Chapter 2
Past, Present and Future World Energy Use

Problem 2.1 Consider an island with a current population density of 50 people/km^2 (about equal to the present world average). If the annual growth rate is 2%, determine the year in which the population density be equal to 20,000 people/km^2 (approximately the population density of Monaco, the world's most densely populated nation).

Solution As the problem deals with population density, the actual area of the island is not relevant. Solving equation (2.8) for a as a function of R gives

$$a = \ln[1+R/100]$$

Substituting in the annual growth rate of 2% gives

$$a = \ln[1+0.02] = 1.98\times10^{-2}\ \mathrm{y}^{-1}$$

From equation (2.4) the quantity as a function of time (relative to the present value) is

$$N(t)/N_0 = \exp(at)$$

Solving for t gives

$$t = (1/a)\times\ln[N(t)/N_0]$$

In this problem N is taken to be the population in 1 km^2 of land area so

$$N(t)/N_0 = 20{,}000/50 = 400.$$

Using the above value for a and solving for t gives

$$t = (\ln 400)/(1.98\times10^{-2}\ \mathrm{y}^{-1}) = 303\ \mathrm{y}$$

Relative to the year 2013, this population density will be reached in the year 2316.

Problem 2.2 A quantity has a doubling time of 110 years. Estimate the annual percent increase in the quantity.

Solution From equation (2.9) the annual rate of increase R is given as

$$R = \frac{100 \ln 2}{t_D}$$

where t_D is the doubling time. If t_D is 110 years then

$$R = \frac{(100) \times (0.693)}{110\text{ y}} = 0.63\%\text{ per year}$$

This is much less than 10% so the approximation given in equation (2.9) is valid.

Problem 2.3 The population of a particular country has a doubling time of 45 years. When will the population be three times its present value?

Solution From equation (2.7) the constant a can be determined from the doubling time as

$$t_D = \frac{1n2}{a}$$

so

$$a = \frac{1n2}{t_D}$$

For t_D=45 years then

$$a = \frac{0.693}{45} = 0.0154\text{y}^{-1}$$

From equation (2.4) the quantity of any time is given in terms of the initial value as

$$N(t) = N_0 \exp(at)$$

so solving for t we get

$$t = \frac{1}{a} \ln\left(\frac{N(t)}{N_0}\right)$$

for $N(t) = 3N_0$ then we get

$$t = \left(\frac{1}{0.0154\text{y}^{-1}}\right)\ln(3) = 71.3\text{years}$$

Problem 2.4 Assume that the historical growth rate of the human population was constant at 1.6% per year. For a population of 7 billion in 2012, determine the time in the past when the human population was 2.

Solution As the annual percentage growth rate is small then we can use the approximation of equation (2.4) to get the doubling time from R so

$$t_D = \left(\frac{100\ln 2}{R}\right) = \frac{(100)\times(0.693)}{1.6} = 43.31\text{years}$$

from equation (2.7) the constant a can be found to be

$$a = \left(\frac{\ln 2}{t_D}\right) = \frac{0.693}{43.31\text{y}} = 0.016\text{y}^{-1}$$

from equation (2.4) we start with an initial population of $N_0 = 2$ at $t = 0$ then $N(t) = 6.7\times10^9$
then from

$$N(t) = N_0 \exp(at)$$

so

$$t = \frac{1}{a}\ln\frac{N(t)}{N_0} = \frac{1}{0.016}\ln\frac{7\times10^9}{2} = 1374\text{y}$$

in the past or at year 2012-1374 = 638 (obviously growth rate was not constant)

Problem 2.5 What is the current average human population density (i.e., people per square kilometer) on earth?

Solution The radius of the Earth is 6378 km (assumed spherical). The total area (including oceans) is $A = 4\pi r^2 = (4)\times(3.14)\times(6378\text{km})^2 = 5.1\times10^8\,\text{km}^2$. The total current population is 6.7×10^9, so the population density is

$$\frac{6.7\times10^{9}}{5.1\times10^{8}\,\mathrm{km}^{2}}=13.1\,\mathrm{people/km}^{2}$$

If only land area is included, the land area on Earth is from various values given on the web range from $1.483\times10^8\,\mathrm{km}^2$ to $1.533\times10^8\,\mathrm{km}^2$. Using $1.5\times10^8\,\mathrm{km}^2$ we find

$$\frac{6.7\times10^{9}}{1.5\times10^{8}\,\mathrm{km}^{2}}=44.7\,\mathrm{people/km}^{2}$$

Problem 2.6 The total world population in 2012 was about 7 billion, and Figure 2.11 shows that at that time the actual world population growth rate was about 1% per year. The figure also shows an anticipated roughly linear decrease in growth rate that extrapolates to zero growth in about the year 2080. Assuming an average growth rate of 0.5% between 2012 and 2080, what would the world population be in 2080? How does this compare with estimates discussed in the text for limits to human population?

Solution If R = 0.5% per year then the doubling time is found from equation (2.9) to be

$$t_D=\frac{10\ln 2}{R}=\frac{(100)\times(0.693)}{0.5}=138.6\,\mathrm{y}$$

using equation (2.7) to get the constant a

$$a=\frac{\ln 2}{t_D}=\frac{0.693}{138.6\,\mathrm{y}}=0.005\,\mathrm{y}^{-1}$$

then equation (2.4) gives

$$N(t)=N_0\exp(at)$$

so from $N_0=7\times10^9$ people and $t=2080-2012=68\,\mathrm{years}$ we find

$$N(t)=\left(7\times10^{9}\right)\exp\left(\left(0.005\,\mathrm{y}^{-1}\right)\times(68\,\mathrm{y})\right)=9.8\times10^{9}\,\mathrm{people}$$

This is consistent with comments in the text which suggest that the limit to human population cannot be much more than 10 billion.

Problem 2.7 The population of a state is 25,600 in the year 1800 and 218,900 in the year 1900. Calculate the expected population in the year 2000 if (a) the growth is linear and (b) the growth is exponential.

Solution If population growth is linear then for 100 years between 1800 and 1900 it grows by $(218.9-25.6)\times 10^3 = 193.3\times 10^3$, so the population would grow by another 193.3×10^3 during the 100 years from 1900 to 2000 for a total of

$$(218.9+193.3)\times 10^3 = 412.2\times 10^3 \text{ people}$$

If the population growth is exponential then from equation (2.4) for $N_0 = 6.7\times 10^9$ in 1800 then for t=100 years, $N(t)$ is 218.9×10^3. From this a can be found to be

$$a = \frac{1}{t}\ln\frac{N(t)}{N_0} = \left(\frac{1}{100\text{y}}\right)\times\ln\left(\frac{218.9\times 10^3}{25.6\times 10^3}\right) = 0.0215\text{y}^{-1}$$

Then using $N_0 = 218.9\times 10^3$ in year 1900 the population at 100y (i.e. in year 2000) is

$$N(t) = (218.9\times 10^3)\times\exp\left((0.0215\text{y}^{-1})\times(100\text{y})\right) = 1.87\times 10^6$$

about 4.5 times the value for linear growth.

Problem 2.8 The population of a country as a function of time is shown in the following table. Is the growth exponential?

year	population (millions)
1700	0.501
1720	0.677
1740	0.891
1760	1.202
1780	1.622
1800	2.163
1820	2.884
1840	3.890
1860	5.176
1880	6.761
1900	8.702
1920	10.23
1940	11.74
1960	13.18
1980	14.45
2000	15.49

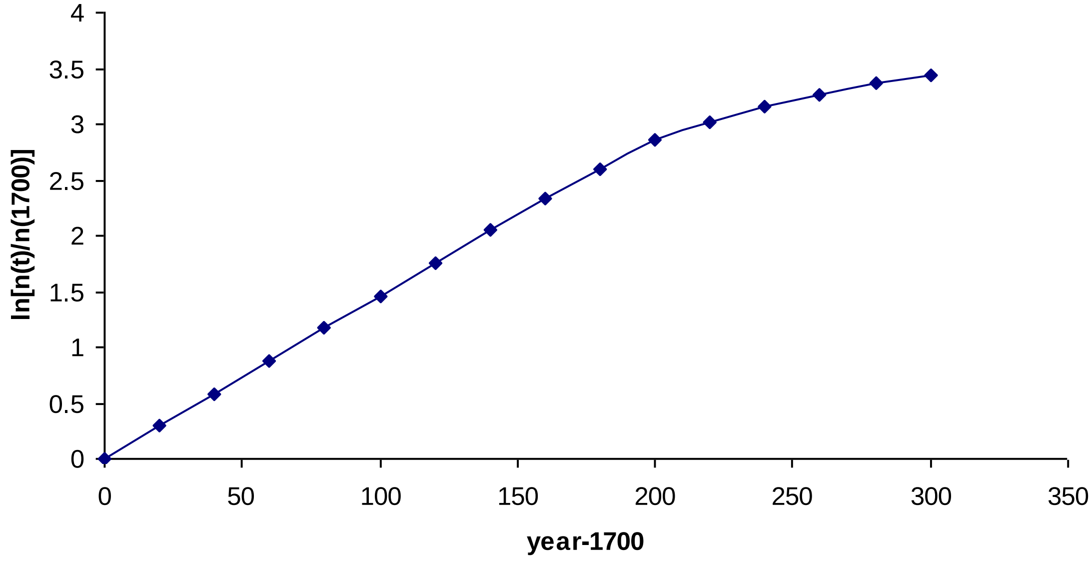
Solution For exponential growth

$$N(t) = N_0\exp(a(t-t_0)) \text{ so } \ln\left(\frac{N(t)}{N_0}\right) = a(t-t_0)$$

and the ln of the related population should be linear in time. Calculating $N(t)/N_0$ from the values above gives the tabulated values. They are plotted as a function of t - t_D as shown

year	population (millions)	year - 1700	ln[*N*(*t*)/*N*(1700)]
1700	0.501	0	0
1720	0.677	20	0.301065
1740	0.891	40	0.575738
1760	1.202	60	0.875136
1780	1.622	80	1.174809
1800	2.163	100	1.462645
1820	2.884	120	1.750327
1840	3.89	140	2.049558
1860	5.176	160	2.335182
1880	6.761	180	2.60232
1900	8.702	200	2.854702
1920	10.23	220	3.016474
1940	11.74	240	3.154151
1960	13.18	260	3.26985
1980	14.45	280	3.361844
2000	15.49	300	3.431344

The graph shows that the ln is linear and hence the population is exponential until ~ 1900 when the increase is less than exponential.



Problem 2.9 Consider a solar photovoltaic system with a total rated output of 10 MW_e and a capacity factor of 29%. If the total installation cost is $35,000,000, calculate the decrease in the cost of electricity per kilowatt-hour if the payback period is 25 years instead of 15 years. Assume a constant interest rate of 5.8%.

Solution From Example 2.3 the contribution to the cost of electricity per kWh due to the capital cost is

$$\frac{1}{Rf\left(8760\text{h/y}\right)}\times\frac{i(1+i)^T}{\left((1+i)^T-1\right)}$$

Using I = 35,000,000, i = 0.058, $R = 10^4$ kW, f =0.29, then for a payback period of 15 years the cost per kWh is

$$\frac{3.5\times10^7}{10^4\times0.29\times8760}\times\frac{0.058\times(1.058)^{15}}{\left((1.058)^{15}-1\right)}=1.378\times0.102=\$0.140/\text{kWh}$$

For a payback period of 25 years the cost is

$$1.378\times\frac{0.058\times(1.058)^{25}}{\left((1.058)^{25}-1\right)}=1.378\times0.0767=\$0.106\,/\,\text{kWh}$$

or a decrease of (0.140-0.106)=$0.034 per kWh.