Chapter 3

3-1 What is the significance of the "critical stress"?

(a) with respect to the structure of the concrete?

A continuous pattern of mortar cracks begins to form. As a result there are few undamaged portions to carry load and the stress-strain curve is highly nonlinear.

(b) with respect to spiral reinforcement?

At the critical stress the lateral strain begins to increase rapidly. This causes the concrete core within the spiral to expand, stretching the spiral. The tension in the spiral is equilibrated by a radial compression in the core. This in turn, biaxially compresses the core, and thus strengthens it.

(c) with respect to strength under sustained loads?

When concrete is subjected to sustained loads greater than the critical stress, it will eventually fail.

3-2 A group of 45 tests on a given type of concrete had a mean strength of 4780 psi and a standard deviation of 525 psi. Does this concrete satisfy the requirements of ACI Code Section 5.3.2 for 4000-psi concrete?

From Eq. 3-3a:

$f'_c = f'_{cr} - 1.34s$

Using $f'_{cr} = 4780$ psi

(for design) $f'_c = 4780 - 1.34 * 525 = 4080$ psi

From Eq. 3-3b:

$f'_c = f'_{cr} - 2.33s + 500$

Using $f'_{cr} = 4780$ psi

(for design) $f'_c = 4780 - 2.33 * 525 + 500 = 4060$ psi

Because both of these exceed 4000 psi, the concrete satisfies the requirements of ACI Code Section 5.3.2 for 4000 psi concrete.

3-3 The concrete containing Type I cement in a structure is cured for 3 days at 70° F followed by 6 days at 40° F. Use the maturity concept to estimate its strength as a fraction of the 28-day strength under standard curing.

Note: $°C = \frac{5}{9}(°F - 32)$, so 70° F = 21.1° C and 40° F = 4.4° C

From Eq. 3-6:

$$M = \sum_{i=1}^{n} (T_i + 10)(t_i)$$
$$= (21.1 + 10)(3) + (4.4 + 10)(6) = 180 \text{ C days}$$

From Fig. 3-8 the compressive strength will be between 0.60 and 0.70 times the 28-day strength under standard curing conditions.

3-4 Use Fig. 3-12a to estimate the compressive strength σ_2 for bi-axially loaded concrete subject to:

(a) $\sigma_1 = 0.0$, $\sigma_2 = f_c'$

(b) $\sigma_1 = 0.75\ f_t'$ in tension, $\sigma_2 = 0.5\ f_c'$

(c) $\sigma_1 = 0.5\ f_c'$ in compression, $\sigma_2 = 1.2\ f_c'$

3-5 The concrete in the core of a spiral is subjected to a uniform confining stress σ_3 of 750 psi. What will the compressive strength, σ_1 be? Assume $f_c' = 4500$ psi.

From Eq. 3-16:

$$\sigma_1 = f_c' + 4.1\sigma_3$$

$$\sigma_1 = 4500 + 4.1 * 750 = 7580 \text{ psi}$$

3-6 What factors affect the shrinkage of concrete?

(a) Relative humidity. Shrinkage increases as the relative humidity decreases, reaching a maximum at RH $\leq$ 40%.

(b) The fraction of the total volume made up of paste. As this fraction increases, shrinkage increases.

(c) The modulus of elasticity of the aggregate. As this increases, shrinkage decreases.

(d) The water/cement ratio. As the water content increases, the aggregate fraction decreases, causing an increase in shrinkage.

(e) The fineness of the cement. Shrinkage increases for finely ground cement that has more surface area to attract and absorb water.

(f) The effective thickness or volume to surface ratio. As this ratio increases, the shrinkage occurs more slowly and the total shrinkage is likely reduced.

(g) Exposure to carbon dioxide tends to increase shrinkage.

3-7 What factors affect the creep of concrete?

(a) The ratio of sustained stress to the strength of the concrete. The creep coefficient, ϕ, is roughly constant up to a stress of 0.5 f_c', but increases above that value.

(b) The humidity of the environment. The amount of creep decreases as the RH increases above 40%.

(c) As the effective thickness or volume to surface ratio increases, the rate at which creep develops decreases.

(d) Concretes with a high paste content creep more that concretes with a large aggregate fraction because only the paste creeps.

3-8 A structure is made from concrete containing Type 1 cement. The average ambient relative humidity is 70 percent. The concrete was moist-cured for 7 days. f_c' = 4000 psi.

(a) Compute the unrestrained shrinkage strain of a rectangular beam with cross-sectional dimensions of 8 in. x 20 in. at 2 years after the concrete was placed.

1. Compute the humidity modification factor from Eq. (3-30a):

$$\gamma_{rh} = 1.40 - 0.01 * RH = 1.4 - 0.01 * 70 = 0.70$$

2. Use Eq. (3-31) to compute the volume/surface area ratio modification factor:

$$\text{Volume per foot of beam} = 12 * 8 * 20 = 1920 \text{ in}^3$$

$$\text{Surface area per foot of beam} = 2 * [(12 * 8) + (12 * 20)] = 672 \text{ in.}^2$$

$$\gamma_{vs} = 1.2^{-0.12V/S} = 1.2^{-0.12*1920/672} = 0.939 \cong 0.94$$

3. Use Eq. (3-29) to compute the ultimate shrinkage strain:

$$(\varepsilon_{sh})_u = \gamma_{rh} * \gamma_{vs} * 780x10^{-6} = 0.70 * 0.94 * 780x10^{-6} = 513x10^{-6} \text{ strain}$$

4. Use Eq. (3-28) to compute the shrinkage strain after 2 years:

$$t = 2 * 365 - 7 = 723 \text{ days}$$

$$(\varepsilon_{sh})_t = \frac{t}{35 + t}(\varepsilon_{sh})_u = \frac{723}{35 + 723} 513x10^{-6} = 489x10^{-6} \cong 490x10^{-6} \text{ strain}$$

(b) Compute the stress dependent (creep) strain in the concrete of a 20 in. x 20 in. x 12 ft column at age 3 years. A compression load of 400 kips was applied to the column at 30 days.

1. Compute the ultimate shrinkage strain coefficient, C_u, using Eqs. (3-36)-(3-39).

$$\lambda_{rh} = 1.27 - 0.0067 * RH = 1.27 - 0.0067 * 70 = 0.80$$

$$\lambda_{to} = 1.25 * t_o^{-0.118} = 1.25 * 30^{-0.118} = 0.84$$

$$\lambda_{vs} = 0.67 * [1 + 1.13^{-0.54*V/S}],$$

Where: $V = 20 \text{ in.} * 20 \text{ in.} * 12 \text{ ft} * 12 \frac{\text{in.}}{\text{ft}} = 57{,}600 \text{ in.}^3$

$S = 4 \text{ sides} * 20 \text{ in.} * 12 \text{ ft} * 12 \frac{\text{in.}}{\text{ft}} = 11{,}520 \text{ in.}^2$

$$\lambda_{vs} = 0.67 * [1 + 1.13^{-0.54*57600/11520}] = 1.15$$

$$C_u = 2.35 * \lambda_{rh} * \lambda_{to} * \lambda_{vs} = 2.35 * 0.80 * 0.84 * 1.15 = 1.82$$

2. Compute the creep coefficient for the time since loading, C_t, using Eq. (3-35).

   $t = 3 * 365 - 30 = 1065$ days

   $$C_t = \frac{t^{0.6}}{10 + t^{0.6}} * C_u = \frac{1065^{0.6}}{10 + 1065^{0.6}} * 1.82 = 1.58$$

3. Compute the total stress-dependent strain, ε_c(total), using Eqs. (3-5), (3-18), and (3-35).

   First, calculate the creep strain since the load was applied:

   $f_{cm} = 1.2 * f_c' = 1.2 * 4000 = 4800$ psi

   $E_c(28) = 57{,}000 * \sqrt{f_{cm}} = 57{,}000 * \sqrt{4800} = 3.95 * 10^6$ psi

   $$\varepsilon_{cc}(t, t_o) = \frac{\sigma_c(t_o)}{E_c(28)} * C_t = \frac{\frac{400{,}000 \text{ lbs}}{20 \text{ in.} * 20 \text{ in.}}}{3.95 * 10^6 \text{ psi}} * 1.58 = 0.4 * 10^{-3} \text{ strain}$$

   Then, calculate the initial strain when the load is applied:

   $$f_c'(t_o) = f_c'(28) * \frac{t_o}{4 + 0.85 * t_o} = 4000 * \frac{30}{4 + 0.85 * 30} = 4070 \text{ psi}$$

   $f_{cm}(t_o) = 1.2 * f_c'(t_o) = 1.2 * 4070 = 4880$ psi

   $E_c(t_o) = 57{,}000 * \sqrt{f_{cm}(t_o)} = 57{,}000 * \sqrt{4880} = 3.98 * 10^6$ psi

   $$\varepsilon_c(t_o) = \frac{\sigma_c(t_o)}{E_c(t_o)} = \frac{\frac{400{,}000 \text{ lbs}}{20 \text{ in.} * 20 \text{ in.}}}{3.98 * 10^6 \text{ psi}} = 0.25 * 10^{-3} \text{ strain}$$

   Thus,

   $$\varepsilon_c(total) = \varepsilon_c(t_o) + \varepsilon_{cc}(t, t_o) = 0.25 * 10^{-3} + 0.4 * 10^{-3} = 0.65 * 10^{-3} \text{ strain}$$