CHAPTER 2

INNTRODUCTION TO LOGIC AND SETS

Assessment 2-1A: Reasoning and Logic: An Introduction

1. (a) **False statement**. A statement is a sentence that is either true or false, but not both.

   (b) **False statement.**

   (c) **Not a statement.**

   (d) **True statement.**

2. (a) **There exists at least one** natural number n such that $n + 8 = 11$.

   (b) **There exists at least one** natural number n such that $n^2 = 4$.

   (c) For **all** natural numbers n, $n + 3 = 3 + n$.

   (d) For **all** natural numbers n, $5n + 4n = 9n$.

3. (a) For **all** natural numbers n, $n + 8 = 11$.

   (b) For **all** natural numbers n, $n^2 = 4$.

   (c) There is **no** natural number x such that $x + 3 = 3 + x$.

   (d) There is **no** natural number x such that $5x + 4x = 9x$.

4. (a) The book **does not have** 500 pages.

   (b) $3 \cdot 5 \neq 15$.

   (c) **Some** dogs **do not have** four legs.

   (d) **No** rectangles are squares.

   (e) **All** rectangles are squares.

   (f) **Some** dogs have fleas.

5. (a) If $n = 4$, or $n = 5$, then $n < 6$ *and* $n > 3$, so the statement is **true**, since it can be shown to work for some natural numbers n..

   (b) If $n = 10$, then $n > 0$; *or* if $n = 1$, then $n < 5$, so the statement is **true**.

6. (a)

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

   (b)

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

   (c) **Yes**. The truth table entries are the same.

   (d) **No**. The truth table entries are not the same.

7. (a)

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	
T	T	T	F	T	
T	F	F	F	F	
F	T	T	T	T	
F	F	T	T	T	

   (b) Answers will vary. Here are two possible examples:1. Let p be "the Bobcats win" and q be "the Bobcats make the playoffs". Then Column 3 would read "If the Bobcats win, then the Bobcats make the playoffs". Column 5 would read "The Bobcats lose or the Bobcats make the playoffs."2. Let p be "it is summer vacation" and q be "I am at home". Then Column 3 would read "If it is summer vacation, then I am at home". Column 5 would read "It is not summer vacation or I am at home."

   (c) In this problem, p is the statement "2 + 3 = 5" and q is the statement "4 + 6 = 10". That would make the statement in this problem be in the form of $\sim p \vee q$. So, it will be logically equivalent to a statement in the form of $p \to q$ or "if 2 + 3 = 5, then 4 + 6 = 10."

8. **(a)** $q \wedge r$. Both q and r are true.

(b) $r \vee \sim q$. r is true or q is not true.

(c) $\sim(q \wedge r)$. q and r are not both true.

(d) $\sim q$. q is not true.

9. **(a)** **False**. The statement is a conjunction. The two parts could be stated as such: p is the statement $2 + 3 = 5$ and q is the statement $4 + 7 = 10$. In this situation, p is true, but q is false. In order for a conjunction to be true, both p and q must be true; otherwise, the conjunction is false.

(b) **True**. This statement is false, since Barack Obama was president in 2013.

(c) **False**. The United States Supreme Court currently has nine justices.

(d) **True**. The only triangles that have three sides of the same length are equilateral triangles. In every case, an equilateral triangle will have two sides the same length as well.

(e) **False**. Isosceles triangles have two sides equal in length, but the third side is not equal to the other two.

10. **(a)** By DeMorgan's Laws, the negation of $p \wedge q$ is $\sim p \vee \sim q$. Therefore, the answer is $2 + 3 \neq 5$ or $4 + 7 \neq 10$.

(b) The president of the United States in 2013 was Barack Obama.

(c) With every seat filled, the Supreme Court of the United States does not have 12 justices.

(d) The triangle has three sides of the same length and the triangle does not have two sides of the same length.

(e) The triangle has two sides of the same length or the triangle does not have three sides of the same length.

In both (d) and (e) above, the negation of a conditional statement $p \rightarrow q$ is $p \wedge \sim q$.

11. **(a)**

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$\sim(p \vee q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	T

Since the truth values for $\sim p \vee \sim q$ are not the same as for $\sim(p \vee q)$, the statements are **not logically equivalent**.

(b)

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Since the truth values for $\sim(p \wedge q)$ are not the same as for $\sim p \wedge \sim q$, the statements are **not logically equivalent**.

12. Dr. No is a male spy. He is not poor, and he is not tall.

13. If p = "it is raining" and q = "the grass is wet":

(a) $\boldsymbol{p \rightarrow q}$. **(b)** $\boldsymbol{\sim p \rightarrow q}$.

(c) $\boldsymbol{p \rightarrow \sim q}$.

(d) $\boldsymbol{p \rightarrow q}$. The hypothesis is "it is raining;" the conclusion is "the grass is wet."

(e) $\boldsymbol{\sim q \rightarrow \sim p}$. **(f)** $\boldsymbol{p \leftrightarrow q}$.

14. **(a)** **Converse**: If a triangle has no two sides of the same length, then the triangle is scalene. **Inverse**: If a triangle is not scalene, then the triangle has (at least) two sides of the same length. **Contrapositive**:If a triangle does not have two sides of the same length, then the triangle is not scalene. Note that this statement is a biconditional.

(b) **Converse:** If an angle is a right angle, then it is not acute. **Inverse**: If an angle is acute, then it is not a right angle. **Contrapositive:** If an angle is not a right angle, then it is acute. Note that the original statement and the contrapositive are not true, while the converse and inverse are true.

(c) **Converse**: If Mary is not a citizen of Cuba, then she is a U.S. citizen. **Inverse**: If Mary is not a U.S. citizen, then she is a citizen of Cuba. **Contrapositive**: If Mary is a citizen of Cuba, then she is not a U.S. citizen. Note that the original statement and the contrapositive are true, while the converse and inverse are not true

(d) **Converse**: If a number is not a natural number, then it is a whole number. **Inverse**: If a number is not a whole number, then it is a natural number. **Contrapositive**: If a number is a natural number, then it is not a whole number.

15. The statements are negations of each other.

p	q	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \to q$	$\sim(p \to q)$
T	T	F	F	T	T	F
T	F	T	T	F	F	T
F	T	T	F	T	T	F
F	F	F	F	T	T	F

16. The contrapositive is logically equivalent: "If a number is not a multiple of 4 then it is not a multiple of 8."

17. **(a)** **Valid.** This is valid by the transitivity property. "All squares are quadrilaterals" is $p \to q$ "all quadrilaterals are polygons" is $q \to r$; and "all squares are polygons" is $p \to r$.

(b) **Invalid.** We do not know what will happen to students who are not freshman. There is no statement "sophomores, juniors, and seniors do not take mathematics."

18.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

19. In the set of teachers, some have IQ's of 145 or more, and some are in Mensa. Notice that those two subsets intersect; their intersection represents the group of teachers who belong in both categories. Therefore, the argument is valid: some teachers with IQ's of 145 or more are in Mensa.

20. **(a)** Since all students in Integrated Mathematics I make A's, and some of those students are in Beta Club, then **some Beta Club students make A's**.

(b) Let $p =$ I study for the final, $q =$ I pass the final, $r =$ I pass the course, $s =$ I look for a teaching job. Then $p \to q,$ if I study for the final, then I will pass the final. $q \to r,$ if I pass the final, then I will pass the course. $r \to s,$ if I pass the course, I will look for a teaching job. So $p \to s,$ **if I study for the final I will look for a teaching job**.

(c) The first statement could be rephrased as "If a triangle is equilateral, then it is isosceles." Let $p =$ equilateral triangle and $q =$ isosceles triangle. So the first statement is $p \to q,$. The second statement is simply p ; then the conclusion should be q or **there exist triangles that are isosceles**.

21. **(a)** If a figure is a square, then it is a rectangle.

(b) If a number is an integer, then it is a rational number.

(c) If a polygon has exactly three sides, then it is a triangle.

22. **(a)** If $\sim p \vee \sim q \equiv \sim(p \wedge q)$, then
$\mathbf{3 \cdot 2 \neq 6}$ ***or*** $\mathbf{1 + 1 = 3}$.

(b) If $\sim p \wedge \sim q \equiv \sim(p \vee q)$, then you cannot pay me now and you cannot pay me later.

Assessment 2-1B

1. **(a)** **Not a statement**. A statement is a sentence that is either true or false.

(b) **Not a statement**. A statement must be either true of false; this could be either.

(c) True statement.

(d) **False statement**. $2 + 3 \neq 8$.

(e) Not a statement.

2. **(a)** For **all** natural numbers n, $n + 0 = n$.

(b) There exists **no** natural number n such that $n + 1 = n + 2$.

(c) **There exists at least one** natural number n such that $3 \cdot (n + 2) = 12$.

(d) **There exists at least one** natural number n such that $n^3 = 8$.

3. **(a)** There is **no** natural number n such that $n + 0 = n$.

(b) **There exists at least one** natural number n such that $n + 1 = n + 2$.

(c) For **all** natural numbers n, $3 \cdot (n + 2) \neq 12$.

(d) For **all** natural numbers n, $n^3 \neq 8$.

4. **(a)** Six is **greater than** or equal to 8. Another way to express this would be to say 6 **is not less than** 8.

(b) **All** cats **have** nine lives. Another way to express this would be to say that **no** cats **do not have** nine lives.

(c) There exists **a** square that is **not** a rectangle. Another way to express this would be to say **some** squares **are not** rectangles.

(d) **All** numbers **are** positive.

(e) **No** people **have** blond hair.

5. **(a)** If $n = 10$, then $n > 5$ and $n > 2$, so the statement is **true**.

(b) x could equal 5, so the statement is **false**.

6. **(a)** $p \vee q$ is false only if both p and q are false, so if p is true the statement is **true** regardless of the truth value of q.

(b) An implication is false only when p is true and q is false, so if p is false then the statement is **true** regardless of the truth value of q.

7. **(a)** $\boldsymbol{q \wedge r}$.

(b) $\boldsymbol{q \wedge \sim r}$.

(c) $\boldsymbol{\sim r \vee \sim q}$.

(d) $\boldsymbol{\sim(q \wedge r)}$.

8. **(a)** **True**. This statement is a disjunction. The two parts could be stated as such: p is the statement "4 + 6 = 10", while q is the statement "2 + 3 = 5". In this situation p is true, and q is true. In order for a disjunction to be true, either p or q (or both) have to be true; the only way a disjunction can be false is if both p and q are false. So therefore, this statement is true.

(b) **False**. If a team has more than 11 players on the field, it is a penalty and the play will not count.

(c) True.

(d) **False**. To see, sketch a drawing where three sides are the same length, but with the two angles where the sides intersect being different measures (in fact, make one a right angle, the other an obtuse angle). You should easily make a quadrilateral with a side length different from the other three.

(e) **True**. If a rectangle has four sides of the same length, then by default it must have three sides the same length. Of course, a rectangle with four equal sides is a square!.

9. **(a)** By DeMorgan's Laws, the negation of the disjunction $p \vee q$ is $\sim p \wedge \sim q$. So, the statement would be $4 + 6 \neq 10$ and $2 + 3 \neq 5$

(b) A National Football League team cannot have more than 11 players on the field while a game is in progress.

(c) The first president of the United States was not George Washington.

(d) A quadrilateral has three sides of the same length and the quadrilateral does not have four sides of the same length.

(e) A rectangle has four sides of the same length and that rectangle does not have three sides of the same length.

In both (d) and (e), the negation of the conditional statement $\boldsymbol{p \to q}$ is $\boldsymbol{p \wedge \sim q}$.

10. **(a)**

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Since the truth values for $\sim(p \vee q)$ are the same as for $\sim p \wedge \sim q$, the statements **are logically equivalent**.

(b)

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Since the truth values for $\sim(p \wedge q)$ are the same as for $\sim p \vee \sim q$, the statements are **logically equivalent**.

11. Ms. Makeover is not single. She has straight blond hair.

12. **(a)** $\boldsymbol{p \to q}$.

(b) $\boldsymbol{\sim p \to q}$.

(c) $\boldsymbol{p \to \sim q}$.

(d) q if p, or $\boldsymbol{p \to q}$.

(e) $\boldsymbol{\sim p \to \sim q}$.

(f) $\sim q \to \sim p$.

13. **(a)** Converse: **If $\boldsymbol{x^2 = 9}$, then $\boldsymbol{x = 3}$**.

Inverse: **If $\boldsymbol{x \neq 3}$, then $\boldsymbol{x^2 \neq 9}$**.

Contrapositive: **If $\boldsymbol{x^2 \neq 9}$, then $\boldsymbol{x \neq 3}$**.

(b) Converse: If classes are canceled, then it snowed.

Inverse: If it does not snow, then classes are not canceled.

Contrapositive: If classes are not canceled, then it did not snow.

14. **No**. This is the inverse; i.e., if it does not rain then lris can either go to the movies or not without making her statement false.

15. **(a)** **Valid**. Use modus ponens: Hypatia was a woman $\to$ all women are mortal $\to$ Hypatia was mortal.

(b) **Valid**. Since *Dirty Harry* was not written by J.K. Rowling, and she wrote all the *Harry Potter* books, then *Dirty Harry* cannot be a *Harry Potter* book.

(c) **Not valid**. There exist some whole numbers that are not natural numbers. It might be easier to understand if the word seven is replaced by a variable "x". So, it reads: Some whole numbers are not natural numbers. "x" is a whole number. Conclusion: "x" is a natural number.

16. **(a)** Since all students in Integrated Mathematics I are in Kappa Mu Epsilon, and Helen is in Integrated Mathematics I, then the conclusion is that **Helen is in Kappa Mu Epsilon**.

(b) Let p = all engineers need mathematics and q = Ron needs mathematics.

Then $p \to q$, or if all engineers need mathematics then Ron needs mathematics.

p is true, but q is false, Ron does not need mathematics.

So Ron is not an engineer.

(c) Since all bicycles have tires and all tires use rubber, then the conclusion is **all bicycles use rubber.**

17. **(a)** **If** a number is a natural number, **then** it is a real number.

(b) **If** a figure is a circle, **then** it is a closed figure.

18. DeMorgan's Laws are that:

$\sim(p \wedge q)$ is the logical equivalent of $\sim p \vee \sim q$.

$\sim(p \vee q)$ is the logical equivalent of $\sim p \wedge \sim q$.

Thus:

(a) The negation is $\mathbf{3 + 5 = 9}$ **or** $\mathbf{3 \cdot 5 \neq 15}$.

(b) The negation is I am not going and she is not going.

19. Therefore, a square is a parallelogram.

Assessment 2-2A: Describing Sets

1. **(a)** Either a list or set-builder notation may be used: $\{\mathbf{a, s, e, m, n, t}\}$ or $\{\boldsymbol{x} \mid \boldsymbol{x}$ **is a letter in the word *assessment*}**.

(b) $\{\mathbf{21, 22, 23, 24, \ldots}\}$ or $\{\boldsymbol{x} \mid \boldsymbol{x}$ **is a natural number and** $\boldsymbol{x} > \mathbf{20}\}$ or $\{\boldsymbol{x} \mid \boldsymbol{x} \in \boldsymbol{N}$ **and** $\boldsymbol{x} > \mathbf{20}\}$.

2. **(a)** $\boldsymbol{P} = \{p, q, r, s\}$.

(b) $\{\mathbf{1, 2}\} \subset \{\mathbf{1, 2, 3}\}$. The symbol $\subset$ refers to a proper subset.

(c) $\{\mathbf{0, 1}\} \not\subseteq \{\mathbf{1, 2, 3}\}$. The symbol $\subseteq$ refers to a subset.

3. **(a)** **Yes**. $\{1, 2, 3, 4, 5\} \sim \{m, n, o, p, q\}$ because both sets have the same number of elements and thus exhibit a one-to-one correspondence.

(b) **Yes**. $\{a, b, c, d, e, f, ..., m\} \sim \{1, 2, 3, \ldots, 13\}$ because both sets have the same number of elements.

(c) **No**. $\{\boldsymbol{x} \mid \boldsymbol{x}$ is a letter in the word *mathematics*$\}$ $\nsim \{1, 2, 3, 4, \ldots, 11\}$; there are only eight unduplicated letters in the word *mathematics*.

4. **(a)** The first element of the first set can be paired with any of the six in the second set, leaving five possible pairings for the second element, four for the third, three for the fourth, two for the fifth, and one for the sixth. Thus there are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{720}$ one-to-one correspondences.

(b) There are $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 = n!$ possible one-to-one correspondences. The first element of the first set can be paired with any of the n elements of the second set; for each of those n ways to make the first pairing, there are $n - 1$ ways the second element of the first set can be paired with any element of the second set; which means there are $n - 2$ ways the third element of the first set can be paired with any element of the third set; and so on. The Fundamental Counting Principle states that the choices can be multiplied to find the total number of correspondences.

5. **(a)** If x must correspond to 5, then y may correspond to any of the four remaining elements of $\{1, 2, 3, 4, 5\}, z$ may correspond to any of the three remaining, etc. Then $1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{24}$ **one-to-one correspondences**.

(b) There would be $1 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = \mathbf{6}$ **one-to-one correspondences**.

(c) The set $\{x, y, z\}$ could correspond to the set $\{1,3,5\}$ in $3 \cdot 2 \cdot 1 = 6$ ways. The set $\{u, v\}$ could correspond with the set $\{2,4\}$ in $2 \cdot 1 = 2$ ways. There would then be $6 \cdot 2 = \mathbf{12}$ **one-to-one correspondences**.

6. **(*i*)** $\boldsymbol{A = C}$. The order of the elements does not matter.

(*ii*) $\boldsymbol{E = H}$; they are both the null set.

(*iii*) $\boldsymbol{I = L}$. Both represent the numbers $1,3,5,7,\ldots$.

7. **(a)** Assume an arithmetic sequence with $a_1 = 201, a_n = 1100,$ and $d = 1.$ Thus $1100 = 201 + (n - 1) \cdot 1;$ solving, $n = 900.$ The cardinal number of the set is therefore **900**.

(b) Assume an arithmetic sequence with $a_1 = 1, a_n = 101,$ and $d = 2.$ Thus $101 = 1 + (n - 1) \cdot 2;$ solving, $n = 51.$ The cardinal number of the set is therefore **51**.

(c) Assume a geometric sequence with $a_1 = 1, a_n = 1024,$ and $r = 2.$ Thus $1024 = 1 \cdot 2^{n-1} \Rightarrow 2^{10} = 2^{n-1} \Rightarrow n - 1 = 10 \Rightarrow n = 11.$ The cardinal number of the set is therefore **11**.

(d) If $k = 1,2,3,\ldots,100,$ the cardinal number of the set $\{x | x = k^3, k = 1,2,3,\ldots,100\} = \mathbf{100}$, since there are 100 elements in the set.

8. $\overline{A}$ represents all elements in U that are not in A, or the set of all college students with at least one grade that is not an A.

9. **(a)** A proper subset must have at least one less element than the set, so the maximum $\boldsymbol{n(B) = 7}$.

(b) Since $B \subset C,$ and $n(B) = 8$ then C could have any number of elements in it, so long as it was greater than eight.

10. **(a)** The sets arc equal, so $\boldsymbol{n(D) = 5}$.

(b) Answers vary.For example, the sets are equal; the sets are also equivalent

11. **(a)** A has 5 elements, thus $2^5 = \mathbf{32}$ **subsets**.

(b) Since A is a subset of A and A is the only subset of A that is not proper, A has $2^5 - 1 =$ **31 proper subsets**.

(c) Let $B = \{b,c,d\}$. Since $B \subset A,$ the subsets of B are all of the subsets of A that do not contain a and $e.$ There are $2^3 = 8$ of these subsets. If we join (union) a and e to each of these subsets there are still **8 subsets**.

Alternative. Start with $\{a,e\}$. For each element $b, c,$ and d there are two options: include the element or don't include the element. So there are $2 \cdot 2 \cdot 2 = 8$ ways to create subsets of A that include a and $e.$

12. If there are n elements in a set, 2^n subsets can be formed. This includes the set itself. So if there are 127 proper subsets, then there are 128 subsets. Since $2^7 = 128,$ the set has **7 elements**.

13. In roster format, $A = \{3,6,9,12,\ldots\}, B = \{6,12,18,24,\ldots\},$ and $C = \{12,24,36,\ldots\}$. Thus, $C \subset A, C \subset B,$ and $B \subset A.$

Alternatively: $12n = 6(2n) = 3(4n).$ Since $2n$ and $4n$ are natural number $\boldsymbol{C \subset A, C \subset B,}$ and $\boldsymbol{B \subset A}$.

14. **(a)** $\notin$. There are no elements in the empty set.

(b) $\in$. $1024 = 2^{10}$ and $10 \in N.$

(c) $\in$. $3(1001) - 1 = 3002$ and $1001 \in N.$

(d) $\notin$. For example, $x = 3$ is not an element because for $3 = 2^n$, $n \notin N.$

15. **(a)** $\not\subseteq$. 0 is not a set so cannot be a subset of the empty set, which has only one subset, $\varnothing$.

(b) $\not\subseteq$. 1024 is an element, not a subset.

(c) $\not\subseteq$. 3002 is an element, not a subset.

(d) $\not\subseteq$. x is an element, not a subset.

16. **(a)** **Yes**. Any set is a subset of itself, so if $A = B$ then $A \subseteq B$.

(b) **No**. A could equal B; then A would be a subset but not a proper subset of B.

(c) **Yes**. Any proper subset is also a subset.

(d) **No**. Consider $A = \{1,2\}$ and $B = \{1,2,3\}$.

17. **(a)** Let $A = \{1,2,3,\ldots,100\}$ and $B = \{1,2,3\}$. Then $n(A) = 100$ and $n(B) = 3$. Since $B \subset A$, $\mathbf{n(B) = 3 < 100 = n(A)}$.

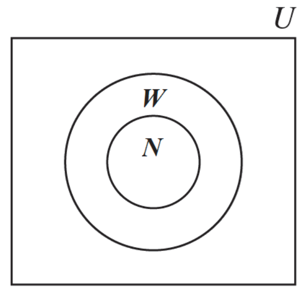
(b) $n(\emptyset) = 0$. Let $A = \{1,2,3\} \Rightarrow n(A) = 3$. $\emptyset \subset A$, which implies that there is at least one more element in A than in $\emptyset$. Thus $\mathbf{0 < 3}$.

18. There are seven senators to choose from and 3 will be chosen. Consider the ways to form subsets with only three members. If we pick the first member, there are 7 senators to choose from. To pick the second member, there are only 6 to choose from, since 1 member has already been chosen. For the third seat, there are 5 to choose from. This yields $7 \cdot 6 \cdot 5$. However, this calculation counts {Able, Brooke, Cox} as a different committee that {Brooke, Able, Cox}. In fact, for any 3 names, there are $3 \cdot 2 \cdot 1$ ways to arrange the names. Thus, the number of unique committees is $7 \cdot 6 \cdot 5 / 3 \cdot 2 \cdot 1 = 7 \cdot 5 = \mathbf{35}$.

19. Answers vary. For example, the set of all odd natural numbers and the set of all even natural numbers are two infinite sets that are equivalent but not equal.. Another possibility is the set of all natural numbers and the set of all whole numbers.

20. Each even natural number $2n$ can be paired with each odd natural number $2n - 1$ in a one-to-one correspondence.

21.



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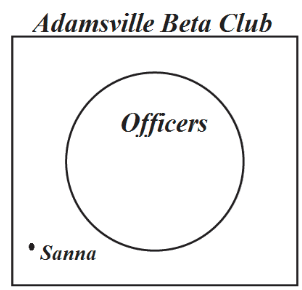


23. Answers vary. Example: All members of the Adamsville Beta Club are officers.

Assessment 2-2B

1. **(a)** Either a list or set-builder notation may be used: $\{a, l, g, e, b, r\}$ or **{*x* | *x* is a letter in the word *algebra*}**

(b) $\{1,2,3,4,5,6,7,8,9\}$ or **{*x* | *x* is a natural number and *x*<10}** or $\{x \mid x \in N \text{ and } x < 10\}$.

2. **(a)** $Q = \{q, r, s\}$

(b) $\{1,3\} = \{3,1\}$. The symbol $=$ refers to the sets being equal (containing the same elements).

(c) $\{1,3\} \not\subset \{1,4,6\}$. The symbol $\not\subset$ refers to “not a proper subset.”

3. **(a)** **Yes.** $\{1,2,3,4\} \sim \{w, c, y, z\}$ because both sets have the same number of elements and thus exhibit a one-to-one correspondence.

(b) **Yes**, because both sets have the same number of elements.

(c) **No**. $\{x|x$ is a letter in the word *geometry*$\}$ $\not\sim \{1,2,3,4,\ldots,8\}$; there are only seven unduplicated letters in geometry.

4. **(a)** The first element of the first set can be paired with any of the eight in the second set, leaving seven possible pairings for the second element, six for the third, five for the fourth, etc. Thus, there are $8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 =$ **40,320** one-to-one correspondences.

(b) There are $\boldsymbol{(n-1)\cdot(n-2)\cdot(n-3)\cdot\ldots\cdot3\cdot2\cdot1}$ possible one-to-one correspondences. The first element of the first set can be paired with any of the $n-1$ elements of the second set; there are $n-2$ ways the second element of the first set can be paired with any element of the second set; which means there are $n-3$ ways the third element of the first set can be paired with any element of the third set; and so on. The Fundamental Counting Principle states that the choices can be multiplied to find the total number of correspondences

5. **(a)** If b must correspond to 3, then a may correspond to any of the three remaining elements of $\{1,2,3,4\}$, and c may correspond to any of the two remaining, etc. Then $1\cdot3\cdot2\cdot1 =$ **6 one-to-one correspondences.**

(b) **T**here would be a $1\cdot1\cdot2\cdot1 =$ **2 one-to-one correspondences.**

(c) The set $\{a,c\}$ could correspond to the set $\{2,4\}$ in $2\cdot1=2$ ways. The set $\{b,d\}$ could correspond with the set $\{1,3\}$ in $2\cdot1=2$ ways. There would then be $2\cdot2 =$ **4 one-to-one correspondences.**

6. $\boldsymbol{A = C}$.

7. **(a)** Assume an arithmetic sequence with $a_1 = 19,\ a_n = 99,$ and $d = 1.$ Thus $99 = 19 + (n-1)\cdot 1;$ solving, $n = 81.$ The cardinal number of the set is therefore **81**.

(b) Assume an arithmetic sequence with $a_1 = 2,\ a_n = 1002,$ and $d = 2.$ Thus $1002 = 2 + (n-1)\cdot 2;$ solving, $n = 501.$ The cardinal number of the set is therefore **501**.

(c) Assume an arithmetic sequence with $a_1 = 1,\ a_n = 99,$ and $d = 2.$ Thus $99 = 1 + (n-1)\cdot 2;$ solving, $n = 50.$ The cardinal number of the set is therefore **50**.

(d) There are no natural numbers that have the property $x = x + 1,$ so the set is empty and the cardinal number is **0**.

8. $\overline{G}$ represents all elements in U that are not in G, or the set of all women who are not alumni of Georgia State University. In set-builder notation, $\overline{G} = \{x|x$ is a woman who is not a graduate of Georgia State University$\}$.

9. **(a)** Since the empty set is a subset of any set, A could be the empty set; then the minimum number of elements in A would be **0**.

(b) **Yes**. Since A is not assumed to be a proper subset, A and B could be equal . Thus, both sets could be empty.

10. To be subsets of each other, the two sets must be **equal** and **equivalent**.

11. **(a)** Since $A = \{1,2,3,4,5,6,7,8,9\}$ has 9 elements, A has $2^9 =$ **512 subsets**.

(b) Since A is a subset of A and not proper, A has $2^9 - 1 =$ **511 subsets**.

12. If there are n elements in a set, 2^n subsets can be formed. Thus, $16 = 2^n$ and $n =$ **4 elements**.

13. In roster format, $A = \{4,7,10,13,16,\ldots\}$, $B = \{7,13,18,\ldots\}$, and $C = \{13,25,38,\ldots\}$. Thus, $C \subset A$, $C \subset B$, and $B \subset A$.

Alternative: $12n + 1 = 6(2n) + 1 = 3(4n) + 1.$ Since $2n$ and $4n$ are natural numbers $\boldsymbol{C \subset A}$, $\boldsymbol{C \subset B}$, and $\boldsymbol{B \subset A}$.

14. **(a)** $\in$. The set containing the empty set has one element; the empty set.

(b) $\in$. $1022 = 2^{10} - 2$ and $10 \in N$.

(c) $\in$. $3(1001) + 1 = 3004$ and $1001 \in N$.

(d) $\in$. 17 is an element of the natural numbers.

15. **(a)** **No**. For example, if $A = \{1\}$ and $B = \{1, 2\}$, then $A \subseteq B$ but $A \neq B$.

(b) **No**. $A \subset B$ implies that A must have at least one less element than B.

(c) **No**. A and B must have the same number of elements, but not necessarily be equal; for example, if $A = \{1, 2\}$ and $B = \{a, b\}$.

(d) **No**. See part (c).

16. **(a)** $n(\emptyset) = 0$. Let $A = \{1, 2\} \Rightarrow n(A) = 2$. $\emptyset \subset A$, which implies that there is at least one more element in A than in $\emptyset$. Thus $\mathbf{0 < 2}$.

(b) Let $A = \{1, 2, 3, \ldots 99\}$ and $B = \{1, 2, 3, \ldots, 100\}$. $n(A) = 99$ and $n(B) = 100$, but $A \subset B$ so $\boldsymbol{n(A) = 99 < 100 = n(B)}$.

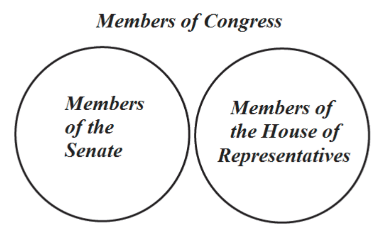
17. **(a)** There are 4 flavors from which to pick the first scoop, leaving 3 from which to pick the second, 2 from which to pick the third, and only 1 from which to pick the last. Thus there are $4 \cdot 3 \cdot 2 \cdot 1 = \mathbf{24}$ **ways to pick the four flavors**.

(b) Each of the four scoops may be picked in four different ways, thus there are $4^4 = \mathbf{256}$ **ways to pick the four flavors**.

18. 6. Note that $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. The Fundamental Counting Principle states that the product of choices gives the number of ways that the one-to-one correspondence can happen; since there are 720 one-to-one correspondences, then there must be six elements in each set.

19. Let every natural number n be paired with the number $n - 1$ from the set of whole numbers. The pairing is a one-to-one correspondence between the set of natural numbers and the set of whole numbers.

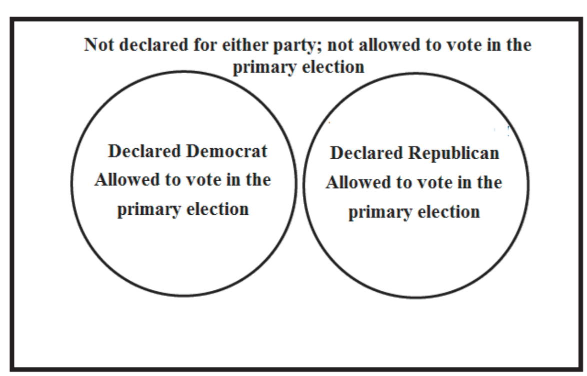
20. The Congress of the United States consists of all members of the Senate, and all members of the House of Representatives; however, no member of the Senate is also a member of the House of Representatives, and vice-versa. Therefore, a Venn diagram consisting of two disjoint circles will work for this problem, as shown below:



21. Since both sets are equal, that means both sets contain all of the same elements, by defintion of equal sets. So, you can infer that every swimmer in the 100 meter butterfly race is from the Maryville Swim Team.

22. For voting in a primary election, some states make voters declare a party affiliation (so that you vote in that parties' primary only). No state would allow a voter to declare as both a Republican and a Democrat, so the Venn diagram would consist of two disjoint circles, as shown below:

Voting in Primary Elections



23. Answers will vary. Some possibilities: Since set B has one more element in it than set A, set A could be a subset of set B. However, the number of elements doesn't necessarily imply that the set with the smaller number of elements is a subset of the set with the larger number of elements, so set A might not be a subset of set B. Similar reasoning could be used to state that set A could be a proper subset of set B, or not. Assuming the sets are finite, the sets are not equivalent nor equal.

24. **(a)** Answers will vary. For example, let the universal set be the whole numbers. Now, let set A consist of all whole numbers greater than ten. That set is infinite. Its complement, $\overline{A}$, would be the natural numbers less than or equal to ten. That set is finite.

(b) Answers will vary. For example, let the universal set be the set of whole numbers. Let A be the set of odd whole numbers. This set is infinite. Then $\overline{A}$ would be the set of even whole numbers, also an infinite set.

Mathematical Connections 2-2: Review Problems

16. **(a)** False. In order for a conjunction to be true, both parts of the conjunction must be true. Since p is false, $p \wedge q$ is false.

(b) False. Since q is true, $\sim q$ is false.

(c) True. Since p is false, $\sim p$ is true. This makes both parts of the conjunction true, so therefore the conjunction $\sim p \wedge q$ is true.

(d) True. In part (a) above, we found out that $p \wedge q$ was false. Therefore, $\sim (p \wedge q)$ must be true.

(e) False. In order for a conjunction to be true, both parts of the conjunction must be true. Since q is true, $\sim q$ is false; so $\sim q \wedge \sim p$ is false.

17. **(a)** False. In order for a conjunction to be true, both parts of the conjunction must be true. Since q is false, $p \wedge q$ is false.

(b) True. Since q is false, $\sim q$ is true.

(c) False. In order for a conjunction to be true, both parts of the conjunction must be true. In this conjunction, neither side is true; so, $\sim p \wedge q$ is false.

(d) True. In part (a) above, we found out that $p \wedge q$ was false. Therefore, $\sim (p \wedge q)$ must be true.

(e) False. In order for a conjunction to be true, both parts of the conjunction must be true. Since p is true, $\sim p$ is false; so $\sim q \wedge \sim p$ is false.

18.

p	q	$\sim q$	$p \vee \sim q$
T	T	F	**T**
T	F	T	**T**
F	T	F	**F**
F	F	T	**T**

Assessment 2-3A: Other Set Operations and Their Properties

1. $A = \{1,3,5,\ldots\}; B = \{2,4,6,\ldots\}$; $C = \{1,3,5,\ldots\}$

(a) $\boldsymbol{A}$ or $\boldsymbol{C}$. Every element in C is either in A or C.

(b) $\boldsymbol{N}$. Every natural number is in either A or B.

(c) $\emptyset$. There are no natural numbers in both A and B.

2. For example, let $U = \{1,2,3,4,5,6,7\}$, $A = \{1,5,6\}, B = \{1,4,5,6,7\}$, and $C = \{1,2,3,4\}$

(a) **Yes**. $A - A$ means the set of all elements that are in A that are also not in A; or more formally, $A - A = \{x | x \in A \text{ and } x \notin A\} = \varnothing$.

(b) **Yes**. $B - A = \{4,7\}$; $\overline{A} = \{2,3,4,7\}$ which means $B \cap \overline{A} = \{4,7\}$ so the sets are equal.

(c) **No**. $B - A = \{4, 7\}$;
$B \cap A = \{1, 5, 6\}$
so the sets are not equal.

(d) **Yes**. $A \cup A = \{1, 5, 6\}$; $A \cup \emptyset = \{1, 5, 6\}$, so the sets are equal. In general, combining a set to itself via set union will not change the elements in the set; combining a set via set union to the empty set will not change the elements in the set.

3. (a) **True**. Let $A = \{1, 2\}$. $A \cup \varnothing = \{1, 2\}$. In general, combining any set with the empty set via set union contributes no extra elements to the set than what is already in the set.

(b) **False**. Let $A = \{1, 2\}$ and $B = \{2, 3\}$.
$A - B = \{1\}; B - A = \{3\}$.

(c) **False**. Let $U = \{1, 2\}; A = \{1\}; B = \{2\}$.
$\overline{A \cap B} = \{1, 2\}; \overline{A} \cap \overline{B} = \varnothing$.

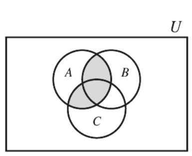
(d) **False**. Let $A = \{1, 2\}; B = \{2, 3, 4\}$.
$(A \cup B) - A = \{1, 2, 3, 4\} - \{1, 2\} = \{3, 4\} \neq B$.

(e) **False**. Let $A = \{1, 2\}; B = \{2, 3\}$.
$(A - B) \cup A = \{1\} \cup \{1, 2\} = \{1, 2\}$;
$(A - B) \cup (B - A) = \{1\} \cup \{3\} = \{1, 3\}$.

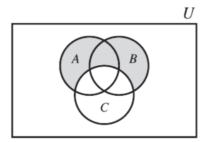
4. (a) If $B \subseteq A$, all elements of B must also be elements of A, but there may be elements of A that are not elements of B, so $\boldsymbol{A \cap B = B}$.

(b) If $B \subseteq A$, all elements of B must also be elements of A, but there may be elements of A that are not elements of B, so $\boldsymbol{A \cup B = A}$.

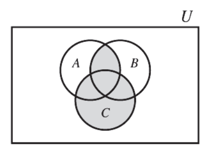
5. (a) $(A \cap B) \cup (A \cap C)$



(b) $(A \cup B) \cap \overline{C}$



(c) $(A \cap B) \cup C$



6. (a) $S \cup \overline{S} = \{x | x \in S \text{ or } x \in \overline{S}\} = \boldsymbol{U}$.

(b) If U is the universe the complement of U can have no elements, thus $\boldsymbol{\overline{U} = \emptyset}$.

(c) There are no elements common to S and $\overline{S}$, so $\boldsymbol{S \cap \overline{S} = \varnothing}$.

(d) Since there are no elements in the empty set there are none common to it and S, so $\boldsymbol{\varnothing \cap S = \varnothing}$.

7. (a) If $A \cap B = \varnothing$ then A and B are disjoint sets and any element in A is not in B, so $A - B = \{x | x \in A \text{ and } x \notin B\} = \boldsymbol{A}$.

(b) Since B is the empty set, there are no elements to remove from A, so $A - B = \boldsymbol{A}$.

(c) If $B = U$ there are no elements in A which are not in B, so $A - B = \varnothing$.

8. **Yes**. By definition, $A - B$ is the set of all elements in A that are not in B. If $A - B = \varnothing$ this means there are no elements in A that are not in B, thus $A \subseteq B$.

More formally, suppose $A \not\subseteq B$. Then there must be an element in A that is not in B, which implies that it is in $A - B$. This implies that $A - B$ is not empty, which is a contradiction. Thus $A \subseteq B$.

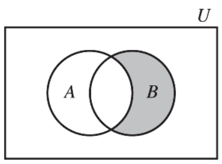
9. Answers may vary.

(a) $\boldsymbol{B \cap \overline{A}}$ or $\boldsymbol{B - A}$; i.e., $\{x | x \in B$ but $x \notin A\}$.

(b) $\overline{\boldsymbol{A \cup B}}$ or $\boldsymbol{\overline{A} \cap \overline{B}}$; i.e., $\{x | x \notin A$ or $B\}$.

(c) $(\boldsymbol{A \cap B}) \cap \overline{\boldsymbol{C}}$ or $(\boldsymbol{A \cap B}) - \boldsymbol{C}$; i.e., $\{x | x \in A$ and B but $x \notin C\}$.

10. (a) $\overline{A}$ is the set of all elements in U that are not in A. $\overline{A} \cap B$ is the set of all elements common to $\overline{A}$ and B:

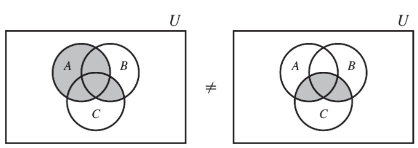


$\overline{A} \cap B$

(b) Answers vary. $\overline{A} \cap B = B - (A \cap B)$. or $\overline{A} \cap B = B - A$. are two common responses.

11. (a) False:

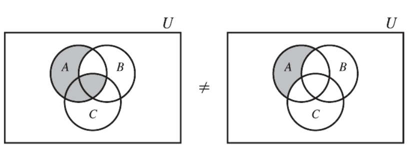
$A \cup (B \cap C) \neq (A \cup B) \cap C$



$A \cup (B \cap C)$ $(A \cup B) \cap C$

(b) False:

$A - (B - C) \neq (A - B) - C$



$A - (B - C)$ $(A - B) - C$

12. (a) Yes. Answers will vary. Note that in general, $\boldsymbol{A \cap B \subseteq A \cup B}$ because all elements of $A \cap B$ are included in $A \cup B$. Example: Let $A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7,8\}$. Then $A \cap B = \{4,5\}$ and $A \cup B = \{1,2,3,4,5,6,7,8\}$.

(b) Yes. Answers will vary. Example: Let $A = \{1,2,3,4,5\}$ and $B = \{4,5,6,7,8\}$, where the universal set is $\{1,2,3,4,5,6,7,8\}$ Then $A - B = \{1,2,3\}$ which means $\overline{A - B} = \{4,5,6,7,8\}$, which does contain two elements of A, namely 4 and 5.

13. (a) (*i*) Greatest $n(A \cup B) = n(A) + n(B) = \mathbf{5}$ if A and B are disjoint.

(*ii*) Greatest $n(A \cap B) = n(B) = \mathbf{2}$ if $B \subseteq A$.

(*iii*) Greatest $n(B - A) = n(B) = \mathbf{2}$ if A and B are disjoint.

(*iv*) Greatest $n(A - B) = n(A) = \mathbf{3}$ if A and B are disjoint.

(b) (*i*) Greatest $n(A \cup B) = \boldsymbol{n + m}$ if A and B are disjoint.

(*ii*) Greatest $n(A \cap B) = \boldsymbol{m}$, if $B \subseteq A$, or $\boldsymbol{n}$, if $A \subseteq B$.

(*iii*) Greatest $n(B - A) = \boldsymbol{m}$ if A and B are disjoint.

(*iv*) Greatest $n(A - B) = \boldsymbol{n}$ if A and B are disjoint.

14. (a) (*i*) Greatest $n(A \cup B \cup C) = n(A) + n(B) + n(C) = 4 + 5 + 6 = \mathbf{15}$, if A, B, and C are disjoint.

(*ii*) Least $n(A \cup B \cup C) = n(C) = \mathbf{6}$, if $A \subset B \subset C$.

(b) (*i*) Greatest $n(A \cap B \cap C) = n(A) = \mathbf{4}$, if $A \subset B \subset C$.

(*ii*) Least $n(A \cap B \cap C) = \mathbf{0}$, if A, B, and C are disjoint.

15. **(a)** 319.2 million non-Muslims. Subtract the Muslim population (2.6 million) away from the total population (321.8 million) of the United States.

(b) 148 million. Subtract the Muslim population (2.6 million) away from the number of people identified as being of some faith (150.6 million).

16. Constructing a Venn diagram will help in visualization:

(a) $B \cap S$ is the set of college **basketball players more than 200 cm tall**.

(b) $\overline{S}$ is the set of humans who are **not college students** or who are **college students less than or equal to 200 cm tall**.

(c) $B \cup S$ is the set of humans who are **college basketball players** or who are **college students taller than 200 cm**.

(d) $\overline{B \cup S}$ is the set of all humans who are **not college basketball players and who are not college students taller than 200 cm**.

(e) $\overline{B} \cap S$ is the set of all **college students taller than 200 cm who are not basketball players**.

(f) $B \cap \overline{S}$ is the set of all **college basketball players less than or equal to 200 cm tall**.

17. Use a three-set Venn diagram, labeling the sets B (for basketball), V (volleyball), and S (soccer):

(*i*) Enter 2 in the region representing $B \cap V \cap S$ (i.e., there were two who played all three sports);

(*ii*) Enter 1 in the region representing $(B \cap V) - S$ (i.e., there was one who played basketball and volleyball but not soccer);

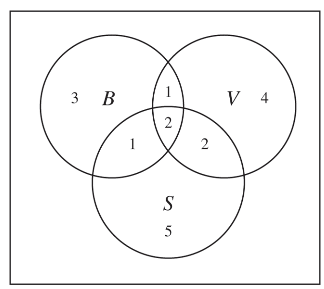
(*iii*) Enter 1 in the region representing $(B \cap S) - V$ (i.e., there was one who played basketball and soccer but not volleyball);

(*iv*) Enter 2 in the region representing $(V \cap S) - B$ (i.e., there were two who played volleyball and soccer but not basketball);

(*v*) Enter $7 - (1 + 1 + 2) = 3$ in the region representing $B - (V \cup S)$ (i.e., of the seven who played basketball, one also played volleyball, one also played soccer, and two also played both volleyball and soccer—leaving three who played basketball only);

(*vi*) Enter $9 - (1 + 2 + 2) = 4$ in the region representing $V - (B \cup S)$ (i.e., of the nine who played volleyball, one also played basketball, two also played soccer, and two also played both basketball and soccer—leaving four who played volleyball only);

(*vii*) Enter $10 - (1 + 2 + 2) = 5$ in the region representing $S - (B \cup V)$ (i.e., of the ten who played soccer, one also played basketball, two also played volleyball, and two also played both basketball and volleyball—leaving five who played soccer only).



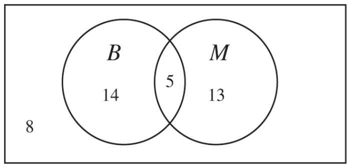
There are then $3 + 4 + 5 + 1 + 1 + 2 + 2 = \mathbf{18}$ who played one or more sports.

18. In the Venn diagram below:

(*i*) There were 5 members who took both biology and mathematics;

(*ii*) Of the 18 who took mathematics 5 also took biology, leaving 13 who took mathematics only;

(iii) 8 took neither course, so of the total of 40 members there were $40 - (5 + 13 + 8) = \mathbf{14}$ who took biology but not mathematics.



19. **(a)** If all bikes needing new tires also need gear repairs, i.e., if $\{TIRES\} \subset \{GEARS\}$, then $n[\{TIRES\} \cap \{GEARS\}] = \mathbf{20}$ **bikes**.

(b) Adding the separate repairs gives $20 + 30 = 50$ bikes, which is the total number of bikes; so it is possible that **zero bikes** needed both repairs, if every bike there needed exactly one repair.

(c) If the maximum number of bikes needed both repairs then all 20 receiving tires would also receive gear work. That would leave 10 additional bikes needing gear work only, leaving **20 bikes** that needed no service.

20. Generate the following Venn diagram in this order:

(i) The 4 who had A, B, and Rh antigens;

(ii) The $5 - 4 = 1$ who had A and B antigens, but who were Rh negative;

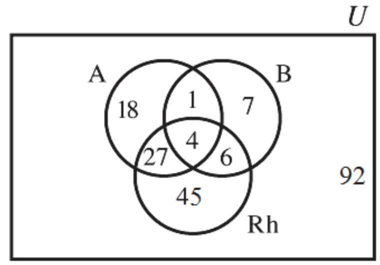
(iii) The $31 - 4 = 27$ who had A antigens and were Rh positive;

(iv) The $10 - 4 = 6$ who had B antigens and were Rh positive;

(v) The $50 - 27 - 4 - 1 = 18$ who had A antigens only;

(vi) The $18 - 6 - 4 - 1 = 7$ who had B antigens only, and;

(vii) The $82 - 27 - 4 - 6 = 45$ who were O positive.



$n(A \cup B \cup \text{Rh}) = 18 + 1 + 7 + 27 + 4 + 6 + 45 = 108$. Thus the set of people who are O-negative is $200 - 108 = \mathbf{92}$.

21. Let M be the set of students taking mathematics, C be the set of students taking chemistry, and P be the set of students taking physics. Generate the following Venn diagram in this order:

(i) The 8 who had all three subjects;

(ii) The $10 - 8 = 2$ students who took mathematics and physics, but not chemistry;

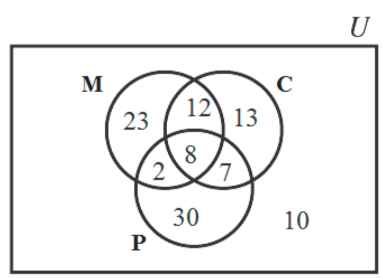
(iii) The $15 - 8 = 7$ students who took chemistry and physics, but not mathematics;

(iv) The $20 - 8 = 12$ students who took mathematics and chemistry, but not physics;

(v) The $45 - 12 - 8 - 2 = 23$ students who took mathematics only;

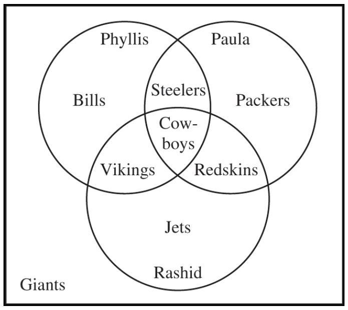
(vi) The $40 - 12 - 8 - 7 = 13$ students who took chemistry only;

(vii) The $47 - 2 - 8 - 7 = 30$ students who took physics only



Combined with the 10 students who didn't take any of the three courses, this Venn diagram yields a total number of students as 105, which contradicts John's reported total of 100. So, given that there must be errors somewhere in John's data gathering, John probably shouldn't be hired for the job.

22. The following Venn diagram helps in isolating the choices:



All picked the Cowboys to win their game, so their opponent cannot be among any of the other choices; the only team not picked was the Giants.

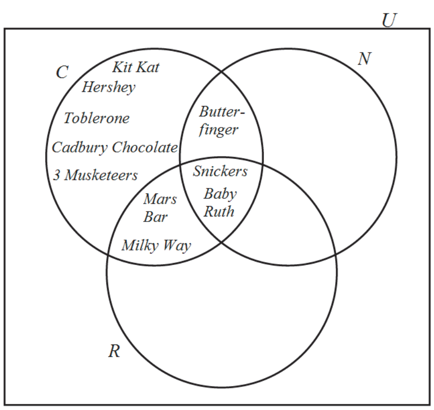
Phyllis and Paula both picked the Steelers, so their opponent cannot be among their other choices. This leaves the Jets.

Phyllis and Rashid both picked the Vikings which leaves the Packers as the only possible opponent.

Paula and Rashid both picked the Redskins which leaves the Bills as the only possible opponent.

Thus we have **Cowboys vs Giants**, **Vikings vs Packers**, **Redskins vs Bills**, **and Jets vs Steelers**.

23. **(a)**



(b) **(*i*)** **none**. Since all candy bars have chocolate, there aren't any in the set of bars without chocolate.

(b) **(*ii*)** **none**. Same reason as (i).

(*iii*) **Chocolate** is the most popular ingredient; **nuts/peanut butter** is the least popular ingredient.

(*iv*) Answers will vary.

24. **57.** Of the 324 first class passengers, you know there were $146 + 4 = 150$ total women and children, and 117 men who were lost. To find the number of men who survived, find $324 - 150 - 117 = 57$

25. **Abby, Harry, and Dick are in one family; Tom, Jane, and Mary are in the other family.** First, note that Abby and Harry have the same characteristics, blue eyes and blond hair. Also note that Mary is their opposite, with brown eyes and brown hair; therefore, Mary is certainly in a different family than Abby and Harry. Now note that Jane and Tom both have blue eyes and brown hair, while Dick has brown eyes and blond hair. Dick cannot be in the same family as Jane and Tom. So, since each family has 3 children, Dick is in the family with Abby and Harry (where they all have blond hair), while Tom, Jane, and Mary are in the other family (where they all have brown hair).

26. If a Cartesian product is the set of all ordered pairs such that the first element of each pair is an element of the first set and the second element of each pair is an element of the second set:

(a) $A \times B = \{(\boldsymbol{x},\boldsymbol{a}),(\boldsymbol{x},\boldsymbol{b}),(\boldsymbol{x},\boldsymbol{c}),(\boldsymbol{y},\boldsymbol{a}),(\boldsymbol{y},\boldsymbol{b}),(\boldsymbol{y},\boldsymbol{c})\}.$

(b) $B \times A = \{(\boldsymbol{a},\boldsymbol{x}),(\boldsymbol{a},\boldsymbol{y}),(\boldsymbol{b},\boldsymbol{x}),(\boldsymbol{b},\boldsymbol{y}),(\boldsymbol{c},\boldsymbol{x}),(\boldsymbol{c},\boldsymbol{y})\}.$

(c) **No**. For sets to be equal, all the elements of one must be elements of the other. But (a, x) for example is not a member of $A \times B$. For ordered pairs to be equal, their first coordinates must be equal and their second coordinates must be equal.

27. **(a)** The first element of each ordered pair is a, so $\boldsymbol{C} = \{\boldsymbol{a}\}$. The second elements in the ordered pairs are, respectively, $b, c, d,$ and e, so $\boldsymbol{D} = \{\boldsymbol{b},\boldsymbol{c},\boldsymbol{d},\boldsymbol{e}\}$.

(b) The first element in the first three ordered pairs is 1; in the second three is 2, so $\boldsymbol{C} = \{\mathbf{1,2}\}$. The second element in the ordered pairs is, respectively, 1, 2, and 3, so $\boldsymbol{D} = \{\mathbf{1,2,3}\}$.

(c) The numbers 0 and 1 appear in each ordered pair, so $\boldsymbol{C} = \boldsymbol{D} = \{\mathbf{0,1}\}$. (The order of the numbers in these sets is irrelevant.)

Assessment 2-3B

1. Given $W = \{0,1,2,3,\ldots\}$ and $N = \{1,2,3,\ldots\}$ then $A = \{1,3,5,7,\ldots\}$ and $B = \{0,2,4,6,\ldots\}$:

(a) $\boldsymbol{B}$, or the set of all elements in W that are not in A.

(b) $\varnothing$. There are no elements common to both A and B.

(c) $\boldsymbol{N}$. All elements are common to both except 0.

2. **(a)** **No**. For example, $A = \{l,i,t\}$ and $B = \{l,i\}$. Then $A - B = \{t\}$ and $B - A = \varnothing$.

(b) **Yes**. By definition $A - B = \{x | x \in A$ and $x \notin B\}$. Since $x \notin B$ describes $\overline{B}$, $A - B = \{x | x \in A$ and $x \in \overline{B}\} = A \cap \overline{B}$.

(c) **No**. By definition, $B - A = \{x | x \in B$ and $x \notin A\}$. So, $B - A$ cannot contain an element that is in A unless it is also in their intersection. But, $B \cup A = A$, if $B \subseteq A$., which means everything in B must be in A. So, the two sets cannot be equal

(d) **Yes**. Since $\varnothing$ is empty, $B \cup \varnothing = B$. Since $B \cap B$ is all the elements common to B and B, $B \cap B = B$.

3. **(a)** **False**. Let $A = \{1,2\}$ and $B = \{2\}$. Then $A - B = \{1\}$, but $A - \varnothing = \{1,2\}$.

(b) **False**. Let $U = \{1,2,3\}, A = \{1,2\}$, and $B = \{2,3\}$. Then $A \cup B = \{1,2,3\}$ and $\overline{A \cup B} = \varnothing$. But $\overline{A} = \{3\}$ and $\overline{B} = \{1\}$, making $\overline{A} \cup \overline{B} = \{1,3\}$.

(c) **False**. Let $A = \{1,2\}, B = \{2,3\}$, and $C = \{1,2,4\}$. Then $B \cup C = \{1,2,3,4\}$ $\Rightarrow A \cap (B \cup C) = \{1,2\}$ but $A \cap B = \{2\} \Rightarrow (A \cap B) \cup C = \{1,2,4\}$.

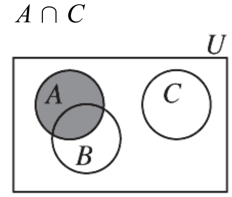
(d) **False**. Let $A = \{1,2\}$ and $B = \{2,3\}$. Then $A - B = \{1\} \Rightarrow (A - B) \cap A = \{1\}$, but $A = \{1,2\}$.

(e) **False.** Let $A = \{1,2,3\}, B = \{1,2,4\}$, and $C = \{1,2,3,4\}$. Then $B \cap C = \{1,2,4\}$, $A - B = \{3\}$, and $A - C = \varnothing$. Thus $A - (B \cap C) = \{3\}$ but $(A - B) \cap (A - C) = \varnothing$.

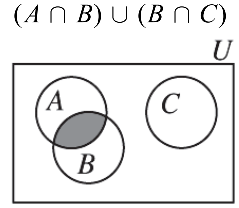
4. **(a)** Since $X \subseteq Y$, there are no elements in X, that are not also in Y so $X - Y = \varnothing$.

(b) If $X \subseteq Y$, then there are no elements in $\overline{Y}$ that are also in X, so $X \cap \overline{Y} = \varnothing$.

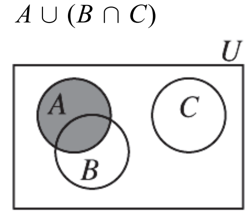
5. **(a)** $A \cap \overline{C}$



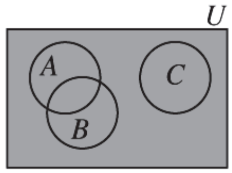
(b) $(A \cap B) \cup (B \cap C)$



(c) $A \cup (B \cap C)$



(d) $A \cup \overline{(B \cap C)}$



6. **(a)** $A \cup U = \boldsymbol{U}$. There are no elements in A that are not also in U.

(b) $U - A = \overline{\boldsymbol{A}}$. The set of all elements in U that are not in A is the complement of A.

(c) $A - \varnothing = \boldsymbol{A}$. There are no elements in $\varnothing$, so there are no elements which are not in A.

(d) $\overline{\varnothing} \cap A = \boldsymbol{A}$. $\overline{\varnothing} = U$, so $U \cap A = A$.

7. **(a)** If $A = B$ there are no elements in one set which are not also in the other, so $B - A = \varnothing$.

(b) If $B \subseteq A$ then all elements of B must also be in A, so $B - A = \{x | x \in B$ and $x \notin A\} = \varnothing$.

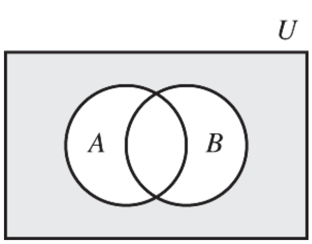
8. Answers may vary.

(a) $\boldsymbol{A \cap C}$; i.e., $\{x | x \in A \text{ and } C\}$.

(b) $\boldsymbol{(A \cup B) \cap C}$ or $\boldsymbol{C - (\overline{A \cup B})}$ or $\boldsymbol{(A \cap C) \cup (B \cap C)}$; i.e., $\{x | x \in A \text{ or } B, \text{ and } x \in C\}$.

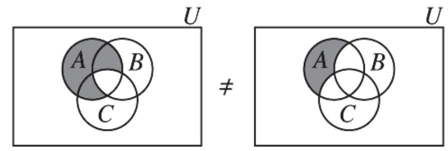
(c) In the diagram, the elements in the shaded region are in B or C but not in A. Thus, the shaded region is **$(B \cup C) - A$ or $(B \cup C) \cap \overline{A}$**

9.



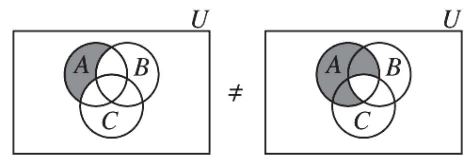
(b) Answers may vary. For example,
$\overline{A \cup B} = \overline{A} \cap \overline{B}$; also
$\overline{A \cup B} = U - (A \cup B)$

10. **(a)** **False**.



$A - (B \cap C)$ $(A - B) \cap (A - C)$

(b) **False**.



$A - (B \cup C)$ $(A - B) \cup (A - C)$

11. **The first description should have fewer people fitting it.** The first description is a subset of the second description, since it includes the fact that the suspect had a beard (in addition to having blond hair and green eyes). There would be less elements of that set, since it would be expected that some blond hair, green-eyed people do not have beards.

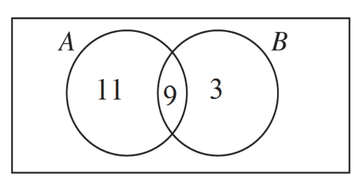
12. **(a)** Use a Venn diagram:

(i) Enter 9 as $n(A \cap B)$;

(ii) $n(B) = 12$, but 9 of these are in $A \cap B$, so there are 3 elements in B but not A;

(iii) $n(A \cup B) = 23$, but 12 are accounted for so there are 11 elements in A but not in B; so

(iv) $\boldsymbol{n(A) = 11 + 9 = 20.}$



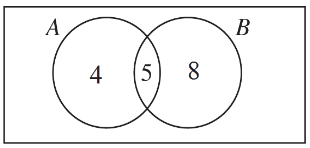
(b) Use a Venn diagram:

(***i***) Enter 5 as $n(A \cap B)$;

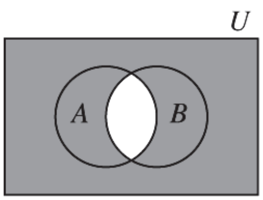
(***ii***) $n(A) = 9,$ but 5 of these are in $A \cap B,$ so there are 4 elements in A but not B;

(***iii***) $n(B) = 13,$ but 5 of these are in $A \cap B,$ so there are 8 elements in B but not in $A,$ so;

(***iv***) $\mathbf{n(A \cup B) = 4 + 5 + 8 = 17.}$



13. (a) $\overline{\boldsymbol{A \cap B}} = \overline{\boldsymbol{A}} \cup \overline{\boldsymbol{B}}$



$\overline{A \cap B} = \overline{A} \cup \overline{B}$

(b) Let $U = \{a,b,c,d\}, A = \{a,b\}, B = \{b,c\}$:

$\overline{A \cap B} = \overline{\{b\}} = \{a,c,d\};$

$\overline{A} \cup \overline{B} = \{c,d\} \cup \{a,d\} = \{a,c,d\}.$

14. (a) The set of all Paxson 8th graders who are **members of the band but not the choir**, or $B - C$.

(b) The set of all Paxson 8th graders who are **members of both the band and the choir**, or $B \cap C$.

(c) The set of all Paxson 8th graders who are **members of the choir but not the band**, or $C - B$.

(d) The set of all Paxson 8th graders who are **neither members of the band nor of the choir**, or $\overline{B \cup C}$.

15. Enter numbers in each region in the following order:

(***i***) $n(A \cap B \cap C) = 3;$

(***ii***) $n(A \cap B) = 10;$ removing the 3 from $(A \cap B \cap C)$ leaves 7 in the region $n(A \cap B \cap \overline{C})$

(***iii***) Similarly, $n(A \cap C \cap \overline{B}) = 8 - 3 = 5;$ and

(***iv***) $n(B \cap C \cap \overline{A}) = 12 - 3 = 9;$

So, now what is left inside the Venn Diagram is to find (a) the number of elements in A, but not in B and C; (b) the number of elements in B, but not in A and C; and (c) the number of elements in C that are not in A and B. Those calculations are shown below

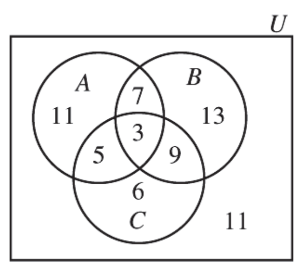
(***v***) $(a) = 26 - 7 - 3 - 5 = 11;$

(***vi***) $(b) = 32 - 7 - 3 - 9 = 13;$

(***vii***) $(c) = 23 - 5 - 3 - 9 = 6;$

Now, $11 + 7 + 3 + 5 + 13 + 9 + 6 = 54$ represents the total number of elements inside the Venn Diagram. Since $n(U) = 65$, find the number of elements outside the Venn Diagram but in the Universe:

(***viii***) $65 - 54 = 11$.



16. In the Venn diagram below:

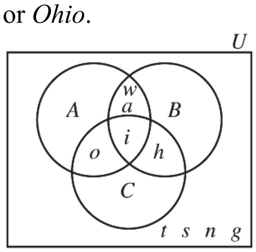
(***i***) "I" is the only letter contained in the set $A \cap B \cap C$ (i.e., the only letter common to *Iowa*, *Hawaii*, and *Ohio*);

(*ii*) "W" and "A" are the only letters contained in the set $(A \cap B) - C$ (i.e., the letters contained in both *Iowa* and *Hawaii* other than "I");

(*iii*) "O" is the only letter contained in the set $(A \cap C) - B$ (i.e., the letter contained in both *Iowa* and *Ohio* other than "I");

(*iv*) "H" is the only letter contained in the set $(B \cap C) - A$ (i.e., the letter contained in both *Hawaii* and *Ohio* other than "I");

(*v*) "T", "S", "N", and "G" are the letters in *Washington* not used in *Iowa*, *Hawaii*, or *Ohio*.



17. **(a)** Elements in regions $\boldsymbol{c, f, g,}$ **and** $\boldsymbol{h}$ represent students who do not take algebra.

(b) Elements in regions $\boldsymbol{b, c, d, e, f,}$ **and** $\boldsymbol{g}$ represent students who take biology or chemistry.

(c) Elements in regions $\boldsymbol{b}$ **and** $\boldsymbol{e}$ represent students who took both algebra and biology.

(d) Since (a) is the part of A with no elements in B or C, we could say **the students at Hellgate who took algebra but did not take biology and did not take chemistry**.

(e) Students at Hellgate who took biology and chemistry but did not take algebra.

(f) $\boldsymbol{(B \cap C) - A}$ **or** $\boldsymbol{(B \cap C) \cap \overline{A}}$

(g) Since regions (d) and (g) represent students who took chemistry, but did not take biology, we write $\boldsymbol{C - B}$ **or** $\boldsymbol{C \cap \overline{B}}$.

(h) Region (g) represents students who took chemistry but did not take biology and did not take algebra. We write $\boldsymbol{C - (A \cup B)}$ **or** $\boldsymbol{C \cap \overline{A \cup B}}$**.**

18. Generate the following Venn diagram in this order:

(*i*) The 50 who used all three cards;

(*ii*) The 60 who used **only** Super and Thrift cards;

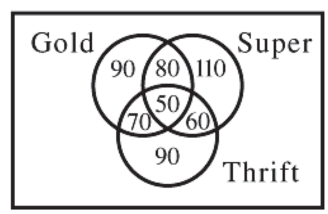
(*iii*) The 70 who used **only** Gold and Thrift cards;

(*iv*) The 80 who used **only** Gold and Super cards;

(*v*) The $290 - (80 + 50 + 70) = 90$ who used **only** Gold cards;

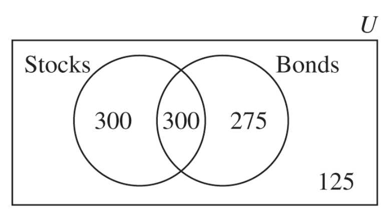
(*vi*) The $300 - (80 + 50 + 60) = 110$ who used **only** Super cards; and

(*vii*) The $270 - (70 + 50 + 60) = 90$ who used **only** Thrift cards.



The diagram indicates only 550 cardholders accounted for, so either there is some other type credit card used by the remaining 50 people or **the editor was right**.

19. Complete the Venn diagram by first completing the region that represents investors who invested in stocks and bonds.



(a) **300**

(b) **875**

(c) **125**

20. **(a)** It is possible that no student took both algebra and biology. **0**

(b) It is possible that all **30** biology students also took algebra.

(c) If all 30 biology students also took algebra, then the numbers of students who took neither is $150 - 90 = \mathbf{60}$ students.

21. **(a)** **146.4 million.** Subtract those who identify themselves as religious Jews (4.2 million) away from the number of people identified as being of some faith (150.6 million).

(b) **171.2 million**. Subtract those who identify themselves as being of some faith (150.6 million) from the total population of the United States (321.8 million).

22. **93.** From the data given, we know there are 44 total Canadian citizens. We also know there were a total of 34 American women and males on the tour. So, all we need to find the number of American girls to complete the problem. Because there are 29 women total, and 17 are American, we know there were 12 Canadian women on the tour. Since the total number of Canadian females is 26, we can deduce that 14 Canadian females were girls. Since there were 29 girls on the trip, we know that 15 girls must be American girls. Therefore $44 + 34 + 15 = \mathbf{93}$

23. **(a)** **False**. These are ordered pairs, thus order is relevant.

(b) **False**. The left side is an ordered pair, while the right side is a set.

24. **(a)** Each of the five elements in A are paired with each of the four in B so there are $5 \cdot 4 = \mathbf{20}$ **elements**.

(b) Each of the m elements in A are paired with each of the n in B so there are $\boldsymbol{m \cdot n}$ **elements**.

(c) $A \times B$ has $m \cdot n$ elements, each of which are paired with the p elements in C so there are $\boldsymbol{m \cdot n \cdot p}$ **elements**.

Mathematical Connections 2-3: Review Problems

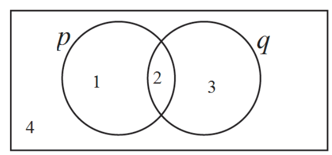
14. The contrapositive of the contrapositive of a statement is the statement itself. If the original statement is $p \to q$, then by definition its contrapositive is $\sim q \to \sim p$. To take the contrapositive of that statement, you would end up with $p \to q$.

15. **(a)** Therefore, Mary will change the lunch menu. (Modus Ponens, or law of detachment)

(b) Therefore, Samuel stays after school.

(c) Therefore, the lake is not frozen. (Modus Tollens)

16. Answers will vary. Consider the numbered Venn diagram below:



The area in numbered region 1 represents the area where p is true but q is false; the area in numbered region 3 represents the area where q is true but p is false; the area in numbered region 2 (the intersection) is where both p and q are true; and the area in numbered region 4 (outside the two circles) is the area where both p and q are false. All cases, therefore, are covered.

17. **(a)** Set-builder notation describes the elements of the set, rather than listing them; i.e., $\{x | x \in N \text{ and } 3 < x < 10\}$, where N represents the set of natural numbers, allows the set to be built.

(b) This set has only three elements; i.e., $\{\mathbf{15, 30, 45}\}$ thus the elements may easily be listed.

18. **(a)** **6**; c, o, m, n, r, and e are the elements in the set. c and m are duplicated once,and o is duplicated twice.

(b) **6**; c, o, m, i, t, e are the elements in the set. $m, t,$ and e are duplicated once.

19. **(a)** These are all the subsets of $\{2,3,4\}$. There are $2^3 = \mathbf{8}$ such subsets.

(b) **8**. Every subset either contains 1 or it does not, so exactly half the $2^4 = 16$ subsets contain 1.

(c) **Twelve** subsets. There are 4 subsets of $\{3,4\}$. Each of these subsets can be appended with 1, 2, or 1 and 2 to each. By the Fundamental Counting Principle then, there are $3 \cdot 4 = 12$ possibilities. (It is also possible to simple systematically list all the possible subsets with 1 or 2 and count them.)

(d) There are **four** subsets containing neither 1 nor 2, since 12 do contain 1 or 2.

(e) **16**. B has $2^5 = 32$ subsets; half contain 5 and half do not.

20. Answers may vary. Some possibilities:

(a) The set of 18 holes on a golf course, and the set of 18 flagsticks on a golf course.

(b) The set of letters in the English alphabet (26) with the set of letters in the Greek alphabet (24).

21. The number of combinations is equivalent to the Cartesian product of $\{SLACKS\}$, $\{SHIRTS\}$, and $\{SWEATERS\}$ so the number of elements is $n(\{SLACKS\}) \cdot n(\{SHIRTS\}) \cdot n(\{SWEATERS\}) =$ $4 \cdot 5 \cdot 3 = \mathbf{60}$ **combinations**.

Chapter 2 Review

1. Answers will vary. Some statements: i). Arizona is a state; ii) The president of the United States is Barack Obama; iii) $2 + 12 = 24$; $2 \times 12 = 24$. Some non-statements: i) Las Vegas is a fun city; ii) Teaching is a tough job; iii) How old are you?; iv) Zane Grey is the best writer ever.

2. In statement (i), every student in the class earned an A, B, or C grade on the final exam; in statement (ii), there was at least one student who earned an A, B, or C grade on the final exam. In statement (i), no student could have received a grade of D or F on the final exam; while in statement (ii), it is possible that a student (or several students) did receive a grade of D or F on the final exam.

3. **(a)** **Yes**. Even though it is false, it is still a statement, by the definition of a statement.

(b) **No**. This sentence is neither true or false, since it depends upon the value of the variable n. Since its truth value cannot be definitely ascertained, it is not a statement.

(c) **Yes**. It can definitely be determined whether or not this statement is true or false.

4. **(a)** **Some women smoke**. The original statement means that there exists no women who smoke. So to negate it, one would have to write a statement that implied there exists at least one woman who smokes.

(b) $3 + 5 = 8$. The original statement is false; to negate it, write a statement that is true.

(c) **Bach wrote some music that was not classical**. The reasoning behind this answer is similar to the answer in part (a). The original statement implies that Bach only wrote classical music. To negate it, a statement must be written to imply that there exists at least one piece of Bach's music that was not classical.

5. The original statement is in the form $p \to q$. Converse $(q \to p)$:If someone will read a tweet, the whole world is tweeting. Inverse $(\sim p \to \sim q)$: If the whole world is not tweeting, no one will read a tweet. Contrapositive $(\sim q \to \sim p)$: If no one will read a tweet, the whole world is not tweeting.

6.

p	q	$\sim p$	$\sim q$	$\sim p \to q$	$\sim q \to p$
T	T	F	F	T	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	F

7. (a)

p	q	$\sim q$	$p \vee \sim q$	$p \vee q$	$(p \vee \sim q) \wedge (p \vee q)$
T	T	F	T	T	T
T	F	T	T	T	T
F	T	F	F	T	F
F	F	T	T	F	F

(b)

p	q	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge \sim q$	$[(p \vee \sim q) \wedge \sim q] \rightarrow p$
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	F	F	T
F	F	T	T	T	F

8. (a) $\therefore$ Alfinia loves Mom and apple pie. This is true by direct reasoning. The first statement could be read as "If a person is a Eurasian, then that person loves Mom and apple pie". We can then make p be the statement "a person is a Eurasian" and q the statement "A person loves Mom and apple pie." So, the first statement is denoted in logical notation as $p \rightarrow q$; the second statement, Alfinia is a Eurasian, is denoted as p. So, it makes sense that the conclusion should be q.

(b) $\therefore$ The Washington Monument will eventually crack. This is true by direct reasoning, similar to (a). The first statement could be written in $p \rightarrow q$ form as "if an object is made of marble and granite, then it will eventually crack."

(c) $\therefore$ Therefore, Josef passed the math for elementary teachers course. The first sentence consists of two statements "Josef passed the math for elementary teachers course" is one statement; "Josef dropped out of school" is the other statement. The "or" between the two statements implies that one or the other is true. The second sentence "Josef did not drop out of school" would imply that the second part of that first sentence isn't true…so the first part must be.

9. Let p be the statement "you passed your classes"; q be the statement "your parents will allow you to go to the dance"; and r be the statement "you sit in the corner." So, the sentences can be written as follows: first sentence: $p \rightarrow q$; second sentence $q \rightarrow \sim r$. Third sentence: $\sim(\sim r) = r$; so the conclusion is $\sim p$. This is a valid argument by transitivity and modus tollens.

10. This argument is valid by modus tollens. Let p be the statement "Bob passes the course" and q be the statement "Bob scored at least a 75 on the final exam". Then the first sentence is $p \rightarrow q$; the second sentence is $\sim q$; therefore the conclusion is $\sim p$.

11. A set with n elements has 2^n subsets. $2^4 = 16$. This includes A. There are $2^4 - 1 =$ **15 proper subsets**.

12. There are 4 elements in the set, thus there are $2^4 = \mathbf{16}$ **subsets**: $\{\}, \{m\}, \{a\}, \{t\}, \{h\}, \{m,a\}, \{m,t\}, \{m,h\}, \{a,t\}, \{a,h\}, \{t,h\}, \{m,a,t\}, \{m,a,h\}, \{m,t,h\}, \{a,t,h\}, \{m,a,t,h\}$.

13. (a) $A \cup B = \{r,a,v,e\} \cup \{a,r,e\} = \{r,a,v,e\} = \boldsymbol{A}$.

(b) $C \cap D = \{l,i,n,e\} \cap \{s,a,l,e\} = \boldsymbol{\{l,e\}}$.

(c) $\overline{D} = \overline{\{s,a,l,e\}} = \boldsymbol{\{u,n,i,v,r\}}$.

(d) $A \cap \overline{D} = \{r,a,v,\mathrm{e}\} \cap \{u,n,i,v,r\} = \boldsymbol{\{r,v\}}$.

(e) $\overline{B \cup C} = \overline{\{a,r,\mathrm{e}\} \cup \{l,i,n,e\}}$
$= \overline{\{a,e,i,l,n,r\}}$
$= \boldsymbol{\{s,u,v\}}$.

(f) $\mathrm{B} \cup \mathrm{C} = \{a,e,i,l,n,r\} \Rightarrow (B \cup C) \cap D$
$= \{a,e,i,l,n,r\} \cap \{s,a,l,\mathrm{e}\}$
$= \boldsymbol{\{a,l,e\}}$.

(g) $\overline{A} \cup B = \{u,n,i,s,l\} \cup \{a,r,e\}$
$= \{u,n,i,e,r,s,a,l\}$.

$C \cap \overline{D} = \{l,i,n,e\} \cap \{u,n,i,v,r\} = \{i,n\}. \Rightarrow (\overline{A} \cup B) \cap (C \cap \overline{D}) = \boldsymbol{\{i,n\}}$.

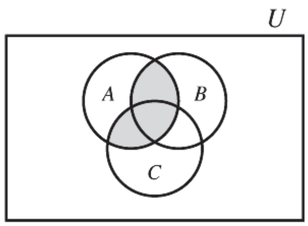
(h) $(C \cap D) \cap A = (\{l,i,n,e\} \cap \{s,a,l,e\}) \cap \{r,a,v,e\} = \{l,e\} \cap \{r,a,v,e\} = \boldsymbol{\{e\}}$.

(i) $n(B - A) = \mathbf{0}$.

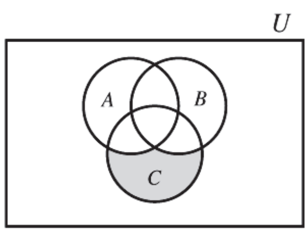
(j) $n(\overline{C}) = n\{u,v,r,s,a\} = \mathbf{5}$.

(k) $n(C \times D) = 4 \cdot 4 = \mathbf{16}$. Each of the four elements in C can be paired with each of the four in D.

14. **(a)** $A \cap (B \cup C)$ includes the elements in A that are common to the union of B and C:



(b) $(\overline{A \cup B}) \cap C$ is the same as $C - (A \cup B)$:

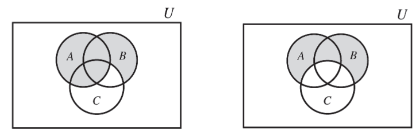


15. Since all 5 letters are distinct, consider seven "slots" in which to put the letters. There are 5 letters which could go in the first slot, then 4 left which could go in the second slot, and so on. So, the number of possible arrangements is then $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

16. **(a)** Answers may vary; **one possible** correspondence is $\boldsymbol{t \leftrightarrow e}$, $\boldsymbol{h \leftrightarrow n}$, **and** $\boldsymbol{e \leftrightarrow d}$.

(b) There are three possible one-to-one correspondences between D and the e in E, two possible between D and the n in E, and then only one possible remaining for the d in E. Thus $3 \cdot 2 \cdot 1 = \mathbf{6}$ one-to-one correspondences are possible.

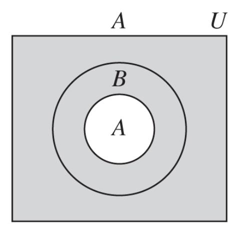
17. $A \cup (B - C) \neq (A \cup B) - C$



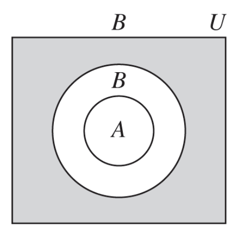
18. **(a)** **False**. Let $U = \{1,2,3,4\}$, $A = \{1,2\}$, and $B = \{1,2,3\}$ Then $\overline{A} = \{3,4\}$ and $\overline{B} = \{4\}$. So $\overline{A} \not\subseteq \overline{B}$.

(b) **True**.

$\overline{A}$



$\overline{B}$



(c) **True**. Since $A \subseteq B$, every element in A is in B, So combining A and B does not include new elements to B.

(d) **True**. Since $A \subseteq B$ and $A \cap B = \{x | x \in A \text{ and } x \in B\}$, the intersection, the elements common to A and B, is A.

(e) **True**. Since $\overline{B} \subseteq \overline{A}$, this property was established in d.

(f) **True,** This is because $\overline{B} \subseteq \overline{A}$, as established in b.

19. **(a)** **False.** For example, let $U = \{1,2,3,4,5\}$; $A = \{1,2,3,4\}$; $B = \{3,4,5\}$ and $C = \{3,4\}$. Then $A - B = \{1,2\} = A - C$; but $B \neq C$

(b) True

20. **(a)** A **.** By the distributive property of set intersection over set union $(A \cap B) \cup (A \cap \overline{B})$ can be written as $A \cap (B \cup \overline{B})$. Since $(B \cup \overline{B}) = U$, we have $A \cap U$, which equals A.

(b) $A \cup \overline{B}$ **.**By the distribution property of set union over set intersection, $(A \cap B) \cup \overline{B}$ can be rewritten as $(A \cup \overline{B}) \cap (B \cup \overline{B})$. Since $(B \cup \overline{B}) = U$, we have $(A \cup \overline{B}) \cap U$, which equals $A \cup \overline{B}$

21. **(a)** **False**. The sets could be disjoint.

(b) **False**. The empty set is not a proper subset of itself.

(c) **False**. $A \sim B$ only requires the same number of elements—not necessarily the same elements.

(d) **False**. The set is in one-to-one correspondence with the set of natural numbers, so it increases without limit.

(e) **False**. For example, the set $\{5,10,15,...\}$ is a proper subset of the natural numbers and is equivalent since there is a one-to-one correspondence.

(f) **False**. Let $B = \{1,2,3\}$ and $A =$ the set of natural numbers.

(g) **True**. If $A \cap B \neq \emptyset$, then the sets are not disjoint.

(h) **False**. The sets may be disjoint but not empty.

22. **(a)** **True**. Venn diagrams show that $A - B$, $B - A$, and $A \cap B$ are all disjoint sets, so $n(A - B) + n(B - A) + n(A \cap B) = n(A \cup B)$.

(b) **True**. Venn diagrams show that $A - B$ and B are disjoint sets, so $n(A - B) + n(B) = n(A \cup B)$. Likewise, Venn diagrams show that $(B - A)$ and A are disjoint, so $n(B - A) + n(A) = n(A \cup B)$.

23. **(a)** **17**, if $P = Q$.

(b) **34**, if P and Q are disjoint.

(c) **0**, if P and Q are disjoint.

(d) **17**, if $P = Q$.

24. $A \times B \times C$ is the set of ordered triples (a,b,c), where $a \in A$, $b \in B$, and $c \in C$. There are 3 possibilities for a, the first entry, 4 possibilities for the second, and 2 for the third.
So $n(A \times B \times C) =$ **24**.

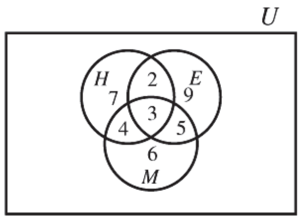
25. $n(\text{Crew}) + n(\text{Swimming}) + n(\text{Soccer}) = 57$. The 2 lettering in all three sports are counted three times, so subtract 2 twice, giving 53. $n(\text{Awards}) = 46$, so $53 - 46 = 7$ were counted twice; i.e., **7 lettered in exactly two sports**.

26. Use the following Venn diagram place values in appropriate areas starting with the fact that 3 students liked all three subjects; 7 liked history and mathematics, but of those 7, three also liked English so 4 is placed in the $\text{H} \cap \text{M}$ region; etc.

(a) A total of **36 students** were in the survey.

(b) **6 students** liked only mathematics.

(c) **5 students** liked English and mathematics but not history.



27. Answers may vary. Possibilities are:

(a) The shaded areas show $\boldsymbol{B \cup (A \cap C)}$

(b) The shaded areas show $\boldsymbol{B - C}$ or $\boldsymbol{B \cap \overline{C}}$

28. 2 slacks $\times$ 3 blouses $\times$ 2 sweaters $=$ **12** outfits.

29. **(a)** Let $A = \{1, 2, 3, \ldots, 13\}$ and $B = \{1, 2, 3\}$. $B \subset A$, so B has fewer elements than A. Then $n(B) < n(A)$ and thus $3 < 13$.

(b) Let $\text{A} = \{1, 2, 3, \ldots, 12\}$ and $B = \{1, 2, 3, \ldots, 9\}$. $B \subset A$, so A has more elements than B. Then $n(A) > n(B)$ and thus $12 > 9$.

Chapter 2
Introduction to Logic and Sets

Activity 1

Box 1

1. No. If there were green gumballs in the jar, it would be labeled correctly, but no jar has a correct label.
2. Red
3. No. If so, the jar labeled GREEN would have GREEN gumballs, but all labels are incorrect.
4. The correct label for the jar labeled Red - Green is Red, the correct label for the jar labeled Red is Green, and the correct label for the jar labeled Green is Red - Green.

Box 2

Greatest amount = \$1.19

Coins = 1 half dollar, 1 quarter, 4 dimes, 4 pennies

Box 3

State	Alabama	Alaska	Oklahoma	Minnesota
Flower	Camellia	Forget Me Not	Mistletoe	Lady's slipper
Bird	Yellowhammer	Willow ptarmigan	Flycatcher	Loon

Clues b, d, and c are the keys to solving the problem.
From b, we know that Alaska and forget me nots go together.
From d, we know that Alabama and yellowhammer go together.
From c, either camellia or lady 's slipper go with Minnesota. If camellia and Minnesota go together, then the state bird can't be the willow ptarmigan (clue e) or the loon (clue c) so it must be the flycatcher. But this would contradict clue a, so Minnesota, lady's slipper, and loons must go together.

Box 4

Two answers are possible. Freddie or Susie can be in either First or Fourth place.

First	Second	Third	Fourth
Yellow	Purple	Green	Red
Freddie or Susie	Liz	Joe	Susie or Freddie

Activity 2

Box 1

2. a. SRS, SRT, SRC, SRR (small, red, rhombus)
   LRS, LRT, LRC, LRR

   b. 8

   c. 32

3. a. SGS, SGT, SGC, SGR, LGS, LGT, LGC, LGR,
   SYS, SYT, SYC, SYR, LYS, LYT, LYC, LYR,
   SBS, SBT, SBC, SBR, LBS, LBT, LBC, LBR

   b. The pieces are outside the loop.

4. a. Sample Responses: the set of blue pieces; the set of circles

   b. You could pair up the pieces in set A with the pieces in set B to make sure there are no pieces left over in either set. That is, you could create a one-to-one correspondence between the elements of the two sets.

   c. The set of pieces that are not large.

Activity 3

1. b. If the loops did not overlap, there would be no way to place the pieces that are Large AND Red.

2. a. (either) **RED or LARGE**

   b. 20

3. a. **RED and LARGE**

   b. 4

4. Sample Response: **Square** and **Circle**

5. Sample Response: **Triangle** and **Not Square** (The triangles are a subset of the pieces that are not square.)

6. a. 12

   b. A

7. **Not Square** and **Blue**
   Since both loops contain pieces that are large and pieces that are small, the labels do not involve size. There are two possibilities for the label for the loop on the left, Not Yellow (since it contains pieces with all the other colors) or Not Square (since it contains only circles, squares, and rhombi). Suppose the correct label is Not Yellow. Then the small red square would go in that loop. Since it isn't, the label for the left loop cannot be Not Yellow. Thus the label must be Not Square. Similarly, the label for the right loop could be Blue or Not Rhombus. However, if the label was Not Rhombus, then the small red square would be in the loop. Since it isn't, the label must be Blue.