1 Introduction

1.1 Equation (a) of the problem statement is used to solve for h as

$$h = \frac{\dot{Q}}{A(T - T_\infty)} \tag{a}$$

The Principle of Dimensional Homogeneity is used to determine the dimensions of the heat transfer coefficient. Using the F-L-T system dimensions of the quantities in Equation (a) are

$$\left[\dot{Q}\right] = \left[\frac{\mathrm{F} \cdot \mathrm{L}}{\mathrm{T}}\right] \tag{b}$$

$$\left[A\right] = \left[\mathrm{L}^2\right] \tag*{(b)}$$

$$\left[T - T_\infty\right] = \left[\Theta\right] \tag{c}$$

Thus from Equations (a)-(d) the dimensions of the heat transfer coefficient are

$$\begin{aligned} \left[h\right] &= \left[\frac{\mathrm{F} \cdot \mathrm{L}}{\mathrm{T} \cdot \Theta \cdot \mathrm{L}^2}\right] \\ &= \left[\frac{\mathrm{F}}{\mathrm{T} \cdot \Theta \cdot \mathrm{L}}\right] \end{aligned} \tag{d}$$

Possible units for the heat transfer coefficient using the SI system are $\frac{\mathrm{N}}{\mathrm{m} \cdot \mathrm{s} \cdot \mathrm{K}}$ while

possible units using the English system are $\frac{\mathrm{lb}}{\mathrm{ft} \cdot \mathrm{s} \cdot \mathrm{R}}$.

1.2 The Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho V D}{\mu} \tag{a}$$

The dimensions of the quantities on the left-hand side of Equation (a) are obtained using Table 1.2 as

$$\left[\rho\right] = \left[\frac{M}{\mathrm{L}^3}\right] \tag{b}$$

$$\left[\mathrm{V}\right] = \left[\frac{\mathrm{L}}{\mathrm{T}}\right] \tag{c}$$

$$\left[\mathrm{D}\right] = \left[\mathrm{L}\right] \tag{d}$$

$$\left[\mu\right] = \left[\frac{\mathrm{M}}{\mathrm{L} \cdot \mathrm{T}}\right] \tag{e}$$

Substituting Equations (b)-(e) in Equation (a) leads to

$$\begin{aligned}[\mathrm{Re}] &= \left[\frac{\frac{\mathrm{M}}{\mathrm{L}^3}\cdot\frac{\mathrm{L}}{\mathrm{T}}\cdot\mathrm{L}}{\frac{\mathrm{M}}{\mathrm{L}\cdot\mathrm{T}}}\right] \\ &= \left[\frac{\mathrm{M}\cdot\mathrm{L}^3\cdot\mathrm{T}}{\mathrm{M}\cdot\mathrm{L}^3\cdot\mathrm{T}}\right] \\ &= [1] \end{aligned} \tag{f}$$

Equation (f) shows that the Reynolds number is dimensionless.

1.3 The capacitance of a capacitor is defined by

$$C = \frac{i}{\frac{dv}{dt}} \tag{a}$$

The dimension of i is that of electric current, which is a basic dimension. The dimensions of electric potential are obtained from Table 1.2 as

$$[v] = \left[\frac{\mathrm{F}\cdot\mathrm{L}}{\mathrm{i}\cdot\mathrm{T}}\right] \tag{b}$$

Thus the dimensions of the time rate of change of electric potential are

$$\left[\frac{dv}{dt}\right] = \left[\frac{\mathrm{F}\cdot\mathrm{L}}{\mathrm{i}\cdot\mathrm{T}^2}\right] \tag{c}$$

Use of Equation (c) in Equation (a) leads to

$$\begin{aligned}[C] &= \left[\frac{\mathrm{i}}{\frac{\mathrm{F}\cdot\mathrm{L}}{\mathrm{i}\cdot\mathrm{T}^2}}\right] \\ &= \left[\frac{\mathrm{i}^2\cdot\mathrm{T}^2}{\mathrm{F}\cdot\mathrm{L}}\right] \end{aligned} \tag{d}$$

1.4 (a) The natural frequency of a mass-spring system is

$$\omega_n = \sqrt{\frac{k}{m}} \tag{a}$$

where m is mass with dimension [M] and k is stiffness with dimensions in the M-L-T system of $\left[\frac{\mathrm{M}}{\mathrm{T}^2}\right]$. Thus the dimensions of natural frequency are

$$[\omega_n] = \left[\left(\frac{\frac{\mathrm{M}}{\mathrm{T}^2}}{\mathrm{M}} \right)^{\frac{1}{2}} \right]$$

$$= \left[\frac{1}{\mathrm{T}} \right] \tag{b}$$

(b) The natural frequency of the system is 100 Hz, which for calculations must be converted to r/s,

$$\omega_n = 20 \frac{\text{cycles}}{\text{s}}$$

$$= \left(20 \frac{\text{cycles}}{\text{s}} \right) \left(2\pi \frac{\text{r}}{\text{cycles}} \right)$$

$$= 125.7 \frac{\text{r}}{\text{s}} \tag{c}$$

Equation (a) is rearranged as

$$k = m\omega_n^2 \tag{d}$$

Substitution of known values into Equation (d) leads to

$$k = (0.1\,\text{kg}) \left(125.7 \frac{\text{r}}{\text{s}} \right)^2$$

$$= 1.58x10^3 \frac{\text{N}}{\text{m}} \tag{e}$$

1.5 (a) The mass of the carbon nanotube is calculated as

$$m = \rho A L = \rho\left(\pi r^2\right) L$$

$$= \left(1300 \frac{\text{kg}}{\text{m}^3} \right) \pi \left(0.34x10^{-9}\ \text{m}\right)^2 \left(80x10^{-9}\ \text{m}\right)$$

$$3.78\text{x}10^{-23}\ \text{kg}$$

(b) Conversion between TPa and psi leads to

$$E = 1.1\,\text{TPa} = 1.1\text{x}10^{12}\ \frac{\text{N}}{\text{m}^2}$$

$$= \left(1.1\text{x}10^{12}\ \frac{\text{N}}{\text{m}^2} \right) \left(0.225 \frac{\text{lb}}{\text{N}} \right) \left(\frac{1\,\text{m}}{3.28\,\text{ft}} \right)^2 \left(\frac{1\,\text{ft}}{12\,\text{in}} \right)^2$$

$$= 1.60x10^8\ \frac{\text{lb}}{\text{in}^2}$$

(c) Calculation of the natural frequency leads to

$$\omega = 22.37\sqrt{\frac{EI}{\rho A L^4}}$$

$$= 22.37\sqrt{\frac{\left(1.1x10^{12}\ \frac{\text{N}}{\text{m}^2}\right)\frac{\pi}{4}\left(0.34x10^{-9}\ \text{m}\right)^4}{\left(1300\ \frac{\text{kg}}{\text{m}^3}\right)\pi\left(0.34x10^{-9}\right)^2\left(80x10^{-9}\ \text{m}\right)^4}}$$

$$= 1.73\text{x}10^{10}\ \frac{\text{r}}{\text{s}}$$

Converting to Hz gives

$$\omega = \left(1.73x10^{10}\ \frac{\text{r}}{\text{s}}\right)\left(\frac{1\,\text{cycle}}{2\pi\ \text{r}}\right)$$

$$= 2.75x10^9\ \text{Hz}$$

1.6 The power of the motor is calculated as

$$P = \frac{900\,\text{kW}\cdot\text{hr}}{24\,\text{hr}}$$

$$= 37.5\,\text{kW} \qquad \text{(a)}$$

The power is converted to English units using the conversions of Table 1.1

$$P = 37.5x10^3\ \text{W}$$

$$= 37.5\text{x}10^3\ \frac{\text{N}\cdot\text{m}}{\text{s}}$$

$$= 37.5x10^3\ \frac{\text{N}\left(\frac{0.225\,\text{lb}}{\text{N}}\right)\cdot\text{m}\left(\frac{3.28\,\text{ft}}{\text{m}}\right)}{\text{s}}$$

$$= 2.77x10^4\ \frac{\text{ft}\cdot\text{lb}}{\text{s}} \qquad \text{(b)}$$

Conversion to horsepower leads to

$$P = 2.77x10^4\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}\left(\frac{1\,\text{hp}}{\frac{550\ \text{ft}\cdot\text{lb}}{\text{s}}}\right)$$

$$= 50.3\,\text{hp} \qquad \text{(c)}$$

1.7 The conversion of density from English units to SI units is

$$\rho = 1.94\,\frac{\text{slugs}}{\text{ft}^3}$$
$$= 1.94\,\frac{\text{slugs}}{\text{ft}^3}\left(\frac{1\,\text{kg}}{0.00685\,\text{slugs}}\right)\left(\frac{3.28\,\text{ft}}{1\,\text{m}}\right)^3$$
$$= 9.99x10^3\,\frac{\text{kg}}{\text{m}^3} \qquad \text{(a)}$$

1.8 The constant acceleration of the train is

$$a = -6\,\frac{\text{m}}{\text{s}^2} \qquad \text{(a)}$$

The velocity is obtained using Equation (a) as

$$v(t) = -6t + C \qquad \text{(b)}$$

The constant of integration is evaluated by requiring

$$v(t=0) = 180\,\frac{\text{km}}{\text{hr}}$$
$$= 180\,\frac{\text{km}}{\text{hr}}\left(\frac{1000\,\text{m}}{\text{km}}\right)\left(\frac{1\,\text{hr}}{3600\,\text{s}}\right)$$
$$= 50\,\frac{\text{m}}{\text{s}} \qquad \text{(c)}$$

Using Equation (c) in Equation (b) leads to

$$v(t) = -6t + 50\,\frac{\text{m}}{\text{s}} \qquad \text{(d)}$$

The train stops when its velocity is zero,

$$0 = -6t + 50$$
$$t = 8.33\,\text{s} \qquad \text{(e)}$$

The distance traveled is obtained by integrating Equation (d) and assuming x(0)=0, leading to

$$x(t) = -3t^2 + 50t \qquad \text{(f)}$$

The distance traveled before the train stops is

$$x(8.33) = -3(8.33)^2 + 50(8.33)$$
$$= 208.3\,\text{m} \qquad \text{(g)}$$

1.9 The differential equation for the angular velocity of a shaft is

$$J\frac{d\omega}{dt} + c_t\omega = T \qquad \text{(a)}$$

Each term in Equation (a) has the same dimensions, those of torque or $[\text{F}\cdot\text{L}]$. The dimensions of angular velocity are $\left[\frac{1}{T}\right]$. Thus the dimensions of c_t are

$$[c_t] = \left[\frac{\mathrm{F \cdot L}}{\frac{1}{\mathrm{T}}}\right]$$
$$= [\mathrm{F \cdot L \cdot T}] \tag{b}$$

1.10 The equation for the torque applied to the armature is

$$T = K_a i_a i_f \tag{a}$$

Equation (a) is rearranged as

$$K_a = \frac{T}{i_a i_f} \tag{b}$$

The dimensions of torque are $[\mathrm{F \cdot L}]$ thus the dimensions of the constant are

$$[K_a] = \left[\frac{\mathrm{F \cdot L}}{\mathrm{i}^2}\right] \tag{c}$$

The equation for the back emf is

$$v = K_v i_f \omega \tag{d}$$

Equation (d) is rearranged as

$$K_v = \frac{v}{i_f \omega} \tag{e}$$

The dimensions of voltage are $\left[\frac{\mathrm{F \cdot L}}{\mathrm{i \cdot T}}\right]$ and the dimensions of angular velocity are $\left[\frac{1}{\mathrm{T}}\right]$. The dimensions of the constant K_v are

$$[K_v] = \left[\frac{\frac{\mathrm{F \cdot L}}{\mathrm{i \cdot T}}}{\mathrm{i}\frac{1}{\mathrm{T}}}\right]$$
$$= \left[\frac{\mathrm{F \cdot L}}{\mathrm{i}^2}\right] \tag{f}$$

It is clear from Equations (c) and (f) that the dimensions of $[K_a]$ and $[K_v]$ are the same.

These dimensions are the same as those of inductance (Table 1.2).

1.11 (a) The dimensions of $\dot{Q}$ are determined from Equation (a)

$$\dot{Q} = \sigma A \varepsilon \left(T^4 - T_b^4\right) \tag{a}$$

$$\left[\frac{\mathrm{F \cdot L}}{\mathrm{L^2 \cdot T \cdot \Theta^4}}\right][L^2][\Theta^4] = \left[\frac{\mathrm{F \cdot L}}{\mathrm{T}}\right] \tag{b}$$

(b) The differential equations governing the temperature in the body is

$$\rho c \frac{dT}{dt} + \sigma\varepsilon(T^4 - T_b^{\,4}) = 0 \tag{c}$$

The perturbation in temperature in the radiating body is defined by

$$T_b = T_{bs} + T_{b1} \tag{d}$$

This leads to a perturbation in the temperature of the receiving body defined as

$$T = T_s + T_1 \tag{e}$$

Substitution of equations (d) and (e) in Equation (c) leads to

$$\rho c \frac{d}{dt}(T_s + T_1) + \sigma\varepsilon\left[(T_s + T_1)^4 - (T_{bs} + T_{b1})^4\right] = 0 \tag{f}$$

Simplifying Equation (f) gives

$$\rho c \frac{dT_1}{dt} + \sigma\varepsilon\left[T_s^4\left(1 + \frac{T_1}{T_s}\right)^4 - T_{bs}^4\left(1 + \frac{T_{b1}}{T_{bs}}\right)^4\right] = 0 \tag{g}$$

Expanding the nonlinear terms, keeping only through the linear terms and noting that $T_s = T_{bs}$

$$\rho c \frac{dT_1}{dt} + \sigma\varepsilon\left[T_s^4\left(4\frac{T_1}{T_s}\right) - T_{bs}^4\left(4\frac{T_{b1}}{T_{bs}}\right)\right] = 0$$

$$\rho c \frac{dT_1}{dt} + 4\sigma\varepsilon T_s^3 T_1 = 4\sigma\varepsilon T_{bs}^3 T_{b1} \tag{h}$$

1.12 The differential equation is linearized by using the small angle assumption which implies $\sin\theta \approx \theta$ and $\cos\theta \approx 1$. Using these approximations in the differential equation leads to the linearized approximation as

$$\frac{1}{3}mL^2\ddot{\theta} + \frac{1}{4}cL^2\dot{\theta} + kL^2\theta = 0 \tag{a}$$

1.13 The differential equation is linearized by using the small angle assumption which implies $\sin\theta \approx \theta$ and $\cos\theta \approx 1$. Using these approximations in the differential equation leads to the linearized approximation as

$$\frac{1}{3}mL^2\ddot{\theta} + \left(mg\frac{L}{2} + \ddot{y}\right)\theta = L\ddot{x} \tag{a}$$

1.14 The nonlinear differential equations governing the concentration of the reactant and temperature are

$$V\frac{dC_A}{dt} + \left(q + \alpha V e^{-E/(RT)}\right)C_A = qC_{Ai} \tag{a}$$

$$\rho q c_p T_i - \rho q c_p T - \dot{Q} + \lambda V \alpha e^{-E/(RT)} C_A = \rho V c_p \frac{dT}{dt} \tag{b}$$

The reactor is operating at a steady-state when a perturbation in flow rate occurs according to

$$q = q_s + q_p(t) \tag{c}$$

The flow rate perturbation induces perturbations in concentration and temperature according to

$$C_A = C_{As} + C_{Ap}(t) \tag{d}$$

$$T = T_s + T_p(t) \tag{e}$$

The steady-state conditions are defined by setting time derivatives to zero in Equation (a) leading to

$$\left(q_s + \alpha V e^{-E/(RT_s)}\right) C_{As} = q_s C_{Ai} \tag{f}$$

$$\rho q c_p T_i - \rho q_s c_p T_s - \dot{Q} + \lambda V \alpha e^{-E/(RT_s)} C_{A_s} = 0 \tag{g}$$

Substitution of Equations (d) and (e) into Equations (a) and (b) leads to

$$V \frac{dC_{Ap}}{dt} + \left(q_s + q_p + \alpha V e^{-E/\left[R(T_s+T_p)\right]}\right)\left(C_{As} + C_{Ap}\right) = \left(q_s + q_p\right) C_{Ai} \tag{h}$$

$$\rho\left(q_s + q_p\right) c_p T_i - \rho\left(q_s + q_p\right) c_p \left(T_s + T_p\right) - \dot{Q} + \lambda V \alpha e^{-E/\left[R(T_s+T_p)\right]}\left(C_{A_s} + C_{Ap}\right) = \rho V c_p \frac{dT_p}{dt} \tag{i}$$

It is noted from Equation (f) of Example (1.6) that a linearization of the exponential terms in Equations (h) and (i) is

$$e^{-\frac{E}{R(T_s+T_p)}} = e^{-\frac{E}{RT_s}} + \frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p \tag{j}$$

Use of Equation (j) in Equations (h) and (i) and rearrangement leads to

$$V \frac{dC_{Ap}}{dt} + \left[q_s + q_p + \alpha V \left(e^{-\frac{E}{RT_s}} + \frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p\right)\right]\left(C_{As} + C_{Ap}\right) = \left(q_s + q_p\right) C_{Ai} \tag{k}$$

$$\rho\left(q_s + q_p\right) c_p T_i - \rho\left(q_s + q_p\right) c_p \left(T_s + T_p\right) - \dot{Q} + \lambda V \alpha \left[e^{-\frac{E}{RT_s}} + \frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p\right]\left(C_{A_s} + C_{Ap}\right) = \rho V c_p \frac{dT_p}{dt} \tag{l}$$

Equations (g) and (h) are used to simplify Equations (k) and (l) to

$$V \frac{dC_{Ap}}{dt} + q_s C_{Ap} + q_p C_{As} + q_p C_{Ap} + \alpha V e^{-\frac{E}{RT_s}} C_{Ap} + \alpha V \left(\frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p\right)\left(C_{As} + C_{Ap}\right) = q_p C_{Ai} \tag{m}$$

$$\rho q_p c_p T_i - \rho c_p \left(q_p T_s + q_s T_p + q_p T_p\right) + \lambda V \alpha e^{-\frac{E}{RT_s}} C_{Ap} + \lambda V \alpha \left[\frac{E}{RT_s^2} e^{-\frac{E}{RT_s}} T_p\right]\left(C_{A_s} + C_{Ap}\right) = \rho V c_p \frac{dT_p}{dt} \tag{n}$$

Neglecting products of perturbations Equations (m) and (n) are rearranged as

$$V\frac{dC_{Ap}}{dt}+q_sC_{Ap}+q_pC_{Ap}+\alpha Ve^{-\frac{E}{RT_s}}C_{Ap}+\alpha V\left(\frac{E}{RT_s^2}e^{-\frac{E}{RT_s}}T_p\right)C_{As}$$

$$=q_pC_{Ai}-\ q_pC_{As} \qquad (\mathbf{o})$$

$$\rho Vc_p\frac{dT_p}{dt}-\rho q_pc_pT_i+\rho c_p\left(q_pT_s+q_sT_p\right)+\lambda V\alpha e^{-\frac{E}{RT_s}}C_{Ap}+\lambda V\alpha\left[\frac{E}{RT_s^2}e^{-\frac{E}{RT_s}}T_p\right]C_{As}=0 \qquad (\mathbf{p})$$

1.15 The specific heat is related to temperature by

$$c_p=A_1+A_2T^{1.5}+A_3T^{2.6} \qquad \text{(a)}$$

The transient temperature is the steady-state temperature plus a perturbation,

$$T=T_s+T_p \qquad \text{(b)}$$

Substituting Equation (b) into Equation (a) leads to

$$c_P=A_1+A_2\left(T_s+T_p\right)^{1.5}+\left(T_s+T_p\right)^{2.6}$$

$$=A_1+A_2T_s^{1.5}\left(1+\frac{T_p}{T_s}\right)^{1.5}+A_3T_s^{2.6}\left(1+\frac{T_p}{T_s}\right)^{2.6} \qquad \text{(c)}$$

Using the binominal expansion to linearize Equation (c) leads to

$$c_p=A_1+A_2T_s^{1.5}\left(1+1.5\frac{T_p}{T_s}\right)+A_3T_s^{2.6}\left(1+2.6\frac{T_p}{T_s}\right) \qquad \text{(d)}$$

The differential equation for the time-dependent temperature is

$$c_p\frac{dT}{dt}+\frac{1}{R}T=\frac{1}{R}T_\infty \qquad \text{(e)}$$

Substituting Equations (b) and (d) into Equation (e) along with $T_\infty=T_{\infty s}+T_{\infty p}$ leads to

$$\left[A_1+A_2T_s^{1.5}\left(1+1.5\frac{T_p}{T_s}\right)+A_3T_s^{2.6}\left(1+2.6\frac{T_p}{T_s}\right)\right]\frac{d}{dt}\left(T_s+T_p\right)+\frac{1}{R}\left(T_s+T_p\right)=\frac{1}{R}\left(T_{\infty s}+T_{\infty p}\right) \qquad \text{(f)}$$

Noting that the steady-state is defined by $\frac{dT_s}{dt}=0$ and $T_s=T_{\infty s}$ reduces Equation (f) to

$$\left[A_1+A_2T_s^{1.5}\left(1+1.5\frac{T_p}{T_s}\right)+A_3T_s^{2.6}\left(1+2.6\frac{T_p}{T_s}\right)\right]\frac{dT_p}{dt}+\frac{1}{R}T_p=\frac{1}{R}T_{\infty p} \qquad \text{(g)}$$

Terms such as $T_p\frac{dT_p}{dt}$ are nonlinear. Equation (g) is linearized by noting that $\left|\frac{T_p}{T_s}\right|<<1$

$$\left(A_1+A_2T_s^{1.5}+A_3T_s^{2.6}\right)\frac{dT_p}{dt}+\frac{1}{R}T_p=\frac{1}{R}T_{\infty p} \qquad \text{(h)}$$

1.16 The force acting on the piston at any instant is

$$F=pA \qquad \text{(a)}$$

where A is the area of the piston head. The pressure is related to the density by

$$p=C\rho^\gamma \qquad \text{(b)}$$

The mass of air in the cylinder is constant and is calculated when the piston is in equilibrium as

$$m = \rho_0 A h \qquad \text{(c)}$$

where ρ_0 is the density of the air in equilibrium. Using Equation (b) in Equation (c) leads to

$$m = \left(\frac{p_0}{C}\right)^{\frac{1}{\gamma}} Ah \qquad \text{(d)}$$

where p_0 is the pressure in the cylinder when the piston is in equilibrium. At any instant the mass is calculated as

$$\begin{aligned} m &= \rho A(h - x) \\ &= \left(\frac{p}{C}\right)^{\frac{1}{\gamma}} A(h - x) \end{aligned} \qquad \text{(e)}$$

Since the mass is constant, Equations (d) and (e) are equated leading to

$$p = p_0 \left(\frac{h}{h - x}\right)^{\gamma} \qquad \text{(f)}$$

Substitution of Equation (f) into Equation (a) leads to

$$F = p_0 A \left(\frac{h}{h - x}\right)^{\gamma} \qquad \text{(g)}$$

(b) Equation (g) is rearranged as

$$F = p_0 A \left(1 - \frac{x}{h}\right)^{-\gamma} \qquad \text{(h)}$$

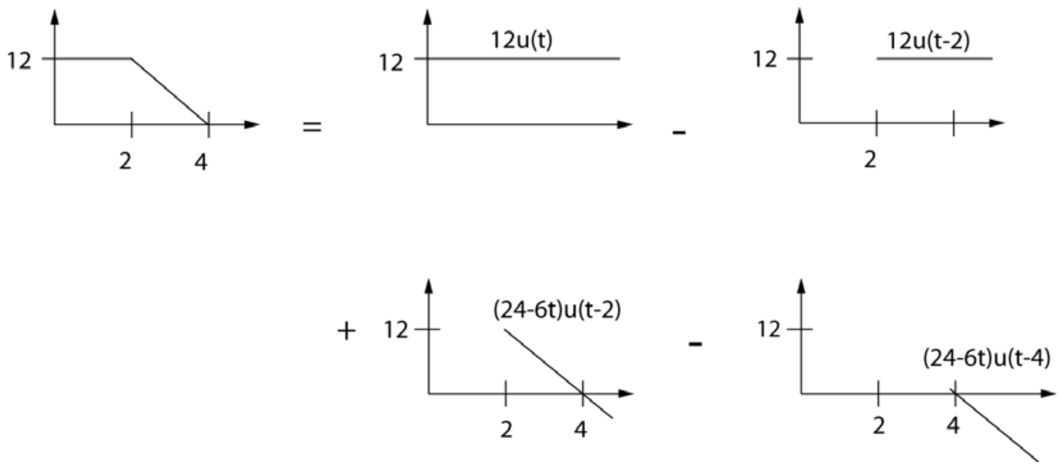
Since $\frac{x}{h} < 1$a binomial expansion can be used on the right-hand side of Equation (h). Using the binomial expansion keeping only through the linear term leads to

$$F = p_0 A + \frac{\gamma p_0 A}{h} x \qquad \text{(e)}$$

The linear stiffness is obtained from Equation (e) as

$$k = \frac{\gamma p_0 A}{h} \qquad \text{(f)}$$

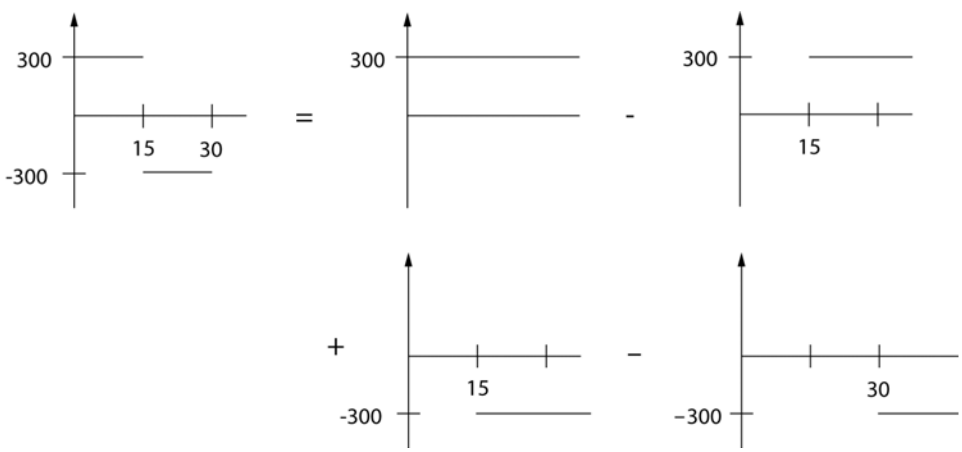
1.17 The appropriate superposition of the voltage in Figure P1.17 is illustrated below



The mathematical representation of the voltage source is

$$v(t) = 12[u(t) - u(t-2)] + (24 - 6t)[u(t-2) - u(t-4)]$$

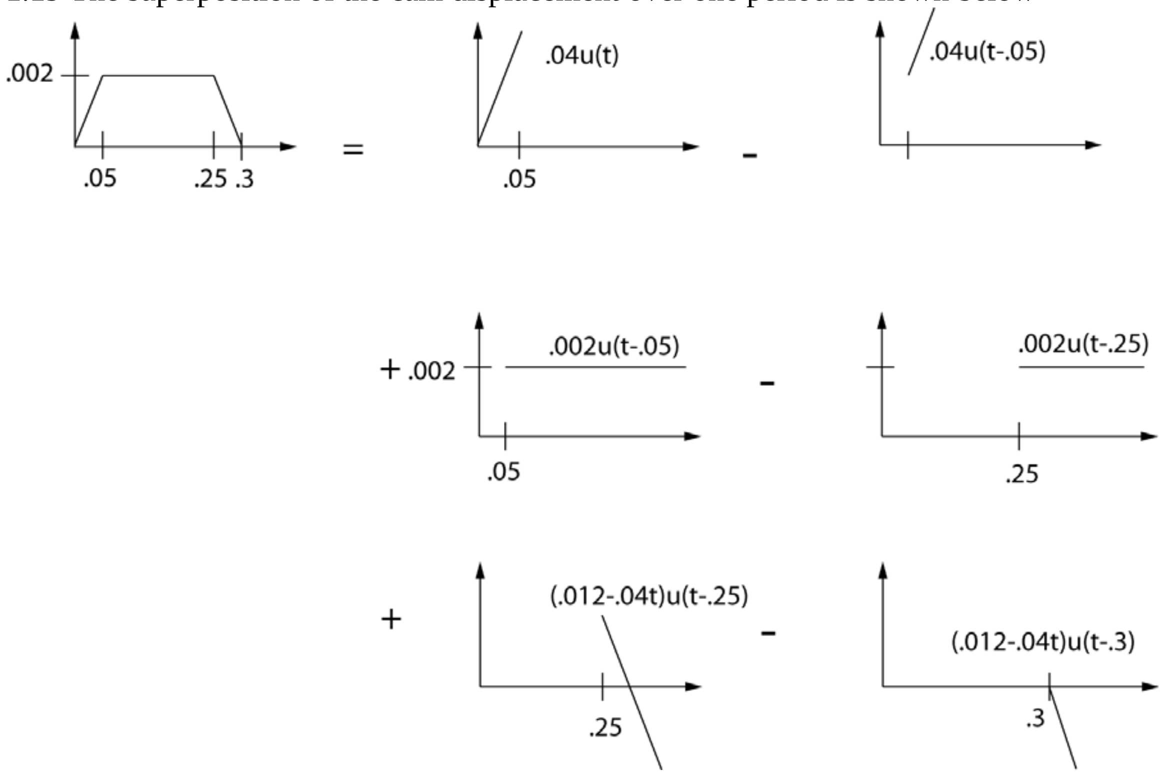
1.18 The superposition of the force of Figure P1.18 is illustrated below.



The mathematical representation of the force is

$$F(t) = 300[u(t) - u(t-15)] - 300[u(t-15) - u(t-30)] \qquad \text{(a)}$$

1.19 The superposition of the cam displacement over one period is shown below



(a) The mathematical representation of the displacement over one period is

$$x(t) = 0.04\big[u(t) - u(t-0.05)\big] + 0.002[u(t-0.05) - u(t-0.25)$$

$$+ (0.012 - 0.04t)[u(t-0.25) - u(t-0.3)] \qquad \textbf{(a)}$$

(b) The period of the cycle is 0.5 s. Thus the displacement over the second period is obtained by replacing t by t+0.5 in Equation (a). The displacement over the kth period is obtained by replacing t by t+(k-1)(0.5) in Equation (a). The total displacement is obtained by summing over all periods

$$x(t) = \sum_{k=1}^{K} \Big\{0.04\big[u(t-.5k+.5) - u(t-.5k+.45)\big] + .002[u(t-0.5k+.45) - u(t-.5k+.25)]$$

$$+ \big(0.02k - 0.008 - 0.04t\big)\big[u(t-0.5k+.25) - u(t-0.5k+0.2]\big]\Big\} \qquad \textbf{(b)}$$

where K is the smallest integer greater than t/(0.05).

1.20 Integration of Newton's second law with respect to time leads to the principle of impulse and momentum

$$I = \int_{t_1}^{t_2} F dt = m(v_2 - v_1) \qquad \text{(a)}$$

where the total impulse applied between t_1 and t_2 is $\int_{t_1}^{t_2} F dt$. The 12 $\text{N} \cdot \text{s}$ impulse is applied instantaneously to the 4-kg particle when it is at rest. Application of the principle of impulse and momentum leads to

$$12\ \mathrm{N\cdot s} = (4\ \mathrm{kg})v_2$$
$$v_2 = \frac{12\ \mathrm{N\cdot s}}{4\ \mathrm{kg}}$$
$$= 3\ \frac{\mathrm{m}}{\mathrm{s}} \qquad \text{(b)}$$

1.21 The equation for the voltage drop across an inductor is

$$v = L\frac{di}{dt} \qquad \text{(a)}$$

Integration of Equation (a) with respect to time leads to

$$\int_0^t v dt = L(i_2 - i_1) \qquad \text{(b)}$$

The initial current is zero. Solving Equation (b) for i_2 leads to

$$i_2 = \frac{\int_0^t v dt}{L}$$
$$= \frac{20\ \mathrm{V\cdot s}}{0.4\ \mathrm{H}}$$
$$= 50\ \mathrm{A} \qquad \text{(c)}$$

1.22 The mathematical representation of the force is

$$F(t) = 100\delta(t) + 150\delta(t-2.5) + 50\delta(t-3.8) \qquad \text{(a)}$$

1.23 The MATLAB file Problem1_23 which determines the steady-state response of a series LRC circuit is listed below

```
% Problem1_23.m
% Steady-state response of seties LRC circuit
clear
disp('Steady-state response of series LRC circuit')
% Input parameters
disp('Input resistance in ohms')
R=input('>> ')
disp('Input capacitance in farads')
C=input('>>')
disp('Input inductance in henrys')
L=input('>> ')
disp('Input source frequency in r/s')
om=input('>> ')
disp('Input source amplitude in V')
V0=input('>> ')
% Calculates parameters
disp('Natural freqeuncy in r/s =')
```

```
omn=1/(L*C)^0.5
disp('Dimensionless damping ratio =')
zeta=R/2*(C/L)^0.5
disp('Phase angle in rad=')
C1=om^2-omn^2;
C2=2*zeta*om*omn;
phi=atan2(C1,C2)
disp('Steady-state amplitude in A =')
C3=V0*om/L;
C4=1/(C1^2+C2^2)^0.5;
I=C3*C4
tf=10*pi/om;
dt=tf/200;
for k=1:201
    t(k)=(k-1)*dt;
    i(k)=I*sin(om*t(k)+phi);
end
plot(t,i)
xlabel('t (s)')
ylabel('i (A)')
title('Steady-state response of series LRC circuit')
str1=['R=',num2str(R),' \Omega'];
str2=['C=',num2str(C),' F'];
str3=['L=',num2str(L),' H'];
str4=['\omega=',num2str(om),' r/s'];
str5=['V_0=',num2str(V0),' V'];
text(0.9*tf,I,str1)
text(0.9*tf,0.8*I,str2)
text(0.9*tf,0.6*I,str3)
text(0.9*tf,0.4*I,str4)
text(0.9*tf,0.2*I,str5)
```

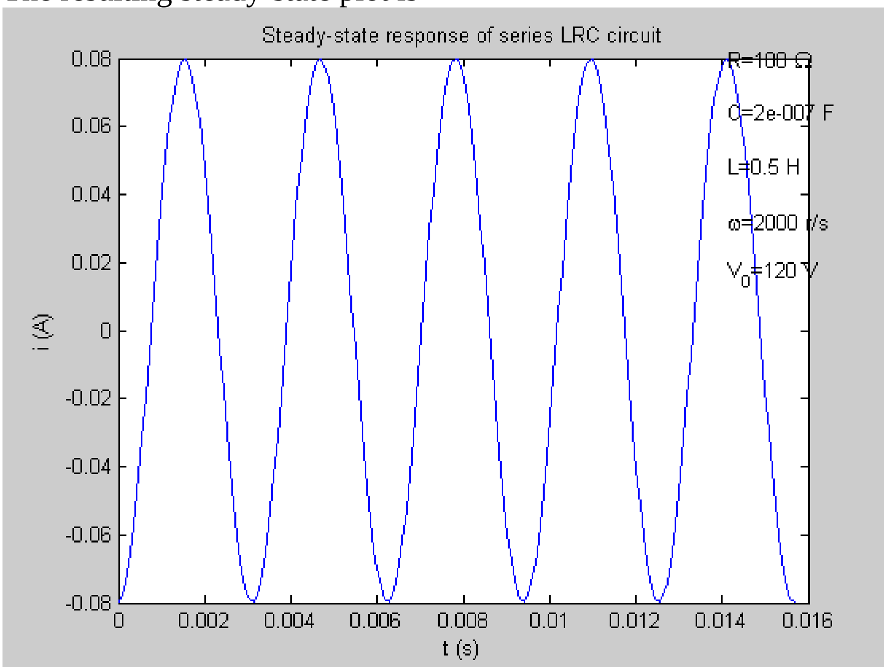
The MATLAB workspace from a sample execution of Problem1_23.m is

```
>> Problem1_23
Steady-state response of series LRC circuit
Input resistance in ohms
>> 100

R =

   100

Input capacitance in farads
>>0.2e-6
```

```
C =

  2.0000e-007

Input inductance in henrys
>> 0.5

L =

    0.5000

Input source frequency in r/s
>> 2000

om =

        2000

Input source amplitude in V
>> 120

V0 =

   120

Natural freqeuncy in r/s =

omn =

  3.1623e+003

Dimensionless damping ratio =

zeta =

    0.0316

Phase angle in rad=

phi =

   -1.5042

Steady-state amplitude in A =

I =
```

```
   0.0798

>>
```

The resulting steady-state plot is



1.24 The MATALB file Prolbem1_24.m is listed below

```
% Problem1_24.m
%(a) Input two five by five matrices
disp('Please input matrix A by row')
for i=1:5
   for j=1:5
      str={['Enter A(',num2str(i),num2str(j),')']};
      disp(str)
      A(i,j)=input('>> ');
   end
end
disp('Please input matrix B by row')
for i=1:5
   for j=1:5
      str={['Enter B(',num2str(i),num2str(j),')']};
      disp(str)
      B(i,j)=input('>> ');
   end
end
A
```

```
B
% (b) =A+B
C=A+B
% (c) D=A*B
D=A*B
% (d) det(A)
detA=det(A)
% eigenvalues and eigenvectors of A
[x,Y]=eigs(A);
disp('Eigenvalues of A')
Y
disp('Matrix of eigenvalues of A')
x
```

A sample output from execution of the file is shown below

```
>> clear
>> Problem1_24
Please input matrix A by row
   'Enter A(11)'

>> 1
   'Enter A(12)'

>> 0
   'Enter A(13)'

>> 12
   'Enter A(14)'

>> -1
   'Enter A(15)'

>> 21
   'Enter A(21)'

>> 14
   'Enter A(22)'

>> -3
   'Enter A(23)'

>> 2
   'Enter A(24)'

>> 0
```

```
   'Enter A(25)'

>> -22
   'Enter A(31)'

>> 11
   'Enter A(32)'

>> 12
   'Enter A(33)'

>> 10
   'Enter A(34)'

>> -4
   'Enter A(35)'

>> 12
   'Enter A(41)'

>> 10
   'Enter A(42)'

>> 11
   'Enter A(43)'

>> 18
   'Enter A(44)'

>> 12
   'Enter A(45)'

>> 21
   'Enter A(51)'

>> 10
   'Enter A(52)'

>> 11
   'Enter A(53)'

>> 31
   'Enter A(54)'

>> 21
   'Enter A(55)'
```

```
>> 11
Please input matrix B by row
   'Enter B(11)'

>> 21
   'Enter B(12)'

>> -21
   'Enter B(13)'

>> 21
   'Enter B(14)'

>> 10
   'Enter B(15)'

>> 9
   'Enter B(21)'

>> 8
   'Enter B(22)'

>> 2
   'Enter B(23)'

>> 2
   'Enter B(24)'

>> 4
   'Enter B(25)'

>> -5
   'Enter B(31)'

>> 16
   'Enter B(32)'

>> 12
   'Enter B(33)'

>> 11
   'Enter B(34)'

>> 18
   'Enter B(35)'
```

```
>> 11
   'Enter B(41)'

>> 21
   'Enter B(42)'

>> 32
   'Enter B(43)'

>> 14
   'Enter B(44)'

>> 19
   'Enter B(45)'

>> 12
   'Enter B(51)'

>> 12
   'Enter B(52)'

>> 9
   'Enter B(53)'

>> -5
   'Enter B(54)'

>> 13
   'Enter B(55)'

>> 21

A =

     1     0    12    -1    21
    14    -3     2     0   -22
    11    12    10    -4    12
    10    11    18    12    21
    10    11    31    21    11
```

B =

```
    21   -21    21    10     9
     8     2     2     4    -5
    16    12    11    18    11
    21    32    14    19    12
    12     9    -5    13    21
```

C =

```
    22   -21    33     9    30
    22    -1     4     4   -27
    27    24    21    14    23
    31    43    32    31    33
    22    20    26    34    32
```

D =

```
        444         280          34         480         570
         38        -474         420        -122        -299
        547        -107         249         418         353
       1090         601         493         969         818
       1367         955         812        1244         859
```

detA =

```
   -1171825
```

Eigenvalues of A

Y =

```
  43.3949                  0                  0                  0                  0
        0          -18.9247                   0                  0                  0
        0                  0          -1.8896 -11.6121i          0                  0
        0                  0                  0          -1.8896 +11.6121i          0
        0                  0                  0                  0            10.3091
```

Matrix of eigenvalues of A

x =

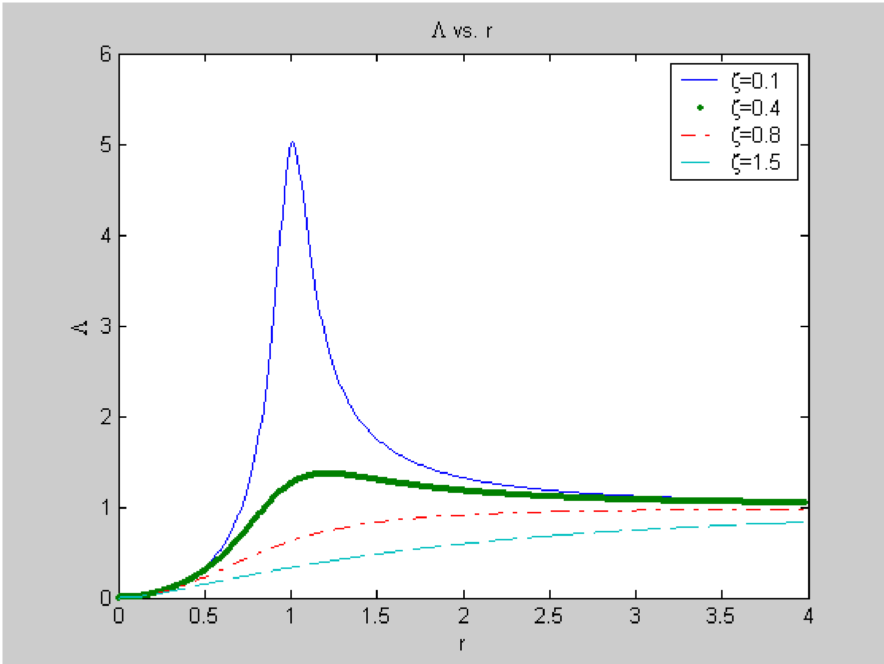
```
  -0.3664            -0.1632            -0.3379 - 0.4681i  -0.3379 + 0.4681i   0.2658
```

0.1869	0.7510	0.7187	0.7187	-0.4106
-0.2133	-0.4463	0.0528 + 0.2510i	0.0528 - 0.2510i	-0.4042
-0.6060	-0.2256	-0.0845 - 0.0753i	-0.0845 + 0.0753i	0.6726
-0.6466	0.3991	-0.2465 + 0.1043i	-0.2465 - 0.1043i	0.3808

1.25 A MATLAB file to calculate and plot $\Lambda(r,\zeta)$ is given below

```
% Plots the function LAMBDA(r,zeta) as a function of r for several values of
% zeta
% Specify four values of zeta
zeta1=0.1;
zeta2=0.4;
zeta3=0.8;
zeta4=1.5;
% Define values of r for calculations
for i=1:400
    r(i)=(i-1)*.01;
% Calculate function
LAMBDA1(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta1*r(i))^2)^0.5;
LAMBDA2(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta2*r(i))^2)^0.5;
LAMBDA3(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta3*r(i))^2)^0.5;
LAMBDA4(i)=r(i)^2/((1-r(i)^2)^2+(2*zeta4*r(i))^2)^0.5;
end
plot(r,LAMBDA1,'-',r,LAMBDA2,'.',r,LAMBDA3,'-.',r,LAMBDA4,'--')
xlabel('r')
ylabel('\Lambda')
str1=['\zeta=',num2str(zeta1)];
str2=['\zeta=',num2str(zeta2)];
str3=['\zeta=',num2str(zeta3)];
str4=['\zeta=',num2str(zeta4)];
legend(str1,str2,str3,str4)
title('\Lambda vs. r')
```

The resulting output from execution of the .m file is the following plot



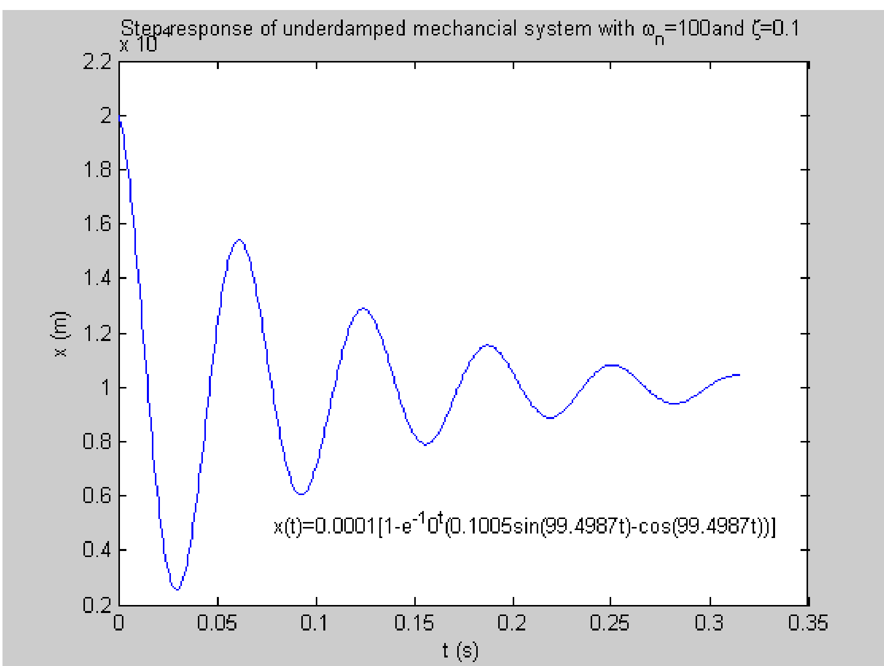
1.26 The MATALB .m file Problem1_26 which determines and plots the step response of an underdamped mechanical system is shown below.

```
% Problem1_26.m
% Step response of an underdamped mechanical system
% Input natural frequency and damping ratio
clear
disp('Step response of underdamped mechanical system')
disp('Please input natural frequency in r/s')
om=input('>> ')
disp('Please input the dimensionless damping ratio')
zeta=input('>> ')
% Damped natural frequency
omd=om*(1-zeta^2)^0.5;
C1=zeta*om/omd;
C2=1/om^2;
C3=zeta*om;
tf=10*pi/omd;
dt=tf/500;
for i=1:501
t(i)=(i-1)*dt;
x(i)=C2*(1-exp(-C3*t(i))*(C1*sin(omd*t(i))-cos(omd*t(i))));
end
plot(t,x)
xlabel('t (s)')
ylabel('x (m)')
```

```
str1=['Step response of underdamped mechancial system with
\omega_n=',num2str(om),'and \zeta=',num2str(zeta)]
title(str1)
str2=['x(t)=',num2str(C2),'[1-e^-^',num2str(C3),'^t(',num2str(C1),'sin(',num2str(omd),'t)-
cos(',num2str(omd),'t))]']
text(tf/4,C2/2,str2)
```

Output from execution of Problem1_26 follows

```
>> Problem1_26
Step response of underdamped mechanical system
Please input natural frequency in r/s
>> 100

om =

100

Please input the dimensionless damping ratio
>> 0.1

zeta =

   0.1000
```



1.27 The perturbation in liquid level is

$$h(t) = qR\left(1 - e^{-t/(RA)}\right) \qquad (a)$$

(a) Since the argument of a transcendental function must be dimensionless the dimensions of the product of resistance and area must be time. Thus the dimensions of resistance must be $\left[\frac{\mathbf{T}}{\mathbf{L}^2}\right]$

(b) Note that the steady-state value of the liquid-level perturbation is qR. The MATLAB file Problem1_27.m which calculates and plots h(t) from t=0 until h is within 1 percent of its steady-state value is given below

```
disp('Please enter resistance in s/m^2 ')
R=input('>> ')
% Final value of h
hf=0.99*q*R;
dt=0.01*R*A;
h1=0;
h(1)=0;
t(1)=0;
i=1;
while h1<hf
   i=i+1;
   t(i)=t(i-1)+dt;
   h(i)=q*R*(1-exp(-t(i)/(R*A)));
   h1=h(i);
end
plot(t,h)
xlabel('t (s)')
ylabel('h (m)')
title('Perturbation flow rate vs time')
str1=['A=',num2str(A),' m^3/s']
str2=['R=',num2str(R),' s/m^2']
str3=['q=',num2str(q),' m^3/s']
text(0.5*t(i),0.5*h(i),str1);
text(0.5*t(i),0.4*h(i),str2);
text(0.5*t(i),0.3*h(i),str3);
```

Sample output from execution of Problem1_27.m is given below

```
>> Please enter tank area in m^2
>> 100

A =

  100

Please enter flow rate in m^3/s
```

Chapter 1

```
>> 0.2

q =

    0.2000

Please enter resistance in s/m^2
>> 15

R =

    15

str1 =

A=100 m^3/s

str2 =

R=15 s/m^2


str3 =

q=0.2 m^3/s

>>
```