CHAPTER 3

Probability

3.2 a. The sample space will include the following simple events:

Single, shore parallel
Planar
Other

b. The probabilities for the simple events are shown in the table below:

Simple Event	**Probabilities**
Single, shore parallel	2/6
Planar	2/6
Other	2/6

c. Since four of the six beach hotspots have either a planar or single shore parallel-nearshore bar condition, we say that the probability is 4/6 = 0.6667.

d. The sample space will include the following simple events:

No dunes/flat, Bluff/scarp, Single dune, and Not observed

e. The probabilities for the simple events are shown in the table below:

Simple Event	**Probabilities**
No dunes/flat	2/6
Bluff/scarp	1/6
Single dune	2/6
Not observed	1/6

f. Since four of the six beach hotspots observed have a beach condition that is not flat, we say that the probability is 4/6 = 0.6667.

3.4 a. 27 of the 83 industrial accidents were caused by faulty engineering and design. We say that the probability of faulty engineering and design causing an industrial accident is 27/83 = 0.3253. About one-third of all industrial accidents are caused by faulty engineering and design.

b. 59 of the 83 industrial accidents were caused by something other than faulty practices and procedures. We say that the probability of this is 59/83 = 0.7108. About 71% of all industrial accidents are caused by something other than faulty practices and procedures.

3.6 a. Let A = {chicken passes inspection with fecal contamination}. $P(A) = 1 / 100 = .01$.

b. Yes. The relative frequency of passing inspection with fecal contamination is 306/32,075 = .0095 ≈ .01.

3.8 a. The sample points would correspond to the possible outcomes: 1, 2, 3, 4, 5, 6, 7, 8, and 9.

b. From the sample, the number of times each of these sample points occurred was different. We would expect the probability of each sample point to the close to the relative frequency of that outcome.

c. Reasonable probabilities would be the relative frequencies of the sample points:

First Digit	Frequency	Relative Frequency
1	109	109/743 = .147
2	75	75/743 = .101
3	77	77/743 = .104
4	99	99/743 = .133
5	72	72/743 = .097
6	117	117/743 = .157
7	89	89/743 = .120
8	62	62/743 = .083
9	43	43/743 = .058
Total	743	1.000

d. P(first digit is 1 or 2) = P(first digit is 1) + P(first digit is 2) = .147 + .101 = .248

3.10 a. There are 9 possible outcomes for this experiment. The outcomes are:

(40,300)(40,350)(40,400)
(45,300)(45,350)(45,400)
(50,300)(50,350)(50,400)

b, The different outcomes for this experiment that are listed above are probably not equally likely to yield the highest electrical resistivity. There are many different factors that affect the electrical resistivity.

3.12 From Exercise 3.2, we have: P(dune is flat) = 2/6 = 0.3333

The rule of complements states: $P(A) = 1 - P(A^c)$

P(dune is not flat) = 1 − P(dune is flat) = 1 − 0.3333 = 0.6667

This answer is the same as in Exercise 3.2**f**.

3.14 A summary for the MTBE data set is shown below:

	MTBE Level		
Aquifer	**Below level**	**Detectable level**	**Total**
Bedrock	138	63	201
Unconsolidated	15	7	22
Total	153	70	223

a. P(Bedrock and Detectable level) = 63/223 = 0.2825

b. P(Bedrock or Detectable level) = P(Bedrock) + P(Detectable level) − P(Both)
= 201/223 + 70/223 − 63/223 = 208/223 = 0.9327

3.16 The Benford's law data set is shown below:

First Digit	Number of Occurrences	Probability
1	109	109/743 = 0.1467
2	75	75/743 = 0.1009
3	77	77/743 = 0.1036
4	99	99/743 = 0.1332
5	72	72/743 = 0.0969
6	117	117/743 = 0.1575
7	89	89/743 = 0.1198
8	62	62/743 = 0.0834
9	43	43/743 = 0.0579
Total	743	1.0000

a. P(first digit > 5) = P(6) + P(7) + P(8) + P(9) = .1575 + .1198 + .0834 + .0579 = 0.4186

b. P(first digit is not 1) = 1 − P(first digit is 1) = 1 − .1467 = 0.853

c. P(first digit is even) = P(2) + P(4) + P(6) + P(8) = .1009 + .1332 + .1575 +.0834
= 0.4750

d. P(odd or less than 7) = P(1, 2, 3, 4, 5, 6, 7, or 9) = 1 − P(first digit is 8) = 1 − .0834
= 0.9166

3.18 a. The simple events in $A \cup B$ are the numbers that are odd or black or both and are:

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 33, 35}

$A \cup B$ consists of 28 of the 38 equally likely simple events.

$$P(A \cup B) = \frac{28}{38}$$

b. The simple events that are in $A \cap C$ are the numbers that are both odd and high and are:

{19, 21, 23, 25, 27, 29, 31, 33, 35}

$$P(A \cap C) = \frac{9}{38}$$

c. The simple events in $B \cup C$ are the numbers that are black or high or both and are:

{2, 4, 6, 8, 10, 11, 13, 15, 17, 19–36}

$$P(B \cup C) = \frac{27}{38}$$

d. The simple events that are in B^c are the numbers that are not black and are:

{0, 00, 1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36}

$$P(B^c) = \frac{20}{38}$$

e. The simple events that are in $A \cap B \cap C$ are the numbers that are odd and black and high and are:

{29, 31, 33, 35}

$$P(A \cap B \cap C) = \frac{4}{38}$$

3.20 a. Select a single steel sheet and measure the type of steel that is used.

b. The simple events are:

cold rolled	cold rolled, high strength
cold rolled, plated	cold rolled, high strength, plated
hot rolled	hot rolled, high strength
hot rolled, plated	

c. Using the table given:

$$P(\text{cold rolled}) = \frac{27}{100} = .27$$

$$P(\text{cold rolled, high strength}) = \frac{12}{100} = .12$$

$$P(\text{cold rolled, plated}) = \frac{30}{100} = .30$$

P(cold rolled, high strength, plated) = $\frac{15}{100}$ = .15

P(hot rolled) = $\frac{8}{100}$ = .08

P(hot rolled, high strength) = $\frac{5}{100}$ = .05

P(hot rolled, plated) = $\frac{3}{100}$ = .03

d. P(hot rolled, high strength) = $\frac{5}{100}$ = .05

e. P(cold rolled) = $\frac{27+12+30+15}{100} = \frac{84}{100}$ = .84

f. P(not plated) = $\frac{27+12+8+5}{100} = \frac{52}{100}$ = .52

3.22 a. Define the following events:

A: {Well is a public well}
B: {Well has a bedrock aquifer}

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{98/223}{120/223} = \frac{98}{223} = 0.4395$$

b. Define the following events:

A: {Well has a bedrock aquifer}
B: {Well has a detected level of MTBE}

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{63/223}{201/223} = \frac{63}{201} = 0.3134$$

3.24 a. P(Extinct) = 38/132 = 0.2879

b. If the first 9 species selected are all extinct, then only 29 extinct species would remain from the 123 bird species left. The probability that the 10th species selected being extinct given that the first 9 were also extinct would be 29/123 = 0.2358.

3.26 a. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$A \cap B$ = {11, 13, 15, 17, 29, 31, 33, 35}

$$P(A \cap B) = \frac{8}{38}$$

$$P(B) = \frac{18}{38}$$

$$P(A \mid B) = \frac{\frac{8}{38}}{\frac{18}{38}} = \frac{8}{18} = .4444$$

b. $$P(B \mid C) = \frac{P(B \cap C)}{P(C)}$$

$B \cap C$ = {20, 22, 24, 26, 28, 29, 31, 33, 35)

$$P(B \cap C) = \frac{9}{38}$$

$$P(C) = \frac{18}{38}$$

$$P(B \mid C) = \frac{\frac{9}{38}}{\frac{18}{38}} = \frac{9}{18} = .5000$$

c. $$P(C \mid A) = \frac{P(A \cap C)}{P(A)}$$

From 3.18**b**, $P(A \cap C) = \frac{9}{38}$

$$P(A) = \frac{18}{38}$$

$$P(C \mid A) = \frac{\frac{9}{38}}{\frac{18}{38}} = \frac{9}{18} = .5000$$

3.28 Define the following events:

A: {Less than 1 latent cancer fatality/year}
B: {Core melt occurs during a year}

From the problem, $P(A \mid B) = .00005$ and $P(B) = 1/100{,}000 = .00001$. The probability that at least 1 latent cancer fatality will occur as a result of a core melt is $P(A^c \cap B) = P(A^c \mid B)P(B)$. Since $P(A \mid B) = .00005$,

$$P(A^c \mid B) = 1 - P(A \mid B) = 1 - .00005 = .99995$$

Thus, $P(A^c \cap B) = .99995(.00001) = .0000099995 \approx .00001$

3.30 a. See the table below. I have numbered the 12 edges.

A 7 1	**A** 10 3	**N** 5
N 8 2	**F** 11 4	**F** 6
N 9	**F** 12	**F**

Edge 1 is A-N	Edge 2 is N-N	Edge 3 is F-A
Edge 4 is F-F	Edge 5 is F-N	Edge 6 is F-F
Edge 7 is A-A	Edge 8 is F-N	Edge 9 is F-N
Edge 10 is A-N	Edge 11 is F-F	Edge 12 is F-F

b. From the list in part **a**, there are only 8 F-edges. They are:

Edge 3 is F-A	Edge 4 is F-F	Edge 5 is F-N
Edge 6 is F-F	Edge 8 is F-N	Edge 9 is F-N
Edge 11 is F-F	Edge 12 is F-F	

A summary of these edges is:

Edge	Frequency
F-A	1
F-N	3
F-F	4
Total	8

c. If an F-edge is selected at random, then the probability of selecting an F-A edge is equal to its relative frequency. Thus, P(F-A) = 1/8 = .125

d. If an F-edge is selected at random, then the probability of selecting an F-N edge is equal to its relative frequency. Thus, P(F-N) = 3/8 = .375

3.32 Define the following events:

A = {Supplier A reach their net profit goal}
B = {Supplier B reach their net profit goal}

We are told that $P(A) = P(B) = 0.95$ and that A and B are independent of one another.

a. $P(A \text{ and } B) = P(A) \times P(B) = 0.95 \times 0.95 = 0.9025$

b. $P(A^c \text{ and } B^c) = P(A^c) \times P(B^c) = 0.05 \times 0.05 = 0.0025$

c. $P(A \text{ or } B) = 1 - P(A^c \text{ and } B^c) = 1 - 0.0025 = .9975$

3.34 Define the following events:

A: {Adirondack lake is acidic}
B: {lake comes by acidity naturally}

From the problem,

$$P(A) = .14 \text{ and } P(B \mid A) = .25$$

$$P(A \cap B) = P(B \mid A)P(A) = .25(.14) = .035$$

3.36 a. Define the following events:

A_1: {Particle 1 is reflected}
A_2: {Particle 2 is reflected}

From the problem, $P(A_1) = .16$ and $P(A_2) = .16$. The event both particles are reflected is $A_1 \cap A_2$. If we assume the events A_1 and A_2 are independent,

$$P(A_1 \cap A_2) = P(A_1)P(A_2) = .16(.16) = .0256$$

b. Let A_3, A_4, and A_5 be defined similarly to A_1 and A_2 above for particles 3, 4, and 5. The event all 5 particles will be absorbed is $A_1^c \cap A_2^c \cap A_3^c \cap A_4^c \cap A_5^c$. Again, assume the events are independent,

$$P(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c \cap A_5^c)$$

$$= P(A_1^c)P(A_2^c)P(A_3^c)P(A_4^c)P(A_5^c)$$

$$= [1 - P(A_1)][1 - P(A_2)][1 - P(A_3)][1 - P(A_4)][1 - P(A_5)]$$

$$= (1-.16)\,(1-.16)\,(1-.16)\,(1-.16)\,(1-.16) = .84^5 = .418$$

c. We must assume the simple events are independent.

3.38 First define the following event:

A: {CVSA correctly determines the veracity of a suspect} $P(A)$ = .498 (from claim)

a. The event that the CVSA us correct for all four suspects is the event $A \cup A \cup A \cup A$.
$P\left(A \cup A \cup A \cup A\right)$=.498(.498)(.498)(.498) = 0.0615

b. The event that the CVSA is incorrect for at least one of the four suspects is the event $\left(A \cup A \cup A \cup A\right)^{c}$.

$P\left((A \cup A \cup A \cup A)^{c}\right) = 1 - P\left(A \cup A \cup A \cup A\right) = 1 - 0.0615 = .9385$

3.40 The two events, trucks in accidents and cars in accidents, are not complementary events. P(cars in accidents ≠ 1 − P(trucks in accidents). To find the correct chance of cars in accidents, we find:

P(cars in accidents) = $P(CC) + P(CT) + P(TC)$
= .64 + .16 + .16 = .96

3.42 Define the following events:

H = {NDE detects a "hit"}
D = {a defect exists in the steel casing}

We are given the following probabilities:

$P(H|D) = .97$
$P(H|D^{c}) = .005$
$P(D) = .01$

$$P(D \mid H) = \frac{P(D)P(H \mid D)}{P(D)P(H \mid D) + P(D^{c})P(H \mid D^{c})} = \frac{.01(.97)}{.01(.97) + .99(.005)} = \frac{.0097}{.01465} = 0.6621$$

3.44 Define the following events:

T: {Athlete illegally uses testosterone}
P: {Test for testosterone is positive}
N: {Athlete does not illegally use testosterone}

a. $P(P \mid T) = \dfrac{50}{100} = .50$

b. $P(P^{c} \mid N) = 1 - P(P \mid N) = 1 - \dfrac{9}{900} = 1 - .01 = .99$

c. $P(T \mid P) = \dfrac{P(P \mid T)P(T)}{P(P)}$

Now, $P(P) = P(P \mid N)P(N) + P(P \mid T)P(T) = .01(.9) + .5(.1) = .009 + .05 = .059$

$$P(T \mid P) = \frac{P(P \mid T)P(T)}{P(P)} = \frac{.5(.1)}{.059} = \frac{.05}{.059} = .847$$

3.46 From Exercise 3.33, we defined the following events:

I: {Intruder}
N: {No intruder}
A: {System A sounds alarm}
B: {System B sounds alarm}

Also, from Exercise 3.33, $P(A \cap B \mid I) = .855$ and $P(A \cap B \mid N) = .02$.

Thus, $P(A \cap B) = P(A \cap B \mid I)P(I) + P(A \cap B \mid N)P(N) = .855(.4) + .02(.6)$
$= .342 + .012 = .354$

$P(I \cap A \cap B) = P(A \cap B \mid I)P(I) = .855(.4) = .342$

$$P(I \mid A \cap B) = \frac{P(I \cap A \cap B)}{P(A \cap B)} = \frac{.342}{.354} = .966$$

3.48 Define the following event:

D: {Chip is defective}

From the Exercise, $P(S_1) = .15$, $P(S_2) = .05$, $P(S_3) = .10$, $P(S_4) = .20$, $P(S_5) = .12$, $P(S_6) = .20$, and $P(S_7) = .18$. Also, $P(D|S_1) = .001$, $P(D|S_2) = .0003$, $P(D|S_3) = .0007$, $P(D|S_4) = .006$, $P(D|S_5) = .0002$, $P(D|S_6) = .0002$, and $P(D|S_7) = .001$.

a. We must find the probability of each supplier given a defective chip.

$$P(S_1 \mid D) = \frac{P(S_1 \cap D)}{P(D)} = \frac{.00015}{.001679} = .0893$$

$$= \frac{P(D \mid S_1)P(S_1)}{P(D \mid S_1)P(S_1) + P(D \mid S_2)P(S_2) + P(D \mid S_3)P(S_3) + P(D \mid S_4)P(S_4) + P(D \mid S_5)P(S_5) + P(D \mid S_6)P(S_6) + P(D \mid S_7)P(S_7)}$$

$$= \frac{.001(.15)}{.001(.15) + .0003(.05) + .0007(.10) + .006(.20) + .0002(.12) + .0002(.02) + .001(.18)}$$

$$= \frac{.00015}{.00015 + .000015 + .00007 + .0012 + .000024 + .00004 + .00018} = \frac{.00015}{.001679} = .0893$$

$$P(S_2 \mid D) = \frac{P(S_2 \cap D)}{P(D)} = \frac{P(D \mid S_2)P(S_2)}{P(D)} = \frac{.0003(.05)}{.001679} = \frac{.000015}{.001679} = .0089$$

$$P(S_3 \mid D) = \frac{P(S_3 \cap D)}{P(D)} = \frac{P(D \mid S_3)P(S_3)}{P(D)} = \frac{.0007(.10)}{.001679} = \frac{.00007}{.001679} = .0417$$

$$P(S_4 \mid D) = \frac{P(S_4 \cap D)}{P(D)} = \frac{P(D \mid S_4)P(S_4)}{P(D)} = \frac{.006(.20)}{.001679} = \frac{.0012}{.001679} = .7147$$

$$P(S_5 \mid D) = \frac{P(S_5 \cap D)}{P(D)} = \frac{P(D \mid S_5)P(S_5)}{P(D)} = \frac{.0002(.12)}{.001679} = \frac{.000024}{.001679} = .0143$$

$$P(S_6 \mid D) = \frac{P(S_6 \cap D)}{P(D)} = \frac{P(D \mid S_6)P(S_6)}{P(D)} = \frac{.0002(.20)}{.001679} = \frac{.00004}{.001679} = .0238$$

$$P(S_7 \mid D) = \frac{P(S_7 \cap D)}{P(D)} = \frac{P(D \mid S_7)P(S_7)}{P(D)} = \frac{.001(.18)}{.001679} = \frac{.00018}{.001679} = .1072$$

Of these probabilities, .7147 is the largest. This implies that if a failure is observed, supplier number 4 was most likely responsible.

b. If the seven suppliers all produce defective chips at the same rate of .0005, then $P(D|S_i)$ =.0005 for all i = 1, 2, 3, … 7 and $P(D)$ = .0005.

For any supplier i, $P(S_i \cap D) = P(D \mid S_i)P(S_i) = .0005P(S_i)$ and

$$P(S_i \mid D) = \frac{P(S_i \cap D)}{P(D)} = \frac{P(D \mid S_i)P(S_i)}{.0005} = \frac{.0005P(S_i)}{.0005} = P(S_i)$$

Thus, if a defective is observed, then it most likely came from the supplier with the largest proportion of sales (probability). In this case, the most likely supplier would be either supplier 4 or supplier 6. Both of these have probabilities of .20.

3.50 a. Using the combinations rule, the number of electrode pairs that can be attached to the ankle is:

$$\binom{6}{2} = \frac{6!}{2!(6-2)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 15$$

b. Using the combinations rule, the number of electrode pairs that can be attached to the knee is:

$$\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10 \cdot 9 \cdot 8 \cdot \cdots \cdot 2 \cdot 1}{2 \cdot 1 \cdot 8 \cdot 7 \cdot \cdots \cdot 2 \cdot 1} = 45$$

c. Using the multiplicative rule, the number of electrode pairs, where one is attached to the knee and one is attached to the ankle is $n_1 \cdot n_2 = 10 \cdot 6 = 60$

3.52 a. There are a total of 3 × 3 = 9 different scenarios possible.

b. Let *O* correspond to "optimist," *M* correspond to "moderate," and *P* correspond to "pessimist." The scenarios are represented by pairs of values, where the first value corresponds to the perspective on abatement and the second value corresponds to the perspective on climate change damage. The scenarios are:

OO, OM, OP, MO, MM, MP, PO, PM, PP

c. We will be picking 5 scenarios from 9 possible scenarios. The number of combinations of 5 scenarios from 9 scenarios is:

$$\binom{9}{5} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!(9-5)!} = 126$$

3.54 a. There were 10 experimental conditions each subject was tested under.

b.

Task *A*, eyes open	Task *A*, eyes closed
Task *B*, eyes open	Task *B*, eyes closed
Task *C*, eyes open	Task *C*, eyes closed
Task *D*, eyes open	Task *D*, eyes closed
Task *E*, eyes open	Task *E*, eyes closed

c. Since two measurements were recorded for each condition listsed above, 10 × 2 = 20 measurements were taken on each subject.

3.56 a. Using the permutations rule,

$$\text{total number of codes} = P_3^N = P_3^{10} = 10(9)(8) = 720$$

b. Using the multiplicative rule,

$$\text{total number of codes} = n_1 \cdot n_2 \cdot n_3 = 10 \cdot 10 \cdot 10 = 1000$$

3.58 Using the combination rule gives us:

$$\binom{20}{5} = \frac{20!}{5!(20-5)!} = \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 15!} = \frac{1{,}860{,}480}{120} = 15{,}504$$

3.60 Let event *A* = hand has four aces.

$$P(A) = \frac{\text{Number of simple events in } A}{\text{Total number of simple events}}$$

Using the combination rule, the total number of simple events is $\binom{52}{5} = \frac{52!}{5!(47!)} = 2{,}598{,}960.$

Since all four aces must be in the hand, there is only one card that can be anything but an ace.

Again, using the combination rule, the number of simple events in *A* is $\binom{48}{1} = \frac{48!}{1!(47!)} = 48.$

$$P(A) = \frac{48}{2{,}598{,}960} = .0000185$$

3.62 Because the probability is so small, we would agree that the stars were not aligned by chance.

3.64 We want to find:

$$P(\text{player wins all three lotteries}) = \frac{1}{1{,}836} \cdot \frac{1}{1{,}365} \cdot \frac{1}{495}$$
$$= \frac{1}{1{,}240{,}539{,}300} = .000000000806$$

Since this is an extremely small probability, we would believe there was some tampering going on.

3.70 a. Using the combinations rule, the number of different samples is:

$$\binom{600}{3} = \frac{600!}{3!(600-3)!} = \frac{600 \cdot 599 \cdot 598 \cdot 597 \cdot \cdots \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 597 \cdot 596 \cdot \cdots \cdot 2 \cdot 1} = 35{,}820{,}200$$

b. If a random sample is employed, then each sample would have a 1/35,820,200 chance of being selected.

3.72 a. The number of ways the bids could have been awarded is:

$$\binom{5}{2} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 3 \cdot 2 \cdot 1} = 10$$

The number of ways in which two large conglomerates were selected is:

$$\binom{3}{2} = \frac{3!}{2!1!} = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 1} = 3$$

$$P(\text{both awarded to large conglomerates}) = \frac{3}{10} = .30$$

b. Since this probability is fairly large, it is not inconsistent with the DOT's claim of random sampling.

3.74 Define the following events:

C: {Committee judges joint acceptable}
I: {Inspector judges joint acceptable}

The sample points of this experiment are:

$C \cap I$
$C \cap I^c$
$C^c \cap I$
$C^c \cap I^c$

a. The probability the inspector judges the joint to be acceptable is:

$$P(I) = P(C \cap I) + P(C^c \cap I) = \frac{101}{153} + \frac{23}{153} = \frac{124}{153} \approx .810$$

The probability the committee judges the joint to be acceptable is:

$$P(C) = P(C \cap I) + P(C \cap I^c) = \frac{101}{153} + \frac{10}{153} = \frac{111}{153} \approx .725$$

b. The probability that both the committee and the inspector judge the joint to be acceptable is:

$$P(C \cap I) = \frac{101}{153} \approx .660$$

The probability that neither judge the joint to be acceptable is:

$$P(C^c \cap I^c) = \frac{19}{153} \approx .124$$

c. The probability the inspector and committee disagree is:

$$P(C \cap I^c) + P(C^c \cap I) = \frac{10}{153} + \frac{23}{153} = \frac{33}{153} \approx .216$$

The probability the inspector and committee agree is:

$$P(C \cap I) + P(C^c \cap I^c) = \frac{101}{153} + \frac{19}{153} = \frac{120}{153} \approx .784$$

3.76 Define the following events:

A: (ATV driver is under age 12}
B: (ATV driver is aged 12–15}
C: (ATV driver is under age 25}

a. P(ATV driver is 15 years old or younger) = $P(A \cup B) = P(A) + P(B) = .14 + .13 = .27$

b. P(ATV driver is 25 years old or older) = $P(C') = 1 - P(C) = 1 - .48 = .52$

c. $P(A \mid C) = \dfrac{P(A \cap C)}{P(C)} = \dfrac{.14}{.48} = .2917$

d. No, the events A and C are not mutually exclusive. $P(A \cap C) = .14 \neq 0$

e. No, the events A and C are not independent.
$P(A)P(C) = .14(.48) = .0672 \neq P(A \cap C) = .14$

3.78 a. Since there are 6 (3 × 2) mold type orientation combinations and 5 tensile specimen distances, the number of measurements will be 6 × 5 = 30.

b. Using the combination rule, $\binom{6}{3} = \dfrac{6!}{3!3!} = \dfrac{6 \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 1 \cdot 3!} = 20$

d. $P(\text{highest three}) = \dfrac{\text{Number of ways highest 3 can be selected}}{\text{Total number of possible selections}} = \dfrac{\binom{3}{3}}{\binom{6}{3}} = \dfrac{1}{20}$

e. P(at least two of highest three) = P(highest three) + P(two of the highest three)

$P(\text{highest three}) = \dfrac{1}{20}$

$P(\text{two of highest three}) = \dfrac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} = \dfrac{3 \cdot 2}{20} = \dfrac{9}{20}$

$P(\text{at least two}) = \dfrac{1}{20} + \dfrac{9}{20} = \dfrac{10}{20}$

3.80 There are three sets of elements: hard disk drives, display stations, and types of interfacing. If we want to draw one item from each set, we use the multiplicative rule to find the total number of combinations. Since there are 2 types of hard disk drives, 4 types of display stations, and 2 types of interfacing, the total number of combinations is 2 × 4 × 2 = 16.

3.82 Let F = battery fails

$P(\text{all three fail}) = P(F \cap F \cap F)$

Considering the batteries as operating independently gives us:

$$P(F \cap F \cap F) = P(F) \cdot P(F) \cdot P(F) = \frac{1}{20} + \frac{1}{20} + \frac{1}{20} = \frac{1}{8000} = .000125$$

3.84 Let A = fuse comes from line 1
D = one of two fuses is defective

We know $P(A) = P(A^c) = .5$
$P(D \mid A) = .1128 = .06(.94) + .94(.06)$
$P(D \mid A^c) = .04875 = .025(.975) + .975(.025)$

We wish to find $P(A \mid D) = \dfrac{P(A \cap D)}{P(D)}$

$$P(A \cap D) = P(A)P(D \mid A) = .5(.1128) = .0564$$

$$\begin{aligned} P(D) &= P(A \cap D) + (A^c \cap D) \\ &= P(A)P(D \mid A) + P(A^c)P(D \mid A^c) \\ &= .5(.1128) + .5(.04875) = .080775 \end{aligned}$$

$$P(A \mid D = \frac{.0564}{.080775} = .6982$$

3.86 To obtain a random sample of 900 intersections, we would use a random number generator that would create random numbers between the values 1 and 500,000. We would select the first 900 unique random numbers and sample the corresponding TV markets at these intersections.

3.88 Define the following events:

A: {Part is supplied by company A}
B: {Part is supplied by company B}
C: {Part is defective}

From the problem, $P(A) = .8$, $P(B) = .2$, $P(C \mid A) = .05$, and $P(C \mid B) = .03$. We know the given part is defective. We want to find the probability it came from company A and the probability it came from company B or $P(A \mid C)$ and $P(B \mid C)$. First, we find:

$$P(A \cap C) = P(C \mid A)P(A) = .05(.8) = .04,$$
$$P(B \cap C) = P(C \mid B)P(B) = .03(.2) = .006, \text{ and}$$
$$P(C) = P(A \cap C) + P(B \cap C) = .04 + .006 = .046$$

$P(A \mid C) = P(A \cap C) / P(C) = .04/.046 = .87$, and
$P(B \mid C) = P(B \cap C) / P(C) = .006/.046 = .13$

Thus, it is more likely the defective part came from company *A* (probability is .87) than from company *B* (probability is .13).

3.90 a. Using the permutation rule,

$$P_3^5 = \frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 60$$

b. We use the permutation rule again to determine how many of the three letter sequences contain no vowels.

$$P_3^3 = \frac{3!}{(3-3)!} = \frac{3 \cdot 2 \cdot 1}{1} = 6$$

The number with at least one vowel must be 60 − 6 = 54.

3.92 Define the following events:

A: {Intruder is detected}
B: {Day is clear}
C: {Day is cloudy)

a. $$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{21/692}{21/692} = 1.000$$

b. $$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{228/692}{234/692} = .9744$$