CHAPTER 2 - Probability

SHORT ANSWER

1. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

a. List all outcomes in the event *A* that all three vehicles go in the same direction.
b. List all outcomes in the event *B* that all three vehicles take different directions.
c. List all outcomes in the event *C* that exactly two of the three vehicles turn right.
d. List all outcomes in the even *D* that exactly tow vehicles go in the same direction.
e. List outcomes in D', $C \cup D$, and $C \cap D$.

ANS:
a. Event *A* = {RRR, LLL, SSS}
b. Event *B* = {RLS, RSL, LRS, LSR, SRL, SLR}
c. Event *C* = {RRL, RRS, RLR, RSE, LRR, SRR}
d. Event *D* = {RRL, RRS, RLR, RSE, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS}
e. Event D' contains outcomes where all cars go the same direction, or they all go different directions:
D' = {RRR, LLL, SSS, RLS, RSL, LRS, LSR, SRL, SLR}
Because Event *D* totally encloses Event *C*, the compound event $C \cup D$ = D. Therefore, $C \cup D$ = {RRL, RRS, RLR, RSR, LRR, SRR, LLR, LLS, LRL, LSL, RLL, SLL, SSR, SSL, SRS, SLS, RSS, LSS}
Using similar reasoning, we see that the compound event $C \cap D = C$. Therefore, $C \cap D$ = {RRL, RRS, RLR, RSR, LRR, SRR}

PTS: 1

2. Each of a sample of four home mortgages is classified as fixed rate (F) or variable rate (V).

a. What are the 16 outcomes in *S*?
b. Which outcomes are in the event that exactly two of the selected mortgages are fixed rate?
c. Which outcomes are in the event that all four mortgages are of the same type?
d. Which outcomes are in the event that at most one of the four is a variable-rate mortgage?
e. What is the union of the events in parts (c) and (d), and what is the intersection of these two events?
f. What are the union and intersection of the two events in parts (b) and (c)?

ANS:
a.

	Home Mortgage Number			
Outcome	1	2	3	4
1	F	F	F	F
2	F	F	F	V
3	F	F	V	F
4	F	F	V	V
5	F	V	F	F
6	F	V	F	V
7	F	V	V	F
8	F	V	V	V
9	V	F	F	F
10	V	F	F	V

11	V	F	V	F
12	V	F	V	V
13	V	V	F	F
14	V	V	F	V
15	V	V	V	F
16	V	V	V	V

b. Outcome numbers 4, 6, 7, 10, 11,13
c. Outcome numbers 1, 16
d. Outcome numbers 1, 2, 3, 5, 9
e. The Union: outcomes 1, 2, 3, 5, 9, 16. The Intersection: outcome 1.
f. The Union: outcomes 1, 4, 6, 7, 10, 11, 13, 16. The Intersection: this cannot happen. (There are no outcomes in common) :$b \cap c = \varnothing$.

PTS: 1

3. A college library has five copies of a certain text on reserve. Two copies (1 and 2) are first printings, and the other three (3, 4, and 5) are second printings. A student examines these books in random order, stopping only when a second printing has been selected. One possible outcome is 4, and another is 125.

a. List the *S*.
b. Let A denote the event that exactly one book must be examined. What outcomes are in A?
c. Let B be the event that book 4 is the one selected. What outcomes are in B?
d. Let C be the event that book 2 is not examined. What outcomes are in C?

ANS:
a.

Outcome Number	Outcome
1	123
2	124
3	125
4	213
5	214
6	215
7	13
8	14
9	15
10	23
11	24
12	25
13	3
14	4
15	5

b. Outcome numbers 13, 14, 15, so A={3, 4, 5}
c. Outcome numbers 2, 5, 8, 11, 14 so B={124, 214, 14, 24, 4}
d. Outcome numbers 7, 8, 9, 13, 14, 15 so C={13, 14, 15, 3, 4, 5}

PTS: 1

4. The Department of Statistics at a state university in California has just completed voting by secret ballot for a department head. The ballot box contains four slips with votes for candidate A and three slips with votes for candidate B. Suppose these slips are removed from the box one by one.

a. List all possible outcomes.

b. Suppose a running tally is kept as slips are removed. For what outcomes does A remain ahead of B throughout the tally?

ANS:
a. S = {BBBAAAA, BBABAAA, BBAABAA, BBAAABA, BBAAAAB, BABBAAA, BABABAA, BABAABA, BABAAAB, BAABBAA, BAABABA, BAABAAB, BAAABBA, BAAABAB, BAAAABB, ABBBAAA, ABBABAA, ABBAABA, ABBAAAB, ABABBAA, ABABABA, ABABAAB, ABAABBA, ABAABAB, ABAAABB, AABBBAA, AABBABA, AABBAAB, AABABBA, AABABAB, AABAABB, AAABBBA, AAABBAB, AAABABB, AAAABBB}

b. {AAAABBB, AAABABB, AAABBAB, AABAABB, AABABAB}

PTS: 1

5. Let A denote the event that the next item checked out at a college library is a math book, and let B be the event that the next item checked out is a history book. Suppose that P(A) = .40 and P(B) = .50.

a. Why is it not the case that P(A) + P(B) = 1?
b. Calculate P(A')
c. Calculate P($A \cup B$).
d. Calculate P($A' \cap B'$).

ANS:
a. The probabilities do not add to 1 because there are other items besides math and history books to be checked out from the library.
b. $P(A') = 1 - P(A) = 1 - .40 = .60$
c. $P(A \cup B) = P(A) + P(B) = .40 + .50 = .90$ (since A and B are mutually exclusive events)
d. $P(A' \cap B') = P[(A \cup B)']$ (De Morgan's law) $= 1 - P(A \cup B) = 1 - .90 = .10$

PTS: 1

6. A large company offers its employees two different health insurance plans and two different dental insurance plans. Plan 1 of each type is relatively inexpensive, but restricts the choice of providers, whereas plan 2 is more expensive but more flexible. The accompanying table gives the percentages of employees who have chosen the various plans:

	Dental Plan	
Health Plan	1	2
1	27%	14%
2	24%	35%

Suppose that an employee is randomly selected and both the health plan and dental plan chosen by the selected employee are determined.

a. What are the four simple events?
b. What is the probability that the selected employee has chosen the more restrictive plan of each type?
c. What is the probability that the employee has chosen the more flexible dental plan?

ANS:
Let H1 and H2 represent the two health plans. Let D1 and D2 represent the two dental plans.

a. The simple events are {H1,D1}, {H1, D2}, {H2,D1}, {H2,D2}.
b. P({H1,D1}) = .27
c. P({D2}) = P({H1,D2}, {H2,D2}) = .14 + .35 = .49

PTS: 1

7. An Economic Department at a state university with five faculty members-Anderson, Box, Cox, Carter, and Davis-must select two of its members to serve on a program review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting five slips of paper in a box, mixing them, and selecting two.

a. What is the probability that both Anderson and Box will be selected? (*Hint*: List the equally likely outcomes.)
b. What is the probability that at least one of the two members whose name begins with C is selected?
c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have at least 15 years' teaching experience at the university?

ANS:
Outcomes:
(A, B), (A, C_1), (A, C_2), (A,D), (B,A), (B, C_1), (B, C_2), (B,D), (C_1,A), (C_1,B), (C_1,C_2), (C_1,D), (C_2,A), (C_2,B), (C_2,C_1), (C_2,D), (D,A), (D,B), (D, C_1), (D, C_2)

a. *P*[(A,B) or (B,A)] = 2 / 20 = .10
b. *P*(at least one C) = 14 / 20 = .70
c. *P*(at least 15 years) = 1 – *P*(at most 14 years)
= 1 – *P*[(3,6) or (6,3) or (3,7) or (7,3) or (3,10) or (10,3) or (6,7) or (7,6)]
= 1 – 8 / 20 = .60

PTS: 1

8. Student Engineers Council at an Indiana college has one student representative from each of the five engineering majors (civil, electrical, industrial, materials, and mechanical). In how many ways can

a. Both a council president and a vice president be selected?
b. A president, a vice president, and a secretary be selected?
c. Two members be selected for the President's Council?

ANS:
a. (5)(4) = 20 (5 choices for president, 4 remain for vice president)
b. (5)(4)(3) = 60
c. $\binom{5}{2} = \frac{5!}{2!3!} = 10$ (No ordering is implied in the choice)

PTS: 1

9. A real estate agent is showing homes to a prospective buyer. There are ten homes in the desired price range listed in the area. The buyer has time to visit only four of them.

a. In how many ways could the four homes be chosen if the order of visiting is considered?
b. In how many ways could the four homes be chosen if the order is disregarded?
c. If four of the homes are new and six have previously been occupied and if the four homes to visit are randomly chosen, what is the probability that all four are new? (The same answer results regardless of whether order is considered.)

ANS:
a. (10)(9)(8)(7) = 5040
b. $\binom{10}{4} = \frac{10!}{4!6!} = 210$

c. P(all 4 are new) = (# of ways of visiting all new)/(# of ways of visiting) = $\dfrac{\binom{4}{4}}{\binom{10}{4}} = \dfrac{1}{210}$

PTS: 1

10. An experimenter is studying the effects of temperature, pressure, and type of catalyst on yield from a certain chemical reaction. Three different temperatures, four different pressures, and five different catalysts are under consideration.

a. If any particular experimental run involves the use of a single temperature, pressure, and catalyst, how many experimental runs are possible?
b. How many experimental runs are there that involve use of the lowest temperature and two lowest pressures?

ANS:
a. $n_1 = 3, n_2 = 4, n_3 = 5$, so $n_1 \cdot n_2 \cdot n_3 = 60$ runs
b. $n_1 = 1$, (just one temperature), $n_2 = 2, n_3 = 5$ implies that there are 10 such runs.

PTS: 1

11. A certain sports car comes equipped with either an automatic or a manual transmission, and the car is available in one of four colors. Relevant probabilities for various combinations of transmission type and color are given in the accompanying table.

		Color		
Transmission Type	White	Blue	Black	Red
A	.13	.10	.11	.11
M	.15	.07	.15	.18

Let A = (automatic transmission), B = {black}, and C = {white}.

a. Calculate $P(A)$, $P(B)$, and $P(A \cap B)$.
b. Calculate both $P(A|B)$ and $P(B|A)$, and explain in context what each of these probabilities represents.

ANS:
a. $P(A) = .13 + .10 + .11 + .11 = .45$,
$P(B) = .11 + .15 = .26$
$P(A \cap B) = .11$
b. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.11}{.26} = .4231$

Knowing that the car is black, the probability that it has an automatic transmission is .4231.

$P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.11}{.45} = .2444$

Knowing that the car has an automatic transmission, the probability that it is black is .2444.

PTS: 1

12. Consider the following information: where A = {Visa Card}, B = {MasterCard}, P(A) = .5, P(B) = .4, and $P(A \cap B) = .25$. Calculate each of the following probabilities.

a. $P(B|A)$

b. $P(B'|A)$
c. $P(A|B)$
d. $P(A'|B)$
e. Given that an individual is selected at random and that he or she has at least one card, what is the probability that he or she has a Visa card?

ANS:

a. $P(B|A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{.25}{.50} = .50$

b. $P(B'|A) = \dfrac{P(A\cap B')}{P(A)} = \dfrac{.25}{.50} = .50$

c. $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{.25}{.40} = .6125$

d. $P(A'|B) = \dfrac{P(A'\cap B)}{P(B)} = \dfrac{.15}{.40} = .3875$

e. $P(A|A\cup B) = \dfrac{P[A\cap(A\cup B)]}{P(A\cup B)} = \dfrac{.50}{.65} = .7692$

PTS: 1

13. A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that P(A) = .625 and P(B) = .05. What is P(B/A)?

ANS:

Since B is contained in A, $A\cap B = B$, then $P(B|A) = \dfrac{P(A\cap B)}{P(A)} = \dfrac{P(B)}{P(A)} = \dfrac{.05}{.625} = .08$

PTS: 1

14. At a certain gas station, 40% of the customers use regular unleaded gas (A_1), 35% use extra unleaded gas (A_2), and 25% use premium unleaded gas (A_3). Of those customers using regular gas, only 30% fill their tanks (event *B*). Of those customers using extra gas, 60% fill their tanks, whereas of those using premium, 50% fill their tanks.

a. What is the probability that the next customer will request extra unleaded gas and fill the tank?
b. What is the probability that the next customer fills the tank?
c. If the next customer fills the tank, what is the probability that regular gas is requested? Extra gas? Premium gas?

ANS:

$P(A_1) = .40,\quad P(A_2) = .35,\quad P(A_3) = .25$
$P(B1/A_1) = .30$
$P(B1/A_2) = .60$
$P(B1/A_3) = .50$
Therefore,
$P(A_1\cap B) = P(A_1)\cdot P(B1A_1) = (.40)(.30) = .12$
$P(A_2\cap B) = P(A_2)\cdot P(B1A_2) = (.35)(.60) = .21$
$P(A_3\cap B) = P(A_3)\cdot P(B1A_3) = (.25)(.50) = .125$

a. $P(A_2 \cap B)$ = .21

b. $P(B)$ = $P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = .12 + .21 + .125 = .455$

c. P($P(A_1|B) = \dfrac{P(A_1 \cap B)}{P(B)} = \dfrac{.12}{.455} = .264$

$$P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{.21}{.455} = .462$$

$$P(A_3|B) = \frac{P(A_3 \cap B)}{P(B)} = \frac{.125}{.455} == .275$$

PTS: 1

15. Suppose that the proportions of blood phenotypes in a particular population are as follows:

A	B	AB	O
.42	.10	.04	.44

Assuming that the phenotypes of two randomly selected individuals are independent of one another, what is the probability that both phenotypes are O? What is the probability that the phenotypes of two randomly selected individuals match?

ANS:
Using subscripts to differentiate between the selected individuals,
$P(O_1 \cap O_2) = P(O_1) \cdot P(O_2)$ = (.44)(.44) = .1936P(two individuals match) = $P(A_1 \cap A_2)$ + $P(B_1 \cap B_2)$ + $P(AB_1 \cap AB_2)$ + $P(O_1 \cap O_2)$ $= .42^2 + .10^2 + .04^2 + .44^2 = .3816$

PTS: 1

16. Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .15, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

ANS:
Let A_1 = older pump fails, A_2 = newer pump fails, and x = $P(A_1 \cap A_2)$. The $P(A_1)$ = .15 + x, $P(A_2)$ = .05 + x, and x = $P(A_1 \cap A_2) = P(A_1) \cdot P(A_2) = (.15 + x)(.05 + x)$. The resulting quadratic equation, $x^2 - .80x + .0075 = 0$, has roots x = .0095 and x = .7905. Hopefully the smaller root is the actual probability of system failure.

PTS: 1

17. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities.

a. P(all of the next three vehicles inspected pass)
b. P(at least one of the next three inspected fail)
c. P(exactly one of the next three inspected passes)
d. P(at most one of the next three vehicles inspected passes)
e. Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass?

ANS:
P(pass) = .70
a. P(three pass) = (.70)(.70)(.70) = .343

b. P(at least one fails) = 1 – P(all pass) = 1 - .343 = .657
c. P(exactly one passes) = (.70)(.30)(.30) + (.30)(.70)(.30) + (.30)(.30)(.70) = .189
d. P (at most one passes) = P(0 passes) + P(one passes) = $(.3)^3$ + .189 = .216
e. P(3 pass | 1 or more pass) = $\dfrac{P(3\text{ pass} \cap \geq 1\text{ pass})}{P(\geq 1\text{ pass})} = \dfrac{P(3\text{ pass})}{P(\geq 1\text{ pass})} = \dfrac{.343}{.973} = .353$

PTS: 1