Chapter 2
Past, Present and Future World Energy Use

2.1 A quantity increases at a rate of 1.5% per year. What is its doubling time?

Solution The doubling time t_D is related to the growth rate R (for small R) by

$$t_D = \frac{100 \ln 2}{R}$$

If R = 1.5 % then the doubling time (in years will be given as

$$t_D = 100 \times \ln(2)/(1.5) = 46.2 \text{ years.}$$

2.2 The population of a particular country was 1.1 million in 1940 and 3.4 million in 2010. Calculate the growth rate (in % per year). The growth rate was constant over that period of time.

Solution For constant growth the population at a time t relative to t=0 is given by

$$N(t) = N_0 \exp(at)$$

In this problem $N(t)/N_0 = 3.4 \times 10^6/1.1 \times 10^6 = 3.1$ and for a time period of 2010-1940 = 70 years we solve for a as

$$a = \frac{1}{t}\ln\left(\frac{N(t)}{N_0}\right)$$

or $a = (1/70) \times (\ln(3.1)) = 0.0162 \text{ y}^{-1}$

Thus the growth rate in percent will be 1.62% per year.

2.3 Consider the earth to be a sphere with a radius of 6378 km. 71% of its surface area is covered with water. The population density in Japan is currently 337 people per km^2. What would the population of the earth be if the population density on land was, on the average, the same as in Japan. Compare this with a current actual world population of about 7 billion.

Solution The total area of the earth (including oceans) is

$$A = 4\pi r^2 = (4)\times(3.14)\times(6378\text{km})^2 = 5.1\times10^8\,\text{km}^2 .$$

If 71% is water then the remaining land area is

$$(5.1 \times 10^8\ \text{km}^2) \times (0.29) = 1.48 \times 10^8\ \text{km}^2.$$

To attain a population density of 337 people per km^2 will, therefore, require a total population of

$$(1.48 \times 10^8\ \text{km}^2) \times (337\ \text{km}^{-2}) = 49.8\ \text{billion}.$$

This is 7 times the current world population and well above virtually all estimates of a maximum sustainable population.

2.4 A country has a constant annual growth rate of 5%. How long will it take for the population to increase by a factor of 10?

Solution The population as a function of time will be given by

$$N(t) = N_0 \exp(at)$$

Solving for t gives

$$t = \frac{1}{a}\ln\left(\frac{N(t)}{N_0}\right)$$

The constant a is related to the growth rate as

$$R = 100 \times (\exp(a) - 1)$$

Solving this for the constant a in terms of the given growth rate gives

$$a = \ln(1 + R/100) = \ln(1.05) = 0.0488\ \text{y}^{-1}$$

Substituting this value and $N(t)/N_0 = 10$ in the above gives the time as

$$t = (1/0.0488\ \text{y}^{-1}) \times \ln(10) = 47.2\ \text{years.}$$