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Chapter 1

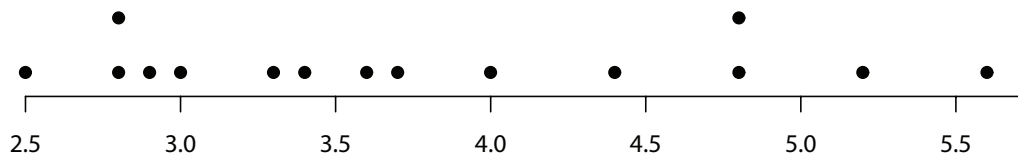
Introduction to Statistics and Data Analysis

1.1 (a) 15.

(b) $\bar{x} = \frac{1}{15}(3.4 + 2.5 + 4.8 + \dots + 4.8) = 3.787$.

(c) Sample median is the 8th value, after the data is sorted from smallest to largest: 3.6.

(d) A dot plot is shown below.



(e) After trimming total 40% of the data (20% highest and 20% lowest), the data becomes:

2.9 3.0 3.3 3.4 3.6
3.7 4.0 4.4 4.8

So, the trimmed mean is

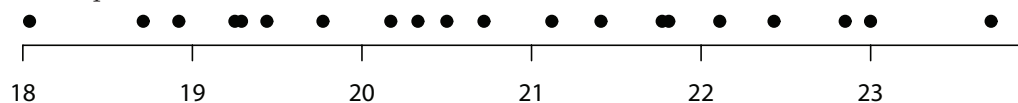
$$\bar{x}_{tr20} = \frac{1}{9}(2.9 + 3.0 + \dots + 4.8) = 3.678.$$

(f) They are about the same.

1.2 (a) Mean=20.7675 and Median=20.610.

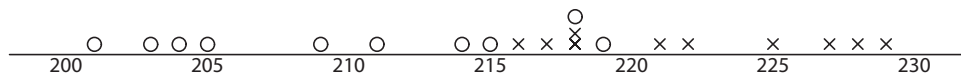
(b) $\bar{x}_{tr10} = 20.743$.

(c) A dot plot is shown below.



(d) No. They are all close to each other.

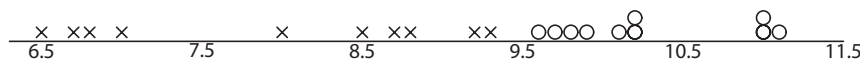
- 1.3 (a) A dot plot is shown below.



In the figure, “x” represents the “No aging” group and “o” represents the “Aging” group.

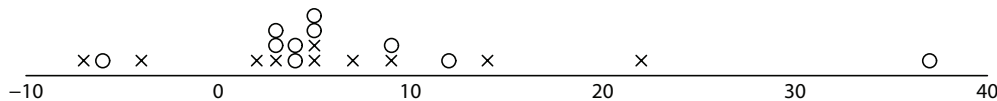
- (b) Yes; tensile strength is greatly reduced due to the aging process.
 (c) $\text{Mean}_{\text{Aging}} = 209.90$, and $\text{Mean}_{\text{No aging}} = 222.10$.
 (d) $\text{Median}_{\text{Aging}} = 210.00$, and $\text{Median}_{\text{No aging}} = 221.50$. The means and medians for each group are similar to each other.
- 1.4 (a) $\bar{X}_A = 7.950$ and $\tilde{X}_A = 8.250$;
 $\bar{X}_B = 10.260$ and $\tilde{X}_B = 10.150$.

- (b) A dot plot is shown below.



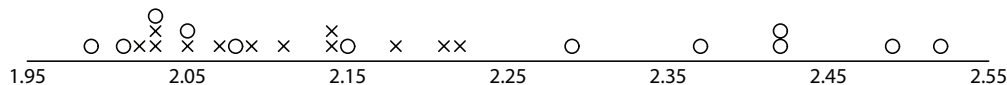
In the figure, “x” represents company *A* and “o” represents company *B*. The steel rods made by company *B* show more flexibility.

- 1.5 (a) A dot plot is shown below.



In the figure, “x” represents the control group and “o” represents the treatment group.

- (b) $\bar{X}_{\text{Control}} = 5.60$, $\tilde{X}_{\text{Control}} = 5.00$, and $\bar{X}_{\text{tr}(10);\text{Control}} = 5.13$;
 $\bar{X}_{\text{Treatment}} = 7.60$, $\tilde{X}_{\text{Treatment}} = 4.50$, and $\bar{X}_{\text{tr}(10);\text{Treatment}} = 5.63$.
 (c) The difference of the means is 2.0 and the differences of the medians and the trimmed means are 0.5, which are much smaller. The possible cause of this might be due to the extreme values (outliers) in the samples, especially the value of 37.
- 1.6 (a) A dot plot is shown below.



In the figure, “x” represents the 20°C group and “o” represents the 45°C group.

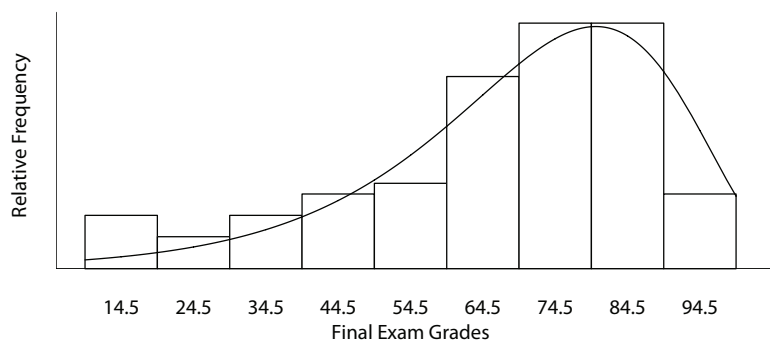
- (b) $\bar{X}_{20^\circ\text{C}} = 2.1075$, and $\bar{X}_{45^\circ\text{C}} = 2.2350$.
 (c) Based on the plot, it seems that high temperature yields more high values of tensile strength, along with a few low values of tensile strength. Overall, the temperature does have an influence on the tensile strength.
 (d) It also seems that the variation of the tensile strength gets larger when the cure temperature is increased.
- 1.7 $s^2 = \frac{1}{15-1}[(3.4 - 3.787)^2 + (2.5 - 3.787)^2 + (4.8 - 3.787)^2 + \dots + (4.8 - 3.787)^2] = 0.94284$;
 $s = \sqrt{s^2} = \sqrt{0.9428} = 0.971$.

Stem	Leaf	Frequency
1	057	3
2	35	2
3	246	3
4	1138	4
5	22457	5
6	00123445779	11
7	01244456678899	14
8	00011223445589	14
9	0258	4

(b) The following is the relative frequency distribution table.

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
10 – 19	14.5	3	0.05
20 – 29	24.5	2	0.03
30 – 39	34.5	3	0.05
40 – 49	44.5	4	0.07
50 – 59	54.5	5	0.08
60 – 69	64.5	11	0.18
70 – 79	74.5	14	0.23
80 – 89	84.5	14	0.23
90 – 99	94.5	4	0.07

(c) A histogram plot is given below.



The distribution skews to the left.

(d) $\bar{X} = 65.48$, $\tilde{X} = 71.50$ and $s = 21.13$.

1.19 (a) A stem-and-leaf plot is shown below.

Stem	Leaf	Frequency
0	22233457	8
1	023558	6
2	035	3
3	03	2
4	057	3
5	0569	4
6	0005	4

(b) The following is the relative frequency distribution table.

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
0.0 – 0.9	0.45	8	0.267
1.0 – 1.9	1.45	6	0.200
2.0 – 2.9	2.45	3	0.100
3.0 – 3.9	3.45	2	0.067
4.0 – 4.9	4.45	3	0.100
5.0 – 5.9	5.45	4	0.133
6.0 – 6.9	6.45	4	0.133

(c) $\bar{X} = 2.797$, $s = 2.227$ and Sample range is $6.5 - 0.2 = 6.3$.

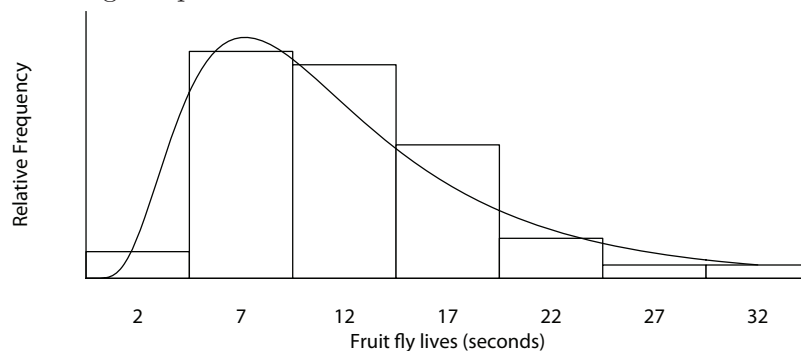
1.20 (a) A stem-and-leaf plot is shown next.

Stem	Leaf	Frequency
0*	34	2
0	56667777777889999	17
1*	0000001223333344	16
1	5566788899	10
2*	034	3
2	7	1
3*	2	1

(b) The relative frequency distribution table is shown next.

Class Interval	Class Midpoint	Frequency, f	Relative Frequency
0 – 4	2	2	0.04
5 – 9	7	17	0.34
10 – 14	12	16	0.32
15 – 19	17	10	0.20
20 – 24	22	3	0.06
25 – 29	27	1	0.02
30 – 34	32	1	0.02

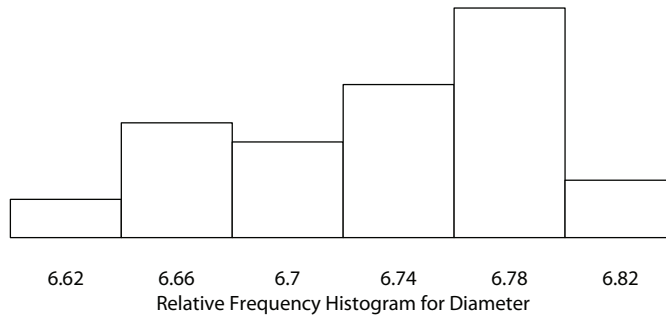
(c) A histogram plot is shown next.



(d) $\tilde{X} = 10.50$.

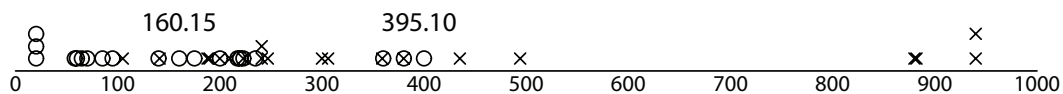
- 1.21 (a) $\bar{X} = 74.02$ and $\tilde{X} = 78$;
 (b) $s = 39.26$.

- 1.22 (a) $\bar{X} = 6.7261$ and $\tilde{X} = 0.0536$.
 (b) A histogram plot is shown next.



- (c) The data appear to be skewed to the left.

- 1.23 (a) A dot plot is shown next.

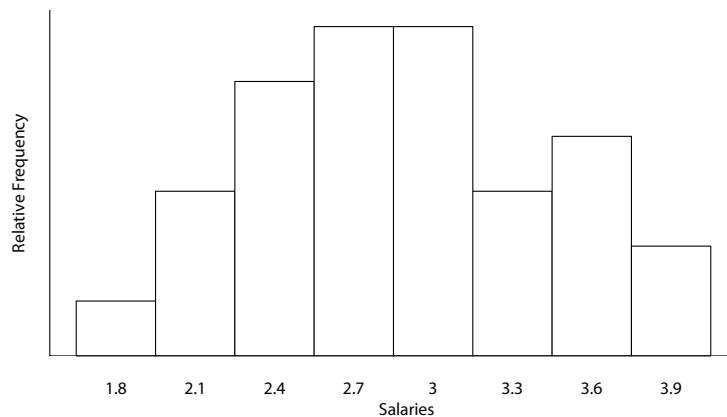


- (b) $\bar{X}_{1980} = 395.1$ and $\bar{X}_{1990} = 160.2$.

- (c) The sample mean for 1980 is over twice as large as that of 1990. The variability for 1990 decreased also as seen by looking at the picture in (a). The gap represents an increase of over 400 ppm. It appears from the data that hydrocarbon emissions decreased considerably between 1980 and 1990 and that the extreme large emission (over 500 ppm) were no longer in evidence.

- 1.24 (a) $\bar{X} = 2.8973$ and $s = 0.5415$.

- (b) A histogram plot is shown next.



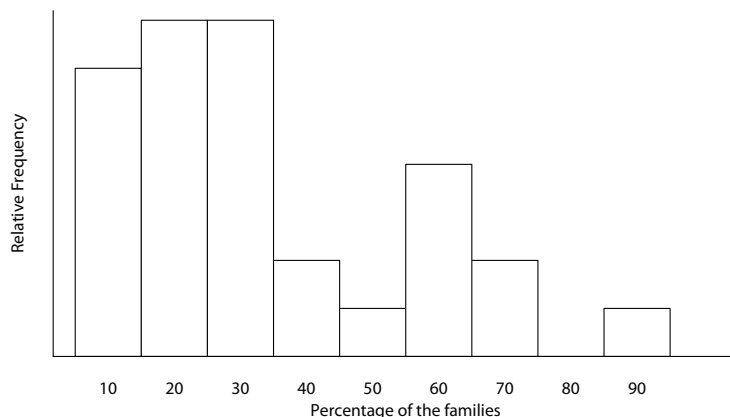
(c) Use the double-stem-and-leaf plot, we have the following.

Stem	Leaf	Frequency
1	(84)	1
2*	(05)(10)(14)(37)(44)(45)	6
2	(52)(52)(67)(68)(71)(75)(77)(83)(89)(91)(99)	11
3*	(10)(13)(14)(22)(36)(37)	6
3	(51)(54)(57)(71)(79)(85)	6

1.25 (a) $\bar{X} = 33.31$;

(b) $\tilde{X} = 26.35$;

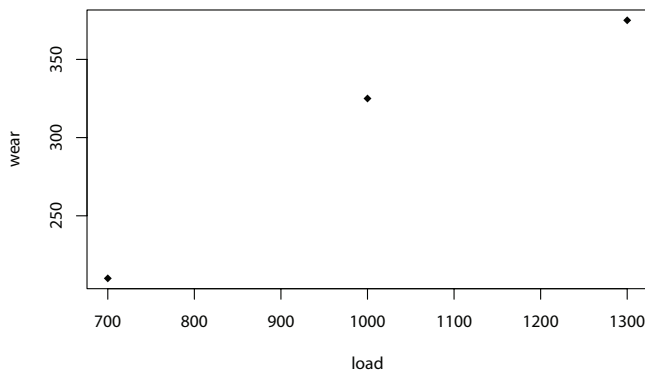
(c) A histogram plot is shown next.



(d) $\bar{X}_{tr(10)} = 30.97$. This trimmed mean is in the middle of the mean and median using the full amount of data. Due to the skewness of the data to the right (see plot in (c)), it is common to use trimmed data to have a more robust result.

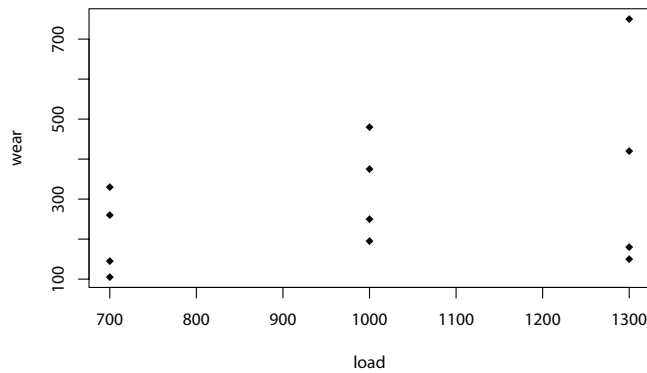
1.26 If a model using the function of percent of families to predict staff salaries, it is likely that the model would be wrong due to several extreme values of the data. Actually if a scatter plot of these two data sets is made, it is easy to see that some outlier would influence the trend.

1.27 (a) The averages of the wear are plotted here.



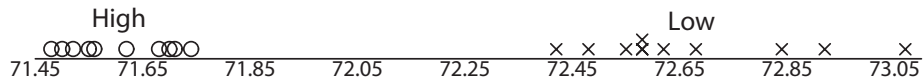
(b) When the load value increases, the wear value also increases. It does show certain relationship.

(c) A plot of wears is shown next.



(d) The relationship between load and wear in (c) is not as strong as the case in (a), especially for the load at 1300. One reason is that there is an extreme value (750) which influence the mean value at the load 1300.

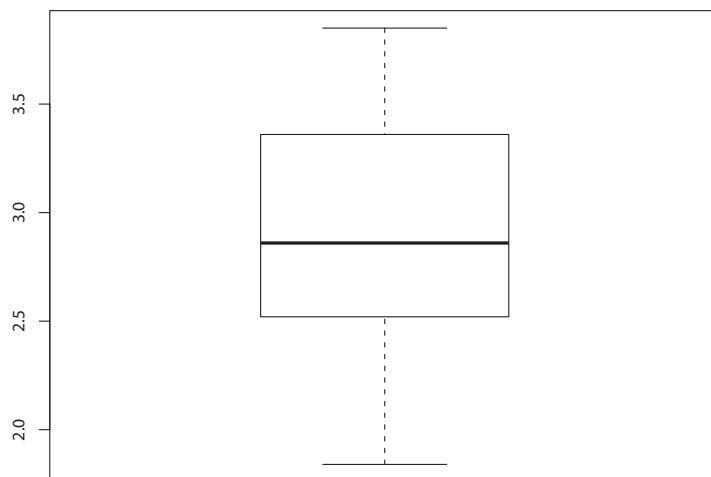
1.28 (a) A dot plot is shown next.



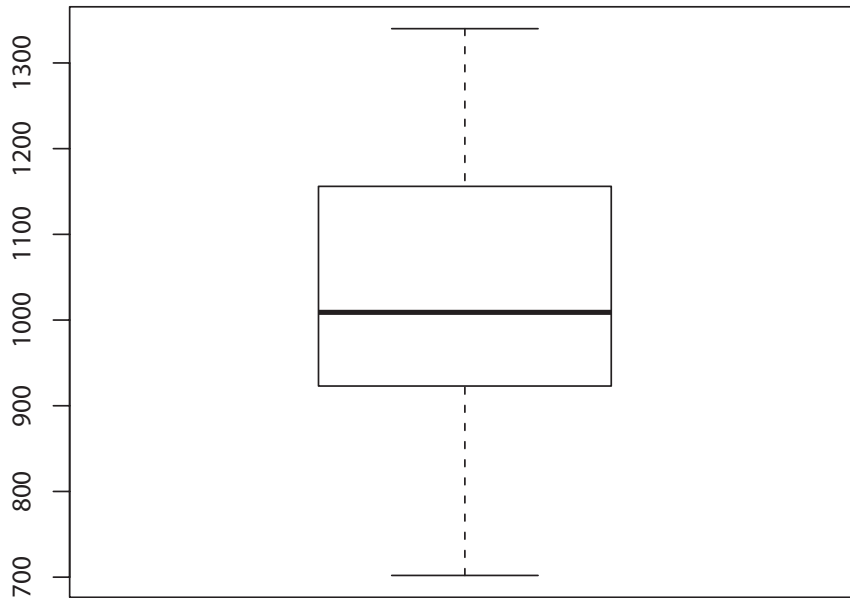
In the figure, “×” represents the low-injection-velocity group and “○” represents the high-injection-velocity group.

(b) It appears that shrinkage values for the low-injection-velocity group is higher than those for the high-injection-velocity group. Also, the variation of the shrinkage is a little larger for the low injection velocity than that for the high injection velocity.

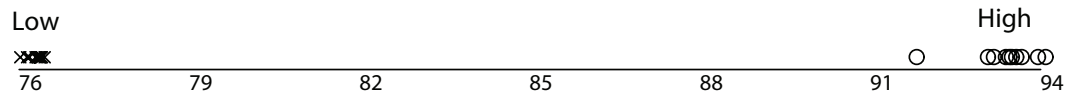
1.29 A box plot is shown next.



1.30 A box plot is shown next.



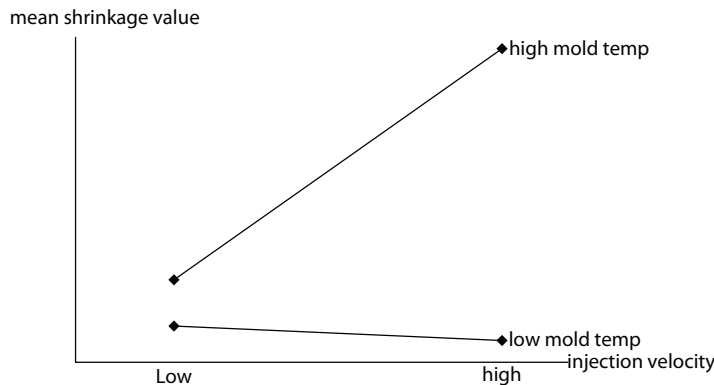
1.31 (a) A dot plot is shown next.



In the figure, “x” represents the low-injection-velocity group and “o” represents the high-injection-velocity group.

- (b) In this time, the shrinkage values are much higher for the high-injection-velocity group than those for the low-injection-velocity group. Also, the variation for the former group is much higher as well.
- (c) Since the shrinkage effects change in different direction between low mode temperature and high mold temperature, the apparent interactions between the mold temperature and injection velocity are significant.

1.32 An interaction plot is shown next.



It is quite obvious to find the interaction between the two variables. Since in this experimental data, those two variables can be controlled each at two levels, the interaction can be inves-

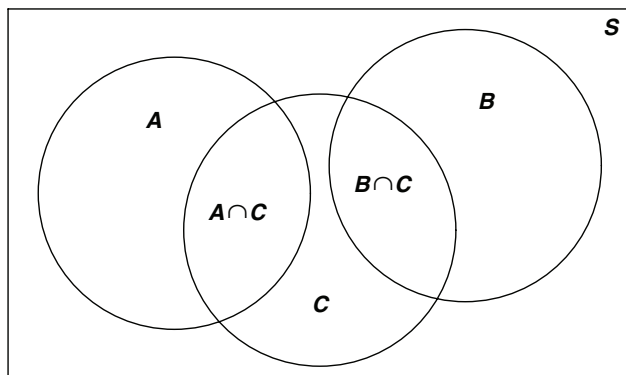
tigated. However, if the data are from an observational studies, in which the variable values cannot be controlled, it would be difficult to study the interactions among these variables.

Chapter 2

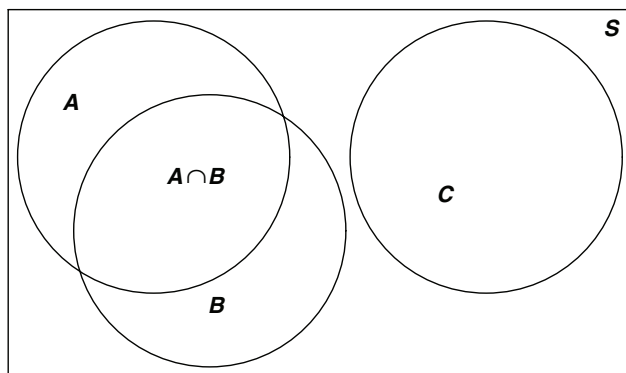
Probability

- 2.1 (a) $S = \{8, 16, 24, 32, 40, 48\}$.
(b) For $x^2 + 4x - 5 = (x + 5)(x - 1) = 0$, the only solutions are $x = -5$ and $x = 1$.
 $S = \{-5, 1\}$.
(c) $S = \{T, HT, HHT, HHH\}$.
(d) $S = \{\text{N. America, S. America, Europe, Asia, Africa, Australia, Antarctica}\}$.
(e) Solving $2x - 4 \geq 0$ gives $x \geq 2$. Since we must also have $x < 1$, it follows that $S = \phi$.
- 2.2 $S = \{(x, y) \mid x^2 + y^2 < 9; x \geq 0, y \geq 0\}$.
- 2.3 (a) $A = \{1, 3\}$.
(b) $B = \{1, 2, 3, 4, 5, 6\}$.
(c) $C = \{x \mid x^2 - 4x + 3 = 0\} = \{x \mid (x - 1)(x - 3) = 0\} = \{1, 3\}$.
(d) $D = \{0, 1, 2, 3, 4, 5, 6\}$. Clearly, $A = C$.
- 2.4 (a) $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.
(b) $S = \{(x, y) \mid 1 \leq x, y \leq 6\}$.
- 2.5 $S = \{1HH, 1HT, 1TH, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T\}$.
- 2.6 $S = \{A_1A_2, A_1A_3, A_1A_4, A_2A_3, A_2A_4, A_3A_4\}$.
- 2.7 $S_1 = \{MMMM, MMMF, MMFM, MFMM, FMMM, MMFF, MFMF, MFFM, FMFM, FFMM, FMMF, MFFF, FMFF, FFMF, FFFM, FFFF\}$.
 $S_2 = \{0, 1, 2, 3, 4\}$.
- 2.8 (a) $A = \{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$.
(b) $B = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6)\}$.

- (c) $C = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.
 (d) $A \cap C = \{(5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$.
 (e) $A \cap B = \phi$.
 (f) $B \cap C = \{(5, 2), (6, 2)\}$.
 (g) A Venn diagram is shown next.



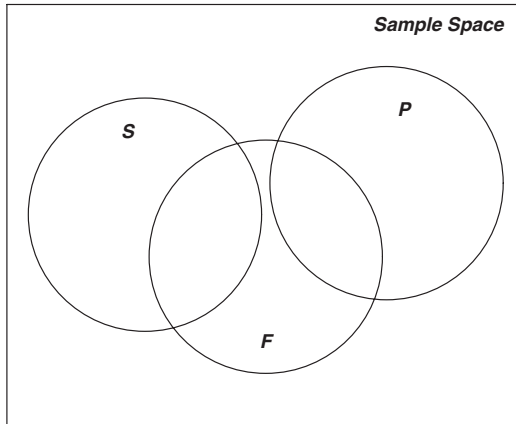
- 2.9 (a) $A = \{1HH, 1HT, 1TH, 1TT, 2H, 2T\}$.
 (b) $B = \{1TT, 3TT, 5TT\}$.
 (c) $A' = \{3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T\}$.
 (d) $A' \cap B = \{3TT, 5TT\}$.
 (e) $A \cup B = \{1HH, 1HT, 1TH, 1TT, 2H, 2T, 3TT, 5TT\}$.
- 2.10 (a) $S = \{FFF, FFN, FNF, NFF, FNN, NFN, NNF, NNN\}$.
 (b) $E = \{FFF, FFN, FNF, NFF\}$.
 (c) The second river was safe for fishing.
- 2.11 (a) $S = \{M_1M_2, M_1F_1, M_1F_2, M_2M_1, M_2F_1, M_2F_2, F_1M_1, F_1M_2, F_1F_2, F_2M_1, F_2M_2, F_2F_1\}$.
 (b) $A = \{M_1M_2, M_1F_1, M_1F_2, M_2M_1, M_2F_1, M_2F_2\}$.
 (c) $B = \{M_1F_1, M_1F_2, M_2F_1, M_2F_2, F_1M_1, F_1M_2, F_2M_1, F_2M_2\}$.
 (d) $C = \{F_1F_2, F_2F_1\}$.
 (e) $A \cap B = \{M_1F_1, M_1F_2, M_2F_1, M_2F_2\}$.
 (f) $A \cup C = \{M_1M_2, M_1F_1, M_1F_2, M_2M_1, M_2F_1, M_2F_2, F_1F_2, F_2F_1\}$.



(g)

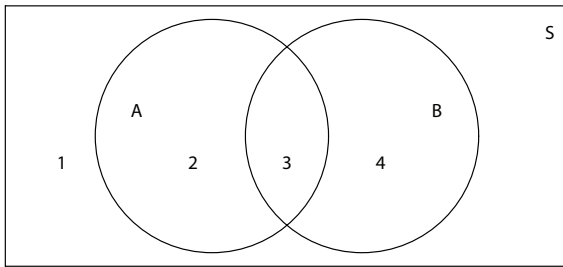
- 2.12 (a) $S = \{ZYF, ZNF, WYF, WNF, SYF, SNF, ZYM\}$.
 (b) $A \cup B = \{ZYF, ZNF, WYF, WNF, SYF, SNF\} = A$.
 (c) $A \cap B = \{WYF, SYF\}$.

2.13 A Venn diagram is shown next.

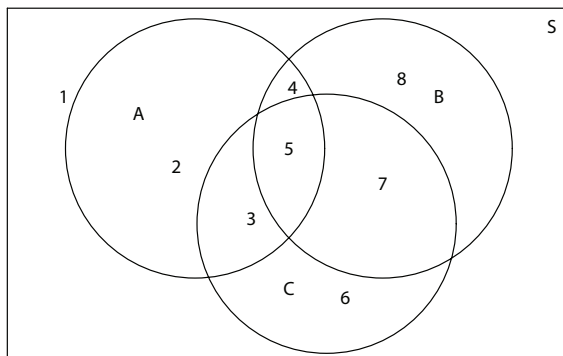


- 2.14 (a) $A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$.
 (b) $A \cap B = \phi$.
 (c) $C' = \{0, 1, 6, 7, 8, 9\}$.
 (d) $C' \cap D = \{1, 6, 7\}$, so $(C' \cap D) \cup B = \{1, 3, 5, 6, 7, 9\}$.
 (e) $(S \cap C)' = C' = \{0, 1, 6, 7, 8, 9\}$.
 (f) $A \cap C = \{2, 4\}$, so $A \cap C \cap D' = \{2, 4\}$.
- 2.15 (a) $A' = \{\text{nitrogen, potassium, uranium, oxygen}\}$.
 (b) $A \cup C = \{\text{copper, sodium, zinc, oxygen}\}$.
 (c) $A \cap B' = \{\text{copper, zinc}\}$ and
 $C' = \{\text{copper, sodium, nitrogen, potassium, uranium, zinc}\}$;
 so $(A \cap B') \cup C' = \{\text{copper, sodium, nitrogen, potassium, uranium, zinc}\}$.
 (d) $B' \cap C' = \{\text{copper, uranium, zinc}\}$.
 (e) $A \cap B \cap C = \phi$.
 (f) $A' \cup B' = \{\text{copper, nitrogen, potassium, uranium, oxygen, zinc}\}$ and
 $A' \cap C = \{\text{oxygen}\}$; so, $(A' \cup B') \cap (A' \cap C) = \{\text{oxygen}\}$.
- 2.16 (a) $M \cup N = \{x \mid 0 < x < 9\}$.
 (b) $M \cap N = \{x \mid 1 < x < 5\}$.
 (c) $M' \cap N' = \{x \mid 9 < x < 12\}$.

2.17 A Venn diagram is shown next.



- (a) From the above Venn diagram, $(A \cap B)'$ contains the regions of 1, 2 and 4.
 (b) $(A \cup B)'$ contains region 1.
 (c) A Venn diagram is shown next.



$(A \cap C) \cup B$ contains the regions of 3, 4, 5, 7 and 8.

- 2.18 (a) Not mutually exclusive.
 (b) Mutually exclusive.
 (c) Not mutually exclusive.
 (d) Mutually exclusive.
- 2.19 (a) The family will experience mechanical problems but will receive no ticket for traffic violation and will not arrive at a campsite that has no vacancies.
 (b) The family will receive a traffic ticket and arrive at a campsite that has no vacancies but will not experience mechanical problems.
 (c) The family will experience mechanical problems and will arrive at a campsite that has no vacancies.
 (d) The family will receive a traffic ticket but will not arrive at a campsite that has no vacancies.
 (e) The family will not experience mechanical problems.
- 2.20 (a) 6;
 (b) 2;
 (c) 2, 5, 6;
 (d) 4, 5, 7, 8.
- 2.21 With $n_1 = 6$ sightseeing tours each available on $n_2 = 3$ different days, the multiplication rule gives $n_1 n_2 = (6)(3) = 18$ ways for a person to arrange a tour.

- 2.22 With $n_1 = 8$ blood types and $n_2 = 3$ classifications of blood pressure, the multiplication rule gives $n_1 n_2 = (8)(3) = 24$ classifications.
- 2.23 Since the die can land in $n_1 = 6$ ways and a letter can be selected in $n_2 = 26$ ways, the multiplication rule gives $n_1 n_2 = (6)(26) = 156$ points in S .
- 2.24 Since a student may be classified according to $n_1 = 4$ class standing and $n_2 = 2$ gender classifications, the multiplication rule gives $n_1 n_2 = (4)(2) = 8$ possible classifications for the students.
- 2.25 With $n_1 = 5$ different shoe styles in $n_2 = 4$ different colors, the multiplication rule gives $n_1 n_2 = (5)(4) = 20$ different pairs of shoes.
- 2.26 Using Theorem 2.8, we obtain the followings.
- (a) There are $\binom{7}{5} = 21$ ways.
 - (b) There are $\binom{5}{3} = 10$ ways.
- 2.27 Using the generalized multiplication rule, there are $n_1 \times n_2 \times n_3 \times n_4 = (4)(3)(2)(2) = 48$ different house plans available.
- 2.28 With $n_1 = 5$ different manufacturers, $n_2 = 3$ different preparations, and $n_3 = 2$ different strengths, the generalized multiplication rule yields $n_1 n_2 n_3 = (5)(3)(2) = 30$ different ways to prescribe a drug for asthma.
- 2.29 With $n_1 = 3$ race cars, $n_2 = 5$ brands of gasoline, $n_3 = 7$ test sites, and $n_4 = 2$ drivers, the generalized multiplication rule yields $(3)(5)(7)(2) = 210$ test runs.
- 2.30 With $n_1 = 2$ choices for the first question, $n_2 = 2$ choices for the second question, and so forth, the generalized multiplication rule yields $n_1 n_2 \cdots n_9 = 2^9 = 512$ ways to answer the test.
- 2.31 Since the first digit is a 5, there are $n_1 = 9$ possibilities for the second digit and then $n_2 = 8$ possibilities for the third digit. Therefore, by the multiplication rule there are $n_1 n_2 = (9)(8) = 72$ registrations to be checked.
- 2.32 (a) By Theorem 2.3, there are $6! = 720$ ways.
- (b) A certain 3 persons can follow each other in a line of 6 people in a specified order is 4 ways or in $(4)(3!) = 24$ ways with regard to order. The other 3 persons can then be placed in line in $3! = 6$ ways. By Theorem 2.1, there are total $(24)(6) = 144$ ways to line up 6 people with a certain 3 following each other.
 - (c) Similar as in (b), the number of ways that a specified 2 persons can follow each other in a line of 6 people is $(5)(2!)(4!) = 240$ ways. Therefore, there are $720 - 240 = 480$ ways if a certain 2 persons refuse to follow each other.
- 2.33 (a) With $n_1 = 4$ possible answers for the first question, $n_2 = 4$ possible answers for the second question, and so forth, the generalized multiplication rule yields $4^5 = 1024$ ways to answer the test.

- (b) With $n_1 = 3$ wrong answers for the first question, $n_2 = 3$ wrong answers for the second question, and so forth, the generalized multiplication rule yields

$$n_1 n_2 n_3 n_4 n_5 = (3)(3)(3)(3)(3) = 3^5 = 243$$

ways to answer the test and get all questions wrong.

- 2.34 (a) By Theorem 2.3, $7! = 5040$.
- (b) Since the first letter must be m , the remaining 6 letters can be arranged in $6! = 720$ ways.
- 2.35 The first house can be placed on any of the $n_1 = 9$ lots, the second house on any of the remaining $n_2 = 8$ lots, and so forth. Therefore, there are $9! = 362,880$ ways to place the 9 homes on the 9 lots.
- 2.36 (a) Any of the 6 nonzero digits can be chosen for the hundreds position, and of the remaining 6 digits for the tens position, leaving 5 digits for the units position. So, there are $(6)(6)(5) = 180$ three digit numbers.
- (b) The units position can be filled using any of the 3 odd digits. Any of the remaining 5 nonzero digits can be chosen for the hundreds position, leaving a choice of 5 digits for the tens position. By Theorem 2.2, there are $(3)(5)(5) = 75$ three digit odd numbers.
- (c) If a 4, 5, or 6 is used in the hundreds position there remain 6 and 5 choices, respectively, for the tens and units positions. This gives $(3)(6)(5) = 90$ three digit numbers beginning with a 4, 5, or 6. If a 3 is used in the hundreds position, then a 4, 5, or 6 must be used in the tens position leaving 5 choices for the units position. In this case, there are $(1)(3)(5) = 15$ three digit number begin with a 3. So, the total number of three digit numbers that are greater than 330 is $90 + 15 = 105$.
- 2.37 The first seat must be filled by any of 5 girls and the second seat by any of 4 boys. Continuing in this manner, the total number of ways to seat the 5 girls and 4 boys is $(5)(4)(4)(3)(3)(2)(2)(1)(1) = 2880$.
- 2.38 (a) $8! = 40320$.
- (b) There are $4!$ ways to seat 4 couples and then each member of a couple can be interchanged resulting in $2^4(4!) = 384$ ways.
- (c) By Theorem 2.3, the members of each gender can be seated in $4!$ ways. Then using Theorem 2.1, both men and women can be seated in $(4!)(4!) = 576$ ways.
- 2.39 (a) Any of the $n_1 = 8$ finalists may come in first, and of the $n_2 = 7$ remaining finalists can then come in second, and so forth. By Theorem 2.3, there $8! = 40320$ possible orders in which 8 finalists may finish the spelling bee.
- (b) The possible orders for the first three positions are ${}_8P_3 = \frac{8!}{5!} = 336$.
- 2.40 By Theorem 2.4, ${}_8P_5 = \frac{8!}{3!} = 6720$.
- 2.41 By Theorem 2.4, ${}_6P_4 = \frac{6!}{2!} = 360$.
- 2.42 By Theorem 2.4, ${}_{40}P_3 = \frac{40!}{37!} = 59,280$.

- 2.43 By Theorem 2.5, there are $4! = 24$ ways.
- 2.44 By Theorem 2.5, there are $7! = 5040$ arrangements.
- 2.45 By Theorem 2.6, there are $\frac{8!}{3!2!} = 3360$.
- 2.46 By Theorem 2.6, there are $\frac{9!}{3!4!2!} = 1260$ ways.
- 2.47 By Theorem 2.8, there are $\binom{8}{3} = 56$ ways.
- 2.48 Assume February 29th as March 1st for the leap year. There are total 365 days in a year. The number of ways that all these 60 students will have different birth dates (i.e, arranging 60 from 365) is ${}_{365}P_{60}$. This is a very large number.
- 2.49 (a) Sum of the probabilities exceeds 1.
 (b) Sum of the probabilities is less than 1.
 (c) A negative probability.
 (d) Probability of both a heart and a black card is zero.
- 2.50 Assuming equal weights
- (a) $P(A) = \frac{5}{18}$;
 (b) $P(C) = \frac{1}{3}$;
 (c) $P(A \cap C) = \frac{7}{36}$.
- 2.51 $S = \{\$10, \$25, \$100\}$ with weights $275/500 = 11/20$, $150/500 = 3/10$, and $75/500 = 3/20$, respectively. The probability that the first envelope purchased contains less than \$100 is equal to $11/20 + 3/10 = 17/20$.
- 2.52 (a) $P(S \cap D') = 88/500 = 22/125$.
 (b) $P(E \cap D \cap S') = 31/500$.
 (c) $P(S' \cap E') = 171/500$.
- 2.53 Consider the events
 S : industry will locate in Shanghai,
 B : industry will locate in Beijing.
- (a) $P(S \cap B) = P(S) + P(B) - P(S \cup B) = 0.7 + 0.4 - 0.8 = 0.3$.
 (b) $P(S' \cap B') = 1 - P(S \cup B) = 1 - 0.8 = 0.2$.
- 2.54 Consider the events
 B : customer invests in tax-free bonds,
 M : customer invests in mutual funds.
- (a) $P(B \cup M) = P(B) + P(M) - P(B \cap M) = 0.6 + 0.3 - 0.15 = 0.75$.
 (b) $P(B' \cap M') = 1 - P(B \cup M) = 1 - 0.75 = 0.25$.
- 2.55 By Theorem 2.2, there are $N = (26)(25)(24)(9)(8)(7)(6) = 47,174,400$ possible ways to code the items of which $n = (5)(25)(24)(8)(7)(6)(4) = 4,032,000$ begin with a vowel and end with an even digit. Therefore, $\frac{n}{N} = \frac{10}{117}$.

- 2.56 (a) Let A = Defect in brake system; B = Defect in fuel system; $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.17 - 0.15 = 0.27$.
 (b) $P(\text{No defect}) = 1 - P(A \cup B) = 1 - 0.27 = 0.73$.
- 2.57 (a) Since 5 of the 26 letters are vowels, we get a probability of $5/26$.
 (b) Since 9 of the 26 letters precede j , we get a probability of $9/26$.
 (c) Since 19 of the 26 letters follow g , we get a probability of $19/26$.
- 2.58 (a) Of the $(6)(6) = 36$ elements in the sample space, only 5 elements $(2,6)$, $(3,5)$, $(4,4)$, $(5,3)$, and $(6,2)$ add to 8. Hence the probability of obtaining a total of 8 is then $5/36$.
 (b) Ten of the 36 elements total at most 5. Hence the probability of obtaining a total of at most is $10/36 = 5/18$.
- 2.59 (a) $\frac{\binom{4}{3}\binom{48}{2}}{\binom{52}{5}} = \frac{94}{54145}$.
 (b) $\frac{\binom{13}{4}\binom{13}{1}}{\binom{52}{5}} = \frac{143}{39984}$.
- 2.60 (a) $\frac{\binom{1}{1}\binom{8}{2}}{\binom{9}{3}} = \frac{1}{3}$.
 (b) $\frac{\binom{5}{2}\binom{3}{1}}{\binom{9}{3}} = \frac{5}{14}$.
- 2.61 (a) $P(M \cup H) = 88/100 = 22/25$;
 (b) $P(M' \cap H') = 12/100 = 3/25$;
 (c) $P(H \cap M') = 34/100 = 17/50$.
- 2.62 (a) 9;
 (b) $1/9$.
- 2.63 (a) 0.32;
 (b) 0.68;
 (c) office or den.
- 2.64 (a) $1 - 0.42 = 0.58$;
 (b) $1 - 0.04 = 0.96$.
- 2.65 $P(A) = 0.2$ and $P(B) = 0.35$
 (a) $P(A') = 1 - 0.2 = 0.8$;
 (b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.2 - 0.35 = 0.45$;
 (c) $P(A \cup B) = 0.2 + 0.35 = 0.55$.
- 2.66 (a) $0.02 + 0.30 = 0.32 = 32\%$;
 (b) $0.32 + 0.25 + 0.30 = 0.87 = 87\%$;
 (c) $0.05 + 0.06 + 0.02 = 0.13 = 13\%$;
 (d) $1 - 0.05 - 0.32 = 0.63 = 63\%$.

- 2.67 (a) $0.12 + 0.19 = 0.31$;
 (b) $1 - 0.07 = 0.93$;
 (c) $0.12 + 0.19 = 0.31$.
- 2.68 (a) $1 - 0.40 = 0.60$.
 (b) The probability that all six purchasing the electric oven or all six purchasing the gas oven is $0.007 + 0.104 = 0.111$. So the probability that at least one of each type is purchased is $1 - 0.111 = 0.889$.
- 2.69 (a) $P(C) = 1 - P(A) - P(B) = 1 - 0.990 - 0.001 = 0.009$;
 (b) $P(B') = 1 - P(B) = 1 - 0.001 = 0.999$;
 (c) $P(B) + P(C) = 0.01$.
- 2.70 (a) $(\$4.50 - \$4.00) \times 50,000 = \$25,000$;
 (b) Since the probability of underfilling is 0.001, we would expect $50,000 \times 0.001 = 50$ boxes to be underfilled. So, instead of having $(\$4.50 - \$4.00) \times 50 = \$25$ profit for those 50 boxes, there are a loss of $\$4.00 \times 50 = \200 due to the cost. So, the loss in profit expected due to underfilling is $\$25 + \$200 = \$250$.
- 2.71 (a) $1 - 0.95 - 0.002 = 0.048$;
 (b) $(\$25.00 - \$20.00) \times 10,000 = \$50,000$;
 (c) $(0.05)(10,000) \times \$5.00 + (0.05)(10,000) \times \$20 = \$12,500$.
- 2.72 $P(A' \cap B') = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1 + P(A \cap B) - P(A) - P(B)$.
- 2.73 (a) The probability that a convict who pushed dope, also committed armed robbery.
 (b) The probability that a convict who committed armed robbery, did not push dope.
 (c) The probability that a convict who did not push dope also did not commit armed robbery.
- 2.74 $P(S | A) = 10/18 = 5/9$.
- 2.75 Consider the events:
M: a person is a male;
S: a person has a secondary education;
C: a person has a college degree.
- (a) $P(M | S) = 28/78 = 14/39$;
 (b) $P(C' | M') = 95/112$.
- 2.76 Consider the events:
A: a person is experiencing hypertension,
B: a person is a heavy smoker,
C: a person is a nonsmoker.
- (a) $P(A | B) = 30/49$;
 (b) $P(C | A') = 48/93 = 16/31$.

- 2.77 (a) $P(M \cap P \cap H) = \frac{10}{68} = \frac{5}{34}$;
 (b) $P(H \cap M \mid P') = \frac{P(H \cap M \cap P')}{P(P')} = \frac{22-10}{100-68} = \frac{12}{32} = \frac{3}{8}$.
- 2.78 (a) $(0.90)(0.08) = 0.072$;
 (b) $(0.90)(0.92)(0.12) = 0.099$.
- 2.79 (a) 0.018;
 (b) $0.22 + 0.002 + 0.160 + 0.102 + 0.046 + 0.084 = 0.614$;
 (c) $0.102/0.614 = 0.166$;
 (d) $\frac{0.102+0.046}{0.175+0.134} = 0.479$.
- 2.80 Consider the events:
C: an oil change is needed,
F: an oil filter is needed.
- (a) $P(F \mid C) = \frac{P(F \cap C)}{P(C)} = \frac{0.14}{0.25} = 0.56$.
 (b) $P(C \mid F) = \frac{P(C \cap F)}{P(F)} = \frac{0.14}{0.40} = 0.35$.
- 2.81 Consider the events:
H: husband watches a certain show,
W: wife watches the same show.
- (a) $P(W \cap H) = P(W)P(H \mid W) = (0.5)(0.7) = 0.35$.
 (b) $P(W \mid H) = \frac{P(W \cap H)}{P(H)} = \frac{0.35}{0.4} = 0.875$.
 (c) $P(W \cup H) = P(W) + P(H) - P(W \cap H) = 0.5 + 0.4 - 0.35 = 0.55$.
- 2.82 Consider the events:
H: the husband will vote on the bond referendum,
W: the wife will vote on the bond referendum.
 Then $P(H) = 0.21$, $P(W) = 0.28$, and $P(H \cap W) = 0.15$.
- (a) $P(H \cup W) = P(H) + P(W) - P(H \cap W) = 0.21 + 0.28 - 0.15 = 0.34$.
 (b) $P(W \mid H) = \frac{P(H \cap W)}{P(H)} = \frac{0.15}{0.21} = \frac{5}{7}$.
 (c) $P(H \mid W') = \frac{P(H \cap W')}{P(W')} = \frac{0.06}{0.72} = \frac{1}{12}$.
- 2.83 Consider the events:
A: the vehicle is a camper,
B: the vehicle has Canadian license plates.
- (a) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.09}{0.28} = \frac{9}{28}$.
 (b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.09}{0.12} = \frac{3}{4}$.
 (c) $P(B' \cup A') = 1 - P(A \cap B) = 1 - 0.09 = 0.91$.

2.84 Define

H : head of household is home,

C : a change is made in long distance carriers.

$$P(H \cap C) = P(H)P(C | H) = (0.4)(0.3) = 0.12.$$

2.85 Consider the events:

A : the doctor makes a correct diagnosis,

B : the patient sues.

$$P(A' \cap B) = P(A')P(B | A') = (0.3)(0.9) = 0.27.$$

2.86 (a) 0.43;

(b) $(0.53)(0.22) = 0.12$;

(c) $1 - (0.47)(0.22) = 0.90$.

2.87 Consider the events:

A : the house is open,

B : the correct key is selected.

$$P(A) = 0.4, P(A') = 0.6, \text{ and } P(B) = \frac{\binom{1}{1}\binom{7}{2}}{\binom{8}{3}} = \frac{3}{8} = 0.375.$$

$$\text{So, } P[A \cup (A' \cap B)] = P(A) + P(A')P(B) = 0.4 + (0.6)(0.375) = 0.625.$$

2.88 Consider the events:

F : failed the test,

P : passed the test.

(a) $P(\text{failed at least one tests}) = 1 - P(P_1P_2P_3P_4) = 1 - (0.99)(0.97)(0.98)(0.99) = 1 - 0.93 = 0.07$,

(b) $P(\text{failed 2 or 3}) = 1 - P(P_2P_3) = 1 - (0.97)(0.98) = 0.0494$.

(c) $100 \times 0.07 = 7$.

(d) 0.25.

2.89 Let A and B represent the availability of each fire engine.

(a) $P(A' \cap B') = P(A')P(B') = (0.04)(0.04) = 0.0016$.

(b) $P(A \cup B) = 1 - P(A' \cap B') = 1 - 0.0016 = 0.9984$.

2.90 (a) $P(A \cap B \cap C) = P(C | A \cap B)P(B | A)P(A) = (0.20)(0.75)(0.3) = 0.045$.

(b) $P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C) = P(C | A \cap B')P(B' | A)P(A) + P(C | A' \cap B')P(B' | A')P(A') = (0.80)(1 - 0.75)(0.3) + (0.90)(1 - 0.20)(1 - 0.3) = 0.564$.

(c) Use similar argument as in (a) and (b), $P(C) = P(A \cap B \cap C) + P(A \cap B' \cap C) + P(A' \cap B \cap C) + P(A' \cap B' \cap C) = 0.045 + 0.060 + 0.021 + 0.504 = 0.630$.

(d) $P(A | B' \cap C) = P(A \cap B' \cap C) / P(B' \cap C) = (0.06)(0.564) = 0.1064$.

2.91 (a) $P(Q_1 \cap Q_2 \cap Q_3 \cap Q_4) = P(Q_1)P(Q_2 | Q_1)P(Q_3 | Q_1 \cap Q_2)P(Q_4 | Q_1 \cap Q_2 \cap Q_3) = (15/20)(14/19)(13/18)(12/17) = 91/323$.

(b) Let A be the event that 4 good quarts of milk are selected. Then

$$P(A) = \frac{\binom{15}{4}}{\binom{20}{4}} = \frac{91}{323}.$$

2.92 $P = (0.95)[1 - (1 - 0.7)(1 - 0.8)](0.9) = 0.8037.$

2.93 This is a parallel system of two series subsystems.

(a) $P = 1 - [1 - (0.7)(0.7)][1 - (0.8)(0.8)(0.8)] = 0.75112.$

(b) $P = \frac{P(A' \cap C \cap D \cap E)}{P_{\text{system works}}} = \frac{(0.3)(0.8)(0.8)(0.8)}{0.75112} = 0.2045.$

2.94 Define S : the system works.

$$P(A' | S') = \frac{P(A' \cap S')}{P(S')} = \frac{P(A')(1 - P(C \cap D \cap E))}{1 - P(S)} = \frac{(0.3)[1 - (0.8)(0.8)(0.8)]}{1 - 0.75112} = 0.588.$$

2.95 Consider the events:

C : an adult selected has cancer,

D : the adult is diagnosed as having cancer.

$P(C) = 0.05$, $P(D | C) = 0.78$, $P(C') = 0.95$ and $P(D | C') = 0.06$. So, $P(D) = P(C \cap D) + P(C' \cap D) = (0.05)(0.78) + (0.95)(0.06) = 0.096.$

2.96 Let S_1, S_2, S_3 , and S_4 represent the events that a person is speeding as he passes through the respective locations and let R represent the event that the radar traps is operating resulting in a speeding ticket. Then the probability that he receives a speeding ticket:

$$P(R) = \sum_{i=1}^4 P(R | S_i)P(S_i) = (0.4)(0.2) + (0.3)(0.1) + (0.2)(0.5) + (0.3)(0.2) = 0.27.$$

2.97 $P(C | D) = \frac{P(C \cap D)}{P(D)} = \frac{0.039}{0.096} = 0.40625.$

2.98 $P(S_2 | R) = \frac{P(R \cap S_2)}{P(R)} = \frac{0.03}{0.27} = 1/9.$

2.99 Consider the events:

A : no expiration date,

B_1 : John is the inspector, $P(B_1) = 0.20$ and $P(A | B_1) = 0.005$,

B_2 : Tom is the inspector, $P(B_2) = 0.60$ and $P(A | B_2) = 0.010$,

B_3 : Jeff is the inspector, $P(B_3) = 0.15$ and $P(A | B_3) = 0.011$,

B_4 : Pat is the inspector, $P(B_4) = 0.05$ and $P(A | B_4) = 0.005$,

$$P(B_1 | A) = \frac{(0.005)(0.20)}{(0.005)(0.20) + (0.010)(0.60) + (0.011)(0.15) + (0.005)(0.05)} = 0.1124.$$

2.100 Consider the events

E : a malfunction by other human errors,

A : station A , B : station B , and C : station C .

$$P(C | E) = \frac{P(E | C)P(C)}{P(E | A)P(A) + P(E | B)P(B) + P(E | C)P(C)} = \frac{(5/10)(10/43)}{(7/18)(18/43) + (7/15)(15/43) + (5/10)(10/43)} = \frac{0.1163}{0.4419} = 0.2632.$$

2.101 Consider the events:

A : a customer purchases latex paint,

A' : a customer purchases semigloss paint,

B : a customer purchases rollers.

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')} = \frac{(0.60)(0.75)}{(0.60)(0.75) + (0.25)(0.30)} = 0.857.$$

2.102 If we use the assumptions that the host would not open the door you picked nor the door with the prize behind it, we can use Bayes rule to solve the problem. Denote by events A , B , and C , that the prize is behind doors A , B , and C , respectively. Of course $P(A) = P(B) = P(C) = 1/3$. Denote by H the event that you picked door A and the host opened door B , while there is no prize behind the door B . Then

$$\begin{aligned} P(A|H) &= \frac{P(H|B)P(B)}{P(H|A)P(A) + P(H|B)P(B) + P(H|C)P(C)} \\ &= \frac{P(H|B)}{P(H|A) + P(H|B) + P(H|C)} = \frac{1/2}{0 + 1/2 + 1} = \frac{1}{3}. \end{aligned}$$

Hence you should switch door.

2.103 Consider the events:

G : guilty of committing a crime,

I : innocent of the crime,

i : judged innocent of the crime,

g : judged guilty of the crime.

$$P(I | g) = \frac{P(g | I)P(I)}{P(g | G)P(G) + P(g | I)P(I)} = \frac{(0.01)(0.95)}{(0.05)(0.90) + (0.01)(0.95)} = 0.1743.$$

2.104 Let A_i be the event that the i th patient is allergic to some type of week.

$$(a) P(A_1 \cap A_2 \cap A_3 \cap A'_4) + P(A_1 \cap A_2 \cap A'_3 \cap A_4) + P(A_1 \cap A'_2 \cap A_3 \cap A_4) + P(A'_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2)P(A_3)P(A'_4) + P(A_1)P(A_2)P(A'_3)P(A_4) + P(A_1)P(A'_2)P(A_3)P(A_4) + P(A'_1)P(A_2)P(A_3)P(A_4) = (4)(1/2)^4 = 1/4.$$

$$(b) P(A'_1 \cap A'_2 \cap A'_3 \cap A'_4) = P(A'_1)P(A'_2)P(A'_3)P(A'_4) = (1/2)^4 = 1/16.$$

2.105 No solution necessary.

$$2.106 (a) 0.28 + 0.10 + 0.17 = 0.55.$$

$$(b) 1 - 0.17 = 0.83.$$

$$(c) 0.10 + 0.17 = 0.27.$$

$$2.107 \text{ The number of hands} = \binom{13}{4} \binom{13}{6} \binom{13}{1} \binom{13}{2}.$$

2.108 (a) $P(M_1 \cap M_2 \cap M_3 \cap M_4) = (0.1)^4 = 0.0001$, where M_i represents that i th person make a mistake.

$$(b) P(J \cap C \cap R' \cap W') = (0.1)(0.1)(0.9)(0.9) = 0.0081.$$

2.109 Let R , S , and L represent the events that a client is assigned a room at the Ramada Inn, Sheraton, and Lakeview Motor Lodge, respectively, and let F represents the event that the plumbing is faulty.

$$(a) P(F) = P(F | R)P(R) + P(F | S)P(S) + P(F | L)P(L) = (0.05)(0.2) + (0.04)(0.4) + (0.08)(0.3) = 0.054.$$

$$(b) P(L | F) = \frac{(0.08)(0.3)}{0.054} = \frac{4}{9}.$$

2.110 Denote by R the event that a patient survives. Then $P(R) = 0.8$.

$$(a) P(R_1 \cap R_2 \cap R'_3) + P(R_1 \cap R'_2 \cap R_3)P(R'_1 \cap R_2 \cap R_3) = P(R_1)P(R_2)P(R'_3) + P(R_1)P(R'_2)P(R_3) + P(R'_1)P(R_2)P(R_3) = (3)(0.8)(0.8)(0.2) = 0.384.$$

$$(b) P(R_1 \cap R_2 \cap R_3) = P(R_1)P(R_2)P(R_3) = (0.8)^3 = 0.512.$$

2.111 Consider events

M : an inmate is a male,

N : an inmate is under 25 years of age.

$$P(M' \cap N') = P(M') + P(N') - P(M' \cup N') = 2/5 + 1/3 - 5/8 = 13/120.$$

2.112 There are $\binom{4}{3}\binom{5}{3}\binom{6}{3} = 800$ possible selections.

2.113 Consider the events:

B_i : a black ball is drawn on the i th drawl,

G_i : a green ball is drawn on the i th drawl.

$$(a) P(B_1 \cap B_2 \cap B_3) + P(G_1 \cap G_2 \cap G_3) = (6/10)(6/10)(6/10) + (4/10)(4/10)(4/10) = 7/25.$$

$$(b) \text{The probability that each color is represented is } 1 - 7/25 = 18/25.$$

2.114 The total number of ways to receive 2 or 3 defective sets among 5 that are purchased is $\binom{3}{2}\binom{9}{3} + \binom{3}{3}\binom{9}{2} = 288$.

2.115 Consider the events:

O : overrun,

A : consulting firm A ,

B : consulting firm B ,

C : consulting firm C .

$$(a) P(C | O) = \frac{P(O | C)P(C)}{P(O | A)P(A) + P(O | B)P(B) + P(O | C)P(C)} = \frac{(0.15)(0.25)}{(0.05)(0.40) + (0.03)(0.35) + (0.15)(0.25)} = \frac{0.0375}{0.0680} = 0.5515.$$

$$(b) P(A | O) = \frac{(0.05)(0.40)}{0.0680} = 0.2941.$$

2.116 (a) 36;

(b) 12;

(c) order is not important.

2.117 (a) $\frac{1}{\binom{36}{2}} = 0.0016$;

$$(b) \frac{\binom{12}{1}\binom{24}{1}}{\binom{36}{2}} = \frac{288}{630} = 0.4571.$$

2.118 Consider the events:

C : a woman over 60 has the cancer,

P : the test gives a positive result.

So, $P(C) = 0.07$, $P(P' | C) = 0.1$ and $P(P | C') = 0.05$.

$$P(C | P') = \frac{P(P' | C)P(C)}{P(P' | C)P(C) + P(P' | C')P(C')} = \frac{(0.1)(0.07)}{(0.1)(0.07) + (1-0.05)(1-0.07)} = \frac{0.007}{0.8905} = 0.00786.$$

2.119 Consider the events:

A : two nondefective components are selected,

N : a lot does not contain defective components, $P(N) = 0.6$, $P(A | N) = 1$,

O : a lot contains one defective component, $P(O) = 0.3$, $P(A | O) = \frac{\binom{19}{2}}{\binom{20}{2}} = \frac{9}{10}$,

T : a lot contains two defective components, $P(T) = 0.1$, $P(A | T) = \frac{\binom{18}{2}}{\binom{20}{2}} = \frac{153}{190}$.

$$(a) P(N | A) = \frac{P(A | N)P(N)}{P(A | N)P(N) + P(A | O)P(O) + P(A | T)P(T)} = \frac{(1)(0.6)}{(1)(0.6) + (9/10)(0.3) + (153/190)(0.1)} \\ = \frac{0.6}{0.9505} = 0.6312;$$

$$(b) P(O | A) = \frac{(9/10)(0.3)}{0.9505} = 0.2841;$$

$$(c) P(T | A) = 1 - 0.6312 - 0.2841 = 0.0847.$$

2.120 Consider events:

D : a person has the rare disease, $P(D) = 1/500$,

P : the test shows a positive result, $P(P | D) = 0.95$ and $P(P | D') = 0.01$.

$$P(D | P) = \frac{P(P | D)P(D)}{P(P | D)P(D) + P(P | D')P(D')} = \frac{(0.95)(1/500)}{(0.95)(1/500) + (0.01)(1 - 1/500)} = 0.1599.$$

2.121 Consider the events:

1: engineer 1, $P(1) = 0.7$, and 2: engineer 2, $P(2) = 0.3$,

E : an error has occurred in estimating cost, $P(E | 1) = 0.02$ and $P(E | 2) = 0.04$.

$$P(1 | E) = \frac{P(E | 1)P(1)}{P(E | 1)P(1) + P(E | 2)P(2)} = \frac{(0.02)(0.7)}{(0.02)(0.7) + (0.04)(0.3)} = 0.5385, \text{ and}$$

$$P(2 | E) = 1 - 0.5385 = 0.4615. \text{ So, more likely engineer 1 did the job.}$$

2.122 Consider the events: D : an item is defective

$$(a) P(D_1 D_2 D_3) = P(D_1)P(D_2)P(D_3) = (0.2)^3 = 0.008.$$

$$(b) P(\text{three out of four are defectives}) = \binom{4}{3}(0.2)^3(1 - 0.2) = 0.0256.$$

2.123 Let A be the event that an injured worker is admitted to the hospital and N be the event that an injured worker is back to work the next day. $P(A) = 0.10$, $P(N) = 0.15$ and $P(A \cap N) = 0.02$. So, $P(A \cup N) = P(A) + P(N) - P(A \cap N) = 0.1 + 0.15 - 0.02 = 0.23$.

2.124 Consider the events:

T : an operator is trained, $P(T) = 0.5$,

M an operator meets quota, $P(M | T) = 0.9$ and $P(M | T') = 0.65$.

$$P(T | M) = \frac{P(M | T)P(T)}{P(M | T)P(T) + P(M | T')P(T')} = \frac{(0.9)(0.5)}{(0.9)(0.5) + (0.65)(0.5)} = 0.581.$$

2.125 Consider the events:

A : purchased from vendor A ,

D : a customer is dissatisfied.

Then $P(A) = 0.2$, $P(A | D) = 0.5$, and $P(D) = 0.1$.

$$\text{So, } P(D | A) = \frac{P(A | D)P(D)}{P(A)} = \frac{(0.5)(0.1)}{0.2} = 0.25.$$

$$2.126 (a) P(\text{Union member} | \text{New company (same field)}) = \frac{13}{13+10} = \frac{13}{23} = 0.5652.$$

$$(b) P(\text{Unemployed} | \text{Union member}) = \frac{2}{40+13+4+2} = \frac{2}{59} = 0.034.$$

2.127 Consider the events:

C : the queen is a carrier, $P(C) = 0.5$,

D : a prince has the disease, $P(D | C) = 0.5$.

$$P(C | D_1 D_2 D_3) = \frac{P(D_1 D_2 D_3 | C)P(C)}{P(D_1 D_2 D_3 | C)P(C) + P(D_1 D_2 D_3 | C')P(C')} = \frac{(0.5)^3(0.5)}{(0.5)^3(0.5) + 1(0.5)} = \frac{1}{9}.$$

