Chapter 2

2.1
Sketch

2.2
Sketch

2.3
Sketch

2.4

From Problem 2.2, phase $= \dfrac{2\pi x}{\lambda} - \omega t$ = constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} - \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = +\omega\left(\frac{\lambda}{2\pi}\right)$$

From Problem 2.3, phase $= \dfrac{2\pi x}{\lambda} + \omega t$ = constant

Then

$$\frac{2\pi}{\lambda} \cdot \frac{dx}{dt} + \omega = 0, \Rightarrow \frac{dx}{dt} = \upsilon_p = -\omega\left(\frac{\lambda}{2\pi}\right)$$

2.5

$$E = h\nu = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

Gold: $E = 4.90 \text{ eV} = (4.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(4.90)(1.6\times10^{-19})} = 2.54\times10^{-5} \text{ cm}$$

or

$\lambda = 0.254\ \mu$m

Cesium: $E = 1.90 \text{ eV} = (1.90)(1.6\times10^{-19})$ J

So,

$$\lambda = \frac{(6.625\times10^{-34})(3\times10^{10})}{(1.90)(1.6\times10^{-19})} = 6.54\times10^{-5} \text{ cm}$$

or

$\lambda = 0.654\ \mu$m

2.6

(a) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{550\times10^{-9}}$

$= 1.205\times10^{-27}$ kg-m/s

$$\upsilon = \frac{p}{m} = \frac{1.2045\times10^{-27}}{9.11\times10^{-31}} = 1.32\times10^{3} \text{ m/s}$$

or $\upsilon = 1.32\times10^{5}$ cm/s

(b) $p = \dfrac{h}{\lambda} = \dfrac{6.625\times10^{-34}}{440\times10^{-9}}$

$= 1.506\times10^{-27}$ kg-m/s

$$\upsilon = \frac{p}{m} = \frac{1.5057\times10^{-27}}{9.11\times10^{-31}} = 1.65\times10^{3} \text{ m/s}$$

or $\upsilon = 1.65\times10^{5}$ cm/s

(c) Yes

2.7

(a) (i)

$$p = \sqrt{2mE} = \sqrt{2(9.11\times10^{-31})(1.2)(1.6\times10^{-19})}$$

$= 5.915\times10^{-25}$ kg-m/s

$$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{5.915\times10^{-25}} = 1.12\times10^{-9} \text{ m}$$

or $\lambda = 11.2\ \text{Å}$

(ii) $p = \sqrt{2(9.11\times10^{-31})(12)(1.6\times10^{-19})}$

$= 1.87\times10^{-24}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{1.8704\times10^{-24}} = 3.54\times10^{-10} \text{ m}$$

or $\lambda = 3.54\ \text{Å}$

(iii) $p = \sqrt{2(9.11\times10^{-31})(120)(1.6\times10^{-19})}$

$= 5.915\times10^{-24}$ kg-m/s

$$\lambda = \frac{6.625\times10^{-34}}{5.915\times10^{-24}} = 1.12\times10^{-10} \text{ m}$$

or $\lambda = 1.12\ \text{Å}$

(b)

$$p=\sqrt{2\left(1.67\times10^{-27}\right)\left(1.2\right)\left(1.6\times10^{-19}\right)}$$

$$=2.532\times10^{-23}\text{ kg-m/s}$$

$$\lambda=\frac{6.625\times10^{-34}}{2.532\times10^{-23}}=2.62\times10^{-11}\text{ m}$$

or $\lambda=0.262\overset{o}{A}$

2.8

$$E_{avg}=\frac{3}{2}kT=\left(\frac{3}{2}\right)(0.0259)=0.03885\text{ eV}$$

Now

$$p_{avg}=\sqrt{2mE_{avg}}$$

$$=\sqrt{2\left(9.11\times10^{-31}\right)(0.03885)\left(1.6\times10^{-19}\right)}$$

or

$$p_{avg}=1.064\times10^{-25}\text{ kg-m/s}$$

Now

$$\lambda=\frac{h}{p}=\frac{6.625\times10^{-34}}{1.064\times10^{-25}}=6.225\times10^{-9}\text{ m}$$

or

$$\lambda=62.25\overset{o}{A}$$

2.9

$$E_p=h\nu_p=\frac{hc}{\lambda_p}$$

Now

$$E_e=\frac{p_e^2}{2m}\text{ and }p_e=\frac{h}{\lambda_e}\Rightarrow E_e=\frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2$$

Set $E_p=E_e$ and $\lambda_p=10\lambda_e$

Then

$$\frac{hc}{\lambda_p}=\frac{1}{2m}\left(\frac{h}{\lambda_e}\right)^2=\frac{1}{2m}\left(\frac{10h}{\lambda_p}\right)^2$$

which yields

$$\lambda_p=\frac{100h}{2mc}$$

$$E_p=E=\frac{hc}{\lambda_p}=\frac{hc}{100h}\cdot2mc=\frac{2mc^2}{100}$$

$$=\frac{2\left(9.11\times10^{-31}\right)\left(3\times10^{8}\right)^2}{100}$$

$$=1.64\times10^{-15}\text{ J}=10.25\text{ keV}$$

2.10

(a) $$p=\frac{h}{\lambda}=\frac{6.625\times10^{-34}}{85\times10^{-10}}$$

$$=7.794\times10^{-26}\text{ kg-m/s}$$

$$\upsilon=\frac{p}{m}=\frac{7.794\times10^{-26}}{9.11\times10^{-31}}=8.56\times10^{4}\text{ m/s}$$

or $\upsilon=8.56\times10^{6}$ cm/s

$$E=\frac{1}{2}m\upsilon^2=\frac{1}{2}\left(9.11\times10^{-31}\right)\left(8.56\times10^{4}\right)^2$$

$$=3.33\times10^{-21}\text{ J}$$

or $$E=\frac{3.334\times10^{-21}}{1.6\times10^{-19}}=2.08\times10^{-2}\text{ eV}$$

(b) $$E=\frac{1}{2}\left(9.11\times10^{-31}\right)\left(8\times10^{3}\right)^2$$

$$=2.915\times10^{-23}\text{ J}$$

or $$E=\frac{2.915\times10^{-23}}{1.6\times10^{-19}}=1.82\times10^{-4}\text{ eV}$$

$$p=m\upsilon=\left(9.11\times10^{-31}\right)\left(8\times10^{3}\right)$$

$$=7.288\times10^{-27}\text{ kg-m/s}$$

$$\lambda=\frac{h}{p}=\frac{6.625\times10^{-35}}{7.288\times10^{-27}}-9.09\times10^{-8}\text{ m}$$

or $\lambda=909\overset{o}{A}$

2.11

(a) $$E=h\nu=\frac{hc}{\lambda}=\frac{\left(6.625\times10^{-34}\right)\left(3\times10^{8}\right)}{1\times10^{-10}}$$

$$=1.99\times10^{-15}\text{ J}$$

Now

$$E=e\cdot V\Rightarrow V=\frac{E}{e}=\frac{1.99\times10^{-15}}{1.6\times10^{-19}}$$

$$V=1.24\times10^{4}\text{ V}=12.4\text{ kV}$$

(b) $$p=\sqrt{2mE}=\sqrt{2\left(9.11\times10^{-31}\right)\left(1.99\times10^{-15}\right)}$$

$$=6.02\times10^{-23}\text{ kg-m/s}$$

Then

$$\lambda=\frac{h}{p}=\frac{6.625\times10^{-34}}{6.02\times10^{-23}}=1.10\times10^{-11}\text{ m}$$

or

$$\lambda=0.11\overset{o}{A}$$

2.12

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-6}}$$
$$= 1.054\times10^{-28}\ \text{kg-m/s}$$

2.13

(a) (i) $\Delta p \Delta x = \hbar$

$$\Delta p = \frac{1.054\times10^{-34}}{12\times10^{-10}} = 8.783\times10^{-26}\ \text{kg-m/s}$$

(ii) $$\Delta E = \frac{dE}{dp}\cdot\Delta p = \frac{d}{dp}\left(\frac{p^2}{2m}\right)\cdot\Delta p$$
$$= \frac{2p}{2m}\cdot\Delta p = \frac{p\Delta p}{m}$$

Now $p = \sqrt{2mE}$
$$= \sqrt{2\left(9\times10^{-31}\right)(16)\left(1.6\times10^{-19}\right)}$$
$$= 2.147\times10^{-24}\ \text{kg-m/s}$$

so $$\Delta E = \frac{\left(2.1466\times10^{-24}\right)\left(8.783\times10^{-26}\right)}{9\times10^{-31}}$$
$$= 2.095\times10^{-19}\ \text{J}$$

or $$\Delta E = \frac{2.095\times10^{-19}}{1.6\times10^{-19}} = 1.31\ \text{eV}$$

(b) (i) $\Delta p = 8.783\times10^{-26}$ kg-m/s

(ii) $$p = \sqrt{2\left(5\times10^{-28}\right)(16)\left(1.6\times10^{-19}\right)}$$
$$= 5.06\times10^{-23}\ \text{kg-m/s}$$
$$\Delta E = \frac{\left(5.06\times10^{-23}\right)\left(8.783\times10^{-26}\right)}{5\times10^{-28}}$$
$$= 8.888\times10^{-21}\ \text{J}$$

or $$\Delta E = \frac{8.888\times10^{-21}}{1.6\times10^{-19}} = 5.55\times10^{-2}\ \text{eV}$$

2.14

$$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-2}} = 1.054\times10^{-32}\ \text{kg-m/s}$$
$$p = m\upsilon \Rightarrow \Delta\upsilon = \frac{\Delta p}{m} = \frac{1.054\times10^{-32}}{1500}$$
$$\Delta\upsilon = 7\times10^{-36}\ \text{m/s}$$

2.15

(a) $\Delta E \Delta t = \hbar$

$$\Delta t = \frac{1.054\times10^{-34}}{(0.8)\left(1.6\times10^{-19}\right)} = 8.23\times10^{-16}\ \text{s}$$

(b) $$\Delta p = \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{1.5\times10^{-10}}$$
$$= 7.03\times10^{-25}\ \text{kg-m/s}$$

2.16

(a) If $\Psi_1(x,t)$ and $\Psi_2(x,t)$ are solutions to Schrodinger's wave equation, then

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_1(x,t)}{\partial x^2} + V(x)\Psi_1(x,t) = j\hbar\frac{\partial\Psi_1(x,t)}{\partial t}$$

and

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2\Psi_2(x,t)}{\partial x^2} + V(x)\Psi_2(x,t) = j\hbar\frac{\partial\Psi_2(x,t)}{\partial t}$$

Adding the two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$
$$+V(x)\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$
$$= j\hbar\frac{\partial}{\partial t}\left[\Psi_1(x,t) + \Psi_2(x,t)\right]$$

which is Schrodinger's wave equation. So $\Psi_1(x,t) + \Psi_2(x,t)$ is also a solution.

(b) If $\Psi_1(x,t)\cdot\Psi_2(x,t)$ were a solution to Schrodinger's wave equation, then we could write

$$\frac{-\hbar^2}{2m}\cdot\frac{\partial^2}{\partial x^2}\left[\Psi_1\cdot\Psi_2\right] + V(x)\left[\Psi_1\cdot\Psi_2\right]$$
$$= j\hbar\frac{\partial}{\partial t}\left[\Psi_1\cdot\Psi_2\right]$$

which can be written as

$$\frac{-\hbar^2}{2m}\left[\Psi_1\frac{\partial^2\Psi_2}{\partial x^2} + \Psi_2\frac{\partial^2\Psi_1}{\partial x^2} + 2\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x)\left[\Psi_1\cdot\Psi_2\right] = j\hbar\left[\Psi_1\frac{\partial\Psi_2}{\partial t} + \Psi_2\frac{\partial\Psi_1}{\partial t}\right]$$

Dividing by $\Psi_1\cdot\Psi_2$, we find

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2} + \frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2} + \frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$
$$+V(x) = j\hbar\left[\frac{1}{\Psi_2}\frac{\partial\Psi_2}{\partial t} + \frac{1}{\Psi_1}\frac{\partial\Psi_1}{\partial t}\right]$$

Since Ψ_1 is a solution, then

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_1}\cdot\frac{\partial^2\Psi_1}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_1}\cdot\frac{\partial\Psi_1}{\partial t}$$

Subtracting these last two equations, we have

$$\frac{-\hbar^2}{2m}\left[\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2}+\frac{2}{\Psi_1\Psi_2}\frac{\partial\Psi_1}{\partial x}\frac{\partial\Psi_2}{\partial x}\right]$$

$$=j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Since Ψ_2 is also a solution, we have

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\Psi_2}\cdot\frac{\partial^2\Psi_2}{\partial x^2}+V(x)=j\hbar\cdot\frac{1}{\Psi_2}\cdot\frac{\partial\Psi_2}{\partial t}$$

Subtracting these last two equations, we obtain

$$\frac{-\hbar^2}{2m}\cdot\frac{2}{\Psi_1\Psi_2}\cdot\frac{\partial\Psi_1}{\partial x}\cdot\frac{\partial\Psi_2}{\partial x}-V(x)=0$$

This equation is not necessarily valid, which means that $\Psi_1\Psi_2$ is, in general, not a solution to Schrodinger's wave equation.

2.17

$$\int_{-1}^{+3}A^2\cos^2\left(\frac{\pi x}{2}\right)dx=1$$

$$A^2\left[\frac{x}{2}+\frac{\sin(\pi x)}{2\pi}\right]\Bigg|_{-1}^{+3}=1$$

$$A^2\left[\frac{3}{2}-\left(\frac{-1}{2}\right)\right]=1$$

so $A^2=\dfrac{1}{2}$

or $|A|=\dfrac{1}{\sqrt{2}}$

2.18

$$\int_{-1/2}^{+1/2}A^2\cos^2(n\pi x)dx=1$$

$$A^2\left[\frac{x}{2}+\frac{\sin(2n\pi x)}{4n\pi}\right]\Bigg|_{-1/2}^{+1/2}=1$$

$$A^2\left[\frac{1}{4}-\left(-\frac{1}{4}\right)\right]=1=A^2\left(\frac{1}{2}\right)$$

or $|A|=\sqrt{2}$

2.19

Note that $\int_0^{\infty}\Psi\cdot\Psi^* dx=1$

Function has been normalized.

(a) Now

$$P=\int_0^{a_o/4}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o/4}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o/4}$$

or

$$P=(-1)\left[\exp\left(\frac{-2a_o}{4a_o}\right)-1\right]=1-\exp\left(\frac{-1}{2}\right)$$

which yields

$P=0.393$

(b)

$$P=\int_{a_o/4}^{a_o/2}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_{a_o/4}^{a_o/2}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_{a_o/4}^{a_o/2}$$

or

$$P=(-1)\left[\exp(-1)-\exp\left(\frac{-1}{2}\right)\right]$$

which yields

$P=0.239$

(c)

$$P=\int_0^{a_o}\left[\sqrt{\frac{2}{a_o}}\exp\left(\frac{-x}{a_o}\right)\right]^2dx$$

$$=\frac{2}{a_o}\int_0^{a_o}\exp\left(\frac{-2x}{a_o}\right)dx$$

$$=\frac{2}{a_o}\left(\frac{-a_o}{2}\right)\exp\left(\frac{-2x}{a_o}\right)\Bigg|_0^{a_o}$$

$$=(-1)[\exp(-2)-1]$$

which yields

$P=0.865$

2.20

$$P=\int\left|\psi(x)\right|^2 dx$$

(a) $$\int_0^{a/4}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{2}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]\Bigg|_0^{a/4}$$

$$=\left(\frac{2}{a}\right)\left[\frac{\left(\frac{a}{4}\right)}{2}+\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{8}+\frac{(1)(a)}{4\pi}\right]$$

or $P=0.409$

(b) $$P=\int_{a/4}^{a/2}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)}\right]\Bigg|_{a/4}^{a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}+\frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)}-\frac{a}{8}-\frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)}\right]$$

$$=2\left[\frac{1}{4}+0-\frac{1}{8}-\frac{1}{4\pi}\right]$$

or $P=0.0908$

(c) $$P=\int_{-a/2}^{+a/2}\left(\frac{2}{a}\right)\cos^2\left(\frac{\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}+\frac{\sin\left(\frac{2\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)}\right]\Bigg|_{-a/2}^{+a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}+\frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)}-\left(\frac{-a}{4}\right)-\frac{\sin(-\pi)}{\left(\frac{4\pi}{a}\right)}\right]$$

or $P=1$

2.21

(a) $$P=\int_0^{a/4}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_0^{a/4}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{8}-\frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=0.25$

(b) $$P=\int_{a/4}^{a/2}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{a/4}^{a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}-\frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)}-\left(\frac{a}{8}\right)+\frac{\sin(\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=0.25$

(c) $$P=\int_{-a/2}^{+a/2}\left(\frac{2}{a}\right)\sin^2\left(\frac{2\pi x}{a}\right)dx$$

$$=\left(\frac{2}{a}\right)\left[\frac{x}{2}-\frac{\sin\left(\frac{4\pi x}{a}\right)}{4\left(\frac{2\pi}{a}\right)}\right]\Bigg|_{-a/2}^{+a/2}$$

$$=\left(\frac{2}{a}\right)\left[\frac{a}{4}-\frac{\sin(2\pi)}{\left(\frac{8\pi}{a}\right)}-\left(\frac{-a}{4}\right)+\frac{\sin(-2\pi)}{\left(\frac{8\pi}{a}\right)}\right]$$

or $P=1$

2.22

(a) (i) $\upsilon_p=\dfrac{\omega}{k}=\dfrac{8\times10^{12}}{8\times10^{8}}=10^4$ m/s

or $\upsilon_p=10^6$ cm/s

$$\lambda=\frac{2\pi}{k}=\frac{2\pi}{8\times10^8}=7.854\times10^{-9}\text{ m}$$

or $\lambda = 78.54\ \overset{o}{A}$

(ii) $p = m\upsilon = \left(9.11\times10^{-31}\right)\left(10^4\right)$

$= 9.11\times10^{-27}$ kg-m/s

$E = \frac{1}{2}m\upsilon^2 = \frac{1}{2}\left(9.11\times10^{-31}\right)\left(10^4\right)^2$

$= 4.555\times10^{-23}$ J

or $E = \frac{4.555\times10^{-23}}{1.6\times10^{-19}} = 2.85\times10^{-4}$ eV

(b) (i) $\upsilon_p = \frac{\omega}{k} = \frac{1.5\times10^{13}}{-1.5\times10^9} = -10^4$ m/s

or $\upsilon_p = -10^6$ cm/s

$\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{1.5\times10^9} = 4.19\times10^{-9}$ m

or $\lambda = 41.9\ \overset{o}{A}$

(ii) $p = -9.11\times10^{-27}$ kg-m/s

$E = 2.85\times10^{-4}$ eV

2.23

(a) $\Psi(x,t) = Ae^{-j(kx+\omega t)}$

(b) $E = (0.025)\left(1.6\times10^{-19}\right) = \frac{1}{2}m\upsilon^2$

$= \frac{1}{2}\left(9.11\times10^{-31}\right)\upsilon^2$

so $|\upsilon| = 9.37\times10^4$ m/s $= 9.37\times10^6$ cm/s

For electron traveling in $-x$ direction,

$\upsilon = -9.37\times10^6$ cm/s

$p = m\upsilon = \left(9.11\times10^{-31}\right)\left(-9.37\times10^4\right)$

$= -8.537\times10^{-26}$ kg-m/s

$\lambda = \frac{h}{|p|} = \frac{6.625\times10^{-34}}{8.537\times10^{-26}} = 7.76\times10^{-9}$ m

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.76\times10^{-9}} = 8.097\times10^8\ \text{m}^{-1}$

$\omega = k\cdot|\upsilon| = \left(8.097\times10^8\right)\left(9.37\times10^4\right)$

or $\omega = 7.586\times10^{13}$ rad/s

2.24

(a) $p = m\upsilon = \left(9.11\times10^{-31}\right)\left(5\times10^4\right)$

$= 4.555\times10^{-26}$ kg-m/s

$\lambda = \frac{h}{p} = \frac{6.625\times10^{-34}}{4.555\times10^{-26}} = 1.454\times10^{-8}$ m

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.454\times10^{-8}} = 4.32\times10^8\ \text{m}^{-1}$

$\omega = k\upsilon = \left(4.32\times10^8\right)\left(5\times10^4\right)$

$= 2.16\times10^{13}$ rad/s

(b) $p = \left(9.11\times10^{-31}\right)\left(10^6\right)$

$= 9.11\times10^{-25}$ kg-m/s

$\lambda = \frac{6.625\times10^{-34}}{9.11\times10^{-25}} = 7.27\times10^{-10}$ m

$k = \frac{2\pi}{7.272\times10^{-10}} = 8.64\times10^9\ \text{m}^{-1}$

$\omega = \left(8.64\times10^9\right)\left(10^6\right) = 8.64\times10^{15}$ rad/s

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2.25

$E_n = \frac{\hbar^2 n^2\pi^2}{2ma^2} = \frac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(75\times10^{-10}\right)^2}$

$E_n = n^2\left(1.0698\times10^{-21}\right)$ J

or

$E_n = \frac{n^2\left(1.0698\times10^{-21}\right)}{1.6\times10^{-19}}$

or $E_n = n^2\left(6.686\times10^{-3}\right)$ eV

Then

$E_1 = 6.69\times10^{-3}$ eV

$E_2 = 2.67\times10^{-2}$ eV

$E_3 = 6.02\times10^{-2}$ eV

2.26

(a) $E_n = \frac{\hbar^2 n^2\pi^2}{2ma^2} = \frac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(10\times10^{-10}\right)^2}$

$= n^2\left(6.018\times10^{-20}\right)$ J

or $E_n = \frac{n^2\left(6.018\times10^{-20}\right)}{1.6\times10^{-19}} = n^2(0.3761)$ eV

Then

$E_1 = 0.376$ eV

$E_2 = 1.504$ eV

$E_3 = 3.385$ eV

(b) $\lambda = \frac{hc}{\Delta E}$

$\Delta E = (3.385 - 1.504)\left(1.6\times10^{-19}\right)$

$= 3.01\times10^{-19}$ J

$$\lambda = \frac{\left(6.625\times10^{-34}\right)\left(3\times10^{8}\right)}{3.01\times10^{-19}}$$
$$= 6.604\times10^{-7}\text{ m}$$
or $\lambda = 660.4$ nm

2.27

(a) $E_n = \dfrac{\hbar^2 n^2 \pi^2}{2ma^2}$

$$15\times10^{-3} = \frac{n^2\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(15\times10^{-3}\right)\left(1.2\times10^{-2}\right)^2}$$
$$15\times10^{-3} = n^2\left(2.538\times10^{-62}\right)$$
or $n = 7.688\times10^{29}$

(b) $E_{n+1} \cong 15$ mJ

(c) No

2.28

For a neutron and $n=1$:
$$E_1 = \frac{\hbar^2\pi^2}{2ma^2} = \frac{\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.66\times10^{-27}\right)\left(10^{-14}\right)^2}$$
$$= 3.3025\times10^{-13}\text{ J}$$
or
$$E_1 = \frac{3.3025\times10^{-13}}{1.6\times10^{-19}} = 2.06\times10^{6}\text{ eV}$$
For an electron in the same potential well:
$$E_1 = \frac{\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(9.11\times10^{-31}\right)\left(10^{-14}\right)^2}$$
$$= 6.0177\times10^{-10}\text{ J}$$
or
$$E_1 = \frac{6.0177\times10^{-10}}{1.6\times10^{-19}} = 3.76\times10^{9}\text{ eV}$$

2.29

Schrodinger's time-independent wave equation
$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi(x) = 0$$
We know that
$$\psi(x) = 0 \text{ for } x \ge \frac{a}{2} \text{ and } x \le \frac{-a}{2}$$
We have
$$V(x) = 0 \text{ for } \frac{-a}{2} < x < \frac{+a}{2}$$
so in this region
$$\frac{\partial^2\psi(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi(x) = 0$$
The solution is of the form
$$\psi(x) = A\cos kx + B\sin kx$$
where
$$k = \sqrt{\frac{2mE}{\hbar^2}}$$
Boundary conditions:
$$\psi(x) = 0 \text{ at } x = \frac{-a}{2},\ x = \frac{+a}{2}$$
First mode solution:
$$\psi_1(x) = A_1\cos k_1 x$$
where
$$k_1 = \frac{\pi}{a} \Rightarrow E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$
Second mode solution:
$$\psi_2(x) = B_2\sin k_2 x$$
where
$$k_2 = \frac{2\pi}{a} \Rightarrow E_2 = \frac{4\pi^2\hbar^2}{2ma^2}$$
Third mode solution:
$$\psi_3(x) = A_3\cos k_3 x$$
where
$$k_3 = \frac{3\pi}{a} \Rightarrow E_3 = \frac{9\pi^2\hbar^2}{2ma^2}$$
Fourth mode solution:
$$\psi_4(x) = B_4\sin k_4 x$$
where
$$k_4 = \frac{4\pi}{a} \Rightarrow E_4 = \frac{16\pi^2\hbar^2}{2ma^2}$$

2.30

The 3-D time-independent wave equation in cartesian coordinates for $V(x,y,z) = 0$ is:
$$\frac{\partial^2\psi(x,y,z)}{\partial x^2} + \frac{\partial^2\psi(x,y,z)}{\partial y^2} + \frac{\partial^2\psi(x,y,z)}{\partial z^2} + \frac{2mE}{\hbar^2}\psi(x,y,z) = 0$$
Use separation of variables, so let
$$\psi(x,y,z) = X(x)Y(y)Z(z)$$
Substituting into the wave equation, we obtain

$$YZ\frac{\partial^2 X}{\partial x^2}+XZ\frac{\partial^2 Y}{\partial y^2}+XY\frac{\partial^2 Z}{\partial z^2}+\frac{2mE}{\hbar^2}XYZ=0$$

Dividing by XYZ and letting $k^2=\dfrac{2mE}{\hbar^2}$, we find

(1) $$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}+\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}+\frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2}+k^2=0$$

We may set

$$\frac{1}{X}\cdot\frac{\partial^2 X}{\partial x^2}=-k_x^2 \Rightarrow \frac{\partial^2 X}{\partial x^2}+k_x^2X=0$$

Solution is of the form

$$X(x)=A\sin(k_x x)+B\cos(k_x x)$$

Boundary conditions: $X(0)=0\Rightarrow B=0$

and $X(x=a)=0\Rightarrow k_x=\dfrac{n_x\pi}{a}$

where $n_x=1,2,3....$

Similarly, let

$$\frac{1}{Y}\cdot\frac{\partial^2 Y}{\partial y^2}=-k_y^2 \text{ and } \frac{1}{Z}\cdot\frac{\partial^2 Z}{\partial z^2}=-k_z^2$$

Applying the boundary conditions, we find

$$k_y=\frac{n_y\pi}{a},\quad n_y=1,2,3....$$

$$k_z=\frac{n_z\pi}{a},\quad n_z=1,2,3...$$

From Equation (1) above, we have

$$-k_x^2-k_y^2-k_z^2+k^2=0$$

or

$$k_x^2+k_y^2+k_z^2=k^2=\frac{2mE}{\hbar^2}$$

so that

$$E\rightarrow E_{n_xn_yn_z}=\frac{\hbar^2\pi^2}{2ma^2}\left(n_x^2+n_y^2+n_z^2\right)$$

2.31

(a) $$\frac{\partial^2\psi(x,y)}{\partial x^2}+\frac{\partial^2\psi(x,y)}{\partial y^2}+\frac{2mE}{\hbar^2}\cdot\psi(x,y)=0$$

Solution is of the form:

$$\psi(x,y)=A\sin k_x x\cdot\sin k_y y$$

We find

$$\frac{\partial\psi(x,y)}{\partial x}=Ak_x\cos k_x x\cdot\sin k_y y$$

$$\frac{\partial^2\psi(x,y)}{\partial x^2}=-Ak_x^2\sin k_x x\cdot\sin k_y y$$

$$\frac{\partial\psi(x,y)}{\partial y}=Ak_y\sin k_x x\cdot\cos k_y y$$

$$\frac{\partial^2\psi(x,y)}{\partial y^2}=-Ak_y^2\sin k_x x\cdot\sin k_y y$$

Substituting into the original equation, we find:

(1) $$-k_x^2-k_y^2+\frac{2mE}{\hbar^2}=0$$

From the boundary conditions,

$A\sin k_x a=0$, where $a=40\ \overset{o}{A}$

So $k_x=\dfrac{n_x\pi}{a}$, $n_x=1,2,3,...$

Also $A\sin k_y b=0$, where $b=20\ \overset{o}{A}$

So $k_y=\dfrac{n_y\pi}{b}$, $n_y=1,2,3,...$

Substituting into Eq. (1) above

$$E_{n_xn_y}=\frac{\hbar^2}{2m}\left(\frac{n_x^2\pi^2}{a^2}+\frac{n_y^2\pi^2}{b^2}\right)$$

(b) Energy is quantized - similar to 1-D result. There can be more than one quantum state per given energy - different than 1-D result.

2.32

(a) Derivation of energy levels exactly the same as in the text

(b) $$\Delta E=\frac{\hbar^2\pi^2}{2ma^2}\left(n_2^2-n_1^2\right)$$

For $n_2=2, n_1=1$

Then

$$\Delta E=\frac{3\hbar^2\pi^2}{2ma^2}$$

(i) For $a=4\ \overset{o}{A}$

$$\Delta E=\frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(4\times10^{-10}\right)^2}$$

$$=6.155\times10^{-22}\text{ J}$$

or $$\Delta E=\frac{6.155\times10^{-22}}{1.6\times10^{-19}}=3.85\times10^{-3}\text{ eV}$$

(ii) For $a = 0.5$ cm

$$\Delta E = \frac{3\left(1.054\times10^{-34}\right)^2\pi^2}{2\left(1.67\times10^{-27}\right)\left(0.5\times10^{-2}\right)^2}$$

$$= 3.939\times10^{-36}\text{ J}$$

or

$$\Delta E = \frac{3.939\times10^{-36}}{1.6\times10^{-19}} = 2.46\times10^{-17}\text{ eV}$$

2.33

(a) For region II, $x > 0$

$$\frac{\partial^2\psi_2(x)}{\partial x^2} + \frac{2m}{\hbar^2}(E - V_O)\psi_2(x) = 0$$

General form of the solution is

$$\psi_2(x) = A_2\exp(jk_2x) + B_2\exp(-jk_2x)$$

where

$$k_2 = \sqrt{\frac{2m}{\hbar^2}(E - V_O)}$$

Term with B_2 represents incident wave and term with A_2 represents reflected wave.

Region I, $x < 0$

$$\frac{\partial^2\psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0$$

General form of the solution is

$$\psi_1(x) = A_1\exp(jk_1x) + B_1\exp(-jk_1x)$$

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Term involving B_1 represents the transmitted wave and the term involving A_1 represents reflected wave: but if a particle is transmitted into region I, it will not be reflected so that $A_1 = 0$.

Then

$$\psi_1(x) = B_1\exp(-jk_1x)$$

$$\psi_2(x) = A_2\exp(jk_2x) + B_2\exp(-jk_2x)$$

(b)

Boundary conditions:

(1) $\psi_1(x=0) = \psi_2(x=0)$

(2) $\left.\frac{\partial\psi_1}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi_2}{\partial x}\right|_{x=0}$

Applying the boundary conditions to the solutions, we find

$$B_1 = A_2 + B_2$$

$$k_2A_2 - k_2B_2 = -k_1B_1$$

Combining these two equations, we find

$$A_2 = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)\cdot B_2$$

$$B_1 = \left(\frac{2k_2}{k_2 + k_1}\right)\cdot B_2$$

The reflection coefficient is

$$R = \frac{A_2A_2^*}{B_2B_2^*} = \left(\frac{k_2 - k_1}{k_2 + k_1}\right)^2$$

The transmission coefficient is

$$T = 1 - R \Rightarrow T = \frac{4k_1k_2}{(k_1 + k_2)^2}$$

2.34

$$\psi_2(x) = A_2\exp(-k_2x)$$

$$P = \frac{|\psi(x)|^2}{A_2A_2^*} = \exp(-2k_2x)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(9.11\times10^{-31}\right)(3.5 - 2.8)\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$$

$$k_2 = 4.286\times10^9\text{ m}^{-1}$$

(a) For $x = 5\overset{o}{A} = 5\times10^{-10}$ m

$$P = \exp(-2k_2x)$$

$$= \exp\left[-2\left(4.2859\times10^9\right)\left(5\times10^{-10}\right)\right]$$

$$= 0.0138$$

(b) For $x = 15\overset{o}{A} = 15\times10^{-10}$ m

$$P = \exp\left[-2\left(4.2859\times10^9\right)\left(15\times10^{-10}\right)\right]$$

$$= 2.61\times10^{-6}$$

(c) For $x = 40\overset{o}{A} = 40\times10^{-10}$ m

$$P = \exp\left[-2\left(4.2859\times10^9\right)\left(40\times10^{-10}\right)\right]$$

$$= 1.29\times10^{-15}$$

2.35

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1 - \frac{E}{V_o}\right)\exp(-2k_2a)$$

where $k_2 = \sqrt{\frac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(9.11\times10^{-31}\right)(1.0 - 0.1)\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$$

or $k_2 = 4.860\times10^9$ m^{-1}

(a) For $a = 4\times10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)\exp\left[-2\left(4.85976\times10^9\right)\left(4\times10^{-10}\right)\right]$$

$$= 0.0295$$

(b) For $a = 12\times10^{-10}$ m

$$T \cong 16\left(\frac{0.1}{1.0}\right)\left(1-\frac{0.1}{1.0}\right)\exp\left[-2\left(4.85976\times10^9\right)\left(12\times10^{-10}\right)\right]$$

$$= 1.24\times10^{-5}$$

(c) $J = N_t e\upsilon$, where N_t is the density of transmitted electrons.

$$E = 0.1\,\text{eV} = 1.6\times10^{-20}\ \text{J}$$

$$= \frac{1}{2}m\upsilon^2 = \frac{1}{2}\left(9.11\times10^{-31}\right)\upsilon^2$$

$$\Rightarrow \upsilon = 1.874\times10^5\ \text{m/s} = 1.874\times10^7\ \text{cm/s}$$

$$1.2\times10^{-3} = N_t\left(1.6\times10^{-19}\right)\left(1.874\times10^7\right)$$

$$N_t = 4.002\times10^8\ \text{electrons/cm}^3$$

Density of incident electrons,

$$N_i = \frac{4.002\times10^8}{0.0295} = 1.357\times10^{10}\ \text{cm}^{-3}$$

2.36

$$T \cong 16\left(\frac{E}{V_O}\right)\left(1-\frac{E}{V_O}\right)\exp\left(-2k_2 a\right)$$

(a) For $m = (0.067)m_o$

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

$$= \left\{\frac{2(0.067)\left(9.11\times10^{-31}\right)(0.8-0.2)\left(1.6\times10^{-19}\right)}{\left(1.054\times10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 1.027\times10^9\ \text{m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)$$

$$\times\exp\left[-2\left(1.027\times10^9\right)\left(15\times10^{-10}\right)\right]$$

or

$$T = 0.138$$

(b) For $m = (1.08)m_o$

$$k_2 = \left\{\frac{2(1.08)\left(9.11\times10^{-31}\right)(0.8-0.2)\left(1.6\times10^{-19}\right)}{\left(1.054\times10^{-34}\right)^2}\right\}^{1/2}$$

or

$$k_2 = 4.124\times10^9\ \text{m}^{-1}$$

Then

$$T = 16\left(\frac{0.2}{0.8}\right)\left(1-\frac{0.2}{0.8}\right)$$

$$\times\exp\left[-2\left(4.124\times10^9\right)\left(15\times10^{-10}\right)\right]$$

or

$$T = 1.27\times10^{-5}$$

2.37

$$T \cong 16\left(\frac{E}{V_o}\right)\left(1-\frac{E}{V_o}\right)\exp\left(-2k_2 a\right)$$

where $k_2 = \sqrt{\dfrac{2m(V_o - E)}{\hbar^2}}$

$$= \frac{\sqrt{2\left(1.67\times10^{-27}\right)(12-1)\times10^6\times\left(1.6\times10^{-19}\right)}}{1.054\times10^{-34}}$$

$$= 7.274\times10^{14}\ \text{m}^{-1}$$

(a)

$$T \cong 16\left(\frac{1}{12}\right)\left(1-\frac{1}{12}\right)\exp\left[-2\left(7.274\times10^{14}\right)\left(10^{-14}\right)\right]$$

$$= 1.222\ \exp\left[-14.548\right]$$

$$= 5.875\times10^{-7}$$

(b)

$$T = (10)\left(5.875\times10^{-7}\right)$$

$$= 1.222\ \exp\left[-2\left(7.274\times10^{14}\right)a\right]$$

$$2\left(7.274\times10^{14}\right)a = \ln\left(\frac{1.222}{5.875\times10^{-6}}\right)$$

or $a = 0.842\times10^{-14}$ m

2.38

Region I $(x<0)$, $V=0$;

Region II $(0<x<a)$, $V=V_O$

Region III $(x>a)$, $V=0$

(a) Region I:

$$\psi_1(x) = A_1\exp\left(jk_1x\right) + B_1\exp\left(-jk_1x\right)$$

(incident) (reflected)

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II:

$$\psi_2(x) = A_2 \exp(k_2 x) + B_2 \exp(-k_2 x)$$

where

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

Region III:

$$\psi_3(x) = A_3 \exp(jk_1 x) + B_3 \exp(-jk_1 x)$$

(b)

In Region III, the B_3 term represents a reflected wave. However, once a particle is transmitted into Region III, there will not be a reflected wave so that $B_3 = 0$.

(c) Boundary conditions:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{d\psi_1}{dx} = \frac{d\psi_2}{dx} \Rightarrow$$

$$jk_1 A_1 - jk_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(k_2 a) + B_2 \exp(-k_2 a) = A_3 \exp(jk_1 a)$$

$$\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx} \Rightarrow$$

$$k_2 A_2 \exp(k_2 a) - k_2 B_2 \exp(-k_2 a) = jk_1 A_3 \exp(jk_1 a)$$

The transmission coefficient is defined as

$$T = \frac{A_3 A_3^*}{A_1 A_1^*}$$

so from the boundary conditions, we want to solve for A_3 in terms of A_1. Solving for A_1 in terms of A_3, we find

$$A_1 = \frac{+jA_3}{4k_1 k_2} \left\{ \left(k_2^2 - k_1^2\right) \left[\exp(k_2 a) - \exp(-k_2 a)\right] - 2jk_1 k_2 \left[\exp(k_2 a) + \exp(-k_2 a)\right] \right\} \times \exp(jk_1 a)$$

We then find

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \left\{ \left(k_2^2 - k_1^2\right) \left[\exp(k_2 a) - \exp(-k_2 a)\right]^2 + 4k_1^2 k_2^2 \left[\exp(k_2 a) + \exp(-k_2 a)\right]^2 \right\}$$

We have

$$k_2 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}}$$

If we assume that $V_O >> E$, then $k_2 a$ will be large so that

$$\exp(k_2 a) >> \exp(-k_2 a)$$

We can then write

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \left\{ \left(k_2^2 - k_1^2\right) \left[\exp(k_2 a)\right]^2 + 4k_1^2 k_2^2 \left[\exp(k_2 a)\right]^2 \right\}$$

which becomes

$$A_1 A_1^* = \frac{A_3 A_3^*}{(4k_1 k_2)^2} \left(k_2^2 + k_1^2\right) \exp(2k_2 a)$$

Substituting the expressions for k_1 and k_2, we find

$$k_1^2 + k_2^2 = \frac{2mV_O}{\hbar^2}$$

and

$$k_1^2 k_2^2 = \left[\frac{2m(V_O - E)}{\hbar^2}\right]\left[\frac{2mE}{\hbar^2}\right]$$

$$= \left(\frac{2m}{\hbar^2}\right)^2 (V_O - E)(E)$$

$$= \left(\frac{2m}{\hbar^2}\right)^2 (V_O)\left(1 - \frac{E}{V_O}\right)(E)$$

Then

$$A_1 A_1^* = \frac{A_3 A_3^* \left(\frac{2mV_O}{\hbar^2}\right)^2 \exp(2k_2 a)}{16\left[\left(\frac{2m}{\hbar^2}\right)^2 V_O \left(1 - \frac{E}{V_O}\right)(E)\right]}$$

$$= \frac{A_3 A_3^*}{16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2k_2 a)}$$

Finally,

$$T = \frac{A_3 A_3^*}{A_1 A_1^*} = 16\left(\frac{E}{V_O}\right)\left(1 - \frac{E}{V_O}\right)\exp(-2k_2 a)$$

2.39

Region I: $V = 0$

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_1(x) = 0 \Rightarrow$$

$$\psi_1(x) = A_1 \exp(jk_1 x) + B_1 \exp(-jk_1 x)$$

incident reflected

where

$$k_1 = \sqrt{\frac{2mE}{\hbar^2}}$$

Region II: $V = V_1$

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2m(E - V_1)}{\hbar^2}\psi_2(x) = 0 \Rightarrow$$

$$\psi_2(x) = A_2 \exp(jk_2 x) + B_2 \exp(-jk_2 x)$$

transmitted reflected

where

$$k_2 = \sqrt{\frac{2m(E - V_1)}{\hbar^2}}$$

Region III: $V = V_2$

$$\frac{\partial^2 \psi_3(x)}{\partial x^2} + \frac{2m(E - V_2)}{\hbar^2}\psi_3(x) = 0 \Rightarrow$$

$$\psi_3(x) = A_3 \exp(jk_3 x)$$

transmitted

where

$$k_3 = \sqrt{\frac{2m(E - V_2)}{\hbar^2}}$$

There is no reflected wave in Region III.
The transmission coefficient is defined as:

$$T = \frac{\upsilon_3}{\upsilon_1}\cdot\frac{A_3 A_3^*}{A_1 A_1^*} = \frac{k_3}{k_1}\cdot\frac{A_3 A_3^*}{A_1 A_1^*}$$

From the boundary conditions, solve for A_3 in terms of A_1. The boundary conditions are:

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow$

$$A_1 + B_1 = A_2 + B_2$$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow$$

$$k_1 A_1 - k_1 B_1 = k_2 A_2 - k_2 B_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \exp(jk_2 a) + B_2 \exp(-jk_2 a) = A_3 \exp(jk_3 a)$$

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x} \Rightarrow$$

$$k_2 A_2 \exp(jk_2 a) - k_2 B_2 \exp(-jk_2 a) = k_3 A_3 \exp(jk_3 a)$$

But $k_2 a = 2n\pi \Rightarrow$

$$\exp(jk_2 a) = \exp(-jk_2 a) = 1$$

Then, eliminating B_1, A_2, B_2 from the boundary condition equations, we find

$$T = \frac{k_3}{k_1}\cdot\frac{4k_1^2}{(k_1 + k_3)^2} = \frac{4k_1 k_3}{(k_1 + k_3)^2}$$

2.40

(a) Region I: Since $V_O > E$, we can write

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_O - E)}{\hbar^2}\psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_2(x) = 0$$

Region III: $V \to \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\psi_1(x) = B_1 \exp(k_1 x)$$

$$\psi_2(x) = A_2 \sin(k_2 x) + B_2 \cos(k_2 x)$$

$$\psi_3(x) = 0$$

where

$$k_1 = \sqrt{\frac{2m(V_O - E)}{\hbar^2}} \quad \text{and} \quad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

or

$$B_2 = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right)B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right)B_2$$

From $B_2 = -A_2 \tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right)B_2 \tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right)\tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \cdot \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E. The energy levels are quantized.

2.41

$$E_n = \frac{-m_o e^4}{(4\pi\epsilon_o)^2 2\hbar^2 n^2} \text{ (J)}$$

$$= \frac{-m_o e^3}{(4\pi\epsilon_o)^2 2\hbar^2 n^2} \text{ (eV)}$$

$$= \frac{-(9.11\times10^{-31})(1.6\times10^{-19})^3}{\left[4\pi(8.85\times10^{-12})\right]^2 2(1.054\times10^{-34})^2 n^2}$$

or

$$E_n = \frac{-13.58}{n^2} \text{ (eV)}$$

$n = 1 \Rightarrow E_1 = -13.58$ eV

$n = 2 \Rightarrow E_2 = -3.395$ eV

$n = 3 \Rightarrow E_3 = -1.51$ eV

$n = 4 \Rightarrow E_4 = -0.849$ eV

2.42

We have

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

and

$$P = 4\pi r^2 \psi_{100} \psi_{100}^*$$

$$= 4\pi r^2 \cdot \frac{1}{\pi} \cdot \left(\frac{1}{a_o}\right)^3 \exp\left(\frac{-2r}{a_o}\right)$$

or

$$P = \frac{4}{(a_o)^3} \cdot r^2 \exp\left(\frac{-2r}{a_o}\right)$$

To find the maximum probability

$$\frac{dP(r)}{dr} = 0$$

$$= \frac{4}{(a_o)^3}\left\{\left(\frac{-2}{a_o}\right)(r^2)\exp\left(\frac{-2r}{a_o}\right) + 2r\exp\left(\frac{-2r}{a_o}\right)\right\}$$

which gives

$$0 = \frac{-r}{a_o} + 1 \Rightarrow r = a_o$$

or $r = a_o$ is the radius that gives the greatest probability.

2.43

ψ_{100} is independent of θ and ϕ, so the wave equation in spherical coordinates reduces to

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi}{\partial r}\right) + \frac{2m_o}{\hbar^2}(E - V(r))\psi = 0$$

where

$$V(r) = \frac{-e^2}{4\pi\epsilon_o\, r} = \frac{-\hbar^2}{m_o a_o r}$$

For

$$\psi_{100} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2} \exp\left(\frac{-r}{a_o}\right)$$

Then

$$\frac{\partial \psi_{100}}{\partial r} = \frac{1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{3/2}\left(\frac{-1}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)$$

so

$$r^2 \frac{\partial \psi_{100}}{\partial r} = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} r^2 \exp\left(\frac{-r}{a_o}\right)$$

We then obtain

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \psi_{100}}{\partial r}\right) = \frac{-1}{\sqrt{\pi}} \cdot \left(\frac{1}{a_o}\right)^{5/2} \times\left[2r\exp\left(\frac{-r}{a_o}\right) - \left(\frac{r^2}{a_o}\right)\exp\left(\frac{-r}{a_o}\right)\right]$$

Substituting into the wave equation, we have

$$\frac{-1}{r^2\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{5/2}\left[2r\exp\left(\frac{-r}{a_o}\right)-\frac{r^2}{a_o}\exp\left(\frac{-r}{a_o}\right)\right]$$

$$+\frac{2m_o}{\hbar^2}\left[E+\frac{\hbar^2}{m_o a_o r}\right]$$

$$\times\left(\frac{1}{\sqrt{\pi}}\right)\cdot\left(\frac{1}{a_o}\right)^{3/2}\exp\left(\frac{-r}{a_o}\right)=0$$

where

$$E=E_1=\frac{-m_o e^4}{(4\pi\epsilon_o)^2 2\hbar^2}=\frac{-\hbar^2}{2m_o a_o^2}$$

Then the above equation becomes

$$\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]\left\{\frac{-1}{r^2 a_o}\left[2r-\frac{r^2}{a_o}\right]\right.$$

$$\left.+\frac{2m_o}{\hbar^2}\left(\frac{-\hbar^2}{2m_o a_o}+\frac{\hbar^2}{m_o a_o r}\right)\right\}=0$$

or

$$\frac{1}{\sqrt{\pi}}\cdot\left(\frac{1}{a_o}\right)^{3/2}\left[\exp\left(\frac{-r}{a_o}\right)\right]$$

$$\times\left\{\frac{-2}{a_o r}+\frac{1}{a_o^2}+\left(\frac{-1}{a_o^2}+\frac{2}{a_o r}\right)\right\}=0$$

which gives 0 = 0 and shows that ψ_{100} is indeed a solution to the wave equation.

2.44

All elements are from the Group I column of the periodic table. All have one valence electron in the outer shell.
