## Chapter 02 - Leveling

2.1 Compute the error due to curvature and refraction for the following distances:
(a) 500 ft
$(c+r)=0.0206 \times(500 / 1000) 2=0.005 \mathrm{ft}$.
(b) $4,000 \mathrm{ft}$
$(c+r)=0.0206 \times(4) 2=0 \mathrm{ft}$.
(c) 300 m
$(c+r)=0.0675 \times 0.3002=0.006 \mathrm{~m}$.
(d) 2.2 mi
$(c+r)=0.574 \times 2.22=2.78 \mathrm{ft}$.
(e) $2,800 \mathrm{~m}$
$(c+r)=0.0675 \times(2.8) 2=0.529 \mathrm{~m}$
(f) 3 km
$(c+r)=0.0675 \times(3) 2=0.608 \mathrm{~m}$.
2.2 Determine the rod readings indicated on the foot and metric rod illustrations in Figure 2-32. The foot readings are to the closest 0.01 ft , and the metric readings are to the closest one-half or one-third cm.

| Rod A | i 1.90 | ii 1.73 | iii 1.57 | iv 1.21 | v $1.03(1.04)$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\operatorname{Rod} B$ | i 1.185 | ii 1.150 | iii 1.040 | iv 1.000 | v 0.930 |
| $\operatorname{Rod} C$ | i 3.06 | ii $2.85(2.84)$ | iii $2.57(2.56)$ | iv 2.21 | v 1.92 |
| $\operatorname{Rod} D$ | i 1.145 | ii 1.065 | iii 1.000 | iv 0.935 | v 0.880 |

2.3 An offshore drilling rig is being towed out to sea. What is the maximum distance away that the navigation lights can still be seen by an observer standing at the shoreline? The observer's eye height is $5^{\prime} 0^{\prime \prime}$ and the uppermost navigation light is 147 ft . above the water.

$$
\begin{array}{ll}
5.00=.574 K^{2}{ }^{2}, & K 1=V 5.00 / .574=2.95 \text { miles } \\
147=.574 \mathrm{~K}^{2}, & \mathrm{~K} 2=\mathrm{V} 147 / .574=16.00 \mathrm{miles} \\
\text { Maximum visibility distance }=18.95 \text { miles }
\end{array}
$$

2.4 Prepare a set of level notes for the survey in Figure 2-33. Show the arithmetic check.

| STATION | BS | HI | IS | FS | ELEVATION |
| :--- | :--- | :---: | :---: | :---: | :---: |
| BM \#50 | 1.27 | 390.34 |  |  | 389.07 |
| TP \#1 | 2.33 | 387.76 |  | 4.91 | 385.43 |
| TP \#2 |  |  |  | 6.17 | 381.59 |
|  | $389.07+3.60=392.67-11.08=381.59$ check |  |  |  |  |

2.5 Prepare a set of profile leveling notes for the survey in Figure 2-34. In addition to computing all elevations, show the arithmetic check and the resulting error in closure.

| STATION | BS | HI | IS | FS | ELEVATION |
| :--- | :--- | :--- | :--- | :--- | :--- |
| BM \#61 | 4.72 | 401.46 |  |  | 396.74 |
| $0+00$ |  |  | 4.42 |  | 397.04 |
| $0+50$ |  |  | 4.30 |  | 394.16 |
| TP \#1 | 5.11 | 404.56 |  | 2.01 | 399.45 |
| $1+00$ |  |  | 4.66 |  | 399.90 |
| $1+50$ |  |  | 3.98 |  | 400.58 |
| $1+75$ |  |  | 1.20 |  | 403.36 |
| TP \#2 |  |  |  | 1.80 | 402.76 |
|  | BS $=9.83$ |  |  | FS $=3.81$ |  |

$E=-0.02 m$ [small error - no need for adjustments]
$396.74+9.83=406.57-3.81=402.76$ check
2.6 Complete the set of differential leveling notes in Table 2-5, and perform the arithmetic check.

| STATION | BS | HI | FS | ELEVATION |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: |
| BM 100 | 2.71 | 317.59 |  | 314.88 |  |  |
| TP 1 | 3.62 | 316.33 | 4.88 | 312.71 |  |  |
| TP 2 | 3.51 | 315.87 | 3.97 | 312.36 |  |  |
| TP 3 | 3.17 | 316.23 | 2.81 | 313.06 |  |  |
| TP 4 | 1.47 | 316.08 | 1.62 | 314.61 |  |  |
| BM 100 |  |  | 1.21 | 314.87 |  |  |
|  | BS $=4.48$ |  |  | FS $=14.49$ |  |  |
|  | $314.88+14.48-14.49=314.87$, check |  |  |  |  |  |

2.7 If the loop distance in Problem 2.6 is 1,000 ft, at what order of survey do the results qualify? Use Table 2-1 or Table 2-2.

Error of closure $=0.01 \mathrm{ft}$. ; for 1000 ft ., second order (see Table 2.2) permits $.035 \mathrm{~V} 1000 / 5280=0.015$; therefore, results qualify for second order accuracy.
2.8 Reduce the set of differential leveling notes in Table 2-6, and perform the arithmetic check

| STATION | BS | HI | IS | FS | ELEVATION |
| :--- | :--- | :--- | :--- | :--- | :--- |
| BM 20 | 8.27 | 186.04 |  |  | 177.77 |
| TP 1 | 9.21 | 192.65 |  | 2.60 | 183.44 |
| $0+00$ |  |  | 11.3 |  | 181.4 |
| $0+50$ |  |  | 9.6 |  | 183.1 |
| $0+61.48$ |  |  | 8.71 |  | 246.65 |
| $1+00$ |  |  | 6.1 |  | 249.3 |
| TP 2 | 7.33 | 195.32 |  | 4.66 | 187.99 |
| $1+50$ |  |  | 5.8 |  | 252.2 |
| 2+00 |  |  | 4.97 |  | 253.06 |
| BM 21 |  |  |  | 3.88 | 191.44 |
|  | BS =24.81 |  |  | FS $=11.14$ |  |
|  |  |  | $177.7+24.81-11.14=191.44$ Check! |  |  |

2.9 If the distance leveled in Problem 2.8 is $1,000 \mathrm{ft}$, for what order of survey do the results qualify if the elevation of BM 21 is known to be 191.40? See Tables 2-1 and 2-2.

Error of closure $=0.04 \mathrm{ft}$. ; for 1000 ft. , third order (see Table 2.2) permits $\pm 0.10 \mathrm{v} 1000 / 5280=0.044$; therefore results qualify for third order accuracy.
2.10 Reduce the set of profile notes in Table 2-7, and perform the arithmetic check.

| STATION | BS | HI | IS | FS | ELEVATION |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BM 22 | 1.203 | 182.425 |  |  | 181.222 |
| 0+00 |  |  |  |  |  |
| G |  |  | 1.211 |  | 181.214 |
| 10M LT., |  |  | 1.430 |  | 180.995 |
| 10M RT., |  |  | 1.006 |  | 181.419 |
| 0+20 |  |  |  |  |  |
| 10M LT., |  |  | 2.93 |  | 179.50 |
| 7.3M LT. |  |  | 2.53 |  | 179.90 |
| 4M LT. |  |  | 2.301 |  | 180.124 |
| G |  |  | 2.381 |  | 180.044 |
| 4M RT. |  |  | 2.307 |  | 180.118 |
| 7.8M RT. |  |  | 2.41 |  | 180.02 |
| 10M RT. |  |  | 2.78 |  | 179.65 |
| 0+40 |  |  |  |  |  |
| 10M LT. |  |  | 3.98 |  | 178.45 |
| 6.2 M LT . |  |  | 3.50 |  | 178.9 |
| 4M LT. |  |  | 3.103 |  | 179.322 |
| ¢ |  |  | 3.187 |  | 179.238 |
| 4M RT. |  |  | 3.100 |  | 179.325 |
| 6.8M RT. |  |  | 3.37 |  | 179.06 |
| 10M RT. |  |  | 3.87 |  | 178.56 |
| TP 1 |  |  |  | 2.773 | 179.65 |

2.11 Reduce the set of municipal cross-section notes in Table 2-8.

| STATION | BS | HI | IS | FS | ELEVATION |
| :---: | :---: | :---: | :---: | :---: | :---: |
| BM 41 | 4.11 | 307.104 |  |  | 302.994 |
| TP 13 | 4.10 | 310.314 |  | 0.89 | 306.214 |
| 12+00 |  |  |  |  |  |
| 50 ft . lt. |  |  | 3.9 |  | 306.4 |
| 18.3 ft. It. |  |  | 4.6 |  | 305.7 |
| G |  |  | 6.33 |  | 303.98 |
| 20.1 ft. rt. |  |  | 7.9 |  | 302.4 |
| 50 ft . rt. |  |  | 8.2 |  | 302.1 |
| 13+00 |  |  |  |  |  |
| 50 ft . lt. |  |  | 5.0 |  | 305.3 |
| 19.6 ft . lt |  |  | 5.7 |  | 304.6 |
| G |  |  | 7.54 |  | 302.77 |
| 20.7 ft. rt. |  |  | 7.9 |  | 302.4 |
| 50 ft . rt. |  |  | 8.4 |  | 301.9 |
| TP 14 | 7.39 | 316.584 |  | 1.12 | 309.194 |
| BM S. 22 |  |  |  | 2.41 | 314.174 |
|  | $B S=15.60$ |  |  | FS $=4.42$ |  |
| $302.994+15.60-4.42=314.174$ check! |  |  |  |  |  |

2.12 Complete the set of highway cross-section notes in Table 2-9.

| STATION | BS | HI | FS | ELEV. | LEFT |  | E | RIGHT |  |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| BM 37 | 7.20 | 385.17 |  | 377.97 |  |  |  |  |  |
|  |  |  |  |  | 50 | 26.7 |  | 28.4 | 50 |
| $5+50$ |  |  |  |  | 4.6 | 3.8 | 3.7 | 3.0 | 2.7 |
|  |  |  |  |  | 380.6 | 381.4 | 381.5 | 382.2 | 382.5 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 50 | 24.1 |  | 25.0 | 50 |
| $6+00$ |  |  |  |  | 4.0 | 4.2 | 3.1 | 2.7 | 2.9 |
|  |  |  |  |  | 381.2 | 381.0 | 382.1 | 382.5 | 382.3 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 50 | 26.4 |  | 23.8 | 50 |
| 6+50 |  |  |  |  | 3.8 | 3.7 | 2.6 | 1.7 | 1.1 |
|  |  |  |  |  | 381.4 | 381.5 | 382.6 | 383.5 | 384.1 |
| TP 1 |  |  | 6.71 | 378.46 |  |  |  |  |  |

2.13 Complete the set of highway cross-section notes in Table 2-10.

| STATION | BS | HI | FS | ELEV. | LEFT |  | ¢ | RIGHT |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| BM 107 | 7.71 | 406.87 |  | 399.16 |  |  |  |  |  |
|  |  |  |  |  | 60 | 28 |  | 32 | 60 |
| 80+50 |  |  |  |  | 9.7 | 8.0 | 5.7 | 4.3 | 4.0 |
|  |  |  |  |  | 397.2 | 398.9 | 401.2 | 402.6 | 402.9 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 60 | 25 |  | 30 | 60 |
| 81+00 |  |  |  |  | 10.1 | 9.7 | 6.8 | 6.0 | 5.3 |
|  |  |  |  |  | 396.8 | 397.2 | 400.1 | 400.9 | 401.6 |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | 60 | 27 |  | 33 | 60 |
| 81+50 |  |  |  |  | 11.7 | 11.0 | 9.2 | 8.3 | 8.0 |
|  |  |  |  |  | 395.2 | 395.9 | 397.7 | 398.6 | 398.9 |
| TP 1 |  |  | 10.17 | 396.70 |  |  |  |  |  |

2.14 A level is set up midway between two wood stakes that are about 300 ft apart. The rod reading on stake $A$ is 8.72 ft , and it is 5.61 ft on stake $B$. The level is then moved to point $B$ and set up about 6 ft or 2 m away. $A$ reading of 5.42 ft is taken on the rod at $B$. The level is then sighted on the rod held on stake $A$, where a Reading of 8.57 ft is noted.
(a) What is the correct difference in elevation between the tops of stakes $A$ and $B$ ?
(b) If the level had been in perfect adjustment, what reading would have been observed at $A$ from the second setup?
(c) What is the line-of-sight error in 300 ft ?
(d) Describe how you would eliminate the line-of-sight error from the telescope.
a) True difference $=8.72-5.61=3.11 \mathrm{ft}$.
b) Correct rod reading $=5.42+3.11=8.53 \mathrm{ft}$.; on A
c) Error is +0.04 in 300 ft ., or $.00001 \mathrm{ft} / \mathrm{ft}$
d) Cross hair adjusted downward from 8.57 to read 8.53 , on A
2.15 A pre-engineering baseline was run down a very steep hill (see Figure 2-35). Rather than measure horizontally downhill with the steel tape, the surveyor measures the vertical angle with a theodolite and the slope distance with a $200-f t$ steel tape. The vertical angle is $-21^{\circ} 26^{\prime}$ turned to a point on a plumbed range pole that is 4.88 ft above the ground. The slope distance from the theodolite to the point on the range pole is 148.61 ft. The theodolite's optical center is 4.66 ft above the upper baseline station at $110+71.25$.
(a) If the elevation of the upper station is 318.71, what is the elevation of the lowerstation?
(b) What is the stationing chainage of the lower station?
a) $\mathrm{V}=148.61 \operatorname{Sin} 21026$ ' $=54.30 \mathrm{ft}$ Elevation of lower station $=318.71+4.66-54.30-4.88=264.19 \mathrm{ft}$.
b) $\mathrm{H}=148.61 \operatorname{Cos}\left(21^{\circ} 26^{\prime}\right)=138.33 \mathrm{ft} \quad$ lower station at $110+71.25+138.33=112+09.58$
2.16 You must establish the elevation of point $B$ from point $A$ (elevation 216.612 m ). $A$ and $B$ are on opposite sides of a 12-lane highway. Reciprocal leveling is used, with the following results:

Setup at A side of highway:
Rod reading on $A=0.673 \mathrm{~m}$
Rod readings on $B=2.416$ and 2.418 m
Setup at $B$ side of highway:
Rod reading on $B=2.992 \mathrm{~m}$
Rod readings on $A=1.254$ and 1.250 m
(a) What is the elevation of point $B$ ?
(b) What is the leveling error?
a) First elevation difference $=2.417-0.673=1.744$

Second elevation difference $=2.992-1.252=1.740$
Average elevation difference $=1.742$
Elevation B $=216.612-1.742=214.870$
b) The leveling error is 0.004 m
2.17 Reduce the set of differential leveling notes inTable 2-11, and perform the arithmetic check.
(a) Determine the order of accuracy (seeTable 2-1 or Table 2-2).
(b) Adjust the elevation of BM K110. The length of the level run was 780 m , with setups that are equally spaced. The elevation of BM 132 is 187.536 m.

| STATION | BS | HI | FS | ELEVATION |
| :--- | :--- | :--- | :--- | :--- |
| BM 130 | 0.702 | 189.269 |  | 188.567 |
| TP 1 | 0.970 | 189.128 | 1.111 | 188.158 |
| TP 2 | 0.559 | 189.008 | 0.679 | 188.449 |
| TP 3 | 1.744 | 187.972 | 2.780 | 186.228 |
| BM K110 | 1.973 | 188.277 | 1.668 | 186.304 |
| TP 4 | 1.927 | 188.416 | 1.788 | 186.489 |
| BM 132 |  |  | 0.888 | 187.528 |
|  | BS $=7.875$ |  | FS $=8.914$ |  |
|  | $188.567+7.875-8.914=187.528$, check |  |  |  |

a) error $=187.536-187.528=-0.008 \mathrm{~m}$.

Using specifications from Table 2.1, Third order accuracy, allowable error $=.012 \mathrm{~V} .780=0.011 \mathrm{~m}$. This error of 0.008 thus qualifies for third order accuracy (in both Tables 2.1 and 2.2)
b)

| STATION | CUMULATIVE <br> DISTANCE | ELEVATION | CORRECTION | ADJUSTED <br> ELEVATION |
| :--- | :---: | :---: | :---: | :---: |
| BM 130 |  | 188.567 |  | 188.567 |
| TP 1 | 130 | 188.158 | $130 / 780 \times .008=+.001$ | 188.159 |
| TP 2 | 260 | 188.449 | $260 / 780 \times .008=+.003$ | 188.452 |
| TP 3 | 390 | 186.228 | $390 / 780 \times .008=+.004$ | 186.232 |
| BM K110 | 520 | 186.304 | $520 / 780 \times .008=+.005$ | 186.309 |
| TP 4 | 650 | 186.489 | $650 / 780 \times .008=+.007$ | 186.496 |
| BM 132 <br> C $=187.536-187.528=-0.008$ | 780 | 187.528 | $780 / 780 \times .008=+.008$ | 186.536 |

The adjusted elevation of BM K110 is 186.309 m

