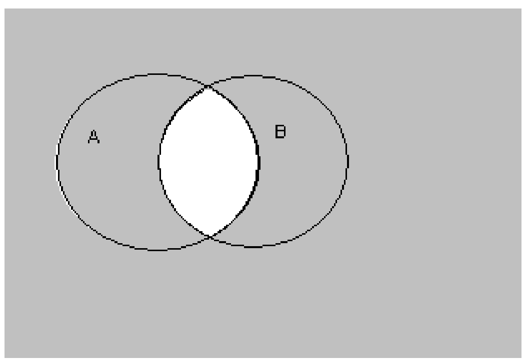
9.

a. In the diagram on the left, the shaded area is $(A \cup B)'$. On the right, the shaded area is A', the striped area is B', and the intersection $A' \cap B'$ occurs where there is both shading <u>and</u> stripes. These two diagrams display the same area.





b. In the diagram below, the shaded area represents $(A \cap B)'$. Using the right-hand diagram from (a), the <u>union</u> of A' and B' is represented by the areas that have either shading <u>or</u> stripes (or both). Both of the diagrams display the same area.



10.

a. Many examples exist; e.g., A = {Chevy, Buick}, B = {Ford, Lincoln}, C = {Toyota} are three mutually exclusive events.

b. No. Let E = {Chevy, Buick}, F = {Buick, Ford}, G = {Toyota}. These events are <u>not</u> mutually exclusive (E and F have an outcome in common), yet there is no outcome common to all three events.

Section 2.2

11.

a. .07.

b. .15 + .10 + .05 = .30.

c. Let A = the selected individual owns shares in a stock fund. Then $P(A) = .18 + .25 = .43$. The desired probability, that a selected customer does not shares in a stock fund, equals $P(A') = 1 - P(A) = 1 - .43 = .57$. This could also be calculated by adding the probabilities for all the funds that are not stocks.

12.

a. $P(A \cup B) = .50 + .40 - .25 = .65$.

b. $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .65 = .35$.

c. The event of interest is $A \cap B'$; from a Venn diagram, we see $P(A \cap B') = P(A) - P(A \cap B) = .50 - .25 = .25$.

13.

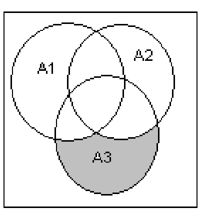
a. $A_1 \cup A_2$ = "awarded either #1 or #2 (or both)": from the addition rule,
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = .22 + .25 - .11 = .36$.

b. $A_1' \cap A_2'$ = "awarded neither #1 or #2": using the hint and part (a),
$P(A_1' \cap A_2') = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 1 - .36 = .64$.

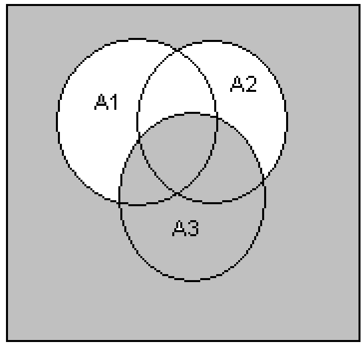
c. $A_1 \cup A_2 \cup A_3$ = "awarded at least one of these three projects": using the addition rule for 3 events,
$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
$.22 + .25 + .28 - .11 - .05 - .07 + .01 = .53$.

d. $A_1' \cap A_2' \cap A_3'$ = "awarded none of the three projects":
$P(A_1' \cap A_2' \cap A_3') = 1 - P(\text{awarded at least one}) = 1 - .53 = .47$.

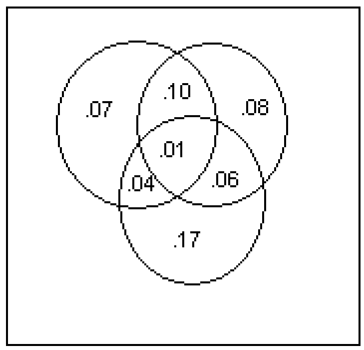
e. $A_1' \cap A_2' \cap A_3$ = "awarded #3 but neither #1 nor #2": from a Venn diagram,
$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) =$
$.28 - .05 - .07 + .01 = .17$. The last term addresses the "double counting" of the two subtractions.



f. $(A_1' \cap A_2') \cup A_3$ = "awarded neither of #1 and #2, or awarded #3": from a Venn diagram, $P((A_1' \cap A_2') \cup A_3)$ = P(none awarded) + $P(A_3)$ = .47 (from **d**) + .28 = 75.



Alternatively, answers to **a-f** can be obtained from probabilities on the accompanying Venn diagram:



14. Let A = an adult consumes coffee and B = an adult consumes carbonated soda. We're told that $P(A) = .55$, $P(B) = .45$, and $P(A \cup B) = .70$.

a. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so $.70 = .55 + .45 - P(A \cap B)$ or $P(A \cap B) = .55 + .45 - .70 = .30$.

b. There are two ways to read this question. We can read "does not (consume at least one)," which means the adult consumes neither beverage. The probability is then P(neither A nor B) = $P(A' \cap B') = 1 - P(A \cup B) = 1 - .70 = .30$.

The other reading, and this is presumably the intent, is "there is at least one beverage the adult does not consume, i.e. $A' \cup B'$. The probability is $P(A' \cup B') = 1 - P(A \cap B) = 1 - .30$ from **a** = .70. (It's just a coincidence this equals $P(A \cup B)$.)

Both of these approaches use *deMorgan's laws*, which say that $P(A' \cap B') = 1 - P(A \cup B)$ and $P(A' \cup B') = 1 - P(A \cap B)$.

15.

a. Let E be the event that at most one purchases an electric dryer. Then E' is the event that at least two purchase electric dryers, and $P(E') = 1 - P(E) = 1 - .428 = .572$.

b. Let A be the event that all five purchase gas, and let B be the event that all five purchase electric. All other possible outcomes are those in which at least one of each type of clothes dryer is purchased. Thus, the desired probability is $1 - [P(A) - P(B)] =$
$1 - [.116 + .005] = .879$.

16.

a. There are six simple events, corresponding to the outcomes *CDP*, *CPD*, *DCP*, *DPC*, *PCD*, and *PDC*. Since the same cola is in every glass, these six outcomes are equally likely to occur, and the probability assigned to each is $\frac{1}{6}$.

b. $P(C \text{ ranked first}) = P(\{CPD, CDP\}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = .333$.

c. $P(C \text{ ranked first and } D \text{ last}) = P(\{CPD\}) = \frac{1}{6}$.

17.

a. The probabilities do not add to 1 because there are other software packages besides SPSS and SAS for which requests could be made.

b. $P(A') = 1 - P(A) = 1 - .30 = .70$.

c. Since A and B are mutually exclusive events, $P(A \cup B) = P(A) + P(B) = .30 + .50 = .80$.

d. By deMorgan's law, $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - .80 = .20$.
In this example, deMorgan's law says the event "neither A nor B" is the complement of the event "either A or B." (That's true regardless of whether they're mutually exclusive.)

18. The only reason we'd need at least two selections to find a 75W bulb is if the <u>first</u> selection was <u>not</u> a 75W bulb. There are 6 + 5 = 11 non-75W bulbs out of 6 + 5 + 4 = 15 bulbs in the box, so the probability of this event is simply $\frac{11}{15}$.

19. Let A be that the selected joint was found defective by inspector A, so $P(A) = \frac{724}{10{,}000}$. Let B be analogous for inspector B, so $P(B) = \frac{751}{10{,}000}$. The event "at least one of the inspectors judged a joint to be defective is $A \cup B$, so $P(A \cup B) = \frac{1159}{10{,}000}$.

a. By deMorgan's law, $P(\text{neither } A \text{ nor } B) = P(A' \cap B') = 1 - \text{P}(\text{A} \cup \text{B}) = 1 - \frac{1159}{10{,}000} = \frac{8841}{10{,}000} = .8841$.

b. The desired event is $B \cap A'$. From a Venn diagram, we see that $P(B \cap A') = P(B) - P(A \cap B)$. From the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ gives $P(A \cap B) = .0724 + .0751 - .1159 = .0316$. Finally, $P(B \cap A') = P(B) - P(A \cap B) = .0751 - .0316 = .0435$.

20.

a. Let S_1, S_2 and S_3 represent day, swing, and night shifts, respectively. Let C_1 and C_2 represent unsafe conditions and unrelated to conditions, respectively. Then the simple events are S_1C_1, S_1C_2, S_2C_1, S_2C_2, S_3C_1, S_3C_2.

b. $P(C_1)$= P({S_1C_1, S_2C_1, S_3C_1})= .10 + .08 + .05 = .23.

c. $P(S_1')$ = 1 – P({S_1C_1, S_1C_2}) = 1 – (.10 + .35) = .55.

21. In what follows, the first letter refers to the auto deductible and the second letter refers to the homeowner's deductible.

a. $P(MH)$ = .10.

b. P(low auto deductible) = P({LN, LL, LM, LH}) = .04 + .06 + .05 + .03 = .18. Following a similar pattern, P(low homeowner's deductible) = .06 + .10 + .03 = .19.

c. P(same deductible for both) = P({LL, MM, HH}) = .06 + .20 + .15 = .41.

d. P(deductibles are different) = 1 – P(same deductible for both) = 1 – .41 = .59.

e. P(at least one low deductible) = P({LN, LL, LM, LH, ML, HL}) = .04 + .06 + .05 + .03 + .10 + .03 = .31.

f. P(neither deductible is low) = 1 – P(at least one low deductible) = 1 – .31 = .69.

22. Let A = motorist must stop at first signal and B = motorist must stop at second signal. We're told that $P(A)$ = .4, $P(B)$ = .5, and $P(A \cup B)$ = .6.

a. From the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, so .6 = .4 + .5 – $P(A \cap B)$, from which $P(A \cap B)$ = .4 + .5 – .6 = .3.

b. From a Venn diagram, $P(A \cap B') = P(A) - P(A \cap B)$ = .4 – .3 = .1.

c. From a Venn diagram, P(stop at exactly one signal) = $P(A \cup B) - P(A \cap B)$ = .6 – .3 = .3. Or, P(stop at exactly one signal) = $P([A \cap B'] \cup [A' \cap B]) = P(A \cap B') + P(A' \cap B) = [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$ = [.4 – .3] + [.5 – .3] = .1 + .2 = .3.

23. Assume that the computers are numbered 1-6 as described and that computers 1 and 2 are the two laptops. There are 15 possible outcomes: (1,2) (1,3) (1,4) (1,5) (1,6) (2,3) (2,4) (2,5) (2,6) (3,4) (3,5) (3,6) (4,5) (4,6) and (5,6).

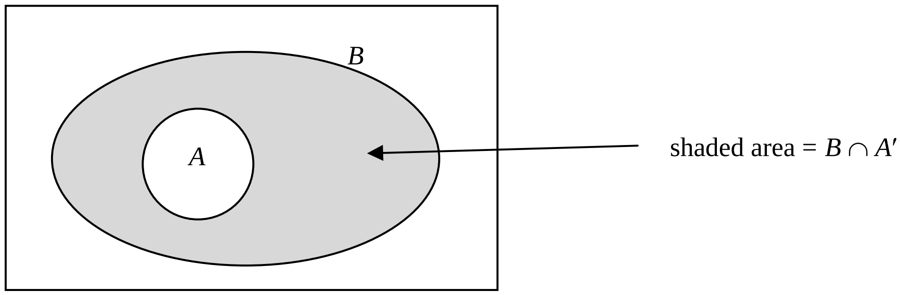
a. P(both are laptops) = P({(1,2)}) = $\frac{1}{15}$ =.067.

b. P(both are desktops) = P({(3,4) (3,5) (3,6) (4,5) (4,6) (5,6)}) = $\frac{6}{15}$ = .40.

c. P(at least one desktop) = 1 – P(no desktops) = 1 – P(both are laptops) = 1 – .067 = .933.

d. P(at least one of each type) = 1 – P(both are the same) = 1 – [P(both are laptops) + P(both are desktops)] = 1 – [.067 + .40] = .533.

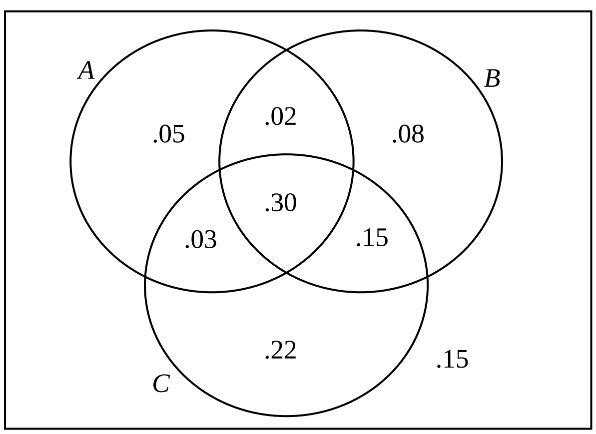
24. Since A is contained in B, we may write $B = A \cup (B \cap A')$, the union of two mutually exclusive events. (See diagram for these two events.) Apply the axioms:
$P(B) = P(A \cup (B \cap A')) = P(A) + P(B \cap A')$ by Axiom 3. Then, since $P(B \cap A') \geq 0$ by Axiom 1, $P(B) = P(A) + P(B \cap A') \geq P(A) + 0 = P(A)$. This proves the statement.



For general events A and B (i.e., not necessarily those in the diagram), it's always the case that $A \cap B$ is contained in A as well as in B, while A and B are both contained in $A \cup B$. Therefore, $P(A \cap B) \leq P(A) \leq P(A \cup B)$ and $P(A \cap B) \leq P(B) \leq P(A \cup B)$.

25. By rearranging the addition rule, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = .40 + .55 - .63 = .32$. By the same method, $P(A \cap C) = .40 + .70 - .77 = .33$ and $P(B \cap C) = .55 + .70 - .80 = .45$. Finally, rearranging the addition rule for 3 events gives
$P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C) = .85 - .40 - .55 - .70 + .32 + .33 + .45 = .30$.

These probabilities are reflected in the Venn diagram below.



a. $P(A \cup B \cup C) = .85$, as given.

b. $P(\text{none selected}) = 1 - P(\text{at least one selected}) = 1 - P(A \cup B \cup C) = 1 - .85 = .15$.

c. From the Venn diagram, $P(\text{only automatic transmission selected}) = .22$.

d. From the Venn diagram, $P(\text{exactly one of the three}) = .05 + .08 + .22 = .35$.

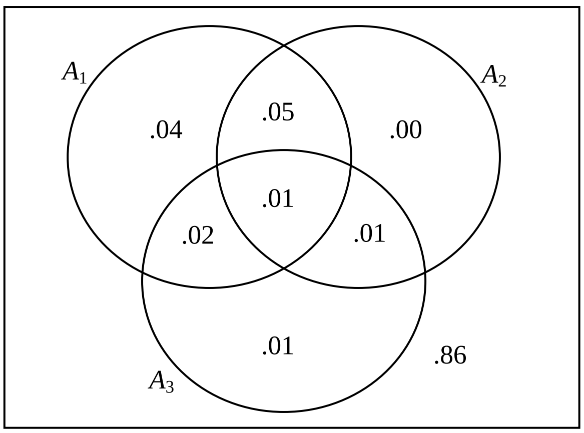
26. These questions can be solved algebraically, or with the Venn diagram below.

a. $P(A_1') = 1 - P(A_1) = 1 - .12 = .88.$

b. The addition rule says $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Solving for the intersection ("and") probability, you get $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .12 + .07 - .13 = .06$.

c. A Venn diagram shows that $P(A \cap B') = P(A) - P(A \cap B)$. Applying that here with $A = A_1 \cap A_2$ and $B = A_3$, you get $P([A_1 \cap A_2] \cap A_3') = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = .06 - .01 = .05$.

d. The event "at most two defects" is the complement of "all three defects," so the answer is just 1 – $P(A_1 \cap A_2 \cap A_3)$ = 1 – .01 = .99.



27. There are 10 equally likely outcomes: {A, B} {A, Co} {A, Cr} {A,F} {B, Co} {B, Cr} {B, F} {Co, Cr} {Co, F} and {Cr, F}.

a. P({A, B}) = $\frac{1}{10}$ = .1.

b. P(at least one C) = P({A, Co} or {A, Cr} or {B, Co} or {B, Cr} or {Co, Cr} or {Co, F} or {Cr, F}) = $\frac{7}{10}$ = .7.

c. Replacing each person with his/her years of experience, P(at least 15 years) = P({3, 14} or {6, 10} or {6, 14} or {7, 10} or {7, 14} or {10, 14}) = $\frac{6}{10}$ = .6.

28. Recall there are 27 equally likely outcomes.

a. P(all the same station) = P((1,1,1) or (2,2,2) or (3,3,3)) = $\frac{3}{27} = \frac{1}{9}$.

b. P(at most 2 are assigned to the same station) = 1 – P(all 3 are the same) = 1 – $\frac{1}{9} = \frac{8}{9}$.

c. P(all different stations) = P((1,2,3) or (1,3,2) or (2,1,3) or (2,3,1) or (3,1,2) or (3,2,1)) = $\frac{6}{27} = \frac{2}{9}$.

Section 2.3

29.

a. There are 26 letters, so allowing repeats there are $(26)(26) = (26)^2 = 676$ possible 2-letter domain names. Add in the 10 digits, and there are 36 characters available, so allowing repeats there are $(36)(36) = (36)^2 = 1296$ possible 2-character domain names.

b. By the same logic as part **a**, the answers are $(26)^3 = 17{,}576$ and $(36)^3 = 46{,}656$.

c. Continuing, $(26)^4 = 456{,}976$; $(36)^4 = 1{,}679{,}616$.

d. P(4-character sequence is already owned) = 1 – P(4-character sequence still available) = 1 – $97{,}786/(36)^4 = .942$.

30.

a. Because order is important, we'll use $P_{3,8} = (8)(7)(6) = 336$.

b. Order doesn't matter here, so we use $\binom{30}{6} = 593{,}775$.

c. The number of ways to choose 2 zinfandels from the 8 available is $\binom{8}{2}$. Similarly, the number of ways to choose the merlots and cabernets are $\binom{10}{2}$ and $\binom{12}{2}$, respectively. Hence, the total number of options (using the Fundamental Counting Principle) equals $\binom{8}{2}\binom{10}{2}\binom{12}{2} = (28)(45)(66) = 83{,}160$.

d. The numerator comes from part **c** and the denominator from part **b**: $\dfrac{83{,}160}{593{,}775} = .140$.

e. We use the same denominator as in part **d**. The number of ways to choose all zinfandel is $\binom{8}{6}$, with similar answers for all merlot and all cabernet. Since these are disjoint events, P(all same) = P(all zin) + P(all merlot) + P(all cab) = $\dfrac{\binom{8}{6}+\binom{10}{6}+\binom{12}{6}}{\binom{30}{6}} = \dfrac{1162}{593{,}775} = .002$.

31.

a. Use the Fundamental Counting Principle: $(9)(27) = 243$.

b. By the same reasoning, there are $(9)(27)(15) = 3645$ such sequences, so such a policy could be carried out for 3645 successive nights, or approximately 10 years, without repeating exactly the same program.

32.

a. Since there are 5 receivers, 4 CD players, 3 speakers, and 4 turntables, the total number of possible selections is (5)(4)(3)(4) = 240.

b. We now only have 1 choice for the receiver and CD player: (1)(1)(3)(4) = 12.

c. Eliminating Sony leaves 4, 3, 3, and 3 choices for the four pieces of equipment, respectively: (4)(3)(3)(3) = 108.

d. From **a**, there are 240 possible configurations. From **c**, 108 of them involve zero Sony products. So, the number of configurations with at least one Sony product is 240 – 108 = 132.

e. Assuming all 240 arrangements are equally likely, $P(\text{at least one Sony}) = \frac{132}{240} = .55$.

Next, P(exactly one component Sony) = P(only the receiver is Sony) + P(only the CD player is Sony) + P(only the turntable is Sony). Counting from the available options gives

$P(\text{exactly one component Sony}) = \dfrac{(1)(3)(3)(3)+(4)(1)(3)(3)+(4)(3)(3)(1)}{240} = \dfrac{99}{240} = .413$.

33.

a. Since there are 15 players and 9 positions, and order matters in a line-up (catcher, pitcher, shortstop, etc. are different positions), the number of possibilities is $P_{9,15}$ = (15)(14)…(7) or 15!/(15–9)! = 1,816,214,440.

b. For each of the starting line-ups in part (a), there are 9! possible batting orders. So, multiply the answer from (a) by 9! to get (1,816,214,440)(362,880) = 659,067,881,472,000.

c. Order still matters: There are $P_{3,5}$ = 60 ways to choose three left-handers for the outfield and $P_{6,10}$ = 151,200 ways to choose six right-handers for the other positions. The total number of possibilities is = (60)(151,200) = 9,072,000.

34.

a. Since order doesn't matter, the number of ways to randomly select 5 keyboards from the 25 available is $\binom{25}{5}$ = 53,130.

b. Sample in two stages. First, there are 6 keyboards with an electrical defect, so the number of ways to select exactly 2 of them is $\binom{6}{2}$. Next, the remaining 5 – 2 = 3 keyboards in the sample must have mechanical defects; as there are 19 such keyboards, the number of ways to randomly select 3 is $\binom{19}{3}$. So, the number of ways to achieve both of these in the sample of 5 is the product of these two counting numbers: $\binom{6}{2}\binom{19}{3}$ = (15)(969) = 14,535.

c. Following the analogy from **b**, the number of samples with exactly 4 mechanical defects is $\binom{19}{4}\binom{6}{1}$, and the number with exactly 5 mechanical defects is $\binom{19}{5}\binom{6}{0}$. So, the number of samples with <u>at least</u> 4 mechanical defects is $\binom{19}{4}\binom{6}{1} + \binom{19}{5}\binom{6}{0}$, and the probability of this event is

$$\frac{\binom{19}{4}\binom{6}{1}+\binom{19}{5}\binom{6}{0}}{\binom{25}{5}} = \frac{34{,}884}{53{,}130} = .657.$$ (The denominator comes from **a**.)

35.

a. Since there are 20 day-shift workers, the number of such samples is $\binom{20}{6}$ = 38,760. With 45 workers total, there are $\binom{45}{6}$ total possible samples. So, the probability of randomly selecting all day-shift workers is $\dfrac{\binom{20}{6}}{\binom{45}{6}} = \dfrac{38{,}760}{8{,}145{,}060}$ = .0048.

b. Following the analogy from **a**, P(all from the same shift) = P(all from day shift) + P(all from swing shift) + P(all from graveyard shift) = $\dfrac{\binom{20}{6}}{\binom{45}{6}} + \dfrac{\binom{15}{6}}{\binom{45}{6}} + \dfrac{\binom{10}{6}}{\binom{45}{6}}$ = .0048 + .0006 + .0000 = .0054.

c. P(at least two shifts represented) = 1 – P(all from same shift) = 1 – .0054 = .9946.

d. There are several ways to approach this question. For example, let A_1 = "day shift is unrepresented," A_2 = "swing shift is unrepresented," and A_3 = "graveyard shift is unrepresented." Then we want $P(A_1 \cup A_2 \cup A_3)$.

$P(A_1)$ = P(day shift unrepresented) = P(all from swing/graveyard) = $\dfrac{\binom{25}{6}}{\binom{45}{6}}$,

since there are 15 + 10 = 25 total employees in the swing and graveyard shifts. Similarly,

$P(A_2) = \dfrac{\binom{30}{6}}{\binom{45}{6}}$ and $P(A_3) = \dfrac{\binom{35}{6}}{\binom{45}{6}}$. Next, $P(A_1 \cap A_2)$ = P(all from graveyard) = $\dfrac{\binom{10}{6}}{\binom{45}{6}}$.

Similarly, $P(A_1 \cap A_3) = \dfrac{\binom{15}{6}}{\binom{45}{6}}$ and $P(A_2 \cap A_3) = \dfrac{\binom{20}{6}}{\binom{45}{6}}$. Finally, $P(A_1 \cap A_2 \cap A_3) = 0$, since at least one shift must be represented. Now, apply the addition rule for 3 events:

$$P(A_1 \cup A_2 \cup A_3) = \frac{\binom{25}{6}}{\binom{45}{6}} + \frac{\binom{30}{6}}{\binom{45}{6}} + \frac{\binom{35}{6}}{\binom{45}{6}} - \frac{\binom{10}{6}}{\binom{45}{6}} - \frac{\binom{15}{6}}{\binom{45}{6}} - \frac{\binom{20}{6}}{\binom{45}{6}} + 0 = .2885.$$

36. There are $\binom{5}{2} = 10$ possible ways to select the positions for *B*'s votes: *BBAAA*, *BABAA*, *BAABA*, *BAAAB*, *ABBAA*, *ABABA*, *ABAAB*, *AABBA*, *AABAB*, and *AAABB*. Only the last two have *A* ahead of *B* throughout the vote count. Since the outcomes are equally likely, the desired probability is 2/10 = .20.

37.

a. By the Fundamental Counting Principle, with $n_1 = 3$, $n_2 = 4$, and $n_3 = 5$, there are (3)(4)(5) = 60 runs.

b. With $n_1 = 1$ (just one temperature), $n_2 = 2$, and $n_3 = 5$, there are (1)(2)(5) = 10 such runs.

c. For each of the 5 specific catalysts, there are (3)(4) = 12 pairings of temperature and pressure. Imagine we separate the 60 possible runs into those 5 sets of 12. The number of ways to select exactly one run from each of these 5 sets of 12 is $\binom{12}{1}^5 = 12^5$.

Since there are $\binom{60}{5}$ ways to select the 5 runs overall, the desired probability is $\dfrac{\binom{12}{1}^5}{\binom{60}{5}} = \dfrac{12^5}{\binom{60}{5}} = .0456$.

38.

a. There are 6 75W bulbs and 9 other bulbs. So, *P*(select exactly 2 75W bulbs) = *P*(select exactly 2 75W bulbs and 1 other bulb) $= \dfrac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \dfrac{(15)(9)}{455} = .2967$.

b. *P*(all three are the same rating) = *P*(all 3 are 40W or all 3 are 60W or all 3 are 75W) =

$$\frac{\binom{4}{3}+\binom{5}{3}+\binom{6}{3}}{\binom{15}{3}} = \frac{4+10+20}{455} = .0747.$$

c. P(one of each type is selected) $= \dfrac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = \dfrac{120}{455} = .2637$.

d. It is necessary to examine at least six bulbs if and only if the first five light bulbs were all of the 40W or 60W variety. Since there are 9 such bulbs, the chance of this event is

$$\frac{\binom{9}{5}}{\binom{15}{5}} = \frac{126}{3003} = .042 .$$

39.

a. We want to choose all of the 5 cordless, and 5 of the 10 others, to be among the first 10 serviced, so the desired probability is $\dfrac{\binom{5}{5}\binom{10}{5}}{\binom{15}{10}} = \dfrac{252}{3003} = .0839$.

b. Isolating one group, say the cordless phones, we want the other two groups (cellular and corded) represented in the last 5 serviced. The number of ways to choose all 5 cordless phones and 5 of the other phones in the first 10 selections is $\binom{5}{5}\binom{10}{5} = \binom{10}{5}$. However, we don't want <u>two</u> types to be eliminated in the first 10 selections, so we must subtract out the ways that either (all cordless and all cellular) or (all cordless and all corded) are selected among the first 10, which is $\binom{5}{5}\binom{5}{5} + \binom{5}{5}\binom{5}{5} = 2$. So, the number of ways to have only cellular and corded phones represented in the last five selections is
$\binom{10}{5} - 2$. We have three types of phones, so the total number of ways to have exactly two types left over is $3 \cdot \left[\binom{10}{5} - 2\right]$, and the probability is $\dfrac{3 \cdot \left[\binom{10}{5} - 2\right]}{\binom{15}{5}} = \dfrac{3(250)}{3003} = .2498$.

c. We want to choose 2 of the 5 cordless, 2 of the 5 cellular, and 2 of the corded phones:

$$\frac{\binom{5}{2}\binom{5}{2}\binom{5}{2}}{\binom{15}{6}} = \frac{1000}{5005} = .1998 .$$

40.

a. If the A's were distinguishable from one another, and similarly for the B's, C's and D's, then there would be 12! possible chain molecules. Six of these are:

$A_1A_2A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $A_1A_3A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_2A_1A_3B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $A_2A_3A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$
$A_3A_1A_2B_2C_3C_1D_3C_2D_1D_2B_3B_1$ $A_3A_2A_1B_2C_3C_1D_3C_2D_1D_2B_3B_1$

These 6 (=3!) differ only with respect to ordering of the 3 A's. In general, groups of 6 chain molecules can be created such that within each group only the ordering of the A's is different. When the A subscripts are suppressed, each group of 6 "collapses" into a single molecule (B's, C's and D's are still distinguishable).
At this point there are (12!/3!) different molecules. Now suppressing subscripts on the B's, C's, and D's in turn gives $\frac{12!}{(3!)^4} = 369,600$ chain molecules.

b. Think of the group of 3 A's as a single entity, and similarly for the B's, C's, and D's. Then there are 4! = 24 ways to order these triplets, and thus 24 molecules in which the A's are contiguous, the B's, C's, and D's also. The desired probability is $\frac{24}{369,600} = .00006494$.

41.

a. (10)(10)(10)(10) = 10^4 = 10,000. These are the strings 0000 through 9999.

b. Count the number of prohibited sequences. There are (i) 10 with all digits identical (0000, 1111, …, 9999); (ii) 14 with sequential digits (0123, 1234, 2345, 3456, 4567, 5678, 6789, and 7890, plus these same seven descending); (iii) 100 beginning with 19 (1900 through 1999). That's a total of 10 + 14 + 100 = 124 impermissible sequences, so there are a total of 10,000 – 124 = 9876 permissible sequences. The chance of randomly selecting one is just $\frac{9876}{10,000}$ = .9876.

c. All PINs of the form 8xx1 are legitimate, so there are (10)(10) = 100 such PINs. With someone randomly selecting 3 such PINs, the chance of guessing the correct sequence is 3/100 = .03.

d. Of all the PINs of the form 1xx1, eleven is prohibited: 1111, and the ten of the form 19x1. That leaves 89 possibilities, so the chances of correctly guessing the PIN in 3 tries is 3/89 = .0337.

42.

a. If Player X sits out, the number of possible teams is $\binom{3}{1}\binom{4}{2}\binom{4}{2}$ = 108. If Player X plays guard, we need one <u>more</u> guard, and the number of possible teams is $\binom{3}{1}\binom{4}{1}\binom{4}{2}$ = 72. Finally, if Player X plays forward, we need one <u>more</u> forward, and the number of possible teams is $\binom{3}{1}\binom{4}{2}\binom{4}{1}$ = 72. So, the total possible number of teams from this group of 12 players is 108 + 72 + 72 = 252.

b. Using the idea in **a**, consider all possible scenarios. If Players X and Y both sit out, the number of possible teams is $\binom{3}{1}\binom{5}{2}\binom{5}{2}$ = 300. If Player X plays while Player Y sits out, the number of possible teams is $\binom{3}{1}\binom{5}{1}\binom{5}{2}+\binom{3}{1}\binom{5}{2}\binom{5}{1}$ = 150 + 150 = 300. Similarly, there are 300 teams with Player X benched and Player Y in. Finally, there are three cases when X and Y both play: they're both guards, they're both forwards, or they split duties. The number of ways to select the rest of the team under these scenarios is $\binom{3}{1}\binom{5}{0}\binom{5}{2}+\binom{3}{1}\binom{5}{2}\binom{5}{0}+\binom{3}{1}\binom{5}{1}\binom{5}{1}$ = 30 + 30 + 75 = 135.

Since there are $\binom{15}{5}$ = 3003 ways to randomly select 5 players from a 15-person roster, the probability of randomly selecting a legitimate team is $\frac{300+300+135}{3003}=\frac{735}{3003}$ = .245.

43. There are $\binom{52}{5}$ = 2,598,960 five-card hands. The number of 10-high straights is (4)(4)(4)(4)(4) = 4^5 = 1024 (any of four 6s, any of four 7s, etc.). So, P(10 high straight) = $\frac{1024}{2,598,960}=.000394$. Next, there ten "types of straight: A2345, 23456, …, 910JQK, 10JQKA. So, P(straight) = $10\times\frac{1024}{2,598,960}=.00394$. Finally, there are only 40 straight flushes: each of the ten sequences above in each of the 4 suits makes (10)(4) = 40. So, P(straight flush) = $\frac{40}{2,598,960}=.00001539$.

44. $\binom{n}{k}=\frac{n!}{k!(n-k)!}=\frac{n!}{(n-k)!k!}=\binom{n}{n-k}$

The number of subsets of size k equals the number of subsets of size $n – k$, because to each subset of size k there corresponds exactly one subset of size $n – k$: the $n – k$ objects not in the subset of size k. The combinations formula counts the number of ways to split n objects into two subsets: one of size k, and one of size $n – k$.

Section 2.4

45.

a. $P(A) = .106 + .141 + .200 = .447$, $P(C) = .215 + .200 + .065 + .020 = .500$, and $P(A \cap C) = .200$.

b. $P(A|C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{.200}{.500} = .400$. If we know that the individual came from ethnic group 3, the probability that he has Type A blood is .40. $P(C|A) = \dfrac{P(A \cap C)}{P(A)} = \dfrac{.200}{.447} = .447$. If a person has Type A blood, the probability that he is from ethnic group 3 is .447.

c. Define D = "ethnic group 1 selected." We are asked for $P(D|B')$. From the table, $P(D \cap B') = .082 + .106 + .004 = .192$ and $P(B') = 1 - P(B) = 1 - [.008 + .018 + .065] = .909$. So, the desired probability is $P(D|B') = \dfrac{P(D \cap B')}{P(B')} = \dfrac{.192}{.909} = .211$.

46. Let A be that the individual is more than 6 feet tall. Let B be that the individual is a professional basketball player. Then $P(A|B)$ = the probability of the individual being more than 6 feet tall, knowing that the individual is a professional basketball player, while $P(B|A)$ = the probability of the individual being a professional basketball player, knowing that the individual is more than 6 feet tall. $P(A|B)$ will be larger. Most professional basketball players are tall, so the probability of an individual in that reduced sample space being more than 6 feet tall is very large. On the other hand, the number of individuals that are pro basketball players is small in relation to the number of males more than 6 feet tall.

47. A Venn diagram appears at the end of this exercise.

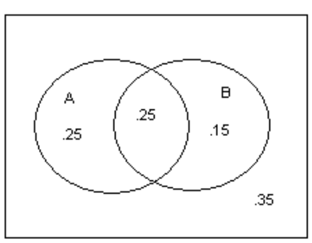
a. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.25}{.50} = .50$.

b. $P(B'|A) = \dfrac{P(A \cap B')}{P(A)} = \dfrac{.25}{.50} = .50$.

c. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.25}{.40} = .6125$.

d. $P(A'|B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{.15}{.40} = .3875$.

e. $P(A|A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A \cup B)} = \dfrac{.50}{.65} = .7692$. It should be clear from the Venn diagram that $A \cap (A \cup B) = A$.



48.

a. $P(A_2 \mid A_1) = \dfrac{P(A_2 \cap A_1)}{P(A_1)} = \dfrac{.06}{.12}$ = .50. The numerator comes from Exercise 26.

b. $P(A_1 \cap A_2 \cap A_3 \mid A_1) = \dfrac{P([A_1 \cap A_2 \cap A_3] \cap A_1)}{P(A_1)} = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \dfrac{.01}{.12}$ = .0833. The numerator simplifies because $A_1 \cap A_2 \cap A_3$ is a subset of A_1, so their intersection is just the smaller event.

c. For this example, you definitely need a Venn diagram. The seven pieces of the partition inside the three circles have probabilities .04, .05, .00, .02, .01, .01, and .01. Those add to .14 (so the chance of no defects is .86).
Let E = "exactly one defect." From the Venn diagram, $P(E)$ = .04 + .00 + .01 = .05. From the addition above, P(at least one defect) = $P(A_1 \cup A_2 \cup A_3)$ = .14. Finally, the answer to the question is
$P(E \mid A_1 \cup A_2 \cup A_3) = \dfrac{P(E \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \dfrac{P(E)}{P(A_1 \cup A_2 \cup A_3)} = \dfrac{.05}{.14}$ = .3571. The numerator simplifies because E is a subset of $A_1 \cup A_2 \cup A_3$.

d. $P(A_3' \mid A_1 \cap A_2) = \dfrac{P(A_3' \cap [A_1 \cap A_2])}{P(A_1 \cap A_2)} = \dfrac{.05}{.06}$ = .8333. The numerator is Exercise 26(c), while the denominator is Exercise 26(b).

49.

a. P(small cup) = .14 + .20 = .34. P(decaf) = .20 + .10 + .10 = .40.

b. P(decaf | small) = $\dfrac{P(\text{small} \cap \text{decaf})}{P(\text{small})} = \dfrac{.20}{.34}$ = .588. 58.8% of all people who purchase a small cup of coffee choose decaf.

c. P(small | decaf) = $\dfrac{P(\text{small} \cap \text{decaf})}{P(\text{decaf})} = \dfrac{.20}{.40}$ = .50. 50% of all people who purchase decaf coffee choose the small size.

50.

a. $P(\mathbf{M} \cap \mathbf{LS} \cap \mathbf{PR})$ = .05, directly from the table of probabilities.

b. $P(\mathbf{M} \cap \mathbf{Pr}) = P(\mathbf{M} \cap \mathbf{LS} \cap \mathbf{PR}) + P(\mathbf{M} \cap \mathbf{SS} \cap \mathbf{PR})$ = .05 + .07 = .12.

c. $P(\mathbf{SS})$ = sum of 9 probabilities in the **SS** table = .56. $P(\mathbf{LS})$ = 1 – .56 = .44.

d. From the two tables, $P(\mathbf{M})$ = .08 + .07 + .12 + .10 + .05 + .07 = .49. $P(\mathbf{Pr})$ = .02 + .07 + .07 + .02 + .05 + .02 = .25.

e. $P(\mathbf{M} | \mathbf{SS} \cap \mathbf{Pl}) = \dfrac{P(\mathbf{M} \cap \mathbf{SS} \cap \mathbf{Pl})}{P(\mathbf{SS} \cap \mathbf{Pl})} = \dfrac{.08}{.04+.08+.03} = .533$.

f. $P(\mathbf{SS} | \mathbf{M} \cap \mathbf{Pl}) = \dfrac{P(\mathbf{SS} \cap \mathbf{M} \cap \mathbf{Pl})}{P(\mathbf{M} \cap \mathbf{Pl})} = \dfrac{.08}{.08+.10} = .444$. $P(\mathbf{LS} | \mathbf{M} \cap \mathbf{Pl}) = 1 - P(\mathbf{SS} | \mathbf{M} \cap \mathbf{Pl})$ = 1 – .444 = .556.

51.

a. If a red ball is drawn from the first box, the composition of the second box becomes eight red and three green. Use the multiplication rule:

$P(\text{R from } 1^{\text{st}} \cap \text{R from } 2^{\text{nd}}) = P(\text{R from } 1^{\text{st}}) \times P(\text{R from } 2^{\text{nd}} | \text{R from } 1^{\text{st}}) = \frac{6}{10} \times \frac{8}{11} = .436$.

b. P(same numbers as originally) = P(both selected balls are the same color) = P(both R) + P(both G) = $\frac{6}{10} \times \frac{8}{11} + \frac{4}{10} \times \frac{4}{11} = .581$.

52. We know that $P(A_1 \cup A_2) = .07$ and $P(A_1 \cap A_2) = .01$, and that $P(A_1) = P(A_2)$ because the pumps are identical. There are two solution methods. The first doesn't require explicit reference to q or r: Let A_1 be the event that #1 fails and A_2 be the event that #2 fails.

Apply the addition rule: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \Rightarrow .07 = 2P(A_1) - .01 \Rightarrow P(A_1) = .04$.

Otherwise, we assume that $P(A_1) = P(A_2) = q$ and that $P(A_1 | A_2) = P(A_2 | A_1) = r$ (the goal is to find q). Proceed as follows: $.01 = P(A_1 \cap A_2) = P(A_1)\, P(A_2 | A_1) = qr$ and $.07 = P(A_1 \cup A_2) =$
$P(A_1 \cap A_2) + P(A_1' \cap A_2) + P(A_1 \cap A_2') = .01 + q(1 - r) + q(1 - r) \Rightarrow q(1 - r) = .03$.

These two equations give $2q - .01 = .07$, from which $q = .04$ (and $r = .25$).

53. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)}$ (since B is contained in A, $A \cap B = B$)

$= \frac{.05}{.60} = .0833$

54.

a. $P(A_2 | A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{.11}{.22} = .50$. If the firm is awarded project 1, there is a 50% chance they will also be awarded project 2.

b. $P(A_2 \cap A_3 | A_1) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.01}{.22} = .0455$. If the firm is awarded project 1, there is a 4.55% chance they will also be awarded projects 2 and 3.

c. $P(A_2 \cup A_3 | A_1) = \frac{P[A_1 \cap (A_2 \cup A_3)]}{P(A_1)} = \frac{P[(A_1 \cap A_2) \cup (A_1 \cap A_3)]}{P(A_1)}$

$= \frac{P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{.15}{.22} = .682$. If the firm is awarded project 1, there is a 68.2% chance they will also be awarded at least one of the other two projects.

d. $P(A_1 \cap A_2 \cap A_3 | A_1 \cup A_2 \cup A_3) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.01}{.53} = .0189$. If the firm is awarded at least one of the projects, there is a 1.89% chance they will be awarded all three projects.

55. Let A = {carries Lyme disease} and B = {carries HGE}. We are told $P(A) = .16$, $P(B) = .10$, and $P(A \cap B \mid A \cup B) = .10$. From this last statement and the fact that $A \cap B$ is contained in $A \cup B$,

$.10 = \frac{P(A \cap B)}{P(A \cup B)} \Rightarrow P(A \cap B) = .10P(A \cup B) = .10[P(A) + P(B) - P(A \cap B)] = .10[.10 + .16 - P(A \cap B)] \Rightarrow$

$1.1P(A \cap B) = .026 \Rightarrow P(A \cap B) = .02364$.

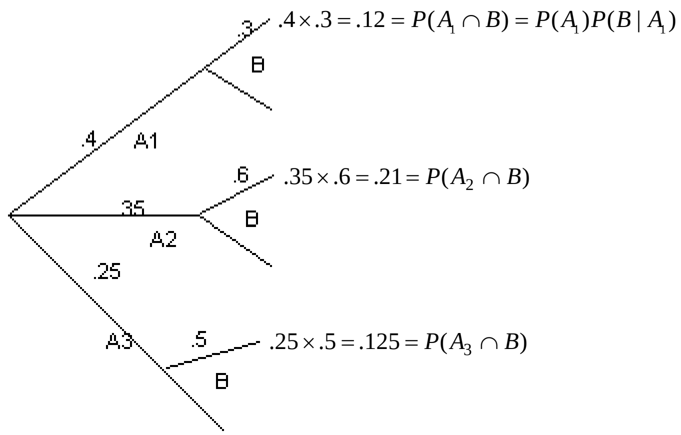
Finally, the desired probability is $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{.02364}{.10} = .2364$.

56. $$P(A \mid B) + P(A' \mid B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A \cap B) + P(A' \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

57. $P(B \mid A) > P(B)$ iff $P(B \mid A) + P(B' \mid A) > P(B) + P(B' \mid A)$ iff $1 > P(B) + P(B' \mid A)$ by Exercise 56 (with the letters switched). This holds iff $1 - P(B) > P(B' \mid A)$ iff $P(B') > P(B' \mid A)$, QED.

58. $P(A \cup B \mid C) = \frac{P[(A \cup B) \cap C)}{P(C)} = \frac{P[(A \cap C) \cup (B \cap C)]}{P(C)} = \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)} = P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$

59. The required probabilities appear in the tree diagram below.

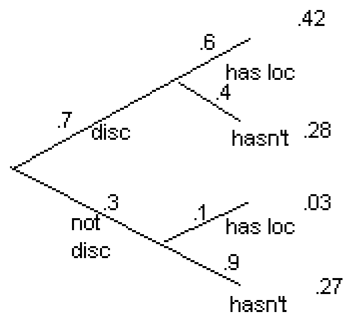


a. $P(A_2 \cap B) = .21$.

b. By the law of total probability, $P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .455$.

c. Using Bayes' theorem, $P(A_1 \mid B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.12}{.455} = .264$; $P(A_2 \mid B) = \frac{.21}{.455} = .462$; $P(A_3 \mid B) = 1 - .264 - .462 = .274$. Notice the three probabilities sum to 1.

60. The tree diagram below shows the probability for the four disjoint options; e.g., P(the flight is discovered and has a locator) = P(discovered)P(locator | discovered) = (.7)(.6) = .42.



a. P(not discovered | has locator) = $\dfrac{P(\text{not discovered} \cap \text{has locator})}{P(\text{has locator})} = \dfrac{.03}{.03+.42} = .067$.

b. P(discovered | no locator) = $\dfrac{P(\text{discovered} \cap \text{no locator})}{P(\text{no locator})} = \dfrac{.28}{.55} = .509$.

61. The initial ("prior") probabilities of 0, 1, 2 defectives in the batch are .5, .3, .2. Now, let's determine the probabilities of 0, 1, 2 defectives in the sample based on these three cases.

- If there are 0 defectives in the batch, clearly there are 0 defectives in the sample.

P(0 def in sample | 0 def in batch) = 1.

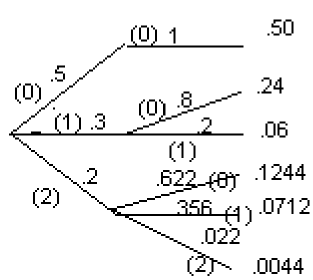
- If there is 1 defective in the batch, the chance it's discovered in a sample of 2 equals 2/10 = .2, and the probability it isn't discovered is 8/10 = .8.

P(0 def in sample | 1 def in batch) = .8, P(1 def in sample | 1 def in batch) = .2.

- If there are 2 defectives in the batch, the chance both are discovered in a sample of 2 equals $\dfrac{2}{10} \times \dfrac{1}{9} = .022$; the chance neither is discovered equals $\dfrac{8}{10} \times \dfrac{7}{9} = .622$; and the chance exactly 1 is discovered equals 1 – (.022 + .622) = .356.

P(0 def in sample | 2 def in batch) = .622, P(1 def in sample | 2 def in batch) = .356,
P(2 def in sample | 2 def in batch) = .022.

These calculations are summarized in the tree diagram below. Probabilities at the endpoints are intersectional probabilities, e.g. P(2 def in batch $\cap$ 2 def in sample) = (.2)(.022) = .0044.



a. Using the tree diagram and Bayes' rule,

P(0 def in batch | 0 def in sample) = $\dfrac{.5}{.5+.24+.1244} = .578$

P(1 def in batch | 0 def in sample) = $\dfrac{.24}{.5+.24+.1244} = .278$

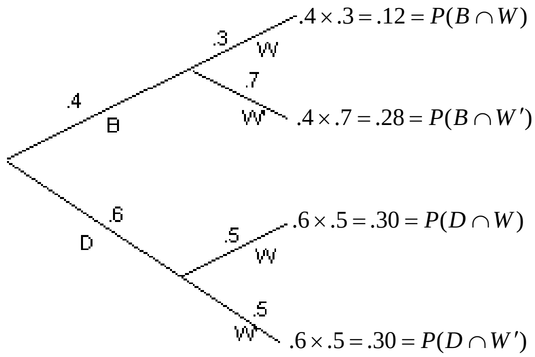
P(2 def in batch | 0 def in sample) = $\dfrac{.1244}{.5+.24+.1244} = .144$

b. P(0 def in batch | 1 def in sample) = 0

P(1 def in batch | 1 def in sample) = $\dfrac{.06}{.06+.0712} = .457$

P(2 def in batch | 1 def in sample) = $\dfrac{.0712}{.06+.0712} = .543$

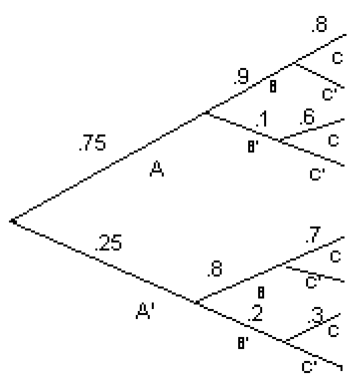
62. Using a tree diagram (B = basic, D = deluxe, W = warranty purchase):



From the diagram, $P(B \mid W) = \dfrac{P(B \cap W)}{P(W)} = \dfrac{.12}{.30+.12} = \dfrac{.12}{.42} = .2857$.

63.

a.



b. From the top path of the tree diagram, $P(A \cap B \cap C) = (.75)(.9)(.8) = .54$.

c. Event $B \cap C$ occurs twice on the diagram: $P(B \cap C) = P(A \cap B \cap C) + P(A' \cap B \cap C) = .54 + (.25)(.8)(.7) = .68$.

d. $P(C) = P(A \cap B \cap C) + P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B' \cap C) = .54 + .045 + .14 + .015 = .74$.

e. Rewrite the conditional probability first: $P(A \mid B \cap C) = \dfrac{P(A \cap B \cap C)}{P(B \cap C)} = \dfrac{.54}{.68} = .7941$.

64. A tree diagram can help. We know that $P(\text{short}) = .6$, $P(\text{medium}) = .3$, $P(\text{long}) = .1$; also, $P(\text{Word} \mid \text{short}) = .8$, $P(\text{Word} \mid \text{medium}) = .5$, $P(\text{Word} \mid \text{long}) = .3$.

a. Use the law of total probability: $P(\text{Word}) = (.6)(.8) + (.3)(.5) + (.1)(.3) = .66$.

b. $P(\text{small} \mid \text{Word}) = \dfrac{P(\text{small} \cap \text{Word})}{P(\text{Word})} = \dfrac{(.6)(.8)}{.66} = .727$. Similarly, $P(\text{medium} \mid \text{Word}) = \dfrac{(.3)(.5)}{.66} = .227$, and $P(\text{long} \mid \text{Word}) = .045$. (These sum to .999 due to rounding error.)

65. A tree diagram can help. We know that $P(\text{day}) = .2$, $P(\text{1-night}) = .5$, $P(\text{2-night}) = .3$; also, $P(\text{purchase} \mid \text{day}) = .1$, $P(\text{purchase} \mid \text{1-night}) = .3$, and $P(\text{purchase} \mid \text{2-night}) = .2$.

Apply Bayes' rule: e.g., $P(\text{day} \mid \text{purchase}) = \dfrac{P(\text{day} \cap \text{purchase})}{P(\text{purchase})} = \dfrac{(.2)(.1)}{(.2)(.1)+(.5)(.3)+(.3)(.2)} = \dfrac{.02}{.23} = .087$.

Similarly, $P(\text{1-night} \mid \text{purchase}) = \dfrac{(.5)(.3)}{.23} = .652$ and $P(\text{2-night} \mid \text{purchase}) = .261$.

66. Let E, C, and L be the events associated with e-mail, cell phones, and laptops, respectively. We are told $P(E) = 40\%$, $P(C) = 30\%$, $P(L) = 25\%$, $P(E \cap C) = 23\%$, $P(E' \cap C' \cap L') = 51\%$, $P(E \mid L) = 88\%$, and $P(L \mid C) = 70\%$.

a. $P(C \mid E) = P(E \cap C)/P(E) = .23/.40 = .575$.

b. Use Bayes' rule: $P(C \mid L) = P(C \cap L)/P(L) = P(C)P(L \mid C)/P(L) = .30(.70)/.25 = .84$.

c. $P(C \mid E \cap L) = P(C \cap E \cap L)/P(E \cap L)$.
For the denominator, $P(E \cap L) = P(L)P(E \mid L) = (.25)(.88) = .22$.
For the numerator, use $P(E \cup C \cup L) = 1 - P(E' \cap C' \cap L') = 1 - .51 = .49$ and write
$P(E \cup C \cup L) = P(C) + P(E) + P(L) - P(E \cap C) - P(C \cap L) - P(E \cap L) + P(C \cap E \cap L)$
$\Rightarrow .49 = .30 + .40 + .25 - .23 - .30(.70) - .22 + P(C \cap E \cap L) \Rightarrow P(C \cap E \cap L) = .20$.
So, finally, $P(C \mid E \cap L) = .20/.22 = .9091$.

67. Let T denote the event that a randomly selected person is, in fact, a terrorist. Apply Bayes' theorem, using $P(T)$ = 1,000/300,000,000 = .0000033:

$P(T \mid +) = \dfrac{P(T)P(+\mid T)}{P(T)P(+\mid T)+P(T')P(+\mid T')} = \dfrac{(.0000033)(.99)}{(.0000033)(.99)+(1-.0000033)(1-.999)}$ = .003289. That is to say, roughly 0.3% of all people "flagged" as terrorists would be actual terrorists in this scenario.

68. Let's see how we can implement the hint. If she's flying airline #1, the chance of 2 late flights is (30%)(10%) = 3%; the two flights being "unaffected" by each other means we can multiply their probabilities. Similarly, the chance of 0 late flights on airline #1 is (70%)(90%) = 63%. Since percents add to 100%, the chance of exactly 1 late flight on airline #1 is 100% – (3% + 63%) = 34%. A similar approach works for the other two airlines: the probability of exactly 1 late flight on airline #2 is 35%, and the chance of exactly 1 late flight on airline #3 is 45%.

The initial ("prior") probabilities for the three airlines are $P(A_1)$ = 50%, $P(A_2)$ = 30%, and $P(A_3)$ = 20%. Given that she had exactly 1 late flight (call that event B), the conditional ("posterior") probabilities of the three airlines can be calculated using Bayes' Rule:

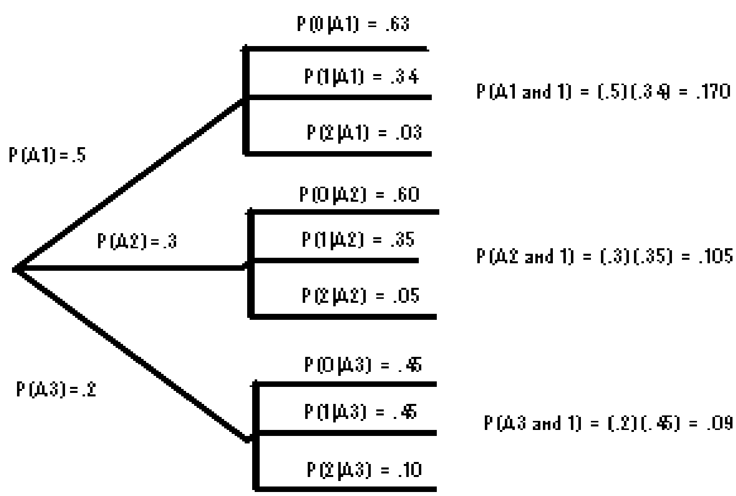
$P(A_1 \mid B) = \dfrac{P(A_1)P(B\mid A_1)}{P(A_1)P(B\mid A_1)+P(A_2)P(B\mid A_2)+P(A_3)P(B\mid A_3)} = \dfrac{(.5)(.34)}{(.5)(.34)+(.3)(.35)+(.2)(.45)} = \dfrac{.170}{.365} =$ .4657;

$P(A_2 \mid B) = \dfrac{P(A_2)P(B\mid A_2)}{P(A_1)P(B\mid A_1)+P(A_2)P(B\mid A_2)+P(A_3)P(B\mid A_3)} = \dfrac{(.3)(.35)}{.365}$ = .2877; and

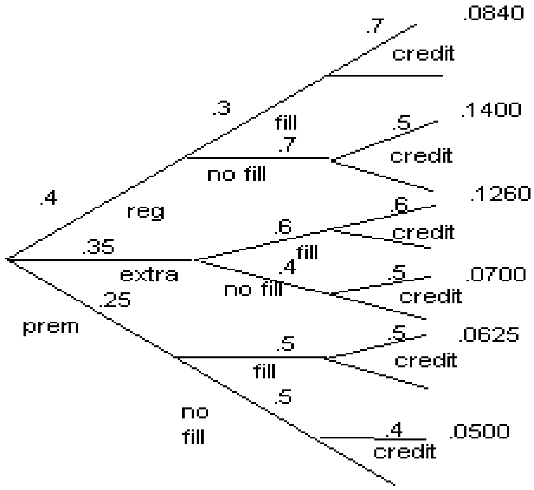
$P(A_3 \mid B) = \dfrac{P(A_3)P(B\mid A_3)}{P(A_1)P(B\mid A_1)+P(A_2)P(B\mid A_2)+P(A_3)P(B\mid A_3)} = \dfrac{(.2)(.45)}{.365}$ = .2466.

Notice that, except for rounding error, these three posterior probabilities add to 1.

The tree diagram below shows these probabilities.



69. The tree diagram below summarizes the information in the exercise (plus the previous information in Exercise 59). Probabilities for the branches corresponding to paying with credit are indicated at the far right. ("extra" = "plus")



a. $P(\text{plus} \cap \text{fill} \cap \text{credit}) = (.35)(.6)(.6) = .1260$.

b. $P(\text{premium} \cap \text{no fill} \cap \text{credit}) = (.25)(.5)(.4) = .05$.

c. From the tree diagram, $P(\text{premium} \cap \text{credit}) = .0625 + .0500 = .1125$.

d. From the tree diagram, $P(\text{fill} \cap \text{credit}) = .0840 + .1260 + .0625 = .2725$.

e. $P(\text{credit}) = .0840 + .1400 + .1260 + .0700 + .0625 + .0500 = .5325$.

f. $P(\text{premium} \mid \text{credit}) = \dfrac{P(\text{premium} \cap \text{credit})}{P(\text{credit})} = \dfrac{.1125}{.5325} = .2113$.

Section 2.5

70. Using the definition, two events A and B are independent if $P(A \mid B) = P(A)$;
$P(A \mid B) = .6125$; $P(A) = .50$; $.6125 \neq .50$, so A and B are not independent.
Using the multiplication rule, the events are independent if $P(A \cap B)=P(A)P(B)$;
$P(A \cap B) = .25$; $P(A)P(B) = (.5)(.4) = .2$. $.25 \neq .2$, so A and B are not independent.

71.

a. Since the events are independent, then A' and B' are independent, too. (See the paragraph below Equation 2.7.) Thus, $P(B'|A') = P(B') = 1 - .7 = .3$.

b. Using the addition rule, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .7 - (.4)(.7) = .82$. Since A and B are independent, we are permitted to write $P(A \cap B) = P(A)P(B) = (.4)(.7)$.

c. $P(AB' \mid A \cup B) = \dfrac{P(AB' \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(AB')}{P(A \cup B)} = \dfrac{P(A)P(B')}{P(A \cup B)} = \dfrac{(.4)(1-.7)}{.82} = \dfrac{.12}{.82} = .146$.

72. $P(A_1 \cap A_2) = .11$ while $P(A_1)P(A_2) = .055$, so A_1 and A_2 are not independent.
$P(A_1 \cap A_3) = .05$ while $P(A_1)P(A_3) = .0616$, so A_1 and A_3 are not independent.
$P(A_2 \cap A_3) = .07$ and $P(A_2)P(A_3) = .07$, so A_2 and A_3 <u>are</u> independent.

73. From a Venn diagram, $P(B) = P(A' \cap B) + P(A \cap B) = P(B) \Rightarrow P(A' \cap B) = P(B) - P(A \cap B)$. If A and B are independent, then $P(A' \cap B) = P(B) - P(A)P(B) = [1 - P(A)]P(B) = P(A')P(B)$. Thus, A' and B are independent.

Alternatively, $P(A' \mid B) = \dfrac{P(A' \cap B)}{P(B)} = \dfrac{P(B) - P(A \cap B)}{P(B)} = \dfrac{P(B) - P(A)P(B)}{P(B)} = 1 - P(A) = P(A')$.

74. Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1)P(O_2) = (.45)(.45) = .2025$.
$P(\text{two individuals match}) = P(A_1 \cap A_2) + P(B_1 \cap B_2) + P(AB_1 \cap AB_2) + P(O_1 \cap O_2) =$
$.40^2 + .11^2 + .04^2 + .45^2 = .3762$.

75. Let event E be the event that an error was signaled incorrectly.
We want $P(\text{at least one signaled incorrectly}) = P(E_1 \cup \ldots \cup E_{10})$. To use independence, we need intersections, so apply deMorgan's law: $= P(E_1 \cup \ldots \cup E_{10}) = 1 - P(E_1' \cap \cdots \cap E_{10}')$. $P(E') = 1 - .05 = .95$, so for 10 independent points, $P(E_1' \cap \cdots \cap E_{10}') = (.95)\ldots(.95) = (.95)^{10}$. Finally, $P(E_1 \cup E_2 \cup \ldots \cup E_{10}) = 1 - (.95)^{10} = .401$. Similarly, for 25 points, the desired probability is $1 - (P(E'))^{25} = 1 - (.95)^{25} = .723$.

76. Follow the same logic as in Exercise 75: If the probability of an event is p, and there are n independent "trials," the chance this event never occurs is $(1 - p)^n$, while the chance of at least one occurrence is $1 - (1 - p)^n$. With $p = 1/9{,}000{,}000{,}000$ and $n = 1{,}000{,}000{,}000$, this calculates to $1 - .9048 = .0952$.

Note: For extremely small values of p, $(1 - p)^n \approx 1 - np$. So, the probability of at least one occurrence under these assumptions is roughly $1 - (1 - np) = np$. Here, that would equal 1/9.

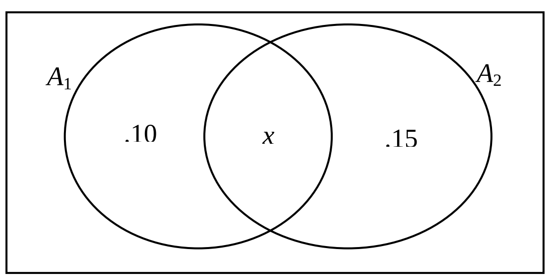
77. Let p denote the probability that a rivet is defective.

a. $.20 = P(\text{seam needs reworking}) = 1 - P(\text{seam doesn't need reworking}) =$
$1 - P(\text{no rivets are defective}) = 1 - P(1^{\text{st}} \text{ isn't def} \cap \ldots \cap 25^{\text{th}} \text{ isn't def}) =$
$1 - (1 - p)\ldots(1 - p) = 1 - (1 - p)^{25}$.
Solve for p: $(1 - p)^{25} = .80 \Rightarrow 1 - p = (.80)^{1/25} \Rightarrow p = 1 - .99111 = .00889$.

b. The desired condition is $.10 = 1 - (1 - p)^{25}$. Again, solve for p: $(1 - p)^{25} = .90 \Rightarrow$
$p = 1 - (.90)^{1/25} = 1 - .99579 = .00421$.

78. $P(\text{at least one opens}) = 1 - P(\text{none open}) = 1 - (.05)^5 = .99999969$.
$P(\text{at least one fails to open}) = 1 - P(\text{all open}) = 1 - (.95)^5 = .2262$.

79. Let A_1 = older pump fails, A_2 = newer pump fails, and $x = P(A_1 \cap A_2)$. The goal is to find x. From the Venn diagram below, $P(A_1) = .10 + x$ and $P(A_2) = .05 + x$. Independence implies that $x = P(A_1 \cap A_2) = P(A_1)P(A_2) = (.10 + x)(.05 + x)$. The resulting quadratic equation, $x^2 - .85x + .005 = 0$, has roots $x = .0059$ and $x = .8441$. The latter is impossible, since the probabilities in the Venn diagram would then exceed 1. Therefore, $x = .0059$.



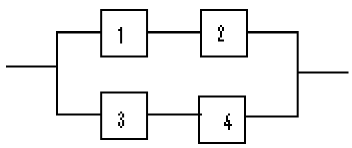
80. Let A_i denote the event that component #i works (i = 1, 2, 3, 4). Based on the design of the system, the event "the system works" is $(A_1 \cup A_2) \cup (A_3 \cap A_4)$. We'll eventually need $P(A_1 \cup A_2)$, so work that out first: $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = (.9) + (.9) - (.9)(.9) = .99$. The third term uses independence of events. Also, $P(A_3 \cap A_4)$ = (.9)(.9) = .81, again using independence.

Now use the addition rule and independence for the system:

$$\begin{aligned} P((A_1 \cup A_2) \cup (A_3 \cap A_4)) &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P((A_1 \cup A_2) \cap (A_3 \cap A_4)) \\ &= P(A_1 \cup A_2) + P(A_3 \cap A_4) - P(A_1 \cup A_2) \times P(A_3 \cap A_4) \\ &= (.99) + (.81) - (.99)(.81) = .9981 \end{aligned}$$

(You could also use deMorgan's law in a couple of places.)

81. Using the hints, let $P(A_i) = p$, and $x = p^2$. Following the solution provided in the example, P(system lifetime exceeds t_0) = $p^2 + p^2 - p^4 = 2p^2 - p^4 = 2x - x^2$. Now, set this equal to .99: $2x - x^2 = .99 \Rightarrow x^2 - 2x + .99 = 0 \Rightarrow x = 0.9$ or $1.1 \Rightarrow p = 1.049$ or .9487. Since the value we want is a probability and cannot exceed 1, the correct answer is p = .9487.



82. A = {(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)} $\Rightarrow P(A) = \frac{6}{36} = \frac{1}{6}$; B = {(1,4)(2,4)(3,4)(4,4)(5,4)(6,4)} $\Rightarrow P(B) = \frac{1}{6}$; and C = {(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)} $\Rightarrow P(C) = \frac{1}{6}$.

$A \cap B$ = {(3,4)} $\Rightarrow P(A \cap B) = \frac{1}{36} = P(A)P(B)$; $A \cap C$ = {(3,4)} $\Rightarrow P(A \cap C) = \frac{1}{36} = P(A)P(C)$; and $B \cap C$ = {(3,4)} $\Rightarrow P(B \cap C) = \frac{1}{36} = P(B)P(C)$. Therefore, these three events are pairwise independent.

However, $A \cap B \cap C$ = {(3,4)} $\Rightarrow P(A \cap B \cap C) = \frac{1}{36}$, while $P(A)P(B)P(C) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$, so $P(A \cap B \cap C) \neq P(A)P(B)P(C)$ and these three events are not mutually independent.

83. We'll need to know P(both detect the defect) = 1 – P(at least one doesn't) = 1 – .2 = .8.

a. P(1st detects $\cap$ 2nd doesn't) = P(1st detects) – P(1st does $\cap$ 2nd does) = .9 – .8 = .1.
Similarly, P(1st doesn't $\cap$ 2nd does) = .1, so P(exactly one does)= .1 + .1= .2.

b. P(neither detects a defect) = 1 – [P(both do) + P(exactly 1 does)] = 1 – [.8+.2] = 0. That is, under this model there is a 0% probability neither inspector detects a defect. As a result, P(all 3 escape) = (0)(0)(0) = 0.

84. Let A_i denote the event that vehicle #i passes inspection (i = 1, 2, 3).

a. $P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2) \times P(A_3) = (.7)(.7)(.7) = (.7)^3$ = .343.

b. This is the complement of part **a**, so the answer is 1 – .343 = .657.

c. $P([A_1 \cap A_2' \cap A_3'] \cup [A_1' \cap A_2 \cap A_3'] \cup [A_1' \cap A_2' \cap A_3])$ = (.7)(.3)(.3) + (.3)(.7)(.3) + (.3)(.3)(.7) = $3(.3)^2(.7)$ = .189. Notice that we're using the fact that if events are independent then their complements are also independent.

d. P(at most one passes) = P(zero pass) + P(exactly one passes) = P(zero pass) + .189. For the first probability, P(zero pass) = $P(A_1' \cap A_2' \cap A_3')$ = (.3)(.3)(.3) = .027. So, the answer is .027 + .189 = .216.

e. We'll need the fact that P(at least one passes) = 1 – P(zero pass) = 1 – .027 = .973. Then,
$$P(A_1 \cap A_2 \cap A_3 \mid A_1 \cup A_2 \cup A_3) = \frac{P([A_1 \cap A_2 \cap A_3] \cap [A_1 \cup A_2 \cup A_3])}{P(A_1 \cup A_2 \cup A_3)} = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1 \cup A_2 \cup A_3)} = \frac{.343}{.973} = .3525.$$

85.

a. Let D_1 = detection on 1st fixation, D_2 = detection on 2nd fixation.
P(detection in at most 2 fixations) = $P(D_1) + P(D_1' \cap D_2)$; since the fixations are independent,
$P(D_1) + P(D_1' \cap D_2) = P(D_1) + P(D_1')\,P(D_2) = p + (1 – p)p = p(2 – p)$.

b. Define $D_1, D_2, \ldots, D_n$ as in **a**. Then P(at most n fixations) =
$P(D_1) + P(D_1' \cap D_2) + P(D_1' \cap D_2' \cap D_3) + \ldots + P(D_1' \cap D_2' \cap \cdots \cap D_{n-1}' \cap D_n)$ =
$p + (1 – p)p + (1 – p)^2p + \ldots + (1 – p)^{n–1}p = p[1 + (1 – p) + (1 – p)^2 + \ldots + (1 – p)^{n–1}]$ =
$$p \cdot \frac{1-(1-p)^n}{1-(1-p)} = 1-(1-p)^n .$$
Alternatively, P(at most n fixations) = 1 – P(at least n+1 fixations are required) =
1 – P(no detection in 1st n fixations) = $1 - P(D_1' \cap D_2' \cap \cdots \cap D_n') = 1 – (1 – p)^n$.

c. P(no detection in 3 fixations) = $(1 – p)^3$.

d. P(passes inspection) = P({not flawed} $\cup$ {flawed and passes})
= P(not flawed) + P(flawed and passes)
= .9 + P(flawed) P(passes | flawed) = .9 + $(.1)(1 – p)^3$.

e. Borrowing from **d**, P(flawed | passed) = $\dfrac{P(\text{flawed} \cap \text{passed})}{P(\text{passed})} = \dfrac{.1(1-p)^3}{.9+.1(1-p)^3}$. For p = .5,

$$P(\text{flawed} \mid \text{passed}) = \frac{.1(1-.5)^3}{.9+.1(1-.5)^3} = .0137 .$$

86.

a. $P(A) = \dfrac{2,000}{10,000}$ = .2. Using the law of total probability, $P(B) = P(A)P(B \mid A) + P(A')P(B \mid A')$ =

$(.2)\dfrac{1,999}{9,999} + (.8)\dfrac{2,000}{9,999}$ = **.2** exactly. That is, $P(B) = P(A) = .2$. Finally, use the multiplication rule:

$P(A \cap B) = P(A) \times P(B \mid A) = (.2)\dfrac{1,999}{9,999}$ = .039984. Events A and B are *not* independent, since $P(B)$ =

.2 while $P(B \mid A) = \dfrac{1,999}{9,999} = .19992$, and these are not equal.

b. If A and B were independent, we'd have $P(A \cap B) = P(A) \times P(B) = (.2)(.2) = .04$. This is very close to the answer .039984 from part **a**. This suggests that, for most practical purposes, we could treat events A and B in this example as if they were independent.

c. Repeating the steps in part **a**, you again get $P(A) = P(B) = .2$. However, using the multiplication rule, $P(A \cap B) = P(A) \times P(B \mid A) = \dfrac{2}{10} \times \dfrac{1}{9}$ =.0222. This is very different from the value of .04 that we'd get if A and B were independent!

The critical difference is that the population size in parts **a-b** is huge, and so the probability a second board is green *almost* equals .2 (i.e., 1,999/9,999 = .19992 ≈ .2). But in part **c**, the conditional probability of a green board shifts a lot: 2/10 = .2, but 1/9 = .1111.

87.

a. Use the information provided and the addition rule:
$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \Rightarrow P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = .55 + .65 - .80 = .40$.

b. By definition, $P(A_2 \mid A_3) = \dfrac{P(A_2 \cap A_3)}{P(A_3)} = \dfrac{.40}{.70}$ = .5714. If a person likes vehicle #3, there's a 57.14% chance s/he will also like vehicle #2.

c. No. From **b**, $P(A_2 \mid A_3) = .5714 \neq P(A_2) = .65$. Therefore, A_2 and A_3 are not independent. Alternatively, $P(A_2 \cap A_3) = .40 \neq P(A_2)P(A_3) = (.65)(.70) = .455$.

d. The goal is to find $P(A_2 \cup A_3 \mid A_1')$, i.e. $\dfrac{P([A_2 \cup A_3] \cap A_1')}{P(A_1')}$. The denominator is simply 1 – .55 = .45.
There are several ways to calculate the numerator; the simplest approach using the information provided is to draw a Venn diagram and observe that $P([A_2 \cup A_3] \cap A_1') = P(A_1 \cup A_2 \cup A_3) - P(A_1)$ =

.88 – .55 = .33. Hence, $P(A_2 \cup A_3 \mid A_1')$ = $\dfrac{.33}{.45}$ = .7333.

88.

a. For route #1, P(late) = P(stopped at 2 or 3 or 4 crossings) = 1 – P(stopped at 0 or 1) = 1 – $[(.9)^4 + 4(.9)^3(.1)]$ = .0523. Notice there are four ways the professor could be stopped exactly once (at crossing #1, #2, #3, or #4).

For route #2, P(late) = P(stopped at 1 or 2 crossings) = 1 – P(stopped at none) = $1 - (.9)^4 = 1 - .81 = .19$.

Thus, since .0523 < .19, route #1 should be taken.

b. Apply Bayes' theorem: $P(\text{4-crossing route} \mid \text{late}) = \dfrac{P(\text{4-c route} \cap \text{late})}{P(\text{late})} =$

$$\frac{P(\text{4-c route})P(\text{late} \mid \text{4-c route})}{P(\text{4-c route})P(\text{late} \mid \text{4-c route}) + P(\text{2-c route})P(\text{late} \mid \text{2-c route})} = \frac{(.5)(.0523)}{(.5)(.0523)+(.5)(.19)} = .216.$$

89. The question asks for P(<u>exactly</u> one tag lost | at <u>most</u> one tag lost) = $P((C_1 \cap C_2') \cup (C_1' \cap C_2) \mid (C_1 \cap C_2)')$.
Since the first event is contained in (a subset of) the second event, this equals

$$\frac{P((C_1 \cap C_2') \cup (C_1' \cap C_2))}{P((C_1 \cap C_2)')} = \frac{P(C_1 \cap C_2') + P(C_1' \cap C_2)}{1 - P(C_1 \cap C_2)} = \frac{P(C_1)P(C_2') + P(C_1')P(C_2)}{1 - P(C_1)P(C_2)} \text{ by independence} =$$

$$\frac{\pi(1-\pi) + (1-\pi)\pi}{1-\pi^2} = \frac{2\pi(1-\pi)}{1-\pi^2} = \frac{2\pi}{1+\pi}.$$

Supplementary Exercises

90.

a. $\binom{20}{3}$ = 1140.

b. There are 19 other machinists to choose from, so the answer is $\binom{19}{3}$ = 969.

c. There are 1140 total possible crews. Among them, the number that have <u>none</u> of the best 10 machinists is $\binom{10}{3}$ = 120 (because you're choosing from the remaining 10). So, the number of crews having at least one of the best 10 machinists is 1140 – 120 = 1020.

d. Using parts **a** and **b**, P(best will not work) = $\frac{969}{1140}$ = .85.

91.

a. P(line 1) = $\frac{500}{1500}$ = .333;

P(crack) = $\frac{.50(500)+.44(400)+.40(600)}{1500} = \frac{666}{1500}$ = .444.

b. This is one of the percentages provided: P(blemish | line 1) = .15.

c. P(surface defect) = $\frac{.10(500)+.08(400)+.15(600)}{1500} = \frac{172}{1500}$;

P(line 1 $\cap$ surface defect) = $\frac{.10(500)}{1500} = \frac{50}{1500}$;

so, P(line 1 | surface defect) = $\frac{50/1500}{172/1500} = \frac{50}{172}$ = .291.

92.

a. He will have one type of form left if either 4 withdrawals or 4 course substitutions remain. This means the first six were either 2 withdrawals and 4 subs or 6 withdrawals and 0 subs; the desired probability is $\frac{\binom{6}{2}\binom{4}{4}+\binom{6}{6}\binom{4}{0}}{\binom{10}{6}} = \frac{16}{210}$ =.0762.

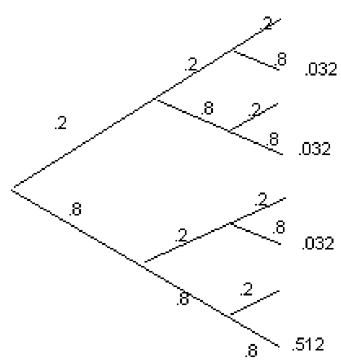
b. He can start with the withdrawal forms or the course substitution forms, allowing two sequences: W-C-W-C or C-W-C-W. The number of ways the first sequence could arise is (6)(4)(5)(3) = 360, and the number of ways the second sequence could arise is (4)(6)(3)(5) = 360, for a total of 720 such possibilities. The <u>total</u> number of ways he could select four forms one at a time is $P_{4,10}$ = (10)(9)(8)(7) = 5040. So, the probability of a perfectly alternating sequence is 720/5040 = .143.

93. Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .626 = P(A) + P(B) - .144$. Apply independence: $P(A \cap B) = P(A)P(B) = .144$.
So, $P(A) + P(B) = .770$ and $P(A)P(B) = .144$.
Let $x = P(A)$ and $y = P(B)$. Using the first equation, $y = .77 - x$, and substituting this into the second equation yields $x(.77 - x) = .144$ or $x^2 - .77x + .144 = 0$. Use the quadratic formula to solve:
$x = \dfrac{.77 \pm \sqrt{(-.77)^2 - (4)(1)(.144)}}{2(1)} = \dfrac{.77 \pm .13}{2} = .32$ or $.45$. Since $x = P(A)$ is assumed to be the larger probability, $x = P(A) = .45$ and $y = P(B) = .32$.

94. The probability of a bit reversal is .2, so the probability of maintaining a bit is .8.

a. Using independence, $P(\text{all three relays correctly send 1}) = (.8)(.8)(.8) = .512$.

b. In the accompanying tree diagram, each .2 indicates a bit reversal (and each .8 its opposite). There are several paths that maintain the original bit: no reversals or exactly two reversals (e.g., $1 \to 1 \to 0 \to 1$, which has reversals at relays 2 and 3). The total probability of these options is $.512 + (.8)(.2)(.2) + (.2)(.8)(.2) + (.2)(.2)(.8) = .512 + 3(.032) = .608$.



c. Using the answer from **b**, $P(1 \text{ sent} \mid 1 \text{ received}) = \dfrac{P(1 \text{ sent} \cap 1 \text{ received})}{P(1 \text{ received})} =$
$\dfrac{P(1 \text{ sent})P(1 \text{ received} \mid 1 \text{ sent})}{P(1 \text{ sent})P(1 \text{ received} \mid 1 \text{ sent}) + P(0 \text{ sent})P(1 \text{ received} \mid 0 \text{ sent})} = \dfrac{(.7)(.608)}{(.7)(.608) + (.3)(.392)} = \dfrac{.4256}{.5432} =$
$.7835$.
In the denominator, $P(1 \text{ received} \mid 0 \text{ sent}) = 1 - P(0 \text{ received} \mid 0 \text{ sent}) = 1 - .608$, since the answer from **b** also applies to a 0 being relayed as a 0.

95.

a. There are 5! = 120 possible orderings, so $P(\text{BCDEF}) = \frac{1}{120} = .0833$.

b. The number of orderings in which F is third equals 4×3×1*×2×1 = 24 (*because F must be here), so $P(\text{F is third}) = \frac{24}{120} = .2$. Or more simply, since the five friends are ordered completely at random, there is a ⅕ chance F is specifically in position three.

c. Similarly, $P(\text{F last}) = \dfrac{4\times3\times2\times1\times1}{120} = .2$.

d. $P(\text{F hasn't heard after 10 times}) = P(\text{not on \#1} \cap \text{not on \#2} \cap \ldots \cap \text{not on \#10}) = \dfrac{4}{5}\times\cdots\times\dfrac{4}{5} = \left(\dfrac{4}{5}\right)^{10} = .1074$.

96. Palmberg equation: $P_d(c) = \dfrac{(c/c^*)^\beta}{1+(c/c^*)^\beta}$

a. $P_d(c^*) = \dfrac{(c^*/c^*)^\beta}{1+(c^*/c^*)^\beta} = \dfrac{1^\beta}{1+1^\beta} = \dfrac{1}{1+1} = .5$.

b. The probability of detecting a crack that is twice the size of the "50-50" size c^* equals $P_d(2c^*) = \dfrac{(2c^*/c^*)^\beta}{1+(2c^*/c^*)^\beta} = \dfrac{2^\beta}{1+2^\beta}$. When $\beta = 4$, $P_d(2c^*) = \dfrac{2^4}{1+2^4} = \dfrac{16}{17} = .9412$.

c. Using the answers from **a** and **b**, $P(\text{exactly one of two detected}) = P(\text{first is, second isn't}) + P(\text{first isn't, second is}) = (.5)(1 - .9412) + (1 - .5)(.9412) = .5$.

d. If $c = c^*$, then $P_d(c) = .5$ irrespective of β. If $c < c^*$, then $c/c^* < 1$ and $P_d(c) \to \dfrac{0}{0+1} = 0$ as $\beta \to \infty$. Finally, if $c > c^*$ then $c/c^* > 1$ and, from calculus, $P_d(c) \to 1$ as $\beta \to \infty$.

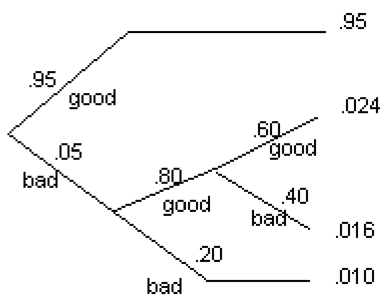
97. When three experiments are performed, there are 3 different ways in which detection can occur on exactly 2 of the experiments: (i) #1 and #2 and not #3; (ii) #1 and not #2 and #3; and (iii) not #1 and #2 and #3. If the impurity is present, the probability of exactly 2 detections in three (independent) experiments is (.8)(.8)(.2) + (.8)(.2)(.8) + (.2)(.8)(.8) = .384. If the impurity is absent, the analogous probability is 3(.1)(.1)(.9) = .027. Thus, applying Bayes' theorem, $P(\text{impurity is present} \mid \text{detected in exactly 2 out of 3})$ $= \dfrac{P(\text{detected in exactly 2} \cap \text{present})}{P(\text{detected in exactly 2})} = \dfrac{(.384)(.4)}{(.384)(.4)+(.027)(.6)} = .905$.

98. There are (6)(6)(6) = 216 possible sequences of selections, all equally likely by the independence assumption. Among those, there are (6)(5)(4) = 120 wherein the 3 contestants all pick different categories. The number of possibilities wherein all 3 contestants choose differently and exactly one of them picks category #1 is (1)(5)(4) + (5)(1)(4) + (5)(4)(1) = 60. Therefore,

P(exactly one selects category #1 | all 3 are different) =

$$\frac{P(\text{exactly one selects category \#1} \cap \text{all 3 are different})}{P(\text{all 3 are different})} = \frac{60/216}{120/216} = \frac{60}{120} = .5.$$

99. Refer to the tree diagram below.



a. P(pass inspection) = P(pass initially $\cup$ passes after recrimping) =
P(pass initially) + P(fails initially $\cap$ goes to recrimping $\cap$ is corrected after recrimping) =
.95 + (.05)(.80)(.60) (following path “bad-good-good” on tree diagram) = .974.

b. P(needed no recrimping | passed inspection) = $\dfrac{P(\text{passed initially})}{P(\text{passed inspection})} = \dfrac{.95}{.974} = .9754$.

100.

a. First, P(both +) = P(carrier $\cap$ both +) + P(not a carrier $\cap$ both +) =
P(carrier)P(both + | carrier) + P(not a carrier)P(both + | not a carrier). Assuming independence of the tests, this equals $(.01)(.90)^2 + (.99)(.05)^2 = .010575$.
Similarly, P(both –) = $(.01)(.10)^2 + (.99)(.95)^2 = .893575$.
Therefore, P(tests agree) = .010575 + .893575 = .90415.

b. From the first part of **a**, P(carrier | both +) = $\dfrac{P(\text{carrier} \cap \text{both } +)}{P(\text{both } +)} = \dfrac{(.01)(.90)^2}{.010575} = .766$.

101. Let A = 1st functions, B = 2nd functions, so $P(B) = .9$, $P(A \cup B) = .96$, $P(A \cap B)=.75$. Use the addition rule:
$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow .96 = P(A) + .9 - .75 \Rightarrow P(A) = .81$.

Therefore, $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{.75}{.81} = .926$.

102. $P(E_1 \cap L) = P(E_1)P(L \mid E_1) = (.40)(.02) = .008$.

103. A tree diagram can also help here.

a. The law of total probability gives $P(L) = \sum P(E_i)P(L \mid E_i) = (.40)(.02) + (.50)(.01) + (.10)(.05) = .018$.

b. $P(E_1' \mid L') = 1 - P(E_1 \mid L') = 1 - \dfrac{P(E_1 \cap L')}{P(L')} = 1 - \dfrac{P(E_1)P(L' \mid E_1)}{1 - P(L)} = 1 - \dfrac{(.40)(.98)}{1 - .018} = .601$.

104. Let B denote the event that a component needs rework. By the law of total probability, $P(B) = \sum P(A_i)P(B \mid A_i) = (.50)(.05) + (.30)(.08) + (.20)(.10) = .069$.

Thus, $P(A_1 \mid B) = \dfrac{(.50)(.05)}{.069} = .362$, $P(A_2 \mid B) = \dfrac{(.30)(.08)}{.069} = .348$, and $P(A_3 \mid B) = .290$.

105. This is the famous "Birthday Problem" in probability.

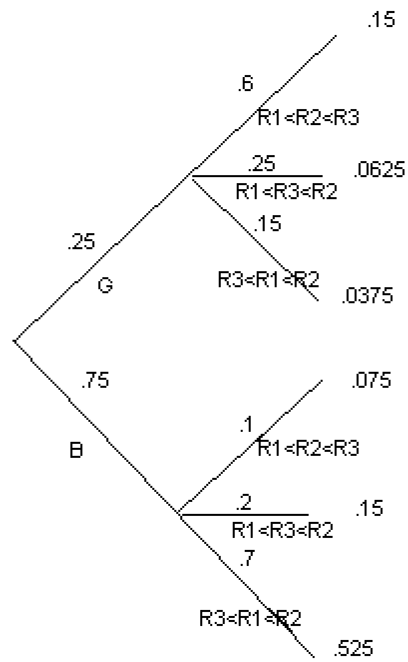
a. There are 365^{10} possible lists of birthdays, e.g. (Dec 10, Sep 27, Apr 1, …). Among those, the number with zero matching birthdays is $P_{10,365}$ (sampling ten birthdays without replacement from 365 days. So, $P(\text{all different}) = \dfrac{P_{10,365}}{365^{10}} = \dfrac{(365)(364)\cdots(356)}{(365)^{10}} = .883$. $P(\text{at least two the same}) = 1 - .883 = .117$.

b. The general formula is $P(\text{at least two the same}) = 1 - \dfrac{P_{k,365}}{365^k}$. By trial and error, this probability equals .476 for $k = 22$ and equals .507 for $k = 23$. Therefore, the smallest k for which k people have at least a 50-50 chance of a birthday match is 23.

c. There are 1000 possible 3-digit sequences to end a SS number (000 through 999). Using the idea from **a**, $P(\text{at least two have the same SS ending}) = 1 - \dfrac{P_{10,1000}}{1000^{10}} = 1 - .956 = .044$.

Assuming birthdays and SS endings are independent, $P(\text{at least one "coincidence"}) = P(\text{birthday coincidence} \cup \text{SS coincidence}) = .117 + .044 - (.117)(.044) = .156$.

106. See the accompanying tree diagram.



a. $P(G \mid R_1 < R_2 < R_3) = \dfrac{.15}{.15+.075} = .67$ while $P(B \mid R_1 < R_2 < R_3) = .33$, so classify the specimen as granite. Equivalently, $P(G \mid R_1 < R_2 < R_3) = .67 > ½$ so granite is more likely.

b. $P(G \mid R_1 < R_3 < R_2) = \dfrac{.0625}{.2125} = .2941 < ½$, so classify the specimen as basalt.

$P(G \mid R_3 < R_1 < R_2) = \dfrac{.0375}{.5625} = .0667 < ½$, so classify the specimen as basalt.

c. P(erroneous classification) = P(B classified as G) + P(G classified as B) =
$P(B)P$(classified as G | B) + $P(G)P$(classified as B | G) =
$(.75)P(R_1 < R_2 < R_3 \mid B) + (.25)P(R_1 < R_3 < R_2 \text{ or } R_3 < R_1 < R_2 \mid G) =$
(.75)(.10) + (.25)(.25 + .15) = .175.

d. For what values of p will $P(G \mid R_1 < R_2 < R_3)$, $P(G \mid R_1 < R_3 < R_2)$, and $P(G \mid R_3 < R_1 < R_2)$ all exceed ½? Replacing .25 and .75 with p and $1 - p$ in the tree diagram,

$P(G \mid R_1 < R_2 < R_3) = \dfrac{.6p}{.6p + .1(1-p)} = \dfrac{.6p}{.1 + .5p} > .5 \text{ iff } p > \dfrac{1}{7}$;

$P(G \mid R_1 < R_3 < R_2) = \dfrac{.25p}{.25p + .2(1-p)} > .5 \text{ iff } p > \dfrac{4}{9}$;

$P(G \mid R_3 < R_1 < R_2) = \dfrac{.15p}{.15p + .7(1-p)} > .5 \text{ iff } p > \dfrac{14}{17}$ (most restrictive). Therefore, one would always classify a rock as granite iff $p > \dfrac{14}{17}$.

107. P(detection by the end of the nth glimpse) = 1 – P(not detected in first n glimpses) =

$1 - P(G_1' \cap G_2' \cap \cdots \cap G_n') = 1 - P(G_1')P(G_2')\cdots P(G_n') = 1 - (1 - p_1)(1 - p_2) \ldots (1 - p_n) = 1 - \prod_{i=1}^{n}(1 - p_i)$.

108.

a. P(walks on 4th pitch) = P(first 4 pitches are balls) = $(.5)^4$ = .0625.

b. P(walks on 6th pitch) = P(2 of the first 5 are strikes $\cap$ #6 is a ball) =

P(2 of the first 5 are strikes)P(#6 is a ball) = $\binom{5}{2}(.5)^2(.5)^3(.5)$ = .15625.

c. Following the pattern from **b**, P(walks on 5th pitch) = $\binom{4}{1}(.5)^1(.5)^3(.5)$ = .125. Therefore, P(batter walks) = P(walks on 4th) + P(walks on 5th) + P(walks on 6th) =
.0625 + .125 + .15625 = .34375.

d. P(first batter scores while no one is out) = P(first four batters all walk) = $(.34375)^4$ = .014.

109.

a. P(all in correct room) = $\frac{1}{4!} = \frac{1}{24}$ = .0417.

b. The 9 outcomes which yield completely incorrect assignments are: 2143, 2341, 2413, 3142, 3412, 3421, 4123, 4321, and 4312, so P(all incorrect) = $\frac{9}{24}$ = .375.

110.

a. P(all full) = $P(A \cap B \cap C)$ = (.6)(.5)(.4) = .12.
P(at least one isn't full) = 1 – P(all full) = 1 – .12 = .88.

b. P(only NY is full) = $P(A \cap B' \cap C') = P(A)P(B')P(C')$ = (.6)(1–.5)(1–.4) = .18.
Similarly, P(only Atlanta is full) = .12 and P(only LA is full) = .08.
So, P(exactly one full) = .18 + .12 + .08 = .38.

111. Note: $s = 0$ means that the very first candidate interviewed is hired. Each entry below is the candidate hired for the given policy and outcome.

Outcome	$s = 0$	$s = 1$	$s = 2$	$s = 3$	**Outcome**	$s = 0$	$s = 1$	$s = 2$	$s = 3$
1234	1	4	4	4	3124	3	1	4	4
1243	1	3	3	3	3142	3	1	4	2
1324	1	4	4	4	3214	3	2	1	4
1342	1	2	2	2	3241	3	2	1	1
1423	1	3	3	3	3412	3	1	1	2
1432	1	2	2	2	3421	3	2	2	1
2134	2	1	4	4	4123	4	1	3	3
2143	2	1	3	3	4132	4	1	2	2
2314	2	1	1	4	4213	4	2	1	3
2341	2	1	1	1	4231	4	2	1	1
2413	2	1	1	3	4312	4	3	1	2
2431	2	1	1	1	4321	4	3	2	1

From the table, we derive the following probability distribution based on s:

s	0	1	2	3
P(hire #1)	$\frac{6}{24}$	$\frac{11}{24}$	$\frac{10}{24}$	$\frac{6}{24}$

Therefore $s = 1$ is the best policy.

112. P(at least one occurs) $= 1 - P$(none occur) $= 1 - (1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4)$.
P(at least two occur) $= 1 - P$(none or exactly one occur) $=$
$1 - [(1 - p_1)(1 - p_2)(1 - p_3)(1 - p_4) + p_1(1 - p_2)(1 - p_3)(1 - p_4) + (1 - p_1)\, p_2(1 - p_3)(1 - p_4) +$
$(1 - p_1)(1 - p_2)p_3(1 - p_4) + (1 - p_1)(1 - p_2)(1 - p_3)p_4]$.

113. $P(A_1) = P$(draw slip 1 or 4) $= ½$; $P(A_2) = P$(draw slip 2 or 4) $= ½$;
$P(A_3) = P$(draw slip 3 or 4) $= ½$; $P(A_1 \cap A_2) = P$(draw slip 4) $= ¼$;
$P(A_2 \cap A_3) = P$(draw slip 4) $= ¼$; $P(A_1 \cap A_3) = P$(draw slip 4) $= ¼$.
Hence $P(A_1 \cap A_2) = P(A_1)P(A_2) = ¼$; $P(A_2 \cap A_3) = P(A_2)P(A_3) = ¼$; and
$P(A_1 \cap A_3) = P(A_1)P(A_3) = ¼$. Thus, there exists pairwise independence. However,
$P(A_1 \cap A_2 \cap A_3) = P$(draw slip 4) $= ¼ \neq ⅛ = P(A_1)P(A_2)P(A_3)$, so the events are not mutually independent.

114. $P(A_1 | A_2 \cap A_3) = \dfrac{P(A_1 \cap A_2 \cap A_3)}{P(A_2 \cap A_3)} = \dfrac{P(A_1)P(A_2)P(A_3)}{P(A_2)P(A_3)} = P(A_1)$.