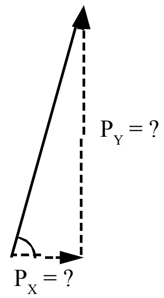
Chapter 2

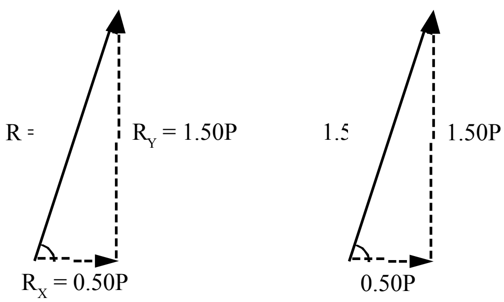
Question 2.1: A force of P defined by the angle $\theta_X = 75°$ to the horizontal acts through a point. What are the components of this force on the x and y axes?



$$\begin{aligned} \cos 75° &= P_X/P \\ P * \cos 75° &= P_X \\ P_X &= 0.26P \end{aligned}$$

$$\begin{aligned} \sin 75° &= P_Y/P \\ P * \sin 75° &= P_Y \\ P_Y &= 0.97P \end{aligned}$$

Question 2.2: The components of a force on the x and y axes are 0.50P and 1.50P, respectively. What are the magnitude and direction of the resultant force?

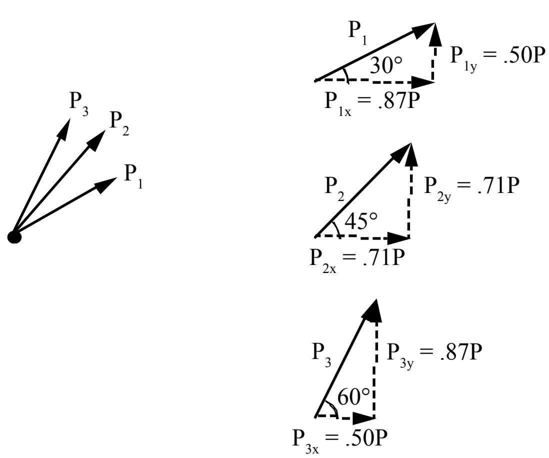


$$\begin{aligned} R &= \text{resultant force} \\ R^2 &= (0.50P)^2 + (1.50P)^2 \\ R &= 1.58P \end{aligned}$$

$$\begin{aligned} \tan \theta &= 1.50P/0.50P \\ \theta &= \tan^{-1} 1.50P/0.50P \\ \theta &= 71.6° \end{aligned}$$

Question 2.3: The following three forces act concurrently through a point: a force P acting to the right at $\theta_X = 30°$ to the horizontal, a force P acting to the right at $\theta_X = 45°$ to the horizontal, and a force P acting to the right at $\theta_X = 60°$ to the horizontal. Find the single resultant force that is equivalent to this three-force system.

Step 1: Find the horizontal and vertical components of each force and the net horizontal and vertical force.



$$\begin{aligned} P_{1x} &= P * \cos 30° \\ P_{1x} &= .87P \end{aligned}$$

$$\begin{aligned} P_{1y} &= P * \sin 30° \\ P_{1y} &= .50P \end{aligned}$$

$$\begin{aligned} P_{2x} &= P * \cos 45° \\ P_{2x} &= .71P \end{aligned}$$

$$\begin{aligned} P_{2y} &= P * \sin 45° \\ P_{2y} &= .71P \end{aligned}$$

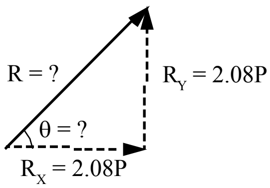
$$\begin{aligned} P_{3x} &= P * \cos 60° \\ P_{3x} &= .50P \end{aligned}$$

$$\begin{aligned} P_{3y} &= P * \sin 60° \\ P_{3y} &= .87P \end{aligned}$$

$$\begin{aligned} R_X = R_Y &= .71P + .50P + .87P \\ R_X = R_Y &= 2.08P \end{aligned}$$

Question 2.3 (continued):

Step 2: Find the magnitude and direction of the resultant force.

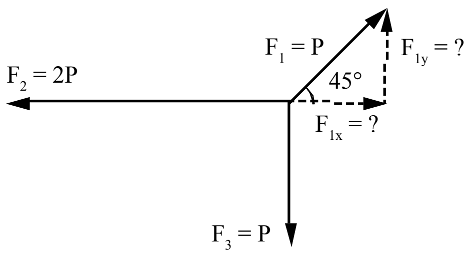


$$\begin{aligned} R^2 &= (2.08P)^2 + (2.08P)^2 \\ R &= 2.93P \end{aligned}$$

$$\begin{aligned} \tan\theta &= R_Y/R_X \\ \tan\theta &= 2.08P/2.08P \\ \theta &= \tan^{-1} 1 \\ \theta &= 45° \end{aligned}$$

Question 2.4: The following three forces act through a point: P at $\theta_x = 45°$, 2P at $\theta_x = 180°$, and P at $\theta_x = 270°$. Find the equivalent resultant force.

Step 1: Find the horizontal and vertical components of each force.



$$\begin{aligned} F_{1x} &= P * \cos 45° \\ F_{1x} &= .71P \\ F_{1y} &= P * \sin 45° \\ F_{1y} &= .71P \end{aligned}$$

$$\begin{aligned} F_{2x} &= -2P \\ F_{2y} &= 0 \end{aligned}$$

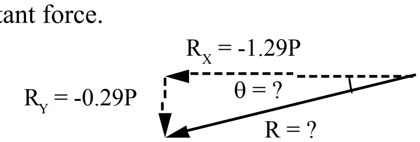
$$\begin{aligned} F_{3x} &= 0 \\ F_{3y} &= -P \end{aligned}$$

Step 2: Find the net horizontal and vertical force.

$$\begin{aligned} R_X &= .71P - 2P \\ R_X &= -1.29P \end{aligned}$$

$$\begin{aligned} R_Y &= .71P - P \\ R_Y &= -0.29P \end{aligned}$$

Step 3: Find the magnitude and direction of the resultant force.



$$\begin{aligned} R^2 &= (-1.29P)^2 + (-0.29P)^2 \\ R &= 1.32P \end{aligned}$$

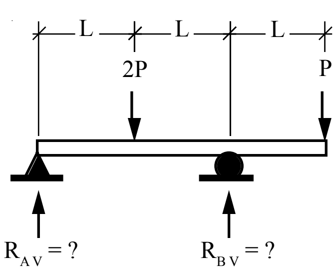
$$\begin{aligned} \tan\theta &= -0.29P / -1.29P \\ \theta &= 12.7° \end{aligned}$$

Summary

resultant force = R = 1.33P acting at 192.7°

Question 2.6: Determine the reactions for the structure shown in Figure 2.59(Q6).

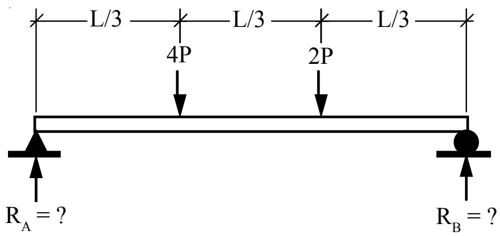
Sum rotational moments about point A. Assume that a counter-clockwise rotational effect is positive.



$$\begin{aligned} \sum M_A &= 0 \\ -(2P * L) + (R_B * 2L) - (P * 3L) &= 0 \\ R_B * 2L &= 2PL + 3PL \\ R_B &= 5PL/2L \\ R_B &= 5P/2 \uparrow \end{aligned}$$

$$\begin{aligned} \sum F_Y &= 0 \\ R_A + R_B - 2P - P &= 0 \\ R_A &= 3P - R_B \\ R_A &= 3P - 5P/2 \\ R_A &= P/2 \uparrow \end{aligned}$$

Question 2.8: Determine the reactions for the structure shown in Figure 2.59(Q8).

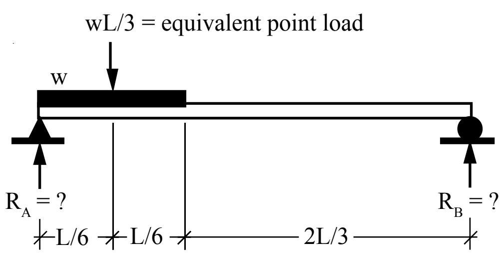


$$\sum M_A = 0$$
$$-(4P * L/3) - (2P * 2L/3) = 0$$
$$(R_B * L) = 0$$
$$R_B * L = (4P * L/3) + (2P * 2L/3)$$
$$R_B * L = 4PL/3 + 4PL/3$$
$$R_B = 8P/3 \uparrow$$

$$\sum F_Y = 0$$
$$R_A + R_B - 4P - 2P = 0$$
$$R_A = 6P - R_B$$
$$R_A = 6P - 8P/3$$
$$R_A = 10P/3 \uparrow$$

Question 2.10: Determine the reactions for the structure shown in Figure 2.59(Q10).

Sum moments about A. Assume that counter-clockwise moments are positive. Convert the uniform load *w* into an equivalent concentrated load for purposes of finding reactions.



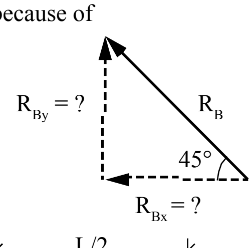
$$\sum M_A = 0$$
$$-(w * L/3 * L/6) + (R_B * L) = 0$$
$$R_B * L = w \times L/3 * L/6$$
$$R_B * L = wL^2/18$$
$$R_B = wL/18 \uparrow$$

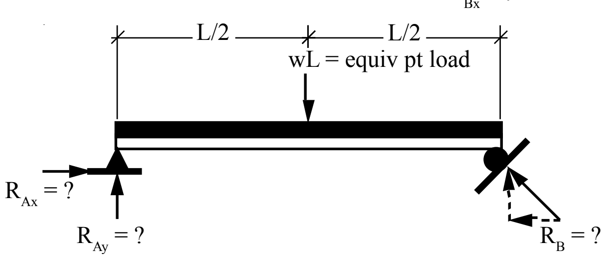
$$\sum F_Y = 0$$
$$R_A + R_B - wL/3 = 0$$
$$R_A = wL/3 - R_B$$
$$R_A = 6wL/18 - wL/18$$
$$R_A = 5wL/18 \uparrow$$

Question 2.12: Determine the reactions for the structure shown in Figure 2.59(Q12).

The angle of the roller on the right determines the direction of the reactive force at B which is then considered in terms of its components. The fact that the reaction at B is inclined means that the reaction at A must also be inclined (the horizontal components of the reactions must sum to zero because of equilibrium in the x direction).

An equivalent point load of *(w)(L)* is used to model the uniform load of *w* acting over the length of the beam.





$$\sum M_A = 0$$
$$(R_{By} * L) - (wL * L/2) = 0$$
$$R_{By} * L = wL * L/2$$
$$R_{By} = wL/2 \uparrow$$

$$\tan 45^\circ = R_{By} / R_{Bx}$$
$$1 = R_{By} / R_{Bx}$$
$$R_{Bx} = R_{By}$$
$$R_{Bx} = wL/2 \leftarrow$$

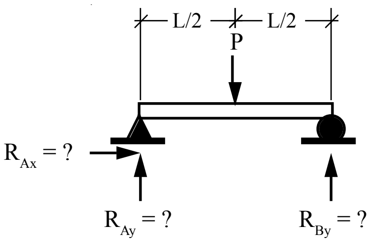
$$\sum F_Y = 0$$
$$R_{Ay} + R_{By} - wL = 0$$
$$R_{Ay} = wL - R_{By}$$
$$R_{Ay} = wL - wL/2$$
$$R_{Ay} = wL/2 \uparrow$$

$$\sum F_X = 0$$
$$R_{Ax} + R_{Bx} = 0$$
$$R_{Ax} = R_{B_x}$$
$$R_{Ax} = wL/2 \rightarrow$$

Question 2.13: Determine the reactions for the four beams shown in Figure 2.59(Q13).

Notice that the three inclined members are identical except for the type of end conditions present. Note how changing the support types radically alters the nature of the reactive forces.

Step 1: Figure 2.33(e)-1

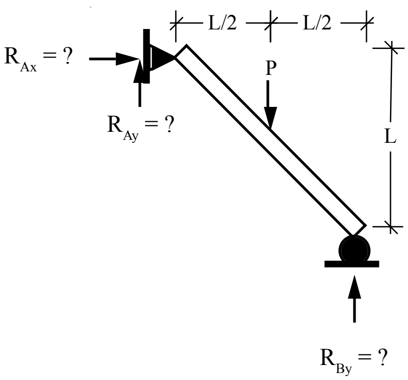


$$\sum M_A = 0$$
$$(R_{By} * L) - (P * L/2) = 0$$
$$R_{By} * L = PL/2$$
$$R_{By} = P/2 \uparrow$$

$$\sum F_Y = 0$$
$$R_{Ay} + R_{By} - P = 0$$
$$R_{Ay} = P - R_{By}$$
$$R_{Ay} = P - P/2$$
$$R_{Ay} = P/2 \uparrow$$

$$\sum F_X = 0$$
$$R_{Ax} = 0$$

Step 2: Figure 2.33(e)-2

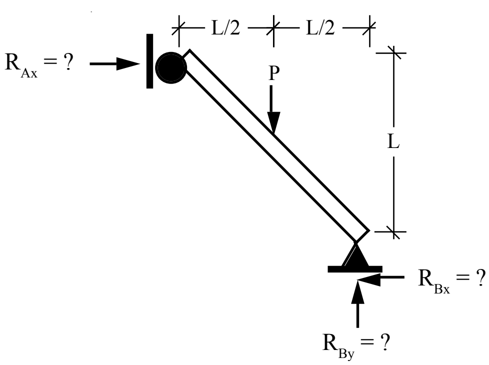


$$\sum M_A = 0$$
$$(R_{By} * L) - (P * L/2) = 0$$
$$R_{By} * L = PL/2$$
$$R_{By} = P/2 \uparrow$$

$$\sum F_Y = 0$$
$$R_{Ay} + R_{By} - P = 0$$
$$R_{Ay} = P - R_{By}$$
$$R_{Ay} = P - P/2$$
$$R_{Ay} = P/2 \uparrow$$

$$\sum F_X = 0$$
$$R_{Ax} = 0$$

Step 3: Figure 2.33(e)-3

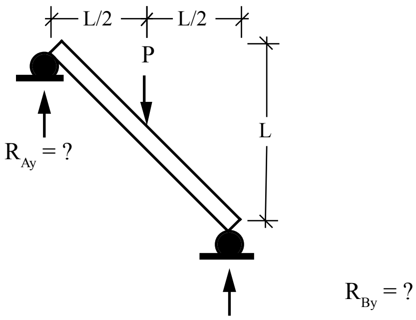


$$\sum M_B = 0$$
$$(P * L/2) - (R_{Ax} * h) = 0$$
$$(R_{Ax} * h) = PL/2$$
$$R_{Ax} = PL/2h \rightarrow$$

$$\sum F_X = 0$$
$$R_{Ax} + R_{Bx} = 0$$
$$R_{Bx} = - R_{Ax}$$
$$R_{Bx} = PL/2h \leftarrow$$

$$\sum F_Y = 0$$
$$R_{By} - P = 0$$
$$R_{By} = P \uparrow$$

Step 4: Figure 2.33(e)-4



$$R_{By} * L = PL/2$$
$$R_{By} = P/2 \uparrow$$

$$\sum F_Y = 0$$

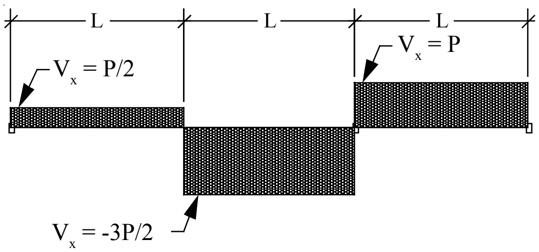
$$R_{Ay} = P - R_{By}$$
$$R_{Ay} = P - P/2$$
$$R_{Ay} = P/2 \uparrow$$

Question 2.15: Draw shear and moment diagrams for the beam analyzed in Question 2.6 [Figure 2.59 (Q6)]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.6).

$$R_A = P/2 \text{ (upward)}$$
$$R_B = 5P/2 \text{ (upward)}$$

Step 2: Draw the shear diagram.



For $0 < x < L$:

$$V_X = P/2$$

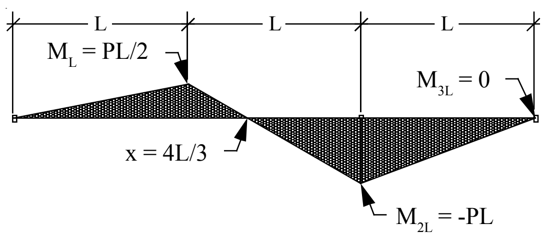
For $L < x < 2L$:

$$V_X = P/2 - 2P$$
$$V_X = -3P/2$$

For $2L < x < 3L$:

$$V_X = P/2 - 2P + 5P/2$$
$$V_X = P$$

Step 3: Draw the moment diagram.



For $0 < x < L$:

$$M_X = (P/2)x$$

When $x = L$:

$$M_L = (P/2)L$$
$$M_L = PL/2$$

For $L < x < 2L$:

$$M_X = (P/2)x - (2P)(x - L)$$

When $M_X = 0$:

$$0 = (P/2)x - (2P)(x - L)$$
$$0 = Px/2 - 2Px + 2PL$$
$$0 = -3Px/2 + 2PL$$
$$3Px/2 = 2PL$$
$$(2/3P)3Px/2 = 2PL(2/3P)$$
$$x = 4L/3$$

When $x = 2L$:

$$M_{2L} = (P/2)2L - (2P)(2L - L)$$
$$M_{2L} = PL - 2PL$$
$$M_{2L} = -PL$$

For $2L < x < 3L$:

$$M_X = (P/2)x - (2P)(x - L) + (5P/2)(x - 2L)$$

Check: when $x = 3L$:

$$M_{3L} = (P/2)3L - (2P)(3L - L) + (5P/2)(3L - 2L)$$
$$M_{3L} = 3PL/2 - (2P)(2L) + (5P/2)L$$
$$M_{3L} = 3PL/2 - 4PL + 5PL/2$$
$$M_{3L} = 8PL/2 - 4PL$$
$$M_{3L} = 0$$

When the shear is positive, the slope to the moment diagram is positive and vice-versa. Also note that when the shear diagram passes through zero the bending moment values are critical. Since only concentrated loads are present, the moment diagram consists of linearly sloped lines only (uniform loadings produce curved lines). The point of zero moment on the bending moment diagram corresponds to a "point of inflection" (reverse curvature) on the deflected shape of the structure (see Section 2.4.4).

Summary

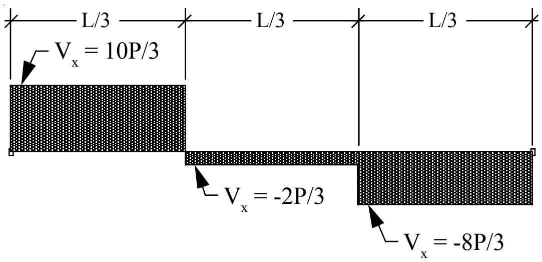
$$V_{MAX} = -3P/2$$
$$M_{MAX} = -PL$$

Question 2.17: Draw shear and moment diagrams for the beam analyzed in Question 2.8 [Figure 2.59]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.8).

$R_A = 10P/3$ (upward)
$R_B = 8P/3$ (upward)

Step 2: Draw the shear diagram.



For $0 < x < L/3$:

$V_X = 10P/3$

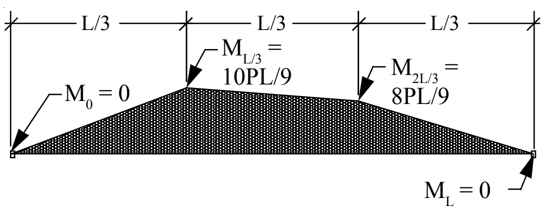
For $L/3 < x < 2L/3$:

$V_X = 10P/3 - 4P$
$V_X = -2P/3$

For $2L/3 < x < L$:

$V_X = 10P/3 - 4P - 2P$
$V_X = -8P/3$

Step 3: Draw the moment diagram.



For $0 < x < L/3$:

$M_X = (10P/3)x$

When $x = L/3$:

$M_{L/3} = (10P/3)(L/3)$
$M_{L/3} = 10PL/9$

For $L/3 < x < 2L/3$:

$M_X = (10P/3)x - 4P(x - L/3)$

When $x = 2L/3$:

$M_{2L/3} = (10P/3)(2L/3) - 4P(L/3)$
$M_{2L/3} = 20PL/9 - 4PL/3$
$M_{2L/3} = 20PL/9 - 12PL/9$
$M_{2L/3} = 8PL/9$

For $2L/3 < x < L$:

$M_X = (10P/3)x - 4P(x - L/3)$
$- 2P(x - 2L/3)$

Check: when $x = L$:

$M_L = (10P/3)L - 4P(L - L/3)$
$- 2P(L - 2L/3)$
$M_L = 10PL/3 - 4P(2L/3)$
$- 2P(L/3)$
$M_L = 10PL/3 - 8PL/3 - 2PL/3$
$M_L = 0$

Summary

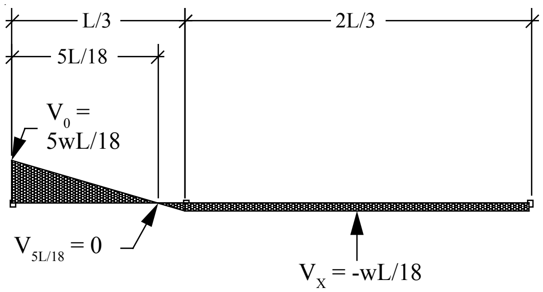
$V_{MAX} = +10P/3$
$M_{MAX} = +10PL/9$

Question 2.19: Draw shear and moment diagrams for the beam analyzed in Question 2.10 [Figure 2.59]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.7).

$$R_A = 5wL/18 \text{ (upward)}$$
$$R_B = wL/18 \text{ (upward)}$$

Step 2: Draw the shear diagram.



For 0 < x < L/3:

$$V_X = 5wL/18 - wx$$

When x = 0:

$$V_X = 5wL/18 - wx$$
$$V_X = 5wL/18$$

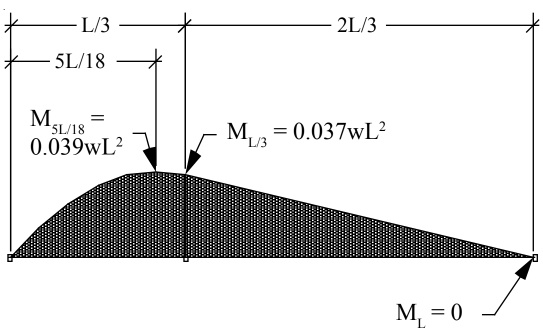
When $V_X = 0$:

$$0 = 5wL/18 - wx$$
$$wx = 5wL/18$$
$$x = 5L/18$$

For L/3 < x < L:

$$V_X = 5wL/18 - w * L/3$$
$$V_X = 5wL/18 - 6wL/18$$
$$V_X = -wL/18$$

Step 3: Draw the moment diagram.



For 0 < x < L/3:

$$M_X = (5wL/18)x - wx(x/2)$$
$$M_X = 5wxL/18 - wx^2/2$$

When x = 5L/18 ($V_x = 0$):

$$M_{5L/18} = (5wL/18)(5L/18) - w(5L/18)^2/2$$
$$M_{5L/18} = 25wL^2/324 - 25wL^2/648$$
$$M_{5L/18} = 25wL^2/648$$
$$M_{5L/18} = 0.039\ wL^2$$

When x = L/3:

$$M_{L/3} = (5wL/18)(L/3) - w(L/3)^2/2$$
$$M_{L/3} = 5wL^2/54 - wL^2/18$$
$$M_{L/3} = 5wL^2/54 - 3wL^2/54$$
$$M_{L/3} = 2wL^2/54$$
$$M_{L/3} = wL^2/27$$
$$M_{L/3} = 0.037wL^2$$

For L/3 < x < L:

$$M_x = (5wL/18)x - (wL/3)(x - L/6)$$

Check: when x = L:

$$M_L = (5wL/18)L - (wL/3)(L - L/6)$$
$$M_L = 5wL^2/18 - (wL/3)(5L/6)$$
$$M_L = 5wL^2/18 - 5wL^2/18$$
$$M_L = 0$$

Summary

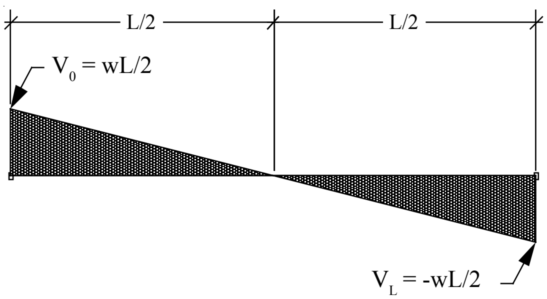
$$V_{MAX} = +5wL/18$$
$$M_{MAX} = +25wL^2/648$$

Question 2.21: Draw shear and moment diagrams for the beam analyzed in Question 2.12 [Figure 2.59]. What is the maximum shear force present? What is the maximum bending moment present?

Step 1: Find the reactions (see Question 2.12).

R_{Ax} = wL/2 (to the right)
R_{Ay} = wL/2 (upward)
R_{Bx} = wL/2 (to the left)
R_{By} = wL/2 (upward)

Step 2: Draw the shear diagram.



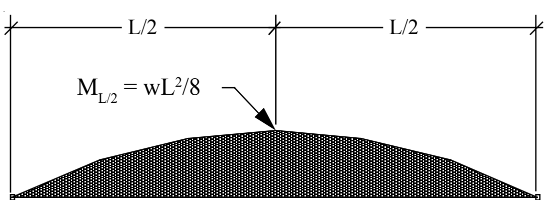
For 0 > x > L:
$V_X = wL/2 - wx$

When x = 0:
$V_0 = wL/2$

When x = L:
$V_L = wL/2 - wL$
$V_L = -wL/2$

When V_X = 0:
$0 = wL/2 - wx$
$wx = wL/2$
$x = L/2$

Step 3: Draw the moment diagram.



For 0 > x > L:
$M_X = (wL/2)x - wx(x/2)$
$M_X = wxL/2 - wx^2/2$

When x = 0:
$M_X = 0$

When x = L/2:
$M_X = (wL/2)(L/2) - w(L/2)(L/4)$
$M_X = wL^2/4 - wL^2/8$
$M_X = wL^2/8$

Check: when x = L:
$M_L = (wL/2)L - wL^2/2$
$M_L = wL^2/2 - wL^2/2$
$M_L = 0$

Summary

$V_{MAX} = \pm wL/2$
$M_{MAX} = +wL^2/8$

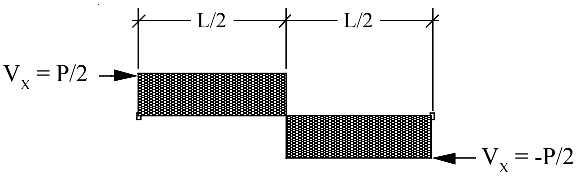
Question 2.22: Draw shear and moment diagrams for the four beams in Question 13 [Figure 2.59]. For the inclined members, the shear and moment diagrams should be drawn with respect to the longitudinal axes of the members. Transverse components of the applied and reactive forces should thus be considered in determining shears and moments. Compare the maximum moments developed in all four beams.

Beam 2.59(Q13a)

Step 1: Find the reactions (see Question 2.13).

$R_{Ax} = 0$
$R_{Ay} = P/2$ (upward)
$R_{By} = P/2$ (upward)

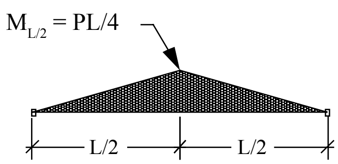
Step 2: Draw the shear diagram.



For $0 < x < L/2$:
$V_X = P/2$

For $L/2 < x < L$:
$V_X = P/2 - P$
$V_X = -P/2$

Step 3: Draw the moment diagram.



For $0 < x < L/2$:
$M_X = (P/2)x$

When $x = L/2$:
$M_X = P/2 * L/2$
$M_X = PL/4$

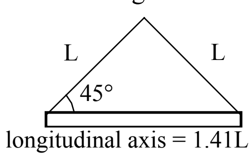
For $L/2 < x < L$:
$M_X = P/2(x) - P(x - L/2)$

Beam 2.59(Q13b)

Step 1: Find the reactions (see Question 2.13).

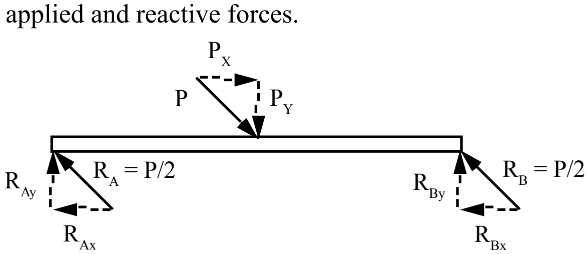
$R_{Ax} = 0$
$R_{Ay} = P/2$ (upward)
$R_{By} = P/2$ (upward)

Step 2: Calculate the longitudinal axis of the member.



$\cos 45° = L/\text{longitudinal axis}$
$\text{longitudinal axis} = L/\cos 45°$
$\text{longitudinal axis} = 1.41L$

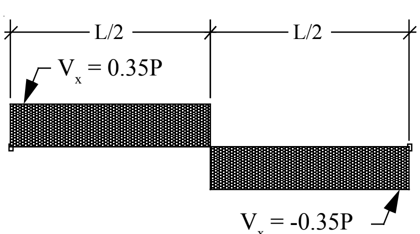
Step 3: Calculate the transverse components of applied and reactive forces.



$P_Y = P * \sin 45°$
$P_Y = 0.71\,P$

$R_{Ay} = R_{By} = P/2 * \sin 45°$
$R_{Ay} = R_{By} = 0.35\,P$

Step 4: Draw the shear diagram.

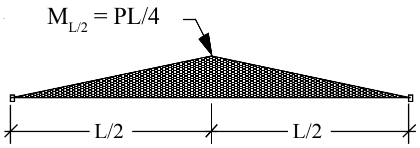


For $0 < x < .71L$:
$V_X = 0.35P$

For $.71L < x < 1.41L$:
$V_X = 0.35P - 0.71P$
$V_X = -0.35P$

Question 2.22 (continued):

Beam 2.59(Q13b) (continued).
Step 5: Draw the moment diagram



For $0 < x < .71L$:

$M_X = 0.35Px$

When $x = .71L$:

$M_{.71L} = 0.35P * .71L$
$M_{.71L} = 0.25PL$
$M_{.71L} = PL/4$

For $.71L < x < 1.41L$:

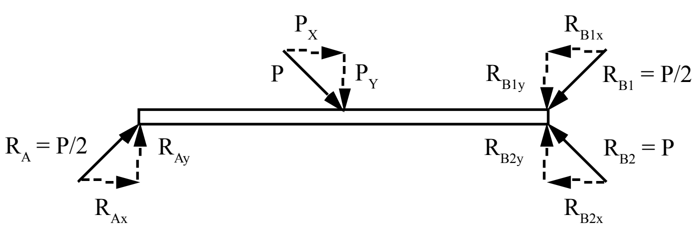
$M_X = 0.35Px - 0.71P(x - .71L)$
$M_X = 0.35Px - 0.71Px + 0.50PL$
$M_X = - 0.35Px + 0.50PL$

Beam 2.59(Q13c)
Step 1: Find the reactions (see Question 2.13).

$R_A = P/2$
$R_{B1} = P/2$
$R_{B2} = P$

longitudinal axis $= 1.41L$

Step 2: Calculate the transverse components of applied and reactive forces.



$P_Y = 0.71\ P$

$R_{Ay} = P/2 * \sin 45°$
$R_{Ay} = 0.35P$ (upward)

$R_{B1y} = P/2 * \sin 45°$
$R_{B1y} = -0.35P$ (downward)

$R_{B2y} = P * \sin 45°$
$R_{B2y} = 0.707P$ (upward)

R_{By} (net reaction) $= 0.35P$ (upward)

Step 3: Draw the shear and moment diagrams.

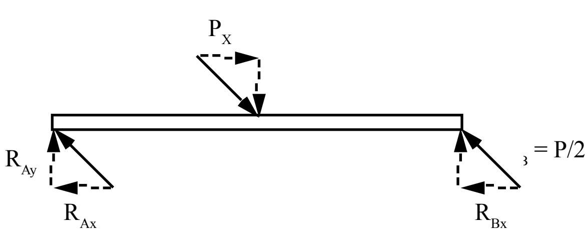
The formulas and diagrams will be the same as those for Beam 2.59(Q13b).

Beam 2.59(Q13d)
Step 1: Find the reactions (see Question 2-13).
Step 2: Calculate the transverse components of applied and reactive forces.

$R_A = P/2$
$R_B = P/2$

$P_Y = 0.71\ P$

$R_{Ay} = R_{By} = P/2 * \sin 45°$
$R_{Ay} = R_{By} = 0.35P$



Step 3: Draw the shear and moment diagrams.

The formulas and diagrams will be the same as those for Beam 2.59(Q13b).

Summary

$M_{MAX} = PL/4$ (for all four beams)

Question 2.24: What is the unit strain present in an aluminum specimen loaded to 10,000 lb/in^2? Assume that $E_a = 11.3 * 10^6$ lb/in^2.

stress / strain = modulus of elasticity
f (stress) = 10,000 lb/in^2
E (modulus of elasticity) = $11.3 * 10^6$ lb/in^2
ε (strain) = ?

$f / \varepsilon = E$
$\varepsilon = f / E$
$\varepsilon = (10{,}000 \text{ lb/in}^2)/(11.3 * 10^6 \text{ lb/in}^2)$
$\varepsilon = 0.000885$ in/in

Question 2.25: What is the unit strain present in a steel specimen loaded to 24,000 lb/in^2? Assume that $E_s = 29.6 * 10^6$ lb/in^2.

stress / strain = modulus of elasticity
f (stress) = 24,000 lb/in^2
E (modulus of elasticity) = $29.6 * 10^6$ lb/in^2
ε (strain) = ?

$f / \varepsilon = E$
$\varepsilon = f / E$
$\varepsilon = (24{,}000 \text{ lb/in}^2)/(29.6 * 10^6 \text{ lb/in}^2)$
$\varepsilon = 0.000811$ in/in

Question 2.26: A 2 in square steel bar is 20 ft long and carries a tension force of 16,000 lb. How much does the bar elongate? Assume that $E_s = 29.6 * 10^6$ lb/in^2.

A (cross-sectional area) = 2 in * 2 in
A = 4 in^2
L (member length) = 20 ft. * 12 in/1 ft
L = 240 in
P (load) = 16,000 lb.
E (modulus of elasticity) = $29.6 * 10^6$ lb/in^2
ΔL (elongation) = ?

$\Delta L = PL/AE$
$\Delta L = (16{,}000 \text{ lb} * 240 \text{ in})/(4 \text{ in}^2 * 29.6 * 10^6 \text{ lb/in}^2)$
$\Delta L = 0.032$ in

Question 2.27: A steel bar that is 20 mm in diameter is 5 m long and carries a tension force of 20kN. How much does the bar elongate? Assume that $E_S = 0.204 * 10^6$ N/mm^2.

A (cross-sectional area) = πr^2
A = $\pi(10 \text{ mm})^2$
A = 314 mm^2
L (member length) = 5 m * 1000 mm/1 m
L = 5000 mm
P (load) = 20 kN *1000 N/1 kN
P = 20 000 N
E (modulus of elasticity) = $0.204 * 10^6$ N/mm^2
ΔL (elongation) = ?

$\Delta L = PL/AE$
$\Delta L = (20\,000 \text{ N} * 5000 \text{ mm})/(314 \text{ mm}^2 * 0.204 * 10^6 \text{ N/mm}^2)$
$\Delta L = 1.56$ mm