For Thought

1. True

2. False, since the range of $y = 4\sin(x)$ is $[-4, 4]$, we get that the range of $y = 4\sin(x) + 3$ is $[-4 + 3, 4 + 3]$ or $[-1, 7]$.

3. True, since the range of $y = \cos(x)$ is $[-1, 1]$, we find that the range of $y = \cos(x) - 5$ is $[-1 - 5, 1 - 5]$ or $[-6, -4]$.

4. False, the phase shift is $-\pi/6$.

5. False, the graph of $y = \sin(x + \pi/6)$ lies $\pi/6$ to the *left* of the graph of $y = \sin(x)$.

6. True, since $\cos(5\pi/6 - \pi/3) = \cos(\pi/2) = 0$ and $\cos(11\pi/6 - \pi/3) = \cos(3\pi/2) = 0$.

7. False, for if $x = \pi/2$ we find $\sin(\pi/2) = 1 \neq \cos(\pi/2 + \pi/2) = \cos(\pi) = -1$.

8. False, the minimum value is -3.

9. True, since the maximum value of $y = -2\cos(x)$ is 2, we get that the maximum value of $y = -2\cos(x) + 4$ is $2 + 4$ or 6.

10. True, since $(-\pi/6 + \pi/3, 0) = (\pi/6, 0)$.

2.1 Exercises

1. $\sin\alpha$, $\cos\alpha$

2. sine

3. sine wave

4. periodic

5. fundamental cycle

6. amplitude

7. phase shift

8. cosine

9. starting, maximum, inflection, minimum, ending

10. maximum, inflection, minimum

11. 0 **12.** 1

13. $\dfrac{\sin(\pi/3)}{\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}.$

14. $\dfrac{\cos(\pi/6)}{\sin(\pi/6)} = \dfrac{\sqrt{3}/2}{1/2} = \sqrt{3}.$

15. 1/2 **16.** 1/2

17. $\dfrac{1}{\cos(\pi/3)} = \dfrac{1}{1/2} = 2.$

18. $\dfrac{1}{\sin(\pi/3)} = \dfrac{1}{\sqrt{3}/2} = \dfrac{2\sqrt{3}}{3}.$

19. 0 **20.** 1

21. 0 **22.** **0**

23. -2 **24.** -2

25. $-\dfrac{\sqrt{2}}{2}$ **26.** 1

27. $(0 + \pi/4, 0) = (\pi/4, 0)$

28. $(\pi/2, 1)$

29. $(\pi/2 + \pi/4, 3) = (3\pi/4, 3)$

30. $(3\pi/4 + \pi/4, -1) = (\pi, -1)$

31. $(-\pi/2 + \pi/4, -1) = (-\pi/4, -1)$

32. $(-\pi/4 + \pi/4, -2) = (0, -2)$

33. $(\pi + \pi/4, 0) = (5\pi/4, 0)$

34. $(2\pi + \pi/4, 0) = (9\pi/4, 0)$

35. $(\pi/3 - \pi/3, 0) = (0, 0)$

36. $(2\pi/3 - \pi/3, -1) = (\pi/3, -1)$

37. $(\pi - \pi/3, 1) = (2\pi/3, 1)$

38. $(2\pi - \pi/3, 4) = (5\pi/3, 4)$

39. $(\pi/2 - \pi/3, -1) = (\pi/6, -1)$

40. $(\pi/4 - \pi/3, 2) = (-\pi/12, 2)$

41. $(-\pi - \pi/3, 1) = (-4\pi/3, 1)$

42. $(-\pi/2 - \pi/3, -1) = (-5\pi/6, -1)$

43. $(\pi + \pi/6, -1 + 2) = (7\pi/6, 1)$

44. $(\pi/6 + \pi/6, -2 + 2) = (\pi/3, 0)$

45. $(\pi/2 + \pi/6, 0 + 2) = (2\pi/3, 2)$

46. $(\pi/3 + \pi/6, -1 + 2) = (\pi/2, 1)$

47. $(-3\pi/2 + \pi/6, 1 + 2) = (-4\pi/3, 3)$

48. $(-\pi/2 + \pi/6, 2 + 2) = (-\pi/3, 4)$

49. $(2\pi + \pi/6, -4 + 2) = (13\pi/6, -2)$

50. $(-\pi + \pi/6, 5 + 2) = (-5\pi/6, 7)$

51. $\left(\frac{\pi + 2\pi}{2}, 0\right) = \left(\frac{3\pi}{2}, 0\right)$

52. $\left(\frac{\pi}{6}, -2\right)$

53. $\left(\frac{0 + \pi/4}{2}, 2\right) = \left(\frac{\pi}{8}, 2\right)$

54. $\left(\frac{3\pi}{4}, 1\right)$

55. $\left(\frac{\pi/6 + \pi/2}{2}, 1\right) = \left(\frac{\pi}{3}, 1\right)$

56. $\left(\frac{3\pi}{8}, 2\right)$

57. $\left(\frac{\pi/3 + \pi/2}{2}, -4\right) = \left(\frac{5\pi}{12}, -4\right)$

58. $\left(\frac{7\pi}{8}, 5\right)$

59. $P(0,0)$, $Q(\pi/4, 2)$, $R(\pi/2, 0)$, $S(3\pi/4, -2)$

60. $P(0,1)$, $Q(\pi/6, 3)$, $R(\pi/3, 1)$, $S(\pi/2, -1)$

61. $P(\pi/4, 0)$, $Q(5\pi/8, 2)$, $R(\pi, 0)$, $S(11\pi/8, -2)$

62. $P(\pi/3, 0)$, $Q(5\pi/6, 3)$, $R(4\pi/3, 0)$, $S(11\pi/6, -3)$

63. $P(0, 2)$, $Q(\pi/12, 3)$, $R(\pi/6, 2)$, $S(\pi/4, 1)$

64. $P(-\pi/8, 0)$, $Q(\pi/8, 2)$, $R(3\pi/8, 0)$, $S(5\pi/8, -2)$

65. Amplitude 2, period 2π, phase shift 0, range $[-2, 2]$

66. Amplitude 4, period 2π, phase shift 0, range $[-4, 4]$

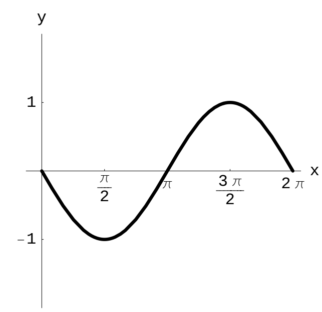
67. Amplitude 1, period 2π, phase shift $\pi/2$, range $[-1, 1]$

68. Amplitude 1, period 2π, phase shift $-\pi/2$, range $[-1, 1]$

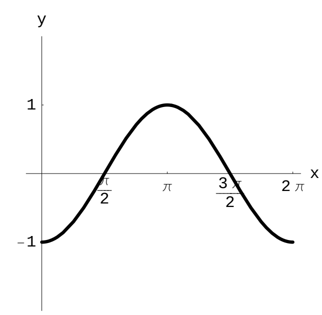
69. Amplitude 2, period 2π, phase shift $-\pi/3$, range $[-2, 2]$

70. Amplitude 3, period 2π, phase shift $\pi/6$, range $[-3, 3]$

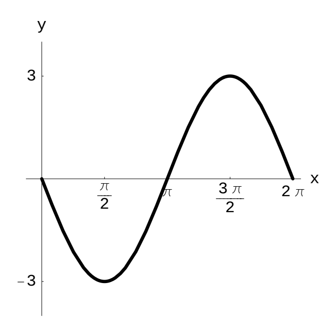
71. Amplitude 1, phase shift 0, range $[-1, 1]$, some points are $(0, 0)$, $\left(\frac{\pi}{2}, -1\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, 1\right)$, $(2\pi, 0)$



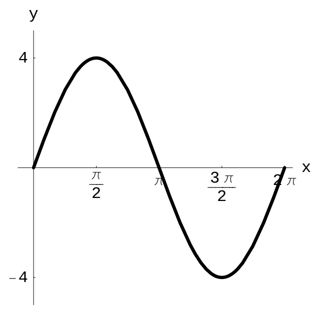
72. Amplitude 1, phase shift 0,range $[-1, 1]$, some points are $(0, -1)$, $(\pi/2, 0)$, $(\pi, 1)$, $(3\pi/2, 0)$, $(2\pi, -1)$



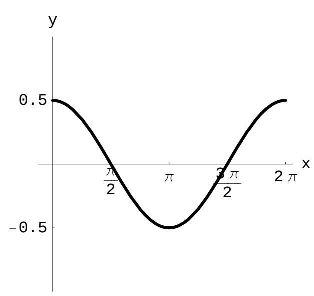
73. Amplitude 3, phase shift 0, range $[-3, 3]$, some points are $(0, 0)$, $\left(\frac{\pi}{2}, -3\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, 3\right)$, $(2\pi, 0)$



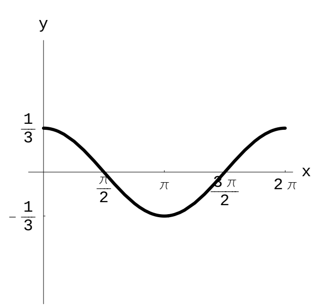
74. Amplitude 4, phase shift 0, range $[-4, 4]$, some points are $(0, 0)$, $\left(\frac{\pi}{2}, 4\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -4\right)$, $(2\pi, 0)$



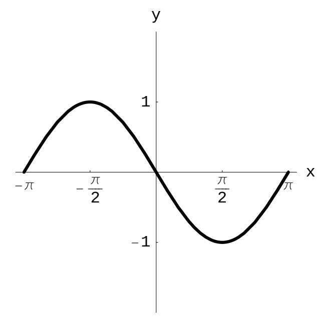
75. Amplitude $1/2$, phase shift 0, range $[-1/2, 1/2]$, some points are $(0, 1/2)$, $(\pi/2, 0)$, $(\pi, -1/2)$ $(3\pi/2, 0)$, $(2\pi, 1/2)$



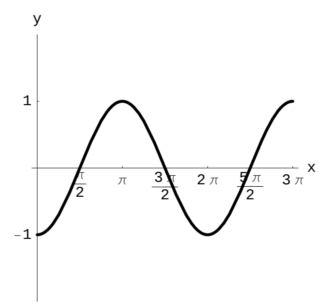
76. Amplitude $1/3$, phase shift 0, range $[-1/3, 1/3]$, some points are $\left(0, \frac{1}{3}\right)$, $\left(\frac{\pi}{2}, 0\right)$, $(\pi, -1/3)$, $(3\pi/2, 0)$, $\left(2\pi, \frac{1}{3}\right)$



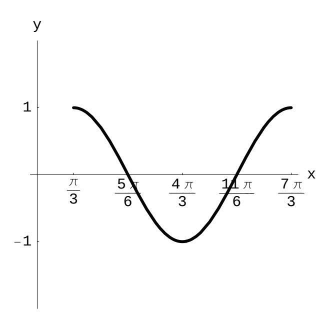
77. Amplitude 1, phase shift $-\pi$, range $[-1, 1]$, some points are $(-\pi, 0)$, $(-\pi/2, 1)$, $(0, 0)$, $\left(\frac{\pi}{2}, -1\right)$, $(\pi, 0)$



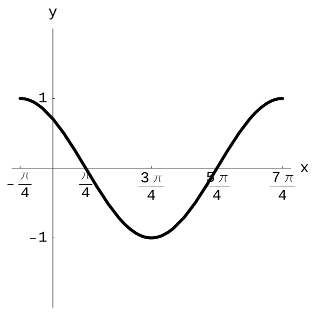
78. Amplitude 1, phase shift π, range $[-1, 1]$, some points are $(0, -1)$, $(\pi/2, 0)$, $(\pi, 1)$, $(3\pi/2, 0)$, $(2\pi, -1)$



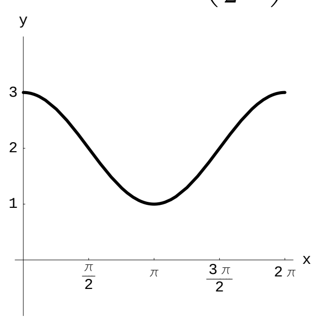
79. Amplitude 1, phase shift $\pi/3$, range $[-1, 1]$, some points are $\left(\frac{\pi}{3}, 1\right)$, $\left(\frac{5\pi}{6}, 0\right)$, $\left(\frac{4\pi}{3}, -1\right)$, $\left(\frac{11\pi}{6}, 0\right)$, $\left(\frac{7\pi}{3}, 1\right)$,



80. Amplitude 1, phase shift $-\pi/4$, range $[-1, 1]$, some points are $\left(-\dfrac{\pi}{4}, 1\right)$, $\left(\dfrac{\pi}{4}, 0\right)$, $\left(\dfrac{3\pi}{4}, -1\right)$, $\left(\dfrac{5\pi}{4}, 0\right)$, $\left(\dfrac{7\pi}{4}, 1\right)$

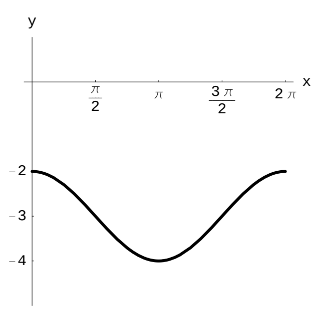


81. Amplitude 1, phase shift 0, range $[1, 3]$, some points are $(0, 3)$, $\left(\dfrac{\pi}{2}, 2\right)$, $(\pi, 1)$ $\left(\dfrac{3\pi}{2}, 2\right)$,

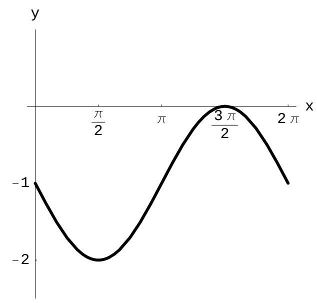


$(2\pi, 3)$

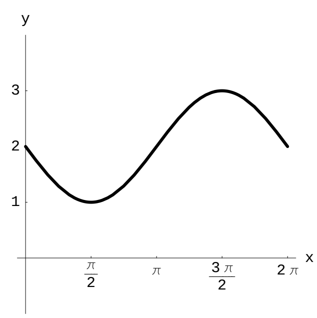
82. Amplitude 1, phase shift 0, range $[-4, -2]$, some points are $(0, -2)$, $\left(\dfrac{\pi}{2}, -3\right)$, $(\pi, -4)$, $\left(\dfrac{3\pi}{2}, -3\right)$, $(2\pi, -2)$



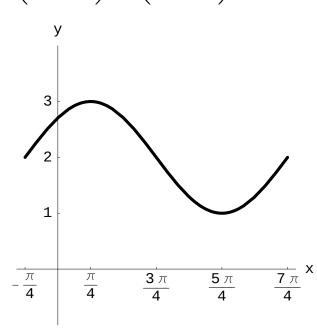
83. Amplitude 1, phase shift 0, range $[-2, 0]$, some points are $(0, -1)$, $\left(\dfrac{\pi}{2}, -2\right)$, $(\pi, -1)$, $\left(\dfrac{3\pi}{2}, 0\right)$, $(2\pi, -1)$



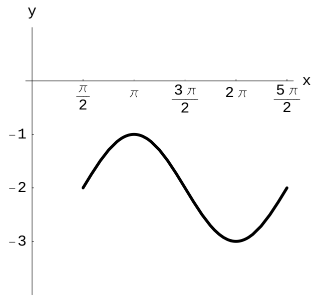
84. Amplitude 1, phase shift 0, range $[1, 3]$, some points are $(0, 2)$, $\left(\dfrac{\pi}{2}, 1\right)$, $(\pi, 2)$, $\left(\dfrac{3\pi}{2}, 3\right)$, $(2\pi, 2)$



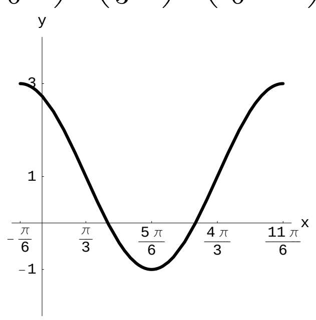
85. Amplitude 1, phase shift $-\pi/4$, range $[1, 3]$, some points are $\left(-\dfrac{\pi}{4}, 2\right)$, $\left(\dfrac{\pi}{4}, 3\right)$, $\left(\dfrac{3\pi}{4}, 2\right)$, $\left(\dfrac{5\pi}{4}, 1\right)$, $\left(\dfrac{7\pi}{4}, 2\right)$



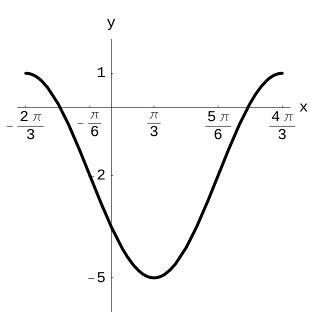
86. Amplitude 1, phase shift $\pi/2$, range $[-3,-1]$, some points are $\left(\frac{\pi}{2},-2\right)$, $(\pi,-1)$, $\left(\frac{3\pi}{2},-2\right)$, $(2\pi,-3)$, $\left(\frac{5\pi}{2},-2\right)$



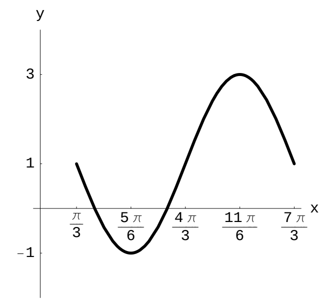
87. Amplitude 2, phase shift $-\pi/6$, range $[-1,3]$, some points are $\left(-\frac{\pi}{6},3\right)$, $\left(\frac{\pi}{3},1\right)$, $\left(\frac{5\pi}{6},-1\right)$, $\left(\frac{4\pi}{3},1\right)$, $\left(\frac{11\pi}{6},3\right)$



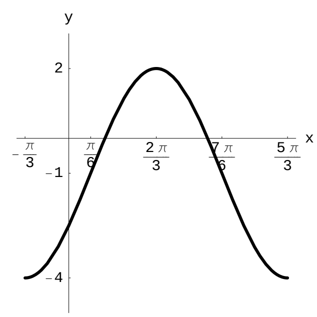
88. Amplitude 3, phase shift $-2\pi/3$, range $[-5,1]$, some points are $\left(-\frac{2\pi}{3},1\right)$, $\left(-\frac{\pi}{6},-2\right)$, $\left(\frac{\pi}{3},-5\right)$, $\left(\frac{5\pi}{6},-2\right)$, and $\left(\frac{4\pi}{3},1\right)$



89. Amplitude 2, phase shift $\pi/3$, range $[-1,3]$, some points are $\left(\frac{\pi}{3},1\right)$, $\left(\frac{5\pi}{6},-1\right)$, $\left(\frac{4\pi}{3},1\right)$, $\left(\frac{11\pi}{6},3\right)$ $\left(\frac{7\pi}{3},1\right)$



90. Amplitude 3, phase shift $-\pi/3$, range $[-4,2]$, some points are $\left(-\frac{\pi}{3},-4\right)$, $\left(\frac{\pi}{6},-1\right)$, $\left(\frac{2\pi}{3},2\right)$, $\left(\frac{7\pi}{6},-1\right)$, $\left(\frac{5\pi}{3},-4\right)$



91. Note the amplitude is 2, phase shift is $\pi/2$, and the period is 2π. Then $A=2$, $C=\pi/2$, and $B=1$. An equation is $y=2\sin\left(x-\frac{\pi}{2}\right)$.

92. Note the amplitude is 2, phase shift is $\pi/3$, period is 2π, and the vertical shift is 1 unit up. Then $A=2$, $C=\pi/3$, $B=1$, and $D=1$.

An equation is $y=2\sin\left(x-\frac{\pi}{3}\right)+1$.

93. Note the amplitude is 2, phase shift $-\pi$, the period is 2π, and the vertical shift is 1 unit up. Then $A=2$, $C=-\pi$, $B=1$, and $D=1$.

An equation is $y=2\sin(x+\pi)+1$.

94. Note the amplitude is 1, phase shift is $-\pi/2$, period is 2π, and the vertical shift is 1 unit up. Then $A=1$, $C=-\pi/2$, $B=1$, and $D=1$.

An equation is $y = \sin\left(x + \dfrac{\pi}{2}\right) + 1$.

95. Note the amplitude is 2, phase shift is $\pi/2$, and the period is 2π. Then $A = 2$, $C = \pi/2$, and $B = 1$. An equation is $y = 2\cos\left(x - \dfrac{\pi}{2}\right)$.

96. Note, $A = -2$, $C = \pi/2$, and $B = 1$.

An equation is $y = -2\cos\left(x - \dfrac{\pi}{2}\right)$.

97. Note, the amplitude is 2, phase shift is π, the period is 2π, and the vertical shift is 1 unit up. Then $A = 2$, $C = \pi$, $B = 1$, and $D = 1$.

An equation is $y = 2\cos(x - \pi) + 1$.

98. Note, the amplitude is 1, phase shift is $-\pi/3$, the period is 2π, and the vertical shift is 1 unit up. Then $A = 1$, $C = -\pi/3$, $B = 1$, and $D = 1$.

An equation is $y = \cos(x + \pi/3) + 1$.

99. $y = \sin\left(x - \dfrac{\pi}{4}\right)$ **100.** $y = \cos\left(x - \dfrac{\pi}{6}\right)$

101. $y = \sin\left(x + \dfrac{\pi}{2}\right)$ **102.** $y = \cos\left(x + \dfrac{\pi}{3}\right)$

103. $y = -\cos\left(x - \dfrac{\pi}{5}\right)$

104. $y = -\sin\left(x + \dfrac{\pi}{7}\right)$

105. $y = -\cos\left(x - \dfrac{\pi}{8}\right) + 2$

106. $y = -\sin\left(x + \dfrac{\pi}{9}\right) - 3$

107. $y = -3\cos\left(x + \dfrac{\pi}{4}\right) - 5$

108. $y = -\dfrac{1}{2}\sin\left(x - \dfrac{\pi}{3}\right) + 4$

109. A determines the stretching, shrinking, or reflection about the x-axis, C is the phase shift, and D is the vertical translation.

110. $f(x) = \sin(x)$ is an odd function since $\sin(-x) = -\sin(x)$, and $f(x) = \cos(x)$ is an even function since $\cos(-x) = \cos(x)$

111. $\pi/6$

112. $315°$

113. $\dfrac{93 \cdot 10^6 \cdot 2\pi}{365(24)} \approx 67{,}000$ mph

114. **a)** -1 **b)** $\dfrac{\sqrt{2}}{2}$ **c)** $\sqrt{3}$

d) Undefined **e)** -2 **f)** Undefined

g) $-\sqrt{3}$ **h)** $-\dfrac{\sqrt{2}}{2}$

115. $\arcsin(0.36) \approx 21.1°$

116. $\sin\alpha = -\sqrt{1 - \cos^2\alpha} = -\sqrt{1 - \dfrac{1}{9}} =$

$-\sqrt{\dfrac{8}{9}} = -\dfrac{2\sqrt{2}}{3}$

117. Let r be the radius of the small circle, and let x be the closest distance from the small circle to the point of tangency of any two circles with radius 1.

By the Pythagorean theorem, we find

$$1 + (x + r)^2 = (1 + r)^2$$

and

$$1 + (1 + 2r + x)^2 = 2^2.$$

The second equation may be written as

$$1 + (r + 1)^2 + 2(r + 1)(r + x) + (r + x)^2 = 4.$$

Using the first equation, the above equation simplifies to

$$(r + 1)^2 + 2(r + 1)(r + x) + (1 + r)^2 = 4$$

or

$$(r + 1)^2 + (r + 1)(r + x) = 2.$$

Since (from first equation, again)

$$x + r = \sqrt{(1 + r)^2 - 1}$$

we obtain

$$(r + 1)^2 + (r + 1)\left(\sqrt{(1 + r)^2 - 1}\right) = 2.$$

Solving for r, we find

$$r = \frac{2\sqrt{3} - 3}{3}.$$

118. Since $\sin x = 3\cos x$, we find

$$\begin{aligned} 1 &= \sin^2 + \cos^2 x \\ 1 &= 9\cos^2 x + \cos^2 x \\ \cos^2 x &= \frac{1}{10}. \end{aligned}$$

Then $\sin x \cos x = 3\cos^2 x = \frac{1}{10}$.

2.1 Pop Quiz

1. Amplitude 5, period 2π, phase shift $-2\pi/3$, range $[-5, 5]$

2. Key points are $(0,0)$, $\left(\frac{\pi}{2}, 3\right)$, $(\pi, 0)$, $\left(\frac{3\pi}{2}, -3\right)$, $(2\pi, 0)$

3. $y = -\cos\left(x - \frac{\pi}{2}\right) + 3$

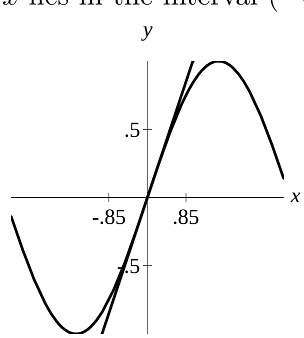
4. $[-2, 6]$

5. Since the period is $5\pi/2 - \pi/2 = 2\pi$, we get $B = 1$. The phase shift is $\pi/2$. The amplitude is 3. An equation is

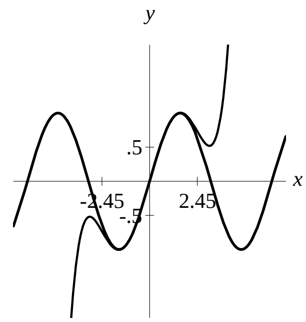
$$y = -3\sin\left(x - \frac{\pi}{2}\right).$$

2.1 Linking Concepts

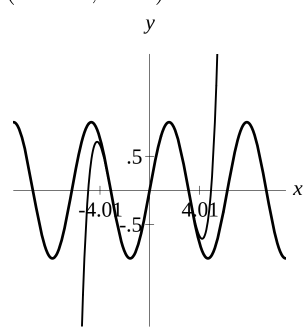
a) From the graphs of $y_1 = x$ and $y_2 = \sin(x)$, we obtain that y_1 and y_2 differ by less than 0.1 if x lies in the interval $(-0.85, 0.85)$.



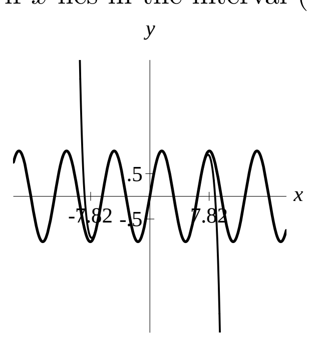
b) From the graphs of $y = x - \frac{x^3}{3!} + \frac{x^5}{5!}$ and $y_2 = \sin(x)$, it follows that y and y_2 differ by less than 0.1 if x lies in the interval $(-2.46, 2.46)$.



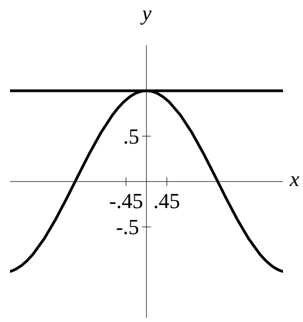
From the graphs of $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$ and $y_2 = \sin(x)$, one obtains that y and y_2 differ by less than 0.1 if x lies in the interval $(-4.01, 4.01)$.



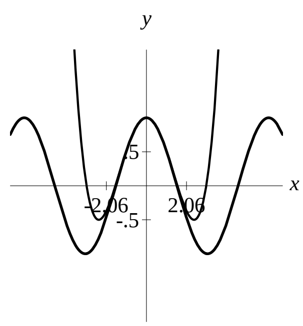
From the graphs of $y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \frac{x^{13}}{13!} - \frac{x^{15}}{15!} + \frac{x^{17}}{17!} - \frac{x^{19}}{19!}$ and $y_2 = \sin(x)$, one finds that y and y_2 differ by less than 0.1 if x lies in the interval $(-7.82, 7.82)$.



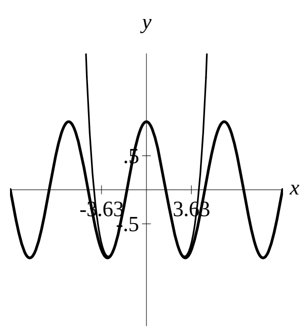
c) From the graphs of $y_1 = 1$ and $y = \cos(x)$, one obtains that y and y_1 differ by less than 0.1 if x lies in the interval $(-0.45, 0.45)$.



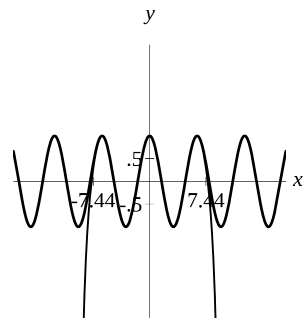
From the graphs of $y = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!}$ and $y_2 = \cos(x)$, one finds y and y_2 differ by less than 0.1 if x lies in the interval $(-2.06, 2.06)$.



From the graphs of $y = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \dfrac{x^8}{8!}$ and $y_2 = \cos(x)$, one derives that y and y_2 differ by less than 0.1 if x lies in the interval $(-3.63, 3.63)$.



From the graphs of $y = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \dfrac{x^6}{6!} + \dfrac{x^8}{8!} - \dfrac{x^{10}}{10!} + \dfrac{x^{12}}{12!} - \dfrac{x^{14}}{14!} + \dfrac{x^{16}}{16!} - \dfrac{x^{18}}{18!}$ and $y_2 = \cos(x)$, one finds that y and y_2 differ by less than 0.1 if x lies in the interval $(-7.44, 7.44)$.



d) To find $\sin(x)$, first let y be the reference angle of x. Note, $0 \le y \le \dfrac{\pi}{2}$.

Next, we obtain that the difference between $f(y) = \sin(y)$ and $f(y) = y - \dfrac{y^3}{3!}$ is less than 0.1 for $0 \le y \le \dfrac{\pi}{2}$. Thus, $\sin(x) = \pm\sin(y) = \pm\left(y - \dfrac{y^3}{3!}\right)$ where the sign on the right side depends on the sign of $\sin(x)$.

For Thought

1. True, since $B = 4$ and the period is $\dfrac{2\pi}{B} = \dfrac{\pi}{2}$.

2. False, since $B = 2\pi$ and the period is $\dfrac{2\pi}{B} = 1$.

3. True, since $B = \pi$ and the period is $\dfrac{2\pi}{B} = 2$.

4. True, since $B = 0.1\pi$ and the period is $\dfrac{2\pi}{0.1\pi} = 20$.

5. False, the phase shift is $-\dfrac{\pi}{12}$.

6. False, the phase shift is $-\dfrac{\pi}{8}$.

7. True, since the period is $P = 2\pi$ the frequency is $\dfrac{1}{P} = \dfrac{1}{2\pi}$.

8. True, since the period is $P = 2$ the frequency is $\dfrac{1}{P} = \dfrac{1}{2}$.

9. False, rather the graphs of $y = \cos(x)$ and $y = \sin\left(x + \dfrac{\pi}{2}\right)$ are identical.

10. True

2.2 Exercises

1. period

2. frequency

3. amplitude

4. phase shift

5. Amplitude 3, period $\frac{2\pi}{4}$ or $\frac{\pi}{2}$, and phase shift 0

6. Amplitude 1, period 4π, and phase shift 0

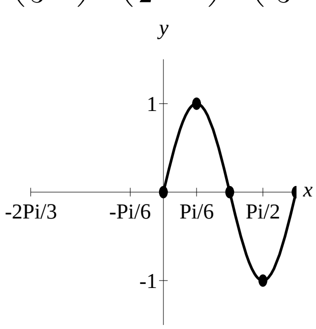
7. Since $y = -2\cos\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$, we get amplitude 2, period $\frac{2\pi}{2}$ or π, and phase shift $-\frac{\pi}{4}$.

8. Since $y = 4\cos\left(3\left(x - \frac{2\pi}{3}\right)\right)$, we get amplitude 4, period $\frac{2\pi}{3}$, and phase shift $\frac{2\pi}{3}$.

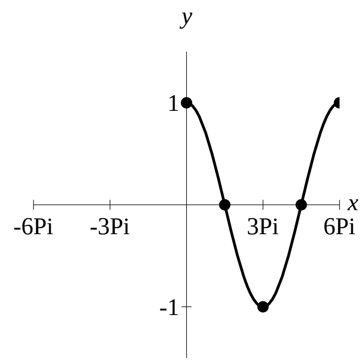
9. Since $y = -2\sin(\pi(x-1))$, we get amplitude 2, period $\frac{2\pi}{\pi}$ or 2, and phase shift 1.

10. Since $y = \sin\left(\frac{\pi}{2}(x+2)\right)$, we get amplitude 1, period $\frac{2\pi}{\pi/2}$ or 4, and phase shift -2.

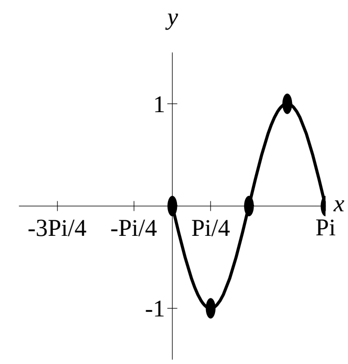
11. Period $2\pi/3$, phase shift 0, range $[-1, 1]$, labeled points are $(0,0)$, $\left(\frac{\pi}{6}, 1\right)$, $\left(\frac{\pi}{3}, 0\right)$, $\left(\frac{\pi}{2}, -1\right)$, $\left(\frac{2\pi}{3}, 0\right)$



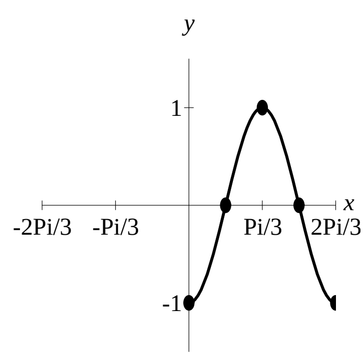
12. Period 6π, phase shift 0, range $[-1, 1]$, labeled points are $(0,1)$, $\left(\frac{3\pi}{2}, 0\right)$, $(3\pi, -1)$, $\left(\frac{9\pi}{2}, 0\right)$, $(6\pi, 1)$



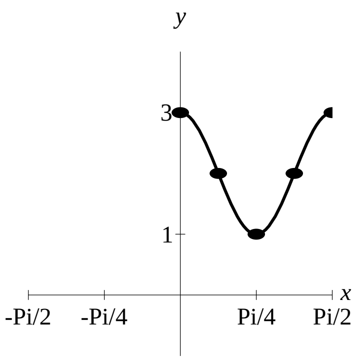
13. Period π, phase shift 0, range $[-1, 1]$, labeled points are $(0,0)$, $\left(\frac{\pi}{4}, -1\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, 1\right)$, $(\pi, 0)$



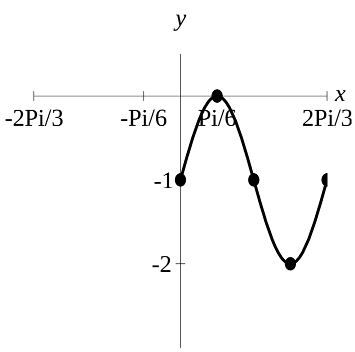
14. Period $2\pi/3$, phase shift 0, range $[-1, 1]$, labeled points are $(0, -1)$, $\left(\frac{\pi}{6}, 0\right)$, $\left(\frac{\pi}{3}, 1\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{2\pi}{3}, -1\right)$



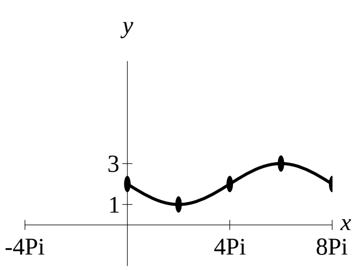
15. Period $\pi/2$, phase shift 0, range $[1,3]$, labeled points are $(0,3)$, $\left(\frac{\pi}{8},2\right)$, $\left(\frac{\pi}{4},1\right)$, $\left(\frac{3\pi}{8},2\right)$, $\left(\frac{\pi}{2},3\right)$



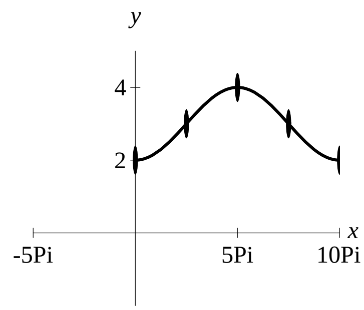
16. Period $2\pi/3$, phase shift 0, range $[-2,0]$, labeled points are $(0,-1)$, $\left(\frac{\pi}{6},0\right)$, $\left(\frac{\pi}{3},-1\right)$, $\left(\frac{\pi}{2},-2\right)$, $\left(\frac{2\pi}{3},-1\right)$



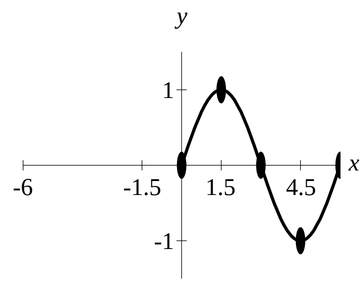
17. Period 8π, phase shift 0, range $[1,3]$, labeled points are $(0,2)$, $(2\pi,1)$, $(4\pi,2)$, $(6\pi,3)$, $(8\pi,2)$



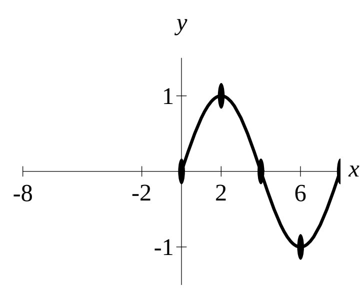
18. Period 10π, phase shift 0, range $[2,4]$, labeled points are $(0,2)$, $\left(\frac{5\pi}{2},3\right)$, $(5\pi,4)$, $\left(\frac{15\pi}{2},3\right)$, $(10\pi,2)$



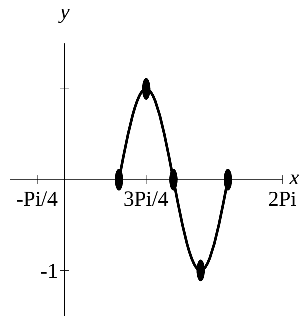
19. Period 6, phase shift 0, range $[-1,1]$, labeled points are $(0,0)$, $(1.5,1)$, $(3,0)$, $(4.5,-1)$, $(6,0)$



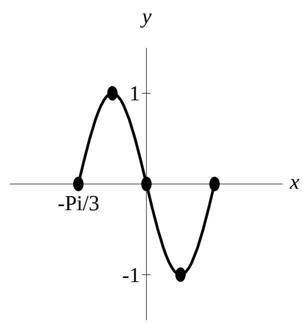
20. Period 8, phase shift 0, range $[-1,1]$, labeled points are $(0,0)$, $(2,1)$, $(4,0)$, $(6,-1)$, $(8,0)$



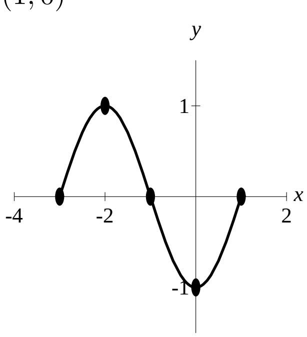
21. Period π, phase shift $\pi/2$, range $[-1, 1]$, labeled points are $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, 1\right)$, $(\pi, 0)$, $\left(\frac{5\pi}{4}, -1\right)$, $\left(\frac{3\pi}{2}, 0\right)$



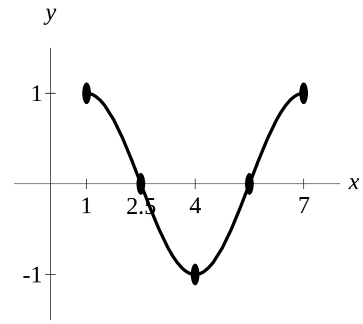
22. Period $2\pi/3$, phase shift $-\pi/3$, range $[-1, 1]$, labeled points are $\left(-\frac{\pi}{3}, 0\right)$, $\left(-\frac{\pi}{6}, 1\right)$, $(0, 0)$, $\left(\frac{\pi}{6}, -1\right)$, $\left(\frac{\pi}{3}, 0\right)$



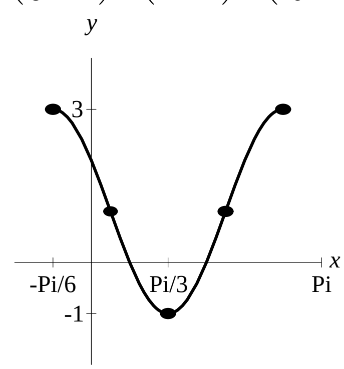
23. Period 4, phase shift -3, range $[-1, 1]$, labeled points are $(-3, 0)$, $(-2, 1)$, $(-1, 0)$, $(0, -1)$, $(1, 0)$



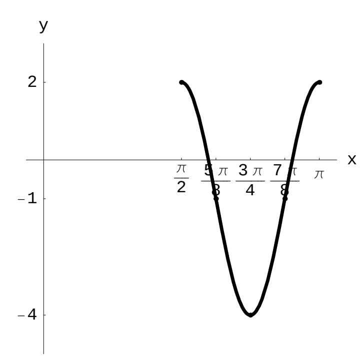
24. Period 6, phase shift 1, range $[-1, 1]$, labeled points are $(1, 1)$, $(2.5, 0)$, $(4, -1)$, $(5.5, 0)$, $(7, 1)$



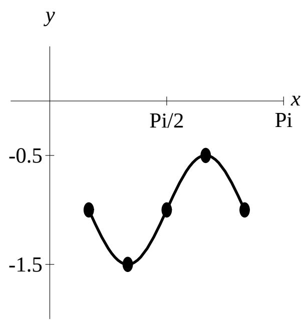
25. Period π, phase shift $-\pi/6$, range $[-1, 3]$, labeled points are $\left(-\frac{\pi}{6}, 3\right)$, $\left(\frac{\pi}{12}, 1\right)$, $\left(\frac{\pi}{3}, -1\right)$, $\left(\frac{7\pi}{12}, 1\right)$, $\left(\frac{5\pi}{6}, 3\right)$



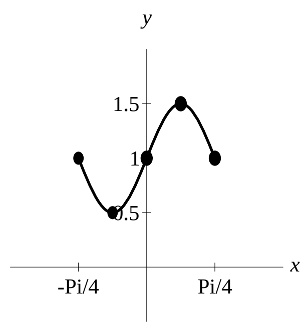
26. Period $\pi/2$, phase shift $\pi/2$, range $[-4, 2]$, labeled points are $\left(\frac{\pi}{2}, 2\right)$, $\left(\frac{5\pi}{8}, -1\right)$, $\left(\frac{3\pi}{4}, -4\right)$, $\left(\frac{7\pi}{8}, -1\right)$, $(\pi, 2)$



27. Period $\dfrac{2\pi}{3}$, phase shift $\dfrac{\pi}{6}$, range $\left[-\dfrac{3}{2},-\dfrac{1}{2}\right]$, labeled points are $\left(\dfrac{\pi}{6},-1\right)$, $\left(\dfrac{\pi}{3},-\dfrac{3}{2}\right)$, $\left(\dfrac{\pi}{2},-1\right)$, $\left(\dfrac{2\pi}{3},-\dfrac{1}{2}\right)$, $\left(\dfrac{5\pi}{6},-1\right)$



28. Period $\pi/2$, phase shift $-\pi/4$, range $[1/2, 3/2]$, labeled points are $\left(-\dfrac{\pi}{4},1\right)$, $\left(-\dfrac{\pi}{8},0.5\right)$, $(0,1)$, $\left(\dfrac{\pi}{8},1.5\right)$, $\left(\dfrac{\pi}{4},1\right)$



29. Note, $A = 2$, period is π and so $B = 2$, phase shift is $C = \dfrac{\pi}{4}$, and $D = 0$ or no vertical shift. Then $y = 2\sin\left(2\left(x-\dfrac{\pi}{4}\right)\right)$.

30. We can choose $A = -1$ with $C = 0$, i.e., no phase shift. The period is 4π and so $B = \dfrac{1}{2}$. There is no vertical shift or $D = 0$. Then $y = -\sin\left(\dfrac{x}{2}\right)$.

31. Note, $A = 3$, period is $\dfrac{4\pi}{3}$ and so $B = \dfrac{3}{2}$, phase shift is $C = -\dfrac{\pi}{3}$, and $D = 3$ since the vertical shift is three units up. Then $y = 3\sin\left(\dfrac{3}{2}\left(x+\dfrac{\pi}{3}\right)\right)+3$.

32. Note, $A = 2$, $B = 2$, phase shift is $C = -\dfrac{\pi}{4}$, and $D = -1$. Then $y = 2\sin\left(2\left(x+\dfrac{\pi}{4}\right)\right)-1$.

33. $\dfrac{\pi}{4}-\dfrac{\pi}{4}=0$

34. $\dfrac{\pi}{2}-\dfrac{\pi}{4}=\dfrac{\pi}{4}$

35. $f\left(g\left(\dfrac{\pi}{4}\right)\right) = f(0) = \sin(0) = 0$

36. $f\left(g\left(\dfrac{\pi}{2}\right)\right) = f\left(\dfrac{\pi}{4}\right) = \sin\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

37. $h\left(f\left(g\left(\dfrac{\pi}{4}\right)\right)\right) = h\left(f\left(0\right)\right) = h\left(\sin\left(0\right)\right) = h(0) = 3\cdot 0 = 0$

38. $h\left(f\left(g\left(\dfrac{\pi}{2}\right)\right)\right) = h\left(f\left(\dfrac{\pi}{4}\right)\right) = h\left(\sin\left(\dfrac{\pi}{4}\right)\right) = h\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{3\sqrt{2}}{2}$

39. $f\left(g\left(x\right)\right) = f\left(x-\dfrac{\pi}{4}\right) = \sin\left(x-\dfrac{\pi}{4}\right)$

40. $f\left(h\left(x\right)\right) = f\left(3x\right) = \sin\left(3x\right)$

41. $h\left(f\left(g\left(x\right)\right)\right) = h\left(f\left(x-\dfrac{\pi}{4}\right)\right) = h\left(\sin\left(x-\dfrac{\pi}{4}\right)\right) = 3\sin\left(x-\dfrac{\pi}{4}\right)$

42. $f\left(h\left(g\left(x\right)\right)\right) = f\left(h\left(x-\dfrac{\pi}{4}\right)\right) = f\left(3\left(x-\dfrac{\pi}{4}\right)\right) = \sin\left(3\left(x-\dfrac{\pi}{4}\right)\right)$

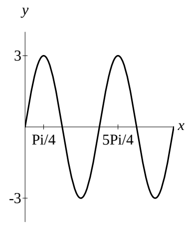
43. 100 cycles/sec since the frequency is the reciprocal of the period

44. 1/2000 cycles per second

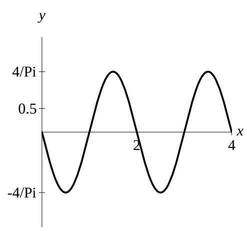
45. Frequency is $\dfrac{1}{0.025} = 40$ cycles per hour

46. Period is $\dfrac{1}{40,000} = 0.000025$ second.

47. Substitute $v_o = 6$, $\omega = 2$, and $x_o = 0$ into $x(t) = \dfrac{v_o}{\omega} \cdot \sin(\omega t) + x_o \cdot \cos(\omega t)$.
Then $x(t) = 3\sin(2t)$.
The amplitude is 3 and the period is π.



48. Substitute $v_o = -4$, $\omega = \pi$, and $x_o = 0$ into $x(t) = \dfrac{v_o}{\omega} \cdot \sin(\omega t) + x_o \cdot \cos(\omega t)$.
Then $x(t) = -\dfrac{4}{\pi}\sin(\pi t)$.
The amplitude is $4/\pi$ and period is 2.



49. 11 years

50. Approximately 1.4 seconds

51. Note, the range of $v = 400\sin(60\pi t) + 900$ is $[-400 + 900, 400 + 900]$ or $[500, 1300]$.
(a) Maximum volume is 1300 cc and mininum volume is 500 cc
(b) The runner takes a breath every 1/30 (which is the period) of a minute. So a runner makes 30 breaths in one minute.

52. (a) Maximum velocity is 8 cm/sec and mininum velocity is 0.
(b) The rodent's heart makes a beat every 1/3 (which is the period) of a second or it makes 180 beats in a minute.

53. Period is 12, amplitude is 15,000, phase-shift is -3, vertical translation is 25,000, a formula for the curve is

$$y = 15,000\sin\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) + 25,000;$$

for April (when $x = 4$), the revenue is

$$15,000\sin\left(\frac{\pi}{6}x + \frac{\pi}{2}\right) + 25,000 \approx \$17,500.$$

54. Period is 12, amplitude is 150, phase-shift is -2, vertical translation is 350, a formula for the curve is

$$y = 150\sin\left(\frac{\pi x}{6} + \frac{\pi}{3}\right) + 350;$$

for November (when $x = 11$), the utility bill is

$$150\sin\left(\frac{\pi x}{6} + \frac{\pi}{3}\right) + 350 \approx \$425$$

55.

a) period is 40, amplitude is 65, an equation for the sine wave is

$$y = 65\sin\left(\frac{\pi}{20}x\right)$$

b) 40 days

c) $65\sin\left(\dfrac{\pi}{20}(36)\right) \approx -38.2$ meters/second

d) The new planet is between Earth and Rho.

56. a) Ganymede's period is 7.155 days, or 7 days and 8 hours; Callisto's period is 16.689 days, 16 days and 17 hours; Io's period is 1.769 days, 1 day and 18 hours; Europa's period is 3.551 days, or 3 days and 13 hours.
To the nearest hour, it would be easiest to find Io's period since it is the satellite with the smallest period.

b) Ios's amplitude is 262,000 miles, Europa's amplitude is 417,000 miles, Ganymede's amplitude is 666,000 miles, Callisto's amplitude is 1,170,000 miles

57. Since the period is $20 = \dfrac{2\pi}{B}$, we get $B = \dfrac{\pi}{10}$.
Also, the amplitude is 1 and the vertical translation is 1. An equation for the swell is

$$y = \sin\left(\frac{\pi}{10}x\right) + 1.$$

58. Since the period is 200, the amplitude is 15, and the vertical translation is 15, an equation for the tsunami is

$$y = 15\sin\left(\frac{\pi}{100}x\right) + 15.$$

59. The sine regression curve is

$$y = 50\sin(0.214x - 0.615) + 48.8$$

or approximately

$$y = 50\sin(0.21x - 0.62) + 48.8$$

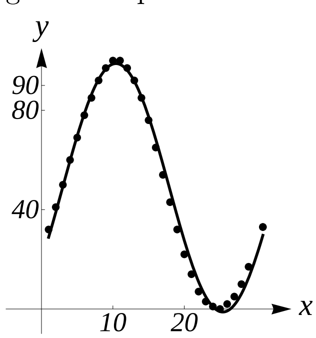
The period is

$$\frac{2\pi}{b} = \frac{2\pi}{0.214} \approx 29.4 \text{ days.}$$

When $x = 39$, we find

$$y = 50\sin(0.214(39) - 0.615) + 48.8 \approx 98\%$$

On February 8, 2020, 98% of the moon is illuminated. Shown below is a graph of the regression equation and the data points.



60. The sine regression curve is

$$y = 95.4\sin(0.514x - 1.84) + 727.0$$

or approximately

$$y = 95.4\sin(0.51x - 1.84) + 727.0$$

The period is

$$\frac{2\pi}{b} = \frac{2\pi}{0.514} \approx 12.2 \text{ months.}$$

When $x = 14$, we obtain

$$95.4\sin(0.51(14) - 1.84) + 727.0 \approx 648 \text{ min.}$$

between sunrise and sunset on Feb. 1, 2021.

62. For instance, one can choose $B = 4$, $C = \dfrac{\pi}{4}$, and $D = 5$.

63. Amplitude $A = \dfrac{1}{2}$,

period $B = \dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$,

phase shift $\dfrac{\pi}{2}$,

period is $[-\dfrac{1}{2} + 3, \dfrac{1}{2} + 3]$ or $[2.5, 3.5]$

64. $y = -\cos(x + \pi) + 2$

65. If x is the height of the tree, then $\tan 30° = h/500$ or

$$h = 500\tan 30° \approx 289 \text{ ft.}$$

66. Let $r = \sqrt{(-3)^2 + 6^2} = \sqrt{45} = 3\sqrt{5}$.

Then $\sin\beta = \dfrac{y}{r} = \dfrac{6}{3\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$,

$\cos\beta = \dfrac{x}{r} = \dfrac{-3}{3\sqrt{5}} = -\dfrac{\sqrt{5}}{5}$, and

$\tan\beta = \dfrac{y}{x} = \dfrac{6}{-3} = -2$

67. Note, the sum of the two angles is $32°37' + 48°39' = 80°76' = 81°16'$.

The third angle is $179°60' - 81°16' = 98°44'$.

68. Since $s = r\alpha$, we find

$$\begin{aligned} 5 &= 60\alpha \\ \alpha &= \frac{1}{12} \text{ radian} \\ \alpha &= \frac{1}{12} \cdot \frac{180°}{\pi} \\ \alpha &\approx 4.8° \end{aligned}$$

69. One possibility is

$$\text{WRONG} = 25938$$

and

$$\text{RIGHT} = 51876.$$

70. Let $x > y$ be the radii of the circles. By the Pythagorean theorem,

$$y^2 + 40^2 = x^2.$$

Then the volume of water in the island is

$$2(\pi x^2 - \pi y^2) = 2\pi(40)^2 \approx 10{,}053 \text{ ft}^3.$$

2.2 Pop Quiz

1. Since we have

$$y = 4\sin\left(2\left(x - \frac{\pi}{3}\right)\right)$$

we obtain amplitude 4, period $2\pi/B = 2\pi/2$ or π, phase shift $\pi/3$.

2. Key points are $(0,0)$, $\left(\frac{\pi}{4}, -3\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, 3\right)$, $(\pi, 0)$

3. The period $2\pi/B = 2\pi/\pi$ or 2. Since the amplitude is 4 and there is a vertical upward shift of 2 units, the range is $[-4+2, 4+2]$ or $[-2, 6]$.

4. Note, $A = 4$ and $C = -\frac{\pi}{6}$. Since the period is

$$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$$

we find $B = 3$. Thus, the curve is

$$y = 4\sin\left(3\left(x + \frac{\pi}{6}\right)\right).$$

5. Note, the period is

$$\frac{2\pi}{B} = \frac{2\pi}{500\pi} = \frac{1}{250}.$$

Since the frequency is the reciprocal of the period, the frequency is

250 cycles/minute.

For Thought

1. True, since $\sec(\pi/4) = \frac{1}{\cos(\pi/4)} = \frac{1}{\sin(\pi/4)}$.

2. True, since $\csc(x) = \frac{1}{\sin(x)}$.

3. True, since $\csc(\pi/2) = \frac{1}{\sin(\pi/2)} = \frac{1}{1} = 1$.

4. False, since $\frac{1}{\cos(\pi/2)}$ or $\frac{1}{0}$ is undefined.

5. True, since $B = 2$ and $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.

6. True, since $B = \pi$ and $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$.

7. False, rather the graphs of $y = 2\csc x$ and $y = \frac{2}{\sin x}$ are identical.

8. True, since the maximum and minimum of $0.5\csc x$ are ± 0.5.

9. True, since $\frac{\pi}{2} + k\pi$ are the zeros of $y = \cos x$ we get that the asymptotes of $y = \sec(2x)$ are $2x = \frac{\pi}{2} + k\pi$ or $x = \frac{\pi}{4} + \frac{k\pi}{2}$. If $k = \pm 1$, we get the asymptotes $x = \pm\frac{\pi}{4}$.

10. True, since if we substitute $x = 0$ in $\frac{1}{\csc(4x)}$ we get $\frac{1}{\csc(0)}$ or $\frac{1}{0}$ which is undefined.

2.3 Exercises

1. domain

2. domain

3. asymptote

4. x-intercepts

5. $\frac{1}{\cos(\pi/3)} = \frac{1}{1/2} = 2$

6. $\frac{1}{\cos(\pi/4)} = \frac{1}{\sqrt{2}/2} = \sqrt{2}$

7. $\frac{1}{\sin(-\pi/4)} = \frac{1}{-1/\sqrt{2}} = -\sqrt{2}$

8. $\frac{1}{\sin(\pi/6)} = \frac{1}{1/2} = 2$

9. Undefined, since $\frac{1}{\cos(\pi/2)} = \frac{1}{0}$

10. Undefined, since $\frac{1}{\cos(3\pi/2)} = \frac{1}{0}$

11. Undefined, since $\frac{1}{\sin(\pi)} = \frac{1}{0}$

12. Undefined, since $\dfrac{1}{\sin(0)} = \dfrac{1}{0}$

13. $\dfrac{1}{\cos 1.56} \approx 92.6$ **14.** $\dfrac{1}{\cos 1.58} \approx -108.7$

15. $\dfrac{1}{\sin 0.01} \approx 100.0$ **16.** $\dfrac{1}{\sin(-0.002)} \approx -500.0$

17. $\dfrac{1}{\sin 3.14} \approx 627.9$ **18.** $\dfrac{1}{\sin 6.28} \approx -313.9$

19. $\dfrac{1}{\cos 4.71} \approx -418.6$ **20.** $\dfrac{1}{\cos 4.72} \approx 131.4$

21. Since $B = 2$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2}$ or π

22. Since $B = 4$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{4}$ or $\dfrac{\pi}{2}$

23. Since $B = \dfrac{3}{2}$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{3/2}$ or $\dfrac{4\pi}{3}$

24. Since $B = \dfrac{1}{2}$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{1/2}$ or 4π

25. Since $B = \pi$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi}$ or 2

26. Since $B = 2\pi$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2\pi}$ or 1

27. $(-\infty, -2] \cup [2, \infty)$

28. $(-\infty, -4] \cup [4, \infty)$

29. $(-\infty, -1/2] \cup [1/2, \infty)$

30. $(-\infty, -1/3] \cup [1/3, \infty)$

31. Since the range of $y = \sec(\pi x - 3\pi)$ is

$$(-\infty, -1] \cup [1, \infty),$$

the range of $y = \sec(\pi x - 3\pi) - 1$ is

$$(-\infty, -1 - 1] \cup [1 - 1, \infty)$$

or equivalently

$$(-\infty, -2] \cup [0, \infty).$$

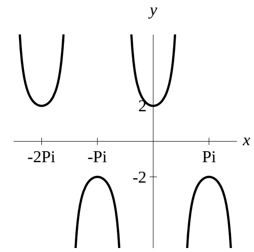
32. Since the range of $y = \sec(3x + \pi/3)$ is $(-\infty, -1] \cup [1, \infty)$, the range of $y = \sec(3x + \pi/3) + 1$ is

$$(-\infty, -1 + 1] \cup [1 + 1, \infty)$$

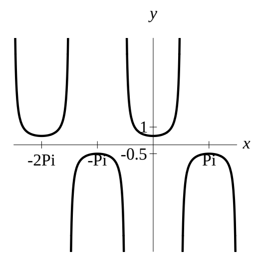
or equivalently

$$(-\infty, 0] \cup [2, \infty).$$

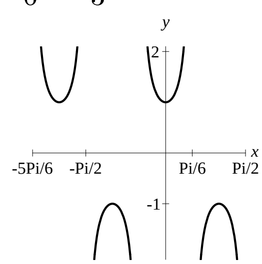
33. period 2π, asymptotes $x = \dfrac{\pi}{2} + k\pi$, range $(-\infty, -2] \cup [2, \infty)$



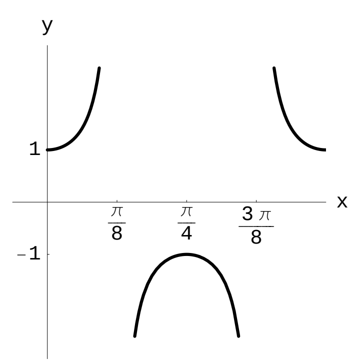
34. period 2π, asymptotes $x = \dfrac{\pi}{2} + k\pi$, range $(-\infty, -1/2] \cup [1/2, \infty)$



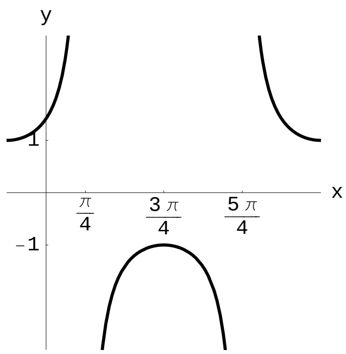
35. period $2\pi/3$, asymptotes $3x = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{\pi}{6} + \dfrac{k\pi}{3}$, range $(-\infty, -1] \cup [1, \infty)$



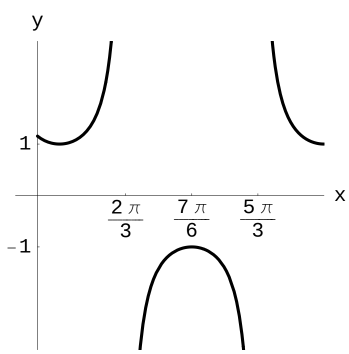
36. period $\dfrac{2\pi}{4} = \dfrac{\pi}{2}$,asymptotes $4x = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{\pi}{8} + \dfrac{k\pi}{4}$, range $(-\infty, -1] \cup [1, \infty)$



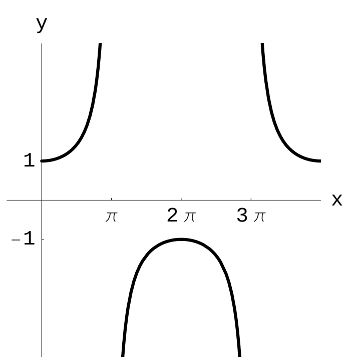
37. period $\dfrac{2\pi}{1} = 2\pi$,asymptotes $x + \dfrac{\pi}{4} = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{\pi}{4} + k\pi$, range $(-\infty, -1] \cup [1, \infty)$



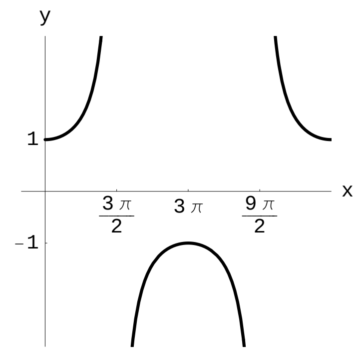
38. period $\dfrac{2\pi}{1} = 2\pi$, asymptotes $x - \dfrac{\pi}{6} = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{2\pi}{3} + k\pi$, range $(-\infty, -1] \cup [1, \infty)$



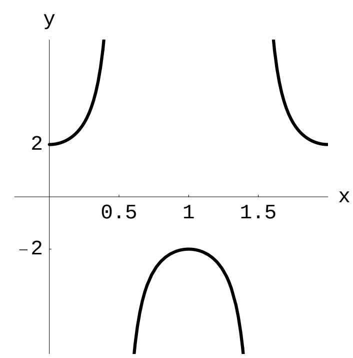
39. period $\dfrac{2\pi}{1/2} = 4\pi$, asymptotes $\dfrac{x}{2} = \dfrac{\pi}{2} + k\pi$ or $x = \pi + 2k\pi$, range $(-\infty, -1] \cup [1, \infty)$



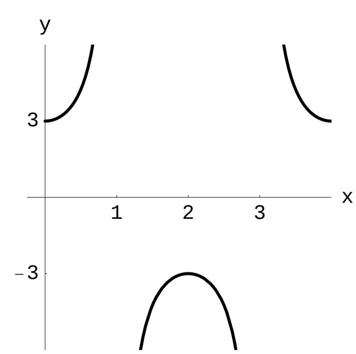
40. period $\dfrac{2\pi}{1/3} = 6\pi$, asymptotes $\dfrac{x}{3} = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{3\pi}{2} + 3k\pi$, range $(-\infty, -1] \cup [1, \infty)$



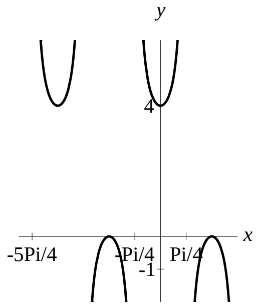
41. period $\dfrac{2\pi}{\pi} = 2$, asymptotes $\pi x = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{1}{2} + k$, range $(-\infty, -2] \cup [2, \infty)$



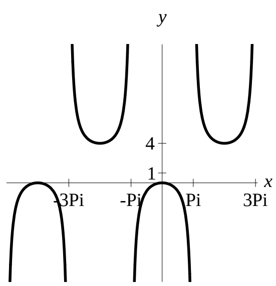
42. period $\dfrac{2\pi}{\pi/2} = 4$, asymptotes $\dfrac{\pi x}{2} = \dfrac{\pi}{2} + k\pi$ or $x = 1 + 2k$, range $(-\infty, -3] \cup [3, \infty)$



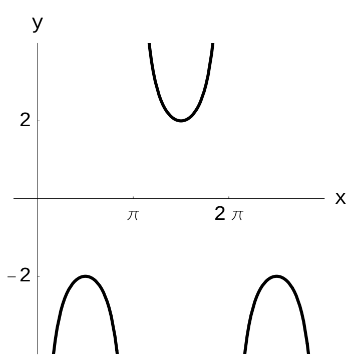
43. period $\dfrac{2\pi}{2} = \pi$, asymptotes $2x = \dfrac{\pi}{2} + k\pi$ or $x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}$, and since the range of $y = 2\sec(2x)$ is $(-\infty, -2] \cup [2, \infty)$ then the range of $y = 2 + 2\sec(2x)$ is $(-\infty, -2+2] \cup [2+2, \infty)$ or $(-\infty, 0] \cup [4, \infty)$.



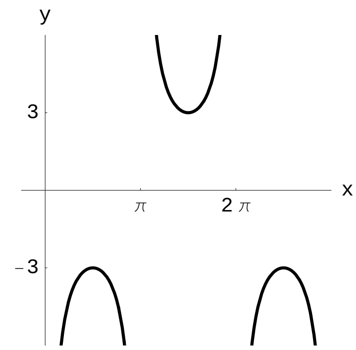
44. period $\dfrac{2\pi}{1/2} = 4\pi$, asymptotes $\dfrac{x}{2} = \dfrac{\pi}{2} + k\pi$ or $x = \pi + 2k\pi$, and since range of $y = -2\sec\left(\dfrac{x}{2}\right)$ is $(-\infty, -2] \cup [2, \infty)$ then the range of $y = 2 - 2\sec\left(\dfrac{x}{2}\right)$ is $(-\infty, -2+2] \cup [2+2, \infty)$ or $(-\infty, 0] \cup [4, \infty)$.



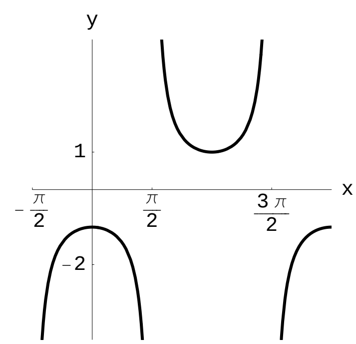
45. period 2π, asymptotes $x = k\pi$, range $(-\infty, -2] \cup [2, \infty)$



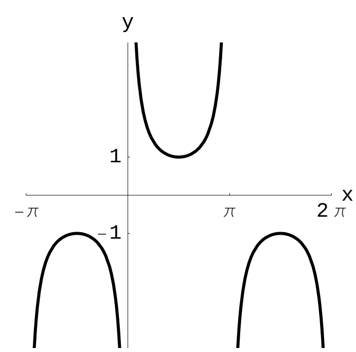
46. period 2π, asymptotes $x = k\pi$, range $(-\infty, -3] \cup [3, \infty)$



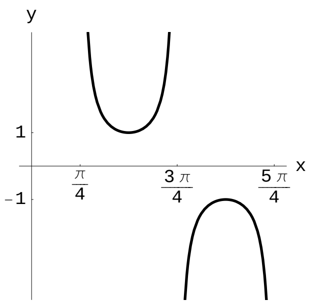
47. period 2π ,asymptotes $x + \dfrac{\pi}{2} = k\pi$ or $x = -\dfrac{\pi}{2} + k\pi$ or $x = \dfrac{\pi}{2} + k\pi$, range $(-\infty, -1] \cup [1, \infty)$



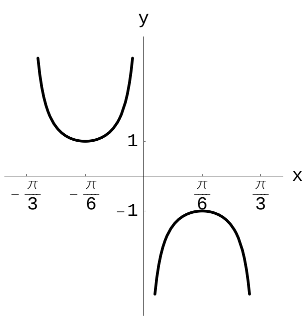
48. period 2π, asymptotes $x - \pi = k\pi$ or $x = \pi + k\pi$ or $x = k\pi$, range $(-\infty, -1] \cup [1, \infty)$



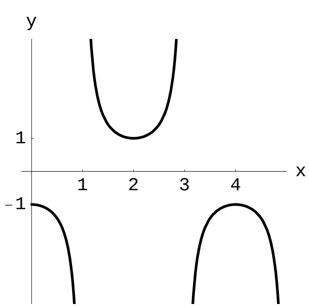
49. period $\dfrac{2\pi}{2}$ or π, asymptotes $2x - \dfrac{\pi}{2} = k\pi$ or $x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}$, range $(-\infty, -1] \cup [1, \infty)$



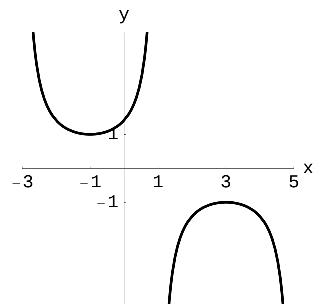
50. period $\dfrac{2\pi}{3}$, asymptotes $3x+\pi = k\pi$ or $x = \dfrac{k\pi}{3}$, range $(-\infty, -1] \cup [1, \infty)$



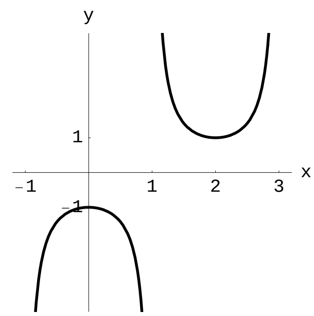
51. period $\dfrac{2\pi}{\pi/2}$ or 4, asymptotes $\dfrac{\pi x}{2} - \dfrac{\pi}{2} = k\pi$ or $x = 1 + 2k$, range $(-\infty, -1] \cup [1, \infty)$



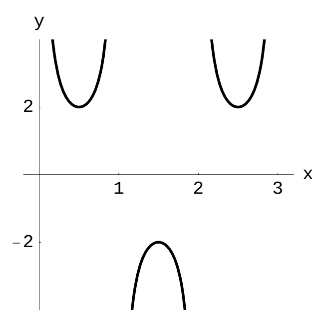
52. period $\dfrac{2\pi}{\pi/4}$ or 8,asymptotes $\dfrac{\pi x}{4} + \dfrac{3\pi}{4} = k\pi$ or $\dfrac{\pi x}{4} = -\dfrac{3\pi}{4} + k\pi$ or $x = -3+4k$ or $x = 1+4k$, range $(-\infty, -1] \cup [1, \infty)$



53. period $\dfrac{2\pi}{\pi/2}$ or 4, asymptotes $\dfrac{\pi x}{2} + \dfrac{\pi}{2} = k\pi$ or $x = -1 + 2k$ or $x = 1 + 2k$, range $(-\infty, -1] \cup [1, \infty)$



54. period $\dfrac{2\pi}{\pi}$ or 2, asymptotes $\pi x - \pi = k\pi$ or $x = k$, range $(-\infty, -2] \cup [2, \infty)$



55. $y = \sec\left(x - \dfrac{\pi}{2}\right) + 1$

56. $y = -\sec(x + \pi) + 2$

57. $y = -\csc(x + 1) + 4$

58. $y = -\csc(x - 2) - 3$

59. Since the zeros of $y = \cos x$ are $x = \dfrac{\pi}{2} + k\pi$, the vertical asymptotes of $y = \sec x$ are $x = \dfrac{\pi}{2} + k\pi$.

60. Since the zeros of $y = \sin x$ are $x = k\pi$, the vertical asymptotes of $y = -\csc x$ are $x = k\pi$.

61. Note, the zeros of $y = \sin x$ are $x = k\pi$. To find the asymptotes of $y = \csc(2x)$, let $2x = k\pi$. The asymptotes are $x = \dfrac{k\pi}{2}$.

62. Note, the zeros of $y = \sin x$ are $x = k\pi$. To find the asymptotes of $y = \csc(4x)$, let $4x = k\pi$. The asymptotes are $x = \dfrac{k\pi}{4}$.

63. Note, the zeros of $y = \cos x$ are $x = \dfrac{\pi}{2} + k\pi$. To find the asymptotes of $y = \sec\left(x - \dfrac{\pi}{2}\right)$, let $x - \dfrac{\pi}{2} = \dfrac{\pi}{2} + k\pi$. Solving for x, we get $x = \pi + k\pi$ or equivalently $x = k\pi$. The asymptotes are $x = k\pi$.

64. Note, the zeros of $y = \cos x$ are $x = \dfrac{\pi}{2} + k\pi$. To find the asymptotes of $y = \sec(x + \pi)$, let $x + \pi = \dfrac{\pi}{2} + k\pi$. Solving for x, we get $x = -\dfrac{\pi}{2} + k\pi$ or equivalently $x = \dfrac{\pi}{2} + k\pi$. The asymptotes are $x = \dfrac{\pi}{2} + k\pi$.

65. Note, the zeros of $y = \sin x$ are $x = k\pi$. To find the asymptotes of $y = \csc(2x - \pi)$, let $2x - \pi = k\pi$. Solving for x, we get $x = \dfrac{\pi}{2} + \dfrac{k\pi}{2}$ or equivalently $x = \dfrac{k\pi}{2}$. The asymptotes are $x = \dfrac{k\pi}{2}$.

66. Note, the zeros of $y = \sin x$ are $x = k\pi$. To find the asymptotes of $y = \csc(4x + \pi)$, let $4x + \pi = k\pi$. Solving for x, we get $x = -\dfrac{\pi}{4} + \dfrac{k\pi}{4}$ or equivalently $x = \dfrac{k\pi}{4}$. The asymptotes are $x = \dfrac{k\pi}{4}$.

67. Note, the zeros of $y = \sin x$ are $x = k\pi$. To find the asymptotes of $y = \dfrac{1}{2}\csc(2x) + 4$, let $2x = k\pi$. Solving for x, we get that the asymptotes are $x = \dfrac{k\pi}{2}$.

68. Note, the zeros of $y = \sin x$ are $x = k\pi$. To find the asymptotes of $y = \dfrac{1}{3}\csc(3x) - 6$, let $3x = k\pi$. Solving for x, we get that the asymptotes are $x = \dfrac{k\pi}{3}$.

69. Note, the zeros of $y = \cos x$ are $x = \dfrac{\pi}{2} + k\pi$. To find the asymptotes of $y = \sec(\pi x + \pi)$, let $\pi x + \pi = \dfrac{\pi}{2} + k\pi$. Solving for x, we get $x = -\dfrac{1}{2} + k$ or equivalently $x = \dfrac{1}{2} + k$. The asymptotes are $x = \dfrac{1}{2} + k$.

70. Note, the zeros of $y = \cos x$ are $x = \dfrac{\pi}{2} + k\pi$. To find the asymptotes of $y = \sec\left(\dfrac{\pi x}{2} - \dfrac{\pi}{2}\right)$, let $\dfrac{\pi x}{2} - \dfrac{\pi}{2} = \dfrac{\pi}{2} + k\pi$. Solving for x, we get $x = 2 + 2k$ or equivalently $x = 2k$. The asymptotes are $x = 2k$.

71. Since the range of $y = A\sec(B(x - C))$ is $(-\infty, -|A|\,] \cup [\,|A|, \infty)$, the range of $y = A\sec(B(x - C)) + D$ is

$$(-\infty, -|A| + D\,] \cup [\,|A| + D, \infty).$$

72. Since the range of $y = A\csc(B(x - C))$ is $(-\infty, -|A|\,] \cup [\,|A|, \infty)$, the range of $y = A\csc(B(x - C)) + D$ is

$$(-\infty, -|A| + D\,] \cup [\,|A| + D, \infty).$$

73. $\sin\alpha = y$, $\cos\alpha = x$

74. sine

75. Note $f(x) = 5\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 3$. The amplitude is $A = 5$, period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2} = \pi$, phase shift is $C = \dfrac{\pi}{2}$, and the range is $[-5 + 3, 5 + 3] = [-2, 8]$.

76. $\dfrac{1}{0.125} = 8$ cycles per second

77. $\beta = 0$

78. **a)** $-30°$ **b)** $120°$ **c)** $-45°$

79. The amplitude of the sine wave is 1/2 since the height of the sine wave is 1. We use a coordinate system such that the sine wave begins at the origin and extends to the right side and the first quadrant. Note, the period of the sine wave is π, which is the diameter of the tube. Then the highest point on the sine wave is $(\pi/2, 1)$. Thus, an equation of the sine wave is

$$y = -\frac{1}{2}\cos(2x) + \frac{1}{2}.$$

80. Let $a < b < c < d < e$ be weights of the five children.

$$\begin{aligned} b+c+d+e &= 354 \\ a+c+d+e &= 314 \\ a+b+d+e &= 277 \\ a+b+c+e &= 265 \\ a+b+c+d &= 254 \end{aligned}$$

The coefficient matrix is

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The inverse of the coefficient matrix is

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -3 & 1 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 & 1 \\ 1 & 1 & -3 & 1 & 1 \\ 1 & 1 & 1 & -3 & 1 \\ 1 & 1 & 1 & 1 & -3 \end{bmatrix}$$

Then

$$A^{-1}\begin{bmatrix} 354 \\ 314 \\ 277 \\ 265 \\ 254 \end{bmatrix} = \begin{bmatrix} 12 \\ 52 \\ 89 \\ 101 \\ 112 \end{bmatrix}$$

The lightest kid weighs 12 lb.

2.3 Pop Quiz

1. $\dfrac{1}{\cos \pi/4} = \dfrac{1}{\sqrt{2}/2} = \dfrac{2}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$

2. Undefined since $\dfrac{1}{\sin \pi}$ or $\dfrac{1}{0}$ is undefined.

3. Since $\cos x = 0$ when $x = \dfrac{\pi}{2} + k\pi$, the asymptotes of $y = \sec(x) + 3$ are

$$x = \frac{\pi}{2} + k\pi$$

where k is an integer.

4. Since $\sin x = 0$ when $x = k\pi$, the asymptotes of $y = \csc(x) - 1$ are

$$x = k\pi$$

where k is an integer.

5. Since $\cos 2x = 0$ when $2x = \dfrac{\pi}{2} + k\pi$ or

$$x = \frac{\pi}{4} + \frac{k\pi}{2},$$

the asymptotes of $y = \sec(2x)$ are

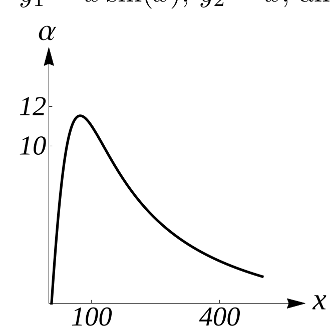
$$x = \frac{\pi}{4} + \frac{k\pi}{2}$$

where k is an integer.

6. $(-\infty - 3] \cup [3, \infty)$

2.3 Linking Concepts

a) Shown below are the graphs of $y_1 = x\sin(x)$, $y_2 = x$, and $y_3 = -x$.



Note, if $x \neq 0$, then $x\sin(x) = x$ has the same solution set as $\sin(x) = 1$. Thus, the exact values of x satisfying $x\sin(x) = x$ are

$$x = 0, \frac{\pi}{2} + 2\pi k$$

where k is an integer.

Similarly, if $x \neq 0$, then $x\sin(x) = -x$ has the same solution set as $\sin(x) = -1$. Thus, the exact values of x satisfying $x\sin(x) = -x$ are

$$x = 0, \frac{3\pi}{2} + 2\pi k$$

where k is an integer.

b) The exact values of x satisfying $x^2\sin(x) = x^2$ are

$$x = 0, \frac{\pi}{2} + 2\pi k$$

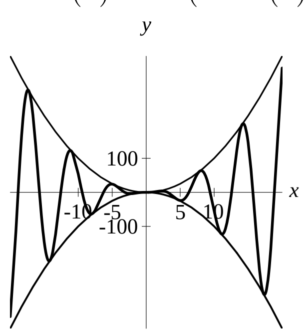
where k is an integer.

The exact values of x satisfying $x^2\sin(x) = -x^2$ are

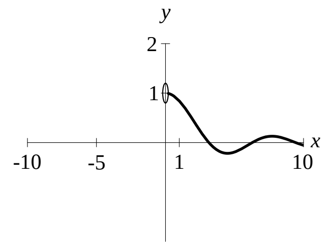
$$x = 0, \frac{3\pi}{2} + 2\pi k$$

where k is an integer.

Shown next are the graphs of $y = x^2\sin(x)$, $y = x^2$, and $y = -x^2$. The points of intersection between $y = x^2\sin(x)$ and $y = x^2$ (respectively, $y = x^2\sin(x)$ and $y = -x^2$) give the exact solutions to $x^2\sin(x) = x^2$ ($x^2\sin(x) = -x^2$, respectively).



c) Given is a graph of $y_1 = \frac{1}{x}\sin(x)$, $0 \leq x \leq 10$.



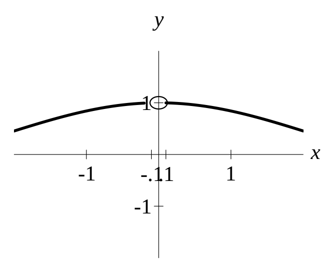
Note, the inequality $-\frac{1}{x} < \frac{1}{x}\sin(x) < \frac{1}{x}$ does not hold true if $x = \frac{\pi}{2}$.

In fact, $\frac{1}{x}\sin(x) = \frac{1}{x}$ if $x = \frac{\pi}{2}$.

d) From the graph of $f(x) = \frac{1}{x}\sin(x)$, as shown in the next column, we note that $f(0)$ is undefined and one can conclude that for each x in $[-0.1, 0.1]$, except when $x = 0$, one has

$$1 > f(x) \geq f(0.1) = f(-0.1) \approx 0.9983.$$

Yes, $0.99 < f(x) < 1$ if x lies in $[-0.1, 0.1]$ and $x \neq 0$.



e) $y = \dfrac{8\sin(3x)}{x}$

For Thought

1. True, since $\tan x = \dfrac{\sin x}{\cos x}$.

2. True, since $\cot x = \dfrac{1}{\tan x}$ provided $\tan x \neq 0$.

3. False, since $\cot\left(\frac{\pi}{2}\right) = 0$ and $\tan\left(\frac{\pi}{2}\right)$ is undefined.

4. True, since $\dfrac{\sin 0}{\cos 0} = \dfrac{0}{1} = 0$.

5. False, since $\dfrac{\sin(\pi/2)}{\cos(\pi/2)}$ or $\dfrac{1}{0}$ is undefined.

6. False, since $\dfrac{\sin(5\pi/2)}{\cos(5\pi/2)}$ or $\dfrac{1}{0}$ is undefined.

7. True

8. False, the range of $y = \cot x$ is $(-\infty, \infty)$.

9. True, since $\tan\left(3 \cdot \left(\pm\dfrac{\pi}{6}\right)\right) = \tan\left(\pm\dfrac{\pi}{2}\right)$ or $\dfrac{\pm 1}{0}$ is undefined.

10. True, since $\cot\left(4 \cdot \left(\pm\dfrac{\pi}{4}\right)\right) = \cot(\pm\pi)$ or $\dfrac{\pm 1}{0}$ is undefined.

2.4 Exercises

1. tangent

2. vertical asymptote

3. domain

4. domain

5. inflection

6. period

7. $\dfrac{\sin(\pi/3)}{\cos(\pi/3)} = \dfrac{\sqrt{3}/2}{1/2} = \dfrac{\sqrt{3}}{2} \cdot \dfrac{2}{1} = \sqrt{3}$

8. $\dfrac{\sin(\pi/4)}{\cos(\pi/4)} = \dfrac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

9. Undefined, since $\dfrac{\sin(\pi/2)}{\cos(\pi/2)}$ has the form $\dfrac{1}{0}$

10. Undefined, since $\dfrac{\sin(3\pi/2)}{\cos(3\pi/2)}$ has the form $\dfrac{-1}{0}$

11. $\dfrac{\sin(\pi)}{\cos(\pi)} = \dfrac{0}{-1} = 0$

12. $\dfrac{\sin(2\pi)}{\cos(2\pi)} = \dfrac{0}{1} = 0$

13. $\dfrac{\cos(\pi/4)}{\sin(\pi/4)} = \dfrac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

14. $\dfrac{\cos(\pi/3)}{\sin(\pi/3)} = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

15. Undefined, since $\dfrac{\cos(0)}{\sin(0)}$ has the form $\dfrac{1}{0}$

16. Undefined, since $\dfrac{\cos(\pi)}{\sin(\pi)}$ has the form $\dfrac{-1}{0}$

17. $\dfrac{\cos(\pi/2)}{\sin(\pi/2)} = \dfrac{0}{1} = 0$

18. $\dfrac{\cos(3\pi/2)}{\sin(3\pi/2)} = \dfrac{0}{-1} = 0$

19. 92.6 **20.** 1255.8

21. −108.6 **22.** −237.9

23. $\dfrac{1}{\tan 0.002} \approx 500.0$ **24.** $\dfrac{1}{\tan 0.003} \approx 333.3$

25. $\dfrac{1}{\tan(-0.002)} \approx -500.0$

26. $\dfrac{1}{\tan(-0.003)} \approx -333.3$

27. Since $B = 8$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{8}$.

28. Since $B = 2$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{2}$.

29. Since $B = \pi$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{\pi} = 1$.

30. Since $B = \dfrac{\pi}{2}$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{\pi/2} = 2$.

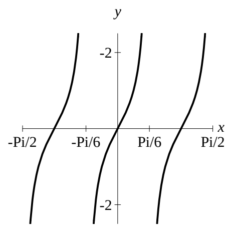
31. Since $B = \dfrac{\pi}{3}$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{\pi/3} = 3$.

32. Since $B = \pi$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{\pi} = 1$.

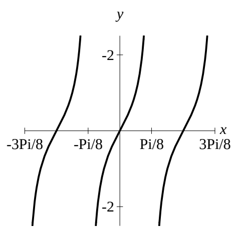
33. Since $B = 3$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{3}$.

34. Since $B = 2$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{2}$.

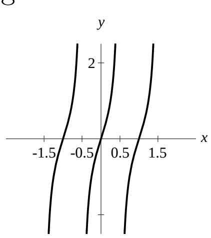
35. $y = \tan(3x)$ has period $\dfrac{\pi}{3}$, and if $3x = \dfrac{\pi}{2} + k\pi$ then the asymptotes are $x = \dfrac{\pi}{6} + \dfrac{k\pi}{3}$ for any integer k.



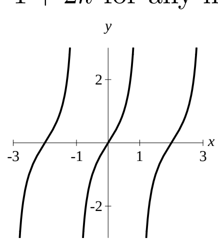
36. $y = \tan(4x)$ has period $\dfrac{\pi}{4}$, and if $4x = \dfrac{\pi}{2} + k\pi$ then the asymptotes are $x = \dfrac{\pi}{8} + \dfrac{k\pi}{4}$ for any integer k.



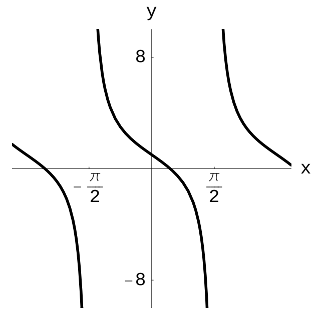
37. $y = \tan(\pi x)$ has period 1, and if $\pi x = \dfrac{\pi}{2} + k\pi$ then the asymptotes are $x = \dfrac{1}{2} + k$ for any integer k.



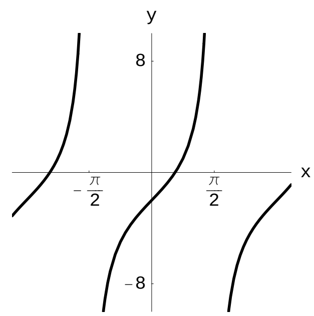
38. $y = \tan(\pi x/2)$ has period 2, and if $\pi x/2 = \dfrac{\pi}{2} + k\pi$ then the asymptotes are $x = 1 + 2k$ for any integer k.



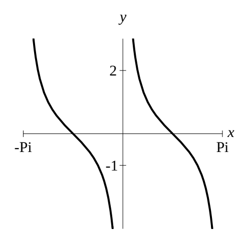
39. $y = -2\tan(x) + 1$ has period π, and the asymptotes are $x = \dfrac{\pi}{2} + k\pi$ where k is an integer.



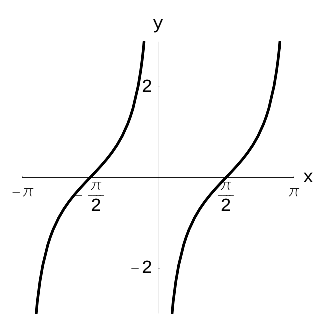
40. $y = 3\tan(x) - 2$ has period π, and the asymptotes are $x = \dfrac{\pi}{2} + k\pi$ where k is an integer.



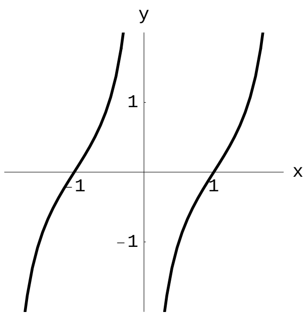
41. $y = -\tan(x - \pi/2)$ has period π, and if $x - \pi/2 = \dfrac{\pi}{2} + k\pi$ then the asymptotes are $x = \pi + k\pi$ or $x = k\pi$ for any integer k.



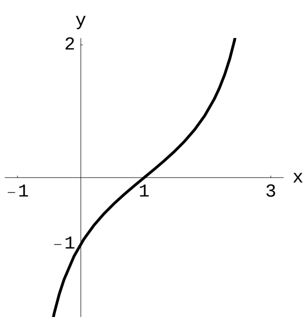
42. $y = \tan(x + \pi/2)$ has period π, and if $x + \dfrac{\pi}{2} = \dfrac{\pi}{2} + k\pi$ then the asymptotes are $x = k\pi$ for any integer k.



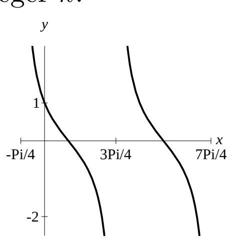
43. $y = \tan\left(\frac{\pi}{2}x - \frac{\pi}{2}\right)$ has period $\frac{\pi}{\pi/2}$ or 2, and if $\frac{\pi}{2}x - \frac{\pi}{2} = \frac{\pi}{2} + k\pi$ then the asymptotes are $x = 2 + 2k$ or $x = 2k$ for any integer k.



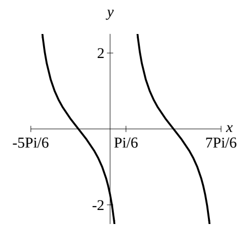
44. $y = \tan\left(\frac{\pi}{4}x + \frac{3\pi}{4}\right)$ has period $\frac{\pi}{\pi/4}$ or 4, and if $\frac{\pi}{4}x + \frac{3\pi}{4} = \frac{\pi}{2} + k\pi$ then the asymptotes are $x = -1 + 4k$ or $x = 3 + 4k$ for any integer k.



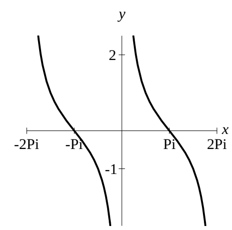
45. $y = \cot(x + \pi/4)$ has period π, and if $x + \frac{\pi}{4} = k\pi$ then the asymptotes are $x = -\frac{\pi}{4} + k\pi$ or $x = \frac{3\pi}{4} + k\pi$ for any integer k.



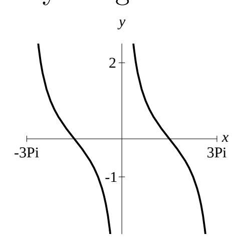
46. $y = \cot(x - \pi/6)$ has period π, and if $x - \frac{\pi}{6} = k\pi$ then the asymptotes are $x = \frac{\pi}{6} + k\pi$ for any integer k.



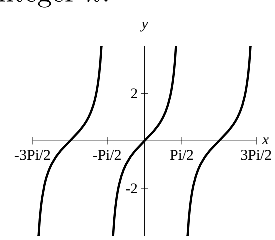
47. $y = \cot(x/2)$ has period 2π, and if $\frac{x}{2} = k\pi$ then the asymptotes are $x = 2k\pi$ for any integer k.



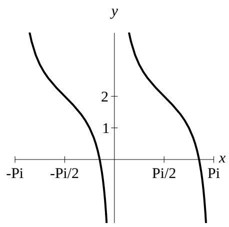
48. $y = \cot(x/3)$ has period 3π, and if $\frac{x}{3} = k\pi$ then the asymptotes are $x = 3k\pi$ for any integer k.



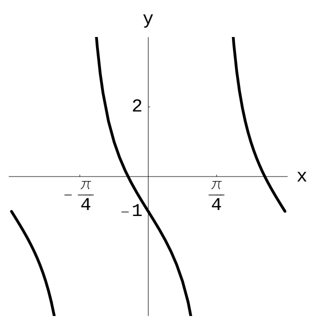
49. $y = -\cot(x + \pi/2)$ has period π, and if $x + \frac{\pi}{2} = k\pi$ then the asymptotes are $x = -\frac{\pi}{2} + k\pi$ or $x = \frac{\pi}{2} + k\pi$ for any integer k.



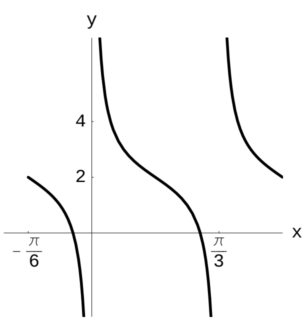
50. $y = 2 + \cot(x)$ has period π, and the asymptotes are $x = k\pi$ for any integer k.



51. $y = \cot(2x - \pi/2) - 1$ has period $\dfrac{\pi}{2}$, and if $2x - \dfrac{\pi}{2} = k\pi$ then the asymptotes are $x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}$ for any integer k.



52. $y = \cot(3x + \pi) + 2$ has period $\dfrac{\pi}{3}$, and if $3x + \pi = k\pi$ then the asymptotes are $x = \dfrac{(k-1)\pi}{3}$ or $x = \dfrac{k\pi}{3}$ for any integer k.



53. $y = 3\tan\left(x - \dfrac{\pi}{4}\right) + 2$

54. $y = \dfrac{1}{2}\tan\left(x + \dfrac{\pi}{2}\right) - 5$

55. $y = -\cot\left(x + \dfrac{\pi}{2}\right) + 1$

56. $y = 2\cot\left(x - \dfrac{\pi}{3}\right) - 2$

57. Note, the period is $\dfrac{\pi}{2}$. So $\dfrac{\pi}{B} = \dfrac{\pi}{2}$ and $B = 2$. The phase shift is $\dfrac{\pi}{4}$ and $A = 1$ since $\left(\dfrac{3\pi}{8}, 1\right)$ is a point on the graph.
An equation is $y = \tan\left(2\left(x - \dfrac{\pi}{4}\right)\right)$.

58. Note, the period is π. So $\dfrac{\pi}{B} = \pi$ and $B = 1$. The phase shift is $\dfrac{\pi}{2}$ and $A = 1$ since $\left(\dfrac{3\pi}{4}, 1\right)$ is a point on the graph.
An equation is $y = \tan\left(x - \dfrac{\pi}{2}\right)$.

59. Note, the period is 2. So $\dfrac{\pi}{B} = 2$ and $B = \dfrac{\pi}{2}$. Since the graph is reflected about the x-axis and $\left(\dfrac{1}{2}, -1\right)$ is a point on the graph, we find $A = -1$. An equation is $y = -\tan\left(\dfrac{\pi}{2}x\right)$.

60. Since the the period is 1, we get $\dfrac{\pi}{B} = 1$ and $B = \pi$. Note, the graph is reflected about the x-axis, $\left(\dfrac{1}{4}, 1\right)$ is a point on the graph, and the phase shift is $C = \dfrac{1}{2}$. Thus, $A = -1$.
An equation is $y = -\tan\left(\pi\left(x - \dfrac{1}{2}\right)\right)$.

61. $f(g(-3)) = f(0) = \tan(0) = 0$

62. $g(f(0)) = g(\tan(0)) = g(0) = 3$

63. Undefined, since $\tan(\pi/2)$ is undefined and $g(h(f(\pi/2))) = g(h(\tan(\pi/2)))$

64. $g(f(h(\pi/6))) = g(f(\pi/3)) = g(\tan(\pi/3)) = g(\sqrt{3}) = \sqrt{3} + 3$

65. $f(g(h(x))) = f(g(2x)) = f(2x + 3) = \tan(2x + 3)$

66. $g(f(h(x))) = g(f(2x)) = g(\tan(2x)) = \tan(2x) + 3$

67. $g(h(f(x))) = g(h(\tan(x))) = g(2\tan(x)) = 2\tan(x) + 3$

68. $h(f(g(x))) = h(f(x + 3)) = h(\tan(x + 3)) = 2\tan(x + 3)$

69. Note $m = \tan(\pi/4) = 1$. Since the line passes through $(2, 3)$, we get $y-3 = 1\cdot(x-2)$. Solving for y, we obtain $y = x + 1$.

70. Note $m = \tan(-\pi/4) = -1$. Since the line passes through $(-1, 2)$, we get $y - 2 = -1 \cdot (x + 1)$. Solving for y, we obtain $y = -x + 1$.

71. Note $m = \tan(\pi/3) = \sqrt{3}$. Since the line passes through $(3, -1)$, we get

$$y + 1 = \sqrt{3}(x - 3).$$

Solving for y, we obtain $y = \sqrt{3}x - 3\sqrt{3} - 1$.

72. Note $m = \tan(\pi/6) = \dfrac{\sqrt{3}}{3}$. Since the line passes through $(-2, -1)$, we get $y + 1 = \dfrac{\sqrt{3}}{3}(x + 2)$. Solving for y, we obtain

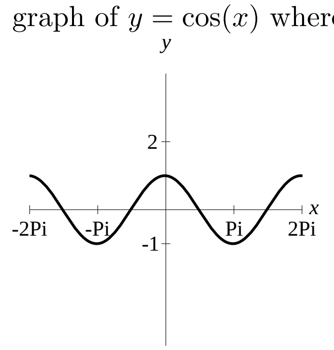
$$y = \frac{\sqrt{3}}{3}x + \frac{2\sqrt{3}}{3} - 1.$$

73.

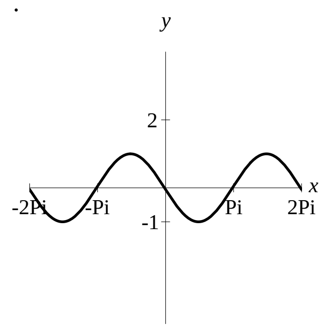
a) Period is about 2.3 years

b) It looks like the graph of a tangent function.

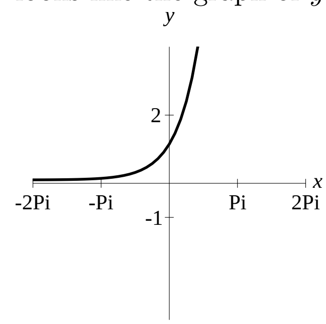
74. a) The graph of y_2 (as shown) looks like the graph of $y = \cos(x)$ where $y_1 = \sin(x)$.



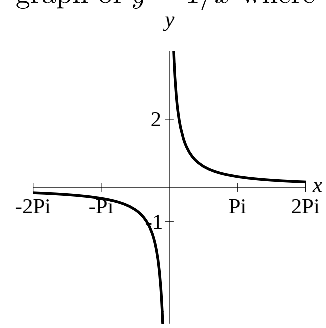
b) The graph of y_2 (as shown) looks like the graph of $y = -\sin(x)$ where $y_1 = \cos(x)$.



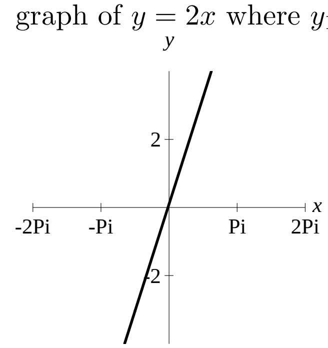
The graph of y_2 (as shown) where $y_1 = e^x$ looks like the graph of $y = e^x$.



The graph of y_2 (as shown) looks like the graph of $y = 1/x$ where $y_1 = \ln(x)$.



The graph of y_2 (as shown) looks like the graph of $y = 2x$ where $y_1 = x^2$.



75. $\csc\alpha = \dfrac{1}{y}$, $\sec\alpha = \dfrac{1}{x}$, $\cot\alpha = \dfrac{x}{y}$

76. Note, $A = 2$ since the y-values of the key points are $0, \pm 2$. Also, $D = 0$

From the first key point $(-\pi/4, 0)$, the phase shift is $C = -\dfrac{\pi}{4}$

Since the difference between the first and last x-values is the period, i.e.,

$$\frac{2\pi}{B} = 3\pi/4 - (-\pi/4) = \pi$$

we find $B = 2$. The equation is

$$y = 2\sin\left(2\left(x + \frac{\pi}{4}\right)\right).$$

77. Period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2} = \pi$.
To find the asymptotes, solve

$$\begin{aligned} 2x - \pi &= \frac{\pi}{2} + m\pi \\ 2x &= \frac{\pi}{2} + (m+1)\pi \\ x &= \frac{\pi}{4} + \frac{(m+1)\pi}{2} \end{aligned}$$

where m is an integer. Let $k = m + 1$. Then the asymptotes are

$$x = \frac{\pi}{4} + \frac{k\pi}{2}$$

The range is $(-\infty, -3] \cup [3, \infty)$.

78. $\cos\beta = -\sqrt{1 - \sin^2\beta} = -\sqrt{1 - (1/4)^2} =$
$-\sqrt{15/16} = -\dfrac{\sqrt{15}}{4}$.

79. Let x be the distance from the building to the boss. Then
$$\tan 28^\circ = \frac{432}{x}$$
and $x = 432/\tan 28^\circ \approx 812$ feet.

80. **a)** $-1/2$ **b)** -1 **c)** 1
d) Undefined **e)** $\dfrac{1}{\sqrt{3}/2} = \dfrac{2\sqrt{3}}{3}$ **f)** -1

81.

a) First, use four tiles to make a 4-by-4 square. Then construct three more 4-by-4 square squares. Now, you have four 4-by-4 squares. Then put these four squares together to make a 8-by-8 square.

b) By elimination, you will not be able to make a 6-by-6 square. There are only a few possibilities and none of them will make a 6-by-6 square.

82. The vertical numbers are 1,4,7, and 9.
The horizontal digits are 2,3,5, and 8.
The number in the corner cell is 6.

2.4 Pop Quiz

1. $-\tan(3\pi/4) = -\dfrac{\sin(3\pi/4)}{\cos(3\pi/4)} = -\dfrac{\sqrt{2}/2}{-\sqrt{2}/2} = 1$

2. $\dfrac{\cos(2\pi/3)}{\sin(2\pi/3)} = \dfrac{-1/2}{\sqrt{3}/2} = -\dfrac{1}{\sqrt{3}} = -\dfrac{\sqrt{3}}{3}$

3. The period is $\dfrac{\pi}{B} = \dfrac{\pi}{3}$

4. Since $\sin 2x = 0$ exactly when $2x = k\pi$ where k is an integer, or $x = k\pi/2$. Then the vertical asymptotes of $y = \cot 2x$ are the vertical lines
$$x = \frac{k\pi}{2}.$$

5. $y = 3\tan\left(x + \dfrac{\pi}{4}\right) - 1$

For Thought

1. False, since the graph of $y = x + \sin x$ does not duplicate itself.

2. True, since the range of $y = \sin x$ is $[-1, 1]$ it follows that $y = x + \sin x$ oscillates about $y = x$.

3. True, since if x is small then the value of y in $y = \dfrac{1}{x}$ is a large number.

4. True, since $y = 0$ is the horizontal asymptote of $y = \dfrac{1}{x}$ and the x-axis is the graph of $y = 0$.

5. True, since the range of $y = \sin x$ is $[-1, 1]$ it follows that $y = \dfrac{1}{x} + \sin x$ oscillates about $y = \dfrac{1}{x}$.

6. True, since $\sin x = 1$ whenever $x = \dfrac{\pi}{2} + k2\pi$ and $\sin x = -1$ whenever $x = \dfrac{3\pi}{2} + k2\pi$, and since $\dfrac{1}{x}$ is approximately zero when x is a large numbers, then $\dfrac{1}{x} + \sin x = 0$ has many solutions in x for 0 is between 1 and -1.

7. False, since $\cos(\pi/6)+\cos(2\cdot\pi/6) = \dfrac{\sqrt{3}+1}{2} > 1$ then 1 is not the maximum of $y = \cos(x) + \cos(2x)$.

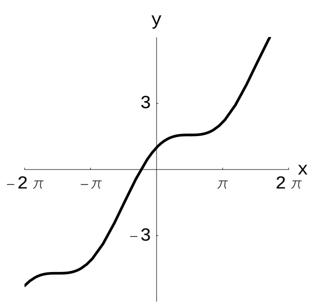
8. False, since $\sin(x) + \cos(x) = \sqrt{2}\sin\left(x + \dfrac{\pi}{4}\right)$ then the maximum value of $\sin(x) + \cos(x)$ is $\sqrt{2}$, and not 2.

9. True, since on the interval $[0, 2\pi]$ we find that $\sin(x) = 0$ for $x = 0, \pi, 2\pi$.

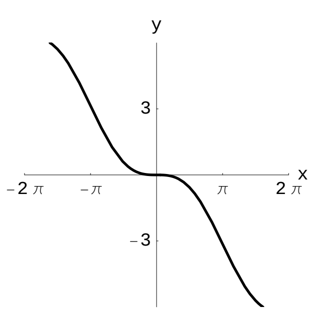
10. True, since $B = \pi$ and the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi}$ or 2.

2.5 Exercises

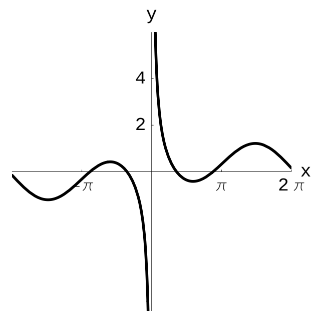
1. For each x-coordinate, the y-coordinate of $y = x + \cos x$ is the sum of the y-coordinates of $y_1 = x$ and $y_2 = \cos x$. Note, the graph below oscillates about $y_1 = x$.



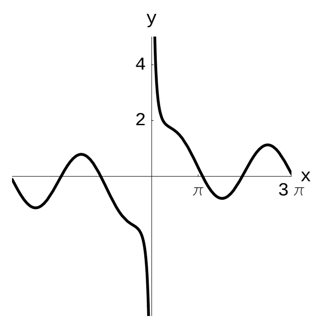
2. For each x-coordinate, the y-coordinate of $y = -x + \sin x$ is the sum of the y-coordinates of $y_1 = -x$ and $y_2 = \sin x$. Note, the graph below oscillates about $y_1 = -x$.



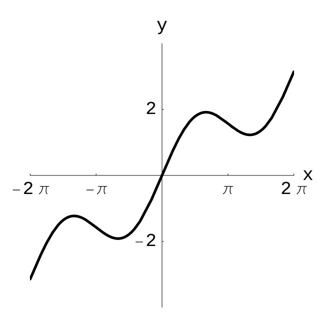
3. For each x-coordinate, the y-coordinate of $y = \dfrac{1}{x} - \sin x$ is obtained by subtracting the y-coordinate of $y_2 = \sin x$ from the y-coordinate of $y_1 = \dfrac{1}{x}$.



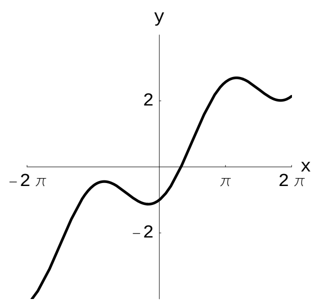
4. For each x-coordinate, the y-coordinate of $y = \dfrac{1}{x} + \sin x$ is the sum of the y-coordinates of $y_1 = \dfrac{1}{x}$ and $y_2 = \sin x$.



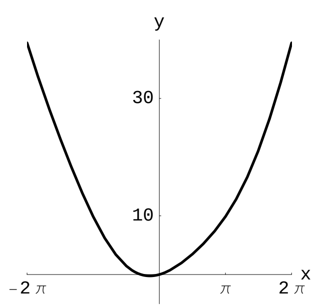
5. For each x-coordinate, the y-coordinate of $y = \dfrac{1}{2}x + \sin x$ is the sum of the y-coordinates of $y_1 = \dfrac{1}{2}x$ and $y_2 = \sin x$.



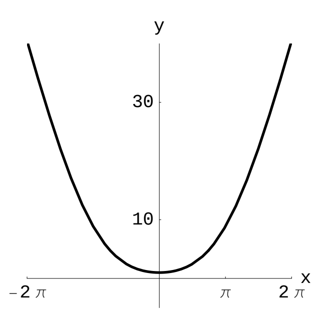
6. For each x-coordinate, the y-coordinate of $y = \frac{1}{2}x - \cos x$ is obtained by subtracting the y-coordinate of $y_2 = \cos x$ from the y-coordinate of $y_1 = \frac{1}{2}x$.



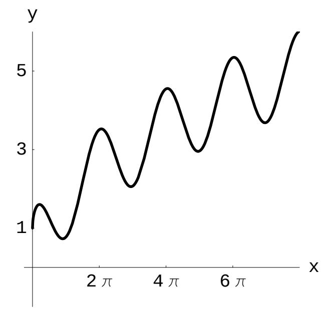
7. For each x-coordinate, the y-coordinate of $y = x^2 + \sin x$ is the sum of the y-coordinates of $y_1 = x^2$ and $y_2 = \sin x$. Note, $y_2 = \sin x$ oscillates about $y_1 = x^2$.



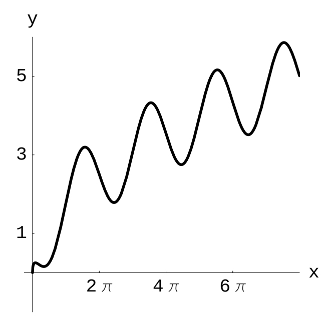
8. For each x-coordinate, the y-coordinate of $y = x^2 + \cos x$ is the sum of the y-coordinates of $y_1 = x^2$ and $y_2 = \cos x$. Note, the graph below oscillates about $y_1 = x^2$.



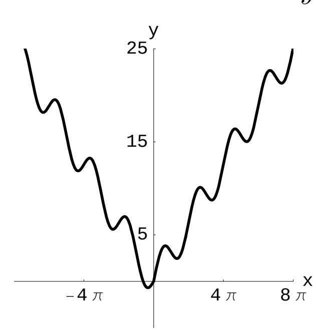
9. For each x-coordinate, the y-coordinate of $y = \sqrt{x} + \cos x$ is the sum of the y-coordinates of $y_1 = \sqrt{x}$ and $y_2 = \cos x$.



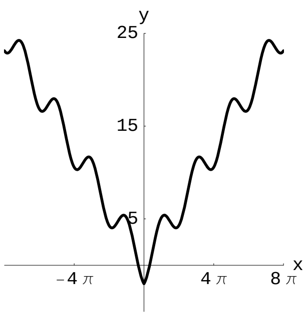
10. For each x-coordinate, the y-coordinate of $y = \sqrt{x} - \sin x$ is obtained by subtracting the y-coordinate of $y_2 = \sin x$ from the y-coordinate of $y_1 = \sqrt{x}$.



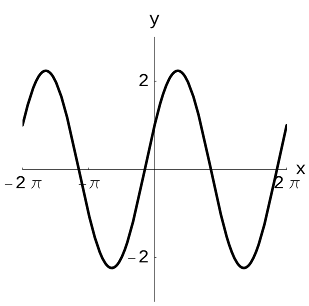
11. For each x-coordinate, the y-coordinate of $y = |x| + 2\sin x$ is the sum of the y-coordinates of $y_1 = |x|$ and $y_2 = 2\sin x$. Note, the graph below oscillates about $y_1 = |x|$.



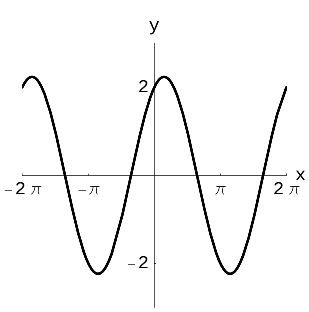
12. For each x-coordinate, the y-coordinate of $y = |x| - 2\cos x$ is obtained by subtracting the y-coordinate of $y_2 = 2\cos x$ from the y-coordinate of $y_1 = |x|$. Note, the graph below oscillates about $y_1 = |x|$.



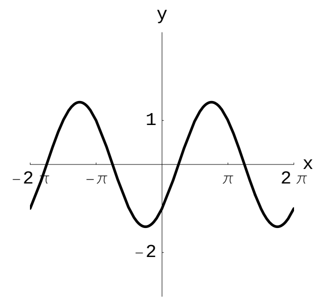
13. For each x-coordinate, the y-coordinate of $y = \cos(x) + 2\sin(x)$ is obtained by adding the y-coordinates of $y_1 = \cos x$ and $y_2 = 2\sin x$. Note, $y = \cos(x) + 2\sin(x)$ is a periodic function since it is the sum of two periodic functions.



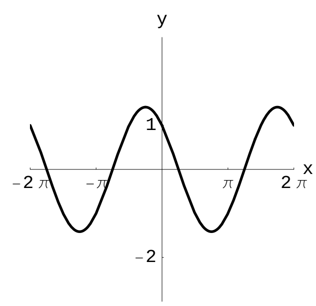
14. For each x-coordinate, the y-coordinate of $y = 2\cos(x) + \sin(x)$ is obtained by adding the y-coordinates of $y_1 = 2\cos x$ and $y_2 = \sin x$. Note, $y = 2\cos(x) + \sin(x)$ is a periodic function since it is the sum of two periodic functions.



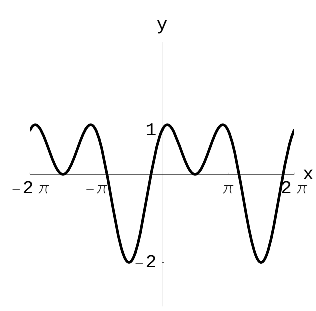
15. For each x-coordinate, the y-coordinate of $y = \sin(x) - \cos(x)$ is obtained by subtracting the y-coordinate of $y_2 = \cos x$ from the y-coordinate of $y_1 = \sin(x)$. Note, $y = \sin(x) - \cos(x)$ is a periodic function since it is the difference of two periodic functions.



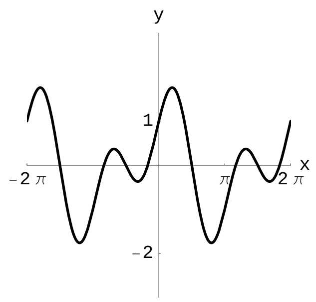
16. For each x-coordinate, the y-coordinate of $y = \cos(x) - \sin(x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin x$ from the y-coordinate of $y_1 = \cos(x)$. Note, $y = \cos(x) - \sin(x)$ is a periodic function since it is the difference of two periodic functions.



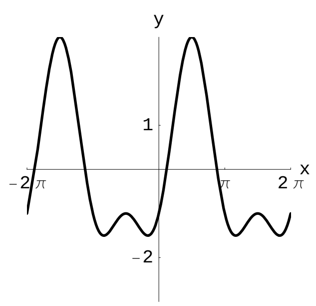
17. For each x-coordinate, the y-coordinate of $y = \sin(x) + \cos(2x)$ is obtained by adding the y-coordinates of $y_1 = \sin x$ and $y_2 = \cos(2x)$. Note, $y = \sin(x) + \cos(2x)$ is a periodic function since it is the sum of two periodic functions.



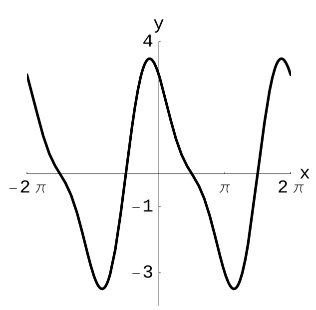
18. For each x-coordinate, the y-coordinate of $y = \cos(x) + \sin(2x)$ is obtained by adding the y-coordinates of $y_1 = \cos x$ and $y_2 = \sin(2x)$. Note, $y = \cos(x) + \sin(2x)$ is a periodic function since it is the sum of two periodic functions.



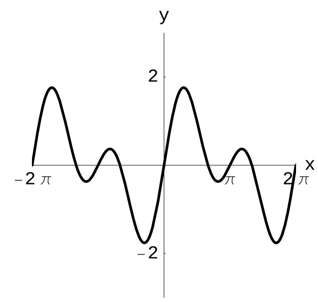
19. For each x-coordinate, the y-coordinate of $y = 2\sin(x) - \cos(2x)$ is obtained by subtracting the y-coordinate of $y_2 = \cos(2x)$ from the y-coordinate of $y_1 = 2\sin(x)$. Note, $y = 2\sin(x) - \cos(2x)$ is a periodic function with period 2π.



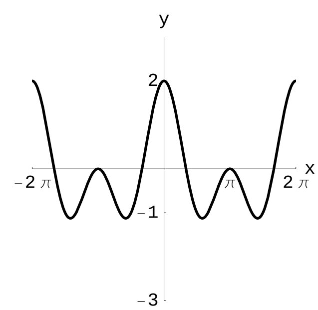
20. For each x-coordinate, the y-coordinate of $y = 3\cos(x) - \sin(2x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin(2x)$ from the y-coordinate of $y_1 = 3\cos(x)$. Note, $y = 3\cos(x) - \sin(2x)$ is a periodic function with period 2π.



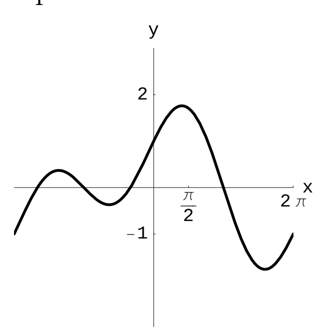
21. For each x-coordinate, the y-coordinate of $y = \sin(x) + \sin(2x)$ is obtained by adding the y-coordinates of $y_1 = \sin x$ and $y_2 = \sin(2x)$. Note, $y = \sin(x) + \sin(2x)$ is a periodic with period 2π.



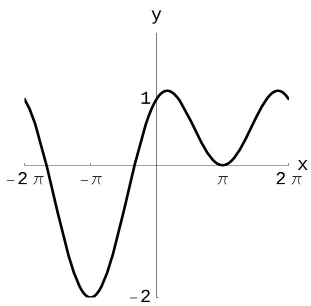
22. For each x-coordinate, the y-coordinate of $y = \cos(x) + \cos(2x)$ is obtained by adding the y-coordinates of $y_1 = \cos x$ and $y_2 = \cos(2x)$. Note, $y = \cos(x) + \cos(2x)$ is a periodic function with period 2π.



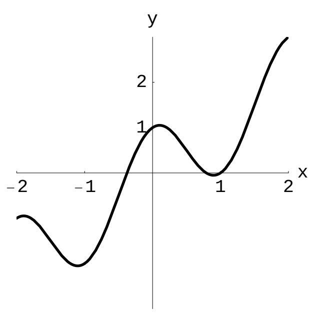
23. For each x-coordinate, the y-coordinate of $y = \sin(x) + \cos\left(\frac{x}{2}\right)$ is obtained by adding the y-coordinates of $y_1 = \sin x$ and $y_2 = \cos\left(\frac{x}{2}\right)$. Note, $y = \sin(x) + \cos\left(\frac{x}{2}\right)$ is a periodic function with period 4π.



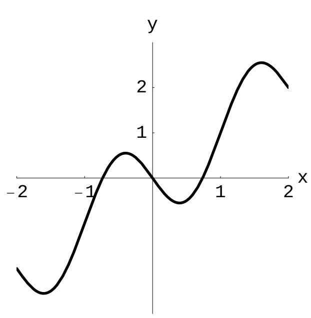
24. For each x-coordinate, the y-coordinate of $y = \cos(x) + \sin\left(\dfrac{x}{2}\right)$ is obtained by adding the y-coordinates of $y_1 = \cos x$ and $y_2 = \sin\left(\dfrac{x}{2}\right)$. Note, $y = \cos(x) + \sin\left(\dfrac{x}{2}\right)$ is a periodic function with period 4π.



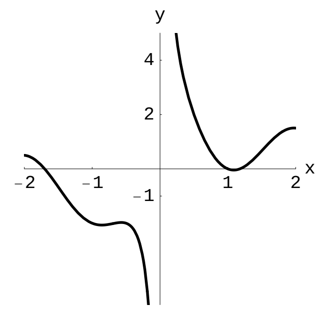
25. For each x-coordinate, the y-coordinate of $y = x + \cos(\pi x)$ is obtained by adding the y-coordinates of $y_1 = x$ and $y_2 = \cos(\pi x)$.



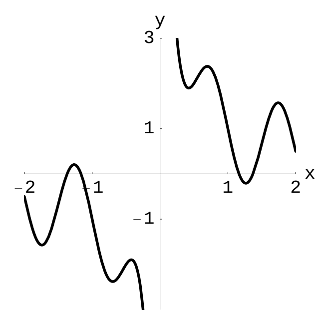
26. For each x-coordinate, the y-coordinate of $y = x - \sin(\pi x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin(\pi x)$ from the y-coordinate of $y_1 = x$.



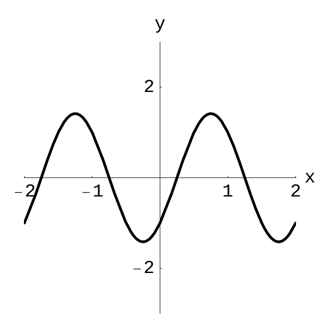
27. For each x-coordinate, the y-coordinate of $y = \dfrac{1}{x} + \cos(\pi x)$ is obtained by adding the y-coordinates of $y_1 = \dfrac{1}{x}$ and $y_2 = \cos(\pi x)$.



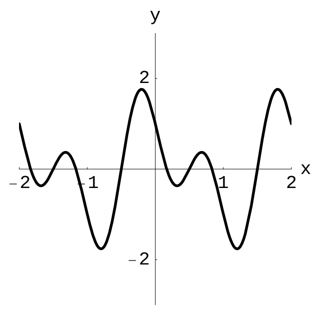
28. For each x-coordinate, the y-coordinate of $y = \dfrac{1}{x} - \sin(2\pi x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin(2\pi x)$ from the y-coordinate of $y_1 = \dfrac{1}{x}$.



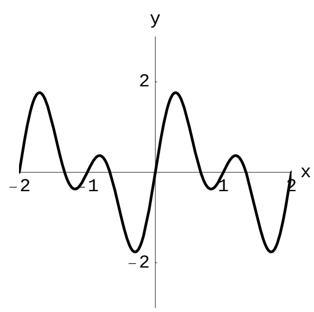
29. For each x-coordinate, the y-coordinate of $y = \sin(\pi x) - \cos(\pi x)$ is obtained by subtracting the y-coordinate of $y_2 = \cos(\pi x)$ from the y-coordinate of $y_1 = \sin(\pi x)$.



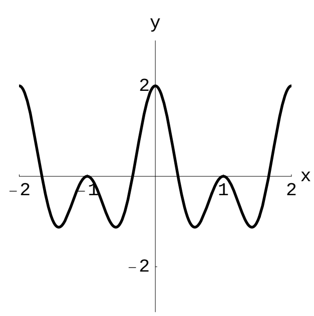
30. For each x-coordinate, the y-coordinate of $y = \cos(\pi x) - \sin(2\pi x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin(2\pi x)$ from the y-coordinate of $y_1 = \cos(\pi x)$.



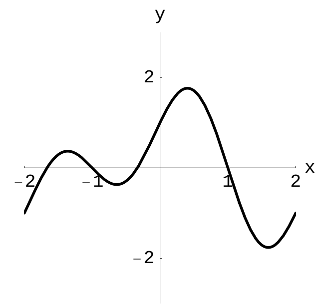
31. For each x-coordinate, the y-coordinate of $y = \sin(\pi x) + \sin(2\pi x)$ is obtained by adding the y-coordinates of $y_1 = \sin(\pi x)$ and $y_2 = \sin(2\pi x)$.



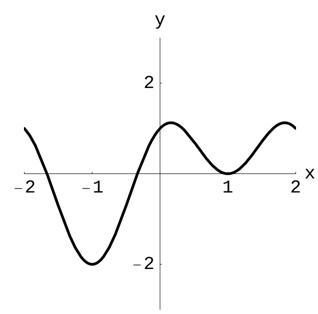
32. For each x-coordinate, the y-coordinate of $y = \cos(\pi x) + \cos(2\pi x)$ is obtained by adding the y-coordinates of $y_1 = \cos(\pi x)$ and $y_2 = \cos(2\pi x)$.



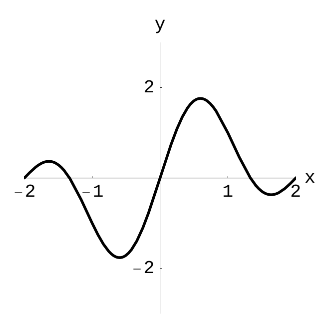
33. For each x-coordinate, the y-coordinate of $y = \cos\left(\frac{\pi}{2}x\right) + \sin(\pi x)$ is obtained by adding the y-coordinates of $y_1 = \cos\left(\frac{\pi}{2}x\right)$ and $y_2 = \sin(\pi x)$.



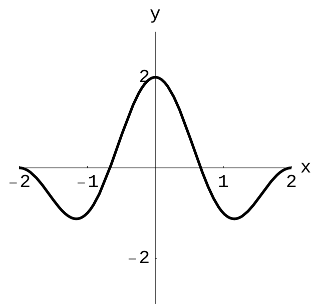
34. For each x-coordinate, the y-coordinate of $y = \sin\left(\frac{\pi}{2}x\right) + \cos(\pi x)$ is obtained by adding the y-coordinates of $y_1 = \sin\left(\frac{\pi}{2}x\right)$ and $y_2 = \cos(\pi x)$.



35. For each x-coordinate, the y-coordinate of $y = \sin(\pi x) + \sin\left(\frac{\pi}{2}x\right)$ is obtained by adding the y-coordinates of $y_1 = \sin(\pi x)$ and $y_2 = \sin\left(\frac{\pi}{2}x\right)$.



36. For each x-coordinate, the y-coordinate of $y = \cos(\pi x) + \cos\left(\frac{\pi}{2}x\right)$ is obtained by adding the y-coordinates of $y_1 = \cos(\pi x)$ and $y_2 = \cos\left(\frac{\pi}{2}x\right)$.



37. c, since the graph of of $y = x + \sin(6x)$ oscillates about the line $y = x$

38. f, since the graph of of $y = \frac{1}{x} + \sin(6x)$ oscillates about the the curve $y = \frac{1}{x}$

39. a, since the graph of of $y = -x + \cos(0.5x)$ oscillates about the line $y = -x$

40. e, since the graph of of $y = \cos x + \cos(10x)$ is periodic and passes through $(0, 2)$

41. d, since the graph of of $y = \sin x + \cos(10x)$ is periodic and passes through $(0, 1)$

42. b, since the graph of of $y = x^2 + \cos(6x)$ oscillates about the parabola $y = x^2$

43. Since $x_\circ = -3$, $v_\circ = 4$, and $\omega = 1$, we obtain

$$\begin{aligned} x(t) &= \frac{v_\circ}{\omega}\sin(\omega t) + x_\circ \cos(\omega t) \\ &= 4\sin t - 3\cos t. \end{aligned}$$

After $t = 3$ sec, the location of the weight is

$$x(3) = 4\sin 3 - 3\cos 3 \approx 3.5 \text{ cm}.$$

The period and amplitude of

$$x(t) = 4\sin t - 3\cos t$$

are $2\pi \approx 6.3$ sec and $\sqrt{4^2 + 3^2} = 5$ cm, respectively.

44. Since $x_\circ = 1$, $v_\circ = -3$, and $\omega = \sqrt{3}$, we find

$$\begin{aligned} x(t) &= \frac{v_\circ}{\omega}\sin(\omega t) + x_\circ \cos(\omega t) \\ &= -\frac{3}{\sqrt{3}}\sin(\sqrt{3}t) + \cos(\sqrt{3}t) \\ &= -\sqrt{3}\sin(\sqrt{3}t) + \cos(\sqrt{3}t). \end{aligned}$$

When $t = 2$ sec, the location of the weight is

$$x(2) = -\sqrt{3}\sin 2\sqrt{3} + \cos 2\sqrt{3} \approx -0.4 \text{ in.}$$

The period and amplitude of

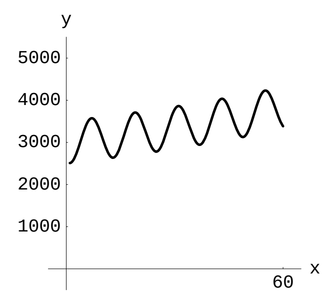
$$x(t) = -\sqrt{3}\sin(\sqrt{3}t) + \cos(\sqrt{3}t)$$

are $2\pi/\sqrt{3} \approx 3.6$ sec and $\sqrt{1^2 + (\sqrt{3})^2} = 2$ in., respectively.

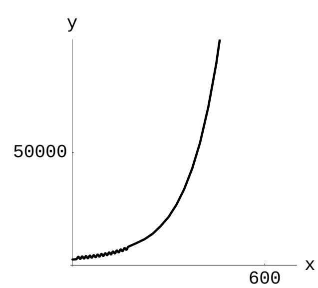
45. a) The graph of

$$P(x) = 1000(1.01)^x + 500\sin\left(\frac{\pi}{6}(x - 4)\right) + 2000$$

for $1 \le x \le 60$ is given below

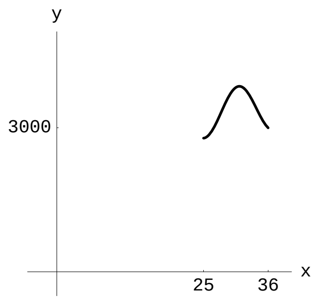


b) The graph for $1 \le x \le 600$ is given below



The graph looks like an exponential function.

46. The graph for 2008, or $25 \leq x \leq 36$ is given below



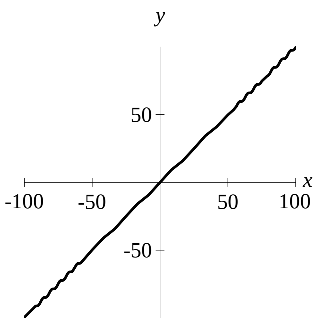
The maximum value in the list below

$$P(25), P(26), ..., P(36)$$

is

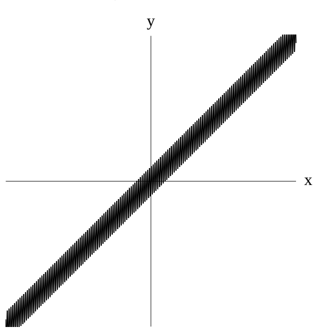
$$P(31) = \$3861.33$$

47. By adding the ordinates of $y = x$ and $y = \sin(x)$, one can obtain the graph of $y = x + \sin(x)$ (which is given below).

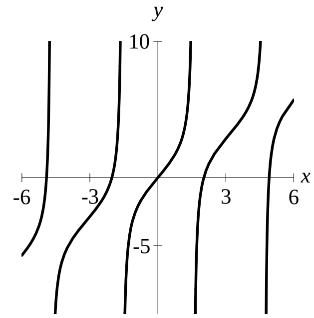


The graph above looks like the graph of $y = x$.

48. By adding the ordinates of $y = x$ and $y = \sin(50x)$, we obtain the graph of $y = x + \sin 50x$. The graph oscillates about the line $y = x$.



49. By adding the ordinates of $y = x$ and $y = \tan(x)$, we obtain the graph of $y = x + \tan(x)$.



The graph above looks like the graph of $y = \tan x$ with increasing vertical translation on each cycle.

51. Since $A = -3$, the amplitude is 3.

Since $\frac{\pi x}{2} - \frac{\pi}{2} = \frac{\pi}{2}(x-1)$, the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi/2} = 4$, and the phase shift is 1.

The range is $[-3+7, 3+7]$ or $[4, 10]$

52. Note, $A = 3$ and $D = 2$ since the maximum and minimum y-values of the key points may be written as $\pm 3 + 2$.

From the second key point $(\pi/2, 5)$, which is the first key point for a cosine wave, the phase shift is $C = \frac{\pi}{2}$

Since the difference between the first and last x-values is the period, i.e.,

$$\frac{2\pi}{B} = 5\pi/4 - \pi/4 = \pi$$

we find $B = 2$. The equation is

$$y = 3\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 2.$$

53. The period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$, and the range is $(-\infty, -5] \cup [5, \infty)$.

54. Solving $2x = k\pi$ where k is an integer, the domain is

$$\left\{x \mid x \neq \frac{k\pi}{2} \text{ for any integer } k\right\}.$$

55. The period is $\dfrac{\pi}{B} = \dfrac{\pi}{\pi} = 1$, and the range is $(-\infty, \infty)$.

56. Solving $2x = \dfrac{\pi}{2} + k\pi$ where k is an integer, we find that the vertical asymptotes are

$$x = \frac{\pi}{4} + \frac{k\pi}{2}$$

where k is any integer.

57. Since there will be no 1's left after the 162nd house, the first house that cannot be numbered correctly is 163.

58. If the exponent is zero, then

$$\begin{aligned} x^2 + x &= 0 \\ x(x+1) &= 0 \\ x &= 0, -1. \end{aligned}$$

If the base is one, then

$$\begin{aligned} \frac{x-5}{3} &= 1 \\ x - 5 &= 3 \\ x &= 8. \end{aligned}$$

If the base equals -1, then

$$\begin{aligned} \frac{x-5}{3} &= -1 \\ x - 5 &= -3 \\ x &= 2 \end{aligned}$$

Notice, $x = 2$ satisfies $\left(\frac{x-5}{3}\right)^{x^2+x} = 1$.

Then the solutions are $x = 0, -1, 8, 2$.

2.5 Pop Quiz

1. *f* **2.** *e* **3.** *d*

4. *a* **5.** *c* **6.** *b*

Review Exercises

1. 1 **2.** $\dfrac{1}{2}$

3. $\dfrac{\sin(\pi/4)}{\cos(\pi/4)} = \dfrac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

4. $\dfrac{\cos(\pi/4)}{\sin(\pi/4)} = \dfrac{\sqrt{2}/2}{\sqrt{2}/2} = 1$

5. $\dfrac{1}{\cos \pi} = \dfrac{1}{-1} = -1$

6. $\dfrac{1}{\sin(\pi/2)} = \dfrac{1}{1} = 1$

7. 0 **8.** $\dfrac{\sin(-3\pi)}{\cos(-3\pi)} = \dfrac{0}{-1} = 0$

9. Since $B = 1$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$.

10. Since $B = 1$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{1} = 2\pi$.

11. Since $B = 2$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{2}$.

12. Since $B = 3$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{3}$.

13. Since $B = \pi$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi} = 2$.

14. Since $B = \dfrac{\pi}{2}$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi/2} = 4$.

15. Since $B = \dfrac{1}{2}$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{1/2} = 4\pi$.

16. Since $B = 3\pi$, the period is $\dfrac{\pi}{3\pi} = \dfrac{1}{3}$.

17. Domain $(-\infty, \infty)$, range $[-2, 2]$

18. Domain $(-\infty, \infty)$, range $[-5, 5]$

19. To find the domain of $y = \tan(2x)$, solve $2x = \dfrac{\pi}{2} + k\pi$, and so the domain is $\left\{x \mid x \neq \dfrac{\pi}{4} + \dfrac{k\pi}{2}\right\}$. The range is $(-\infty, \infty)$.

20. To find the domain of $y = \cot(3x)$, solve $3x = k\pi$, and so the domain is $\left\{x \mid x \neq \dfrac{k\pi}{3}\right\}$. The range is $(-\infty, \infty)$.

21. Since the zeros of $y = \cos(x)$ are $x = \dfrac{\pi}{2} + k\pi$, the domain of $y = \sec(x) - 2$ is $\left\{x \mid x \neq \dfrac{\pi}{2} + k\pi\right\}$. The range is $(-\infty, -3] \cup [-1, \infty)$.

22. By using the zeros of $y = \sin(x)$ and by solving $\frac{x}{2} = k\pi$, we get that the domain of $y = \csc(x/2)$ is $\{x \mid x \neq 2k\pi\}$. The range of $y = \csc(x/2)$ is $(-\infty, -1] \cup [1, \infty)$.

23. By using the zeros of $y = \sin(x)$ and by solving $\pi x = k\pi$, we get that the domain of $y = \cot(\pi x)$ is $\{x \mid x \neq k\}$. The range of $y = \cot(\pi x)$ is $(-\infty, \infty)$.

24. By using the zeros of $y = \cos(x)$ and by solving $\pi x = \frac{\pi}{2} + k\pi$, we get that the domain of $y = \tan(\pi x)$ is $\left\{x \mid x \neq \frac{1}{2} + k\right\}$. The range of $y = \tan(\pi x)$ is $(-\infty, \infty)$.

25. By solving $2x = \frac{\pi}{2} + k\pi$, we get that the asymptotes of $y = \tan(2x)$ are $x = \frac{\pi}{4} + \frac{k\pi}{2}$.

26. By solving $4x = \frac{\pi}{2} + k\pi$, we find that the asymptotes of $y = \tan(4x)$ are $x = \frac{\pi}{8} + \frac{k\pi}{4}$.

27. By solving $\pi x = k\pi$, we find that the asymptotes of $y = \cot(\pi x) + 1$ are $x = k$.

28. By solving $\frac{\pi x}{2} = k\pi$, we obtain that the asymptotes of $y = 3\cot\left(\frac{\pi x}{2}\right)$ are $x = 2k$.

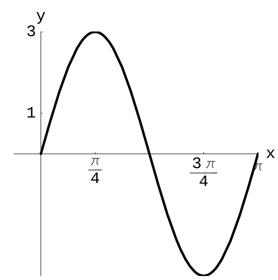
29. Solving $x - \frac{\pi}{2} = \frac{\pi}{2} + k\pi$, we get $x = \pi + k\pi$ or equivalently $x = k\pi$. Then the asymptotes of $y = \sec\left(x - \frac{\pi}{2}\right)$ are $x = k\pi$.

30. Solving $x + \frac{\pi}{2} = \frac{\pi}{2} + k\pi$, we get $x = k\pi$. The asymptotes of $y = \sec\left(x + \frac{\pi}{2}\right)$ are $x = k\pi$.

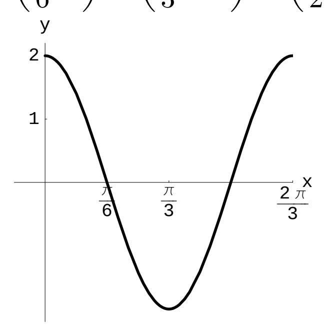
31. Solving $\pi x + \pi = k\pi$, we get $x = -1 + k$ or equivalently $x = k$. Then the asymptotes of $y = \csc(\pi x + \pi)$ are $x = k$.

32. Solving $2x - \pi = k\pi$, we get $x = \frac{\pi}{2} + \frac{k\pi}{2}$ or equivalently $x = \frac{k\pi}{2}$. Then the asymptotes of $y = \csc(2x - \pi)$ are $x = \frac{k\pi}{2}$.

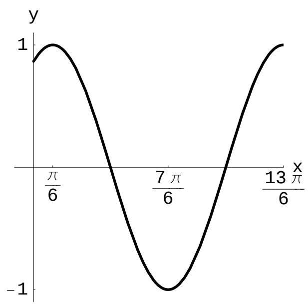
33. Amplitude 3, since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or equivalently π, phase shift is 0, and range is $[-3, 3]$. Five points are $(0, 0)$, $\left(\frac{\pi}{4}, 3\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, -3\right)$, and $(\pi, 0)$.



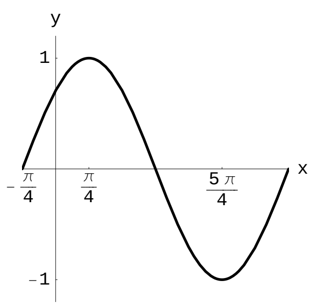
34. Amplitude 2, period $\frac{2\pi}{3}$, phase shift is 0, and range is $[-2, 2]$. Five points are $(0, 2)$, $\left(\frac{\pi}{6}, 0\right)$, $\left(\frac{\pi}{3}, -2\right)$, $\left(\frac{\pi}{2}, 0\right)$, and $\left(\frac{2\pi}{3}, 2\right)$.



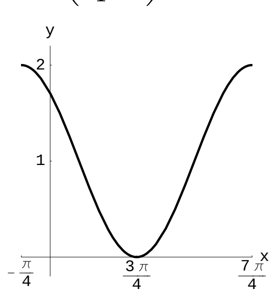
35. Amplitude 1, since $B = 1$ the period is $\frac{2\pi}{B} = \frac{2\pi}{1}$ or equivalently 2π, phase shift is $\frac{\pi}{6}$, and range is $[-1, 1]$. Five points are $\left(\frac{\pi}{6}, 1\right)$, $\left(\frac{2\pi}{3}, 0\right)$, $\left(\frac{7\pi}{6}, -1\right)$, $\left(\frac{5\pi}{3}, 0\right)$, and $\left(\frac{13\pi}{6}, 1\right)$.



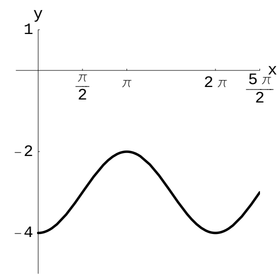
36. Amplitude 1, period 2π, phase shift is $-\frac{\pi}{4}$, and range is $[-1, 1]$. Five points are $\left(\frac{-\pi}{4}, 0\right)$, $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $\left(\frac{5\pi}{4}, -1\right)$, and $\left(\frac{7\pi}{4}, 0\right)$.



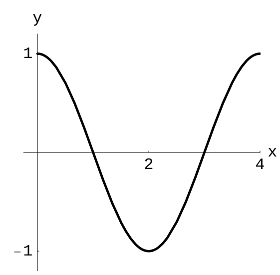
37. Amplitude 1, since $B = 1$ the period is $\frac{2\pi}{B} = \frac{2\pi}{1}$ or equivalently 2π, phase shift is $-\frac{\pi}{4}$, and range is $[-1+1, 1+1]$ or $[0, 2]$. Five points are $\left(-\frac{\pi}{4}, 2\right)$, $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, 0\right)$, $\left(\frac{5\pi}{4}, 1\right)$, and $\left(\frac{7\pi}{4}, 2\right)$.



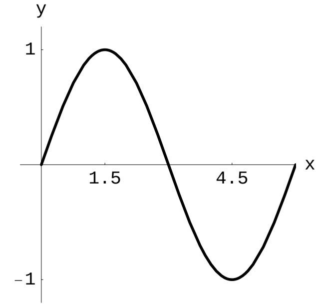
38. Amplitude 1, period 2π, phase shift is $\frac{\pi}{2}$, and range is $[-1-3, 1-3]$ or $[-4, -2]$. Five points are $\left(\frac{\pi}{2}, -3\right)$, $(\pi, -2)$, $\left(\frac{3\pi}{2}, -3\right)$, $(2\pi, -4)$, and $\left(\frac{5\pi}{2}, -3\right)$.



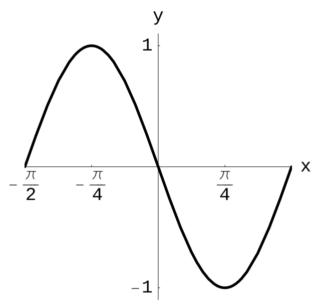
39. Amplitude 1, since $B = \frac{\pi}{2}$ the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi/2}$ or equivalently 4, phase shift is 0, and range is $[-1, 1]$. Five points are $(0, 1)$, $(1, 0)$, $(2, -1)$, $(3, 0)$, and $(4, 1)$.



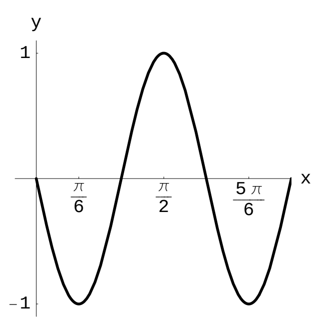
40. Amplitude 1, since $B = \frac{\pi}{3}$ the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi/3}$ or equivalently 6, phase shift is 0, and range is $[-1, 1]$. Five points are $(0, 0)$, $\left(\frac{3}{2}, 1\right)$, $(3, 0)$, $\left(\frac{9}{2}, -1\right)$, and $(6, 0)$.



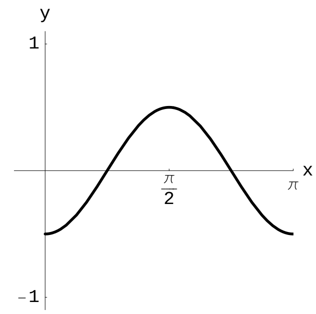
41. Note, $y = \sin\left(2\left(x + \frac{\pi}{2}\right)\right)$. Then amplitude is 1, since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or equivalently π, phase shift is $-\frac{\pi}{2}$, and range is $[-1, 1]$. Five points are $\left(-\frac{\pi}{2}, 0\right)$, $\left(-\frac{\pi}{4}, 1\right)$, $(0, 0)$, $\left(\frac{\pi}{4}, -1\right)$, and $\left(\frac{\pi}{2}, 0\right)$.



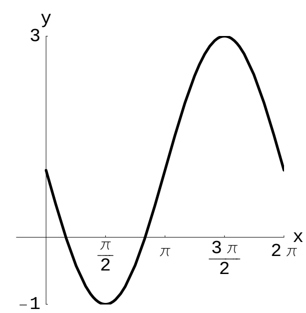
42. Note, $y = \sin\left(3\left(x - \frac{\pi}{3}\right)\right)$. Then amplitude is 1, since $B = 3$ the period is $\frac{2\pi}{B} = \frac{2\pi}{3}$, phase shift is $\frac{\pi}{3}$, and range is $[-1, 1]$. Five points are $\left(\frac{\pi}{3}, 0\right)$, $\left(\frac{\pi}{2}, 1\right)$, $\left(\frac{2\pi}{3}, 0\right)$, $\left(\frac{5\pi}{6}, -1\right)$, and $(\pi, 0)$.



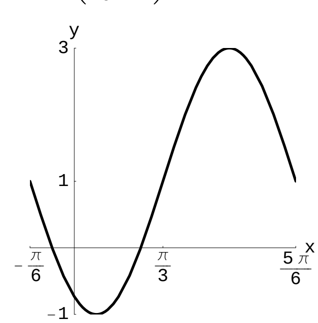
43. Amplitude $\frac{1}{2}$, since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or equivalently π, phase shift is 0, and range is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. Five points are $\left(0, -\frac{1}{2}\right)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, \frac{1}{2}\right)$, $\left(\frac{3\pi}{4}, 0\right)$, and $\left(\pi, -\frac{1}{2}\right)$.



44. Amplitude 2, since $B = 1$ the period is $\frac{2\pi}{B} = \frac{2\pi}{1}$ or equivalently 2π, phase shift is 0, and range is $[-2 + 1, 2 + 1]$ or $[-1, 3]$. Five points are $(0, 1)$, $\left(\frac{\pi}{2}, -1\right)$, $(\pi, 1)$, $\left(\frac{3\pi}{2}, 3\right)$, and $(2\pi, 1)$.

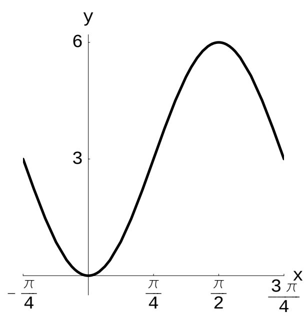


45. Note, $y = -2\sin\left(2\left(x + \frac{\pi}{6}\right)\right) + 1$. Then amplitude is 2, since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or π, phase shift is $-\frac{\pi}{6}$, and range is $[-2 + 1, 2 + 1]$ or $[-1, 3]$. Five points are $\left(-\frac{\pi}{6}, 1\right)$, $\left(\frac{\pi}{12}, -1\right)$, $\left(\frac{\pi}{3}, 1\right)$, $\left(\frac{7\pi}{12}, 3\right)$, and $\left(\frac{5\pi}{6}, 1\right)$.



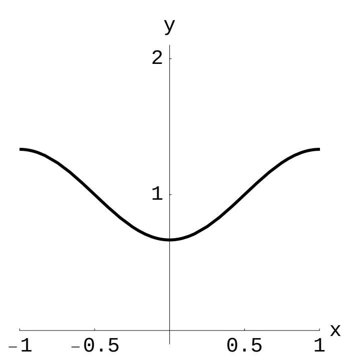
46. Note, $y = -3\sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 3$.

Then amplitude is 3, since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or π, phase shift is $-\frac{\pi}{4}$, and range is $[-3+3, 3+3]$ or $[0,6]$. Five points are $\left(-\frac{\pi}{4}, 3\right)$, $(0,0)$, $\left(\frac{\pi}{4}, 3\right)$, $\left(\frac{\pi}{2}, 6\right)$, and $\left(\frac{3\pi}{4}, 3\right)$.



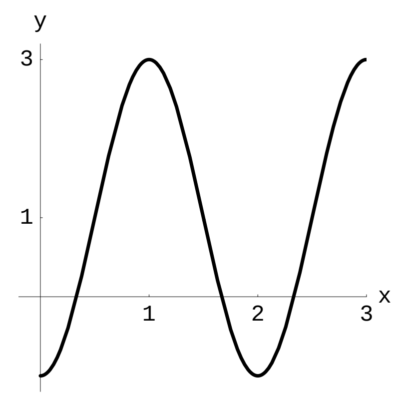
47. Note, $y = \frac{1}{3}\cos(\pi(x+1)) + 1$.

Then amplitude is $\frac{1}{3}$, since $B = \pi$ the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi}$ or 2, phase shift is -1, and range is $\left[-\frac{1}{3}+1, \frac{1}{3}+1\right]$ or $\left[\frac{2}{3}, \frac{4}{3}\right]$. Five points are $\left(-1, \frac{4}{3}\right)$, $\left(-\frac{1}{2}, 1\right)$, $\left(0, \frac{2}{3}\right)$, $\left(\frac{1}{2}, 1\right)$, and $\left(1, \frac{4}{3}\right)$.



48. Note, $y = 2\cos(\pi(x-1)) + 1$.

Then amplitude is 2, since $B = \pi$ the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi}$ or 2, phase shift is 1, and range is $[-2+1, 2+1]$ or $[-1,3]$. Five points are $(1,3)$, $\left(\frac{3}{2}, 1\right)$, $(2,-1)$, $\left(\frac{5}{2}, 1\right)$, and $(3,3)$.



49. Note, $A = 10$, period is 8 and so $B = \frac{\pi}{4}$, and the phase shift can be $C = -2$. Then

$$y = 10\sin\left(\frac{\pi}{4}(x+2)\right).$$

50. Note, $A = 4$, period is 12 and so $B = \frac{\pi}{6}$, and the phase shift can be $C = 3$. Then

$$y = 4\sin\left(\frac{\pi}{6}(x-3)\right).$$

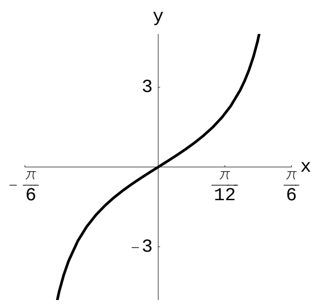
51. Note, $A = 20$, period is 4 and so $B = \frac{\pi}{2}$, and the phase shift can be $C = 1$, and vertical shift is 10 units up or $D = 10$. Then

$$y = 20\sin\left(\frac{\pi}{2}(x-1)\right) + 10.$$

52. Note, $A = 3$, period is 2 and so $B = \pi$, and the phase shift can be $C = \frac{3}{2}$, and vertical shift is 3 units up or $D = 3$. Then

$$y = 3\sin\left(\pi\left(x - \frac{3}{2}\right)\right) + 3.$$

53. Since $B = 3$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{3}$. To find the asymptotes, let $3x = \dfrac{\pi}{2} + k\pi$. Then the asymptotes are $x = \dfrac{\pi}{6} + \dfrac{k\pi}{3}$. The range is $(-\infty, \infty)$.

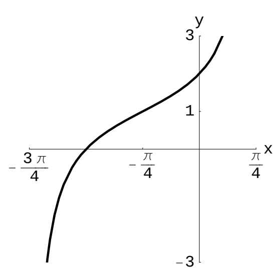


54. Since $B = 1$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{1}$ or π.

To find the asymptotes, let $x + \dfrac{\pi}{4} = \dfrac{\pi}{2} + k\pi$.

Then the asymptotes are $x = \dfrac{\pi}{4} + k\pi$.

The range is $(-\infty, \infty)$.



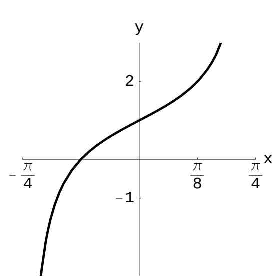
55. Since $B = 2$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{2}$.

To find the asymptotes, let $2x + \pi = \dfrac{\pi}{2} + k\pi$.

Then $x = -\dfrac{\pi}{4} + \dfrac{k\pi}{2}$ or equivalently

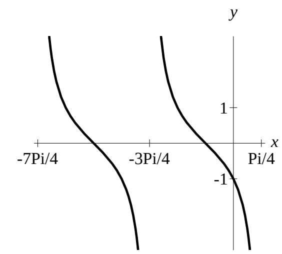
we get $x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}$, which are

the asymptotes. The range is $(-\infty, \infty)$.



56. Since $B = 1$, the period is $\dfrac{\pi}{B} = \dfrac{\pi}{1}$ or π.

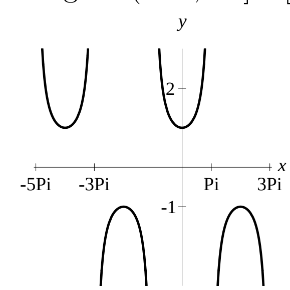
To find the asymptotes, let $x - \dfrac{\pi}{4} = k\pi$. Then $x = \dfrac{\pi}{4} + k\pi$ which are the asymptotes. The range is $(-\infty, \infty)$.



57. Since $B = \dfrac{1}{2}$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{1/2}$ or 4π.

To find the asymptotes, let $\dfrac{1}{2}x = \dfrac{\pi}{2} + k\pi$.

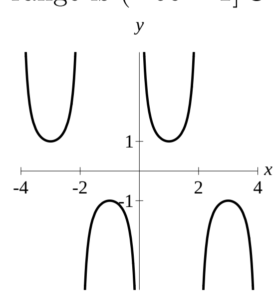
Then $x = \pi + 2k\pi$ which are the asymptotes.
The range is $(-\infty, -1] \cup [1, \infty)$.



58. Since $B = \dfrac{\pi}{2}$, the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi/2}$ or 4.

To find the asymptotes, let $\dfrac{\pi}{2}x = k\pi$.

Then $x = 2k$ which are the asymptotes.
The range is $(-\infty - 1] \cup [1, \infty)$.

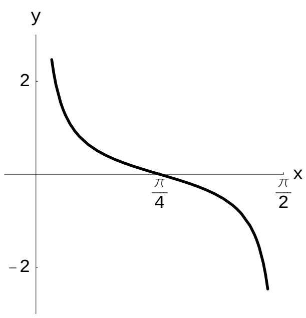


59. Since $B = 2$, the period is $\frac{\pi}{B} = \frac{\pi}{2}$.

To find the asymptotes, let $2x = k\pi$.

Then the asymptotes are $x = \frac{k\pi}{2}$.

The range is $(-\infty, \infty)$.

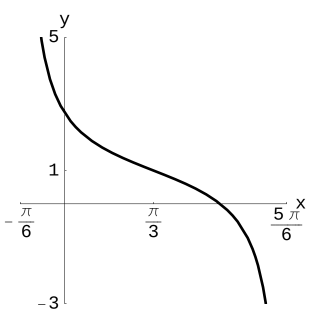


60. Since $B = 1$, the period is $\frac{\pi}{B} = \frac{\pi}{1}$ or π.

To find the asymptotes, let $x - \frac{\pi}{3} = \frac{\pi}{2} + k\pi$.

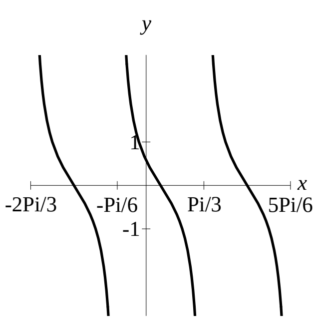
Then the asymptotes are $x = \frac{5\pi}{6} + k\pi$.

The range is $(-\infty, \infty)$.



61. Since $B = 2$, the period is $\frac{\pi}{B} = \frac{\pi}{2}$. To find the asymptotes, let $2x + \frac{\pi}{3} = k\pi$. Then $x = -\frac{\pi}{6} + \frac{k\pi}{2}$ or equivalently $x = \frac{\pi}{3} + \frac{k\pi}{2}$, which are the asymptotes.

The range is $(-\infty, \infty)$.

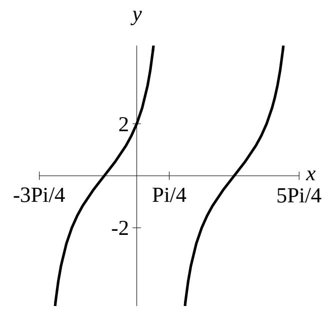


62. Since $B = 1$, the period is $\frac{\pi}{B} = \frac{\pi}{1}$ or π.

To get the asymptotes, let $x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi$.

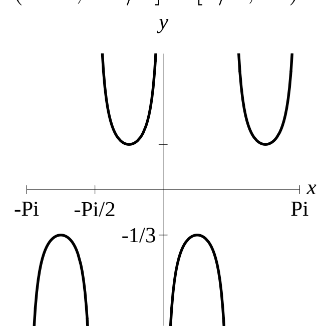
Then $x = \frac{\pi}{4} + k\pi$, which are the asymptotes.

The range is $(-\infty, \infty)$.



63. Since $B = 2$, the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or π.

To obtain the asymptotes, let $2x + \pi = k\pi$ or equivalently $2x = k\pi$. Then the asymptotes are $x = \frac{k\pi}{2}$. The range is $(-\infty, -1/3] \cup [1/3, \infty)$.

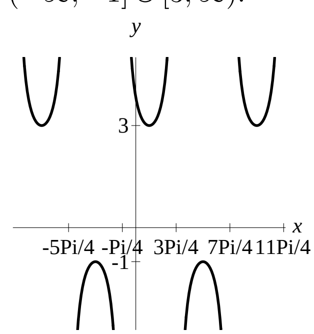


64. Since $B = 1$, the period is $\frac{2\pi}{B} = \frac{2\pi}{1}$ or 2π.

To find the asymptotes, let $x - \frac{\pi}{4} = \frac{\pi}{2} + k\pi$.

Then $x = \frac{3\pi}{4} + k\pi$ which are the asymptotes.

The range is $(-\infty, -2 + 1] \cup [2 + 1, \infty)$ or $(-\infty, -1] \cup [3, \infty)$.

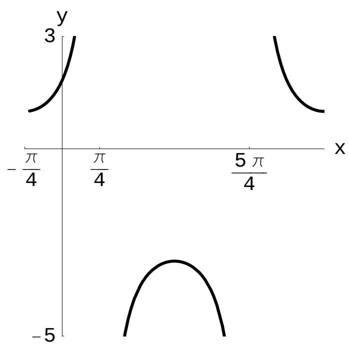


65. Since $B = 1$, the period is $\frac{2\pi}{B} = \frac{2\pi}{1}$ or 2π.

To find the asymptotes, let $x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi$.

Then $x = \frac{\pi}{4} + k\pi$ which are the asymptotes.

The range is $(-\infty, -2-1] \cup [2-1, \infty)$ or $(-\infty, -3] \cup [1, \infty)$.

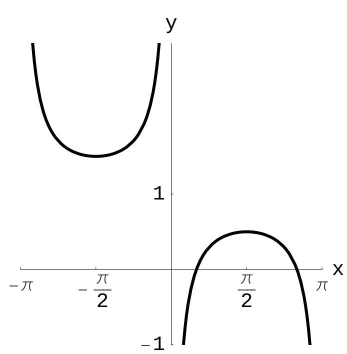


66. Since $B = 1$, the period is $\frac{2\pi}{B} = \frac{2\pi}{1}$ or 2π.

To find the asymptotes, let $x - \pi = k\pi$.

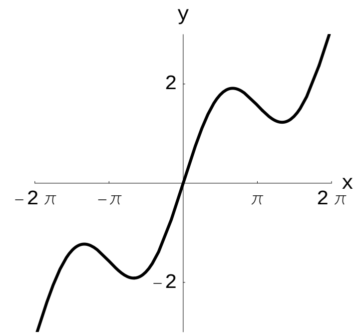
Equivalently, $x = k\pi$, the asymptotes.

The range is $(-\infty, -1/2+1] \cup [1/2+1, \infty)$ or $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$.



67. For each x-coordinate, the y-coordinate of $y = \frac{1}{2}x + \sin(x)$ is obtained by adding the y-coordinates of $y_1 = \frac{1}{2}x$ and $y_2 = \sin x$.

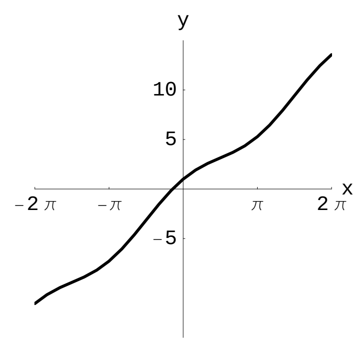
For instance, $\left(\pi/2, \frac{1}{2} \cdot \pi/2 + \sin(\pi/2)\right)$ or $(\pi/2, \pi/4 + 1)$ is a point on the graph of $y = \frac{1}{2}x + \sin(x)$.



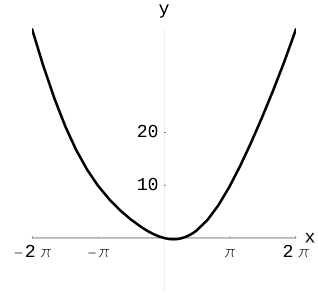
68. For each x-coordinate, the y-coordinate of $y = 2x + \cos(x)$ is obtained by adding the y-coordinates of $y_1 = 2x$ and $y_2 = \cos x$.

For instance, $(\pi/2, 2 \cdot \pi/2 + \cos(\pi/2))$

or $(\pi/2, \pi)$ is a point on the graph of $y = 2x + \cos(x)$.

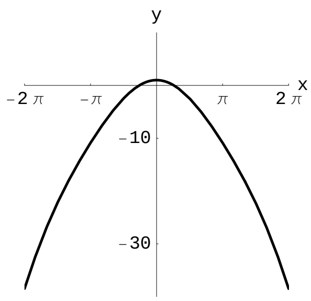


69. For each x-coordinate, the y-coordinate of $y = x^2 - \sin(x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin x$ from the y-coordinate of $y_1 = x^2$. For instance, $\left(\pi, \pi^2 - \sin(\pi)\right)$ or $\left(\pi, \pi^2\right)$ is a point on the graph of $y = x^2 - \sin(x)$.

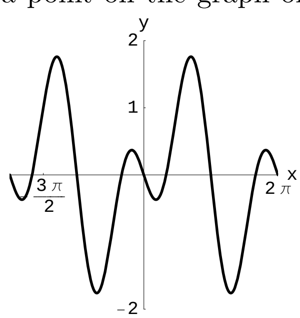


70. For each x-coordinate, the y-coordinate of $y = -x^2 + \cos(x)$ is obtained by adding the y-coordinates of $y_1 = -x^2$ and $y_2 = \cos x$.

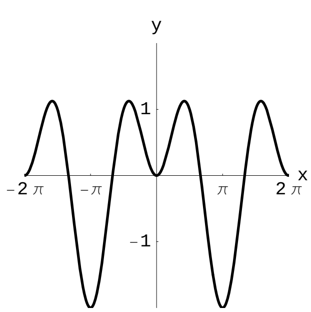
For instance, $(\pi, -\pi^2 + \cos(\pi))$ or $(\pi, -\pi^2 - 1)$ is a point on the graph of $y = -x^2 + \cos(x)$.



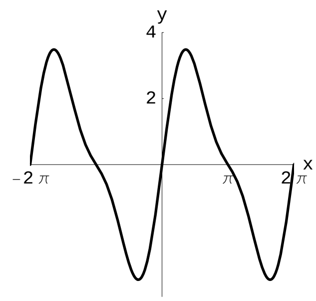
71. For each x-coordinate, the y-coordinate of $y = \sin(x) - \sin(2x)$ is obtained by subtracting the y-coordinate of $y_2 = \sin(2x)$ from the y-coordinate of $y_1 = \sin(x)$. For instance, $(\pi/2, \sin(\pi/2) - \sin(2 \cdot \pi/2))$ or $(\pi/2, 1)$ is a point on the graph of $y = \sin(x) - \sin(2x)$.



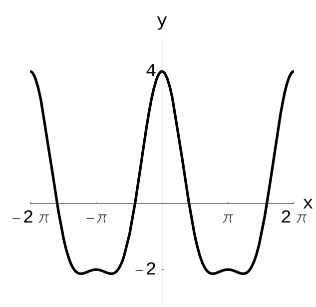
72. For each x-coordinate, the y-coordinate of $y = \cos(x) - \cos(2x)$ is obtained by subtracting the y-coordinate of $y_2 = \cos(2x)$ from the y-coordinate of $y_1 = \cos(x)$. For instance, $(\pi, \cos(\pi) - \cos(2 \cdot \pi))$ or $(\pi, -2)$ is a point on the graph of $y = \cos(x) - \cos(2x)$.



73. For each x-coordinate, the y-coordinate of $y = 3\sin(x) + \sin(2x)$ is obtained by adding the y-coordinates of $y_1 = 3\sin(x)$ and $y_2 = \sin(2x)$. For instance, $(\pi/2, 3\sin(\pi/2) + \sin(2 \cdot \pi/2))$ or $(\pi/2, 3)$ is a point on the graph of $y = 3\sin(x) + \sin(2x)$.



74. For each x-coordinate, the y-coordinate of $y = 3\cos(x) + \cos(2x)$ is obtained by adding the y-coordinates of $y_1 = 3\cos(x)$ and $y_2 = \cos(2x)$. For instance, $(\pi, 3\cos(\pi) + \cos(2 \cdot \pi))$ or $(\pi, -2)$ is a point on the graph of $y = 3\cos(x) + \cos(2x)$.



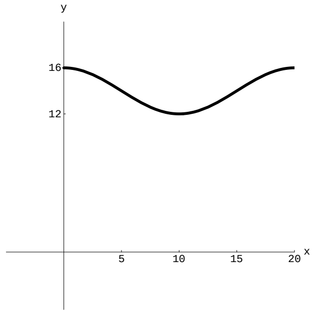
75. The period is $\dfrac{1}{92.3 \times 10^6} \approx 1.08 \times 10^{-8}$ sec

76. The period is $\dfrac{1}{870 \times 10^3} \approx 1.1 \times 10^{-6}$ sec

77. Since the period is 20 minutes, $\dfrac{2\pi}{b} = 20$ or $b = \dfrac{\pi}{10}$. Since the depth is between 12 ft and 16 ft, the vertical upward shift is 14 and $a = 2$. Since the depth is 16 ft at time $t = 0$, one can assume there is a left shift of 5 minutes. An equation is

$$y = 2\sin\left(\frac{\pi}{10}(x + 5)\right) + 14$$

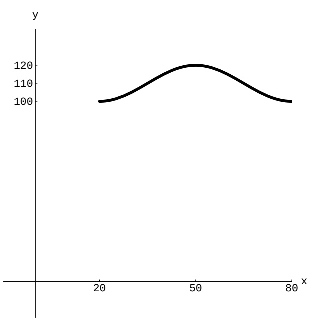
and its graph is given on the next page.



78. Since temperature oscillates between 100°F and 120°F, there is a vertical upward shift of 110 and $a = 10$. Note, the period is 60 minutes. Thus, $\dfrac{2\pi}{b} = 60$ or $b = \dfrac{\pi}{30}$. Since the temperature is 100°F when $t = 20$, we conclude the temperature is 110°F at $t = 35$. Hence, an equation is

$$y = 10\sin\left(\frac{\pi}{30}(x-35)\right) + 110.$$

The graph is given below.



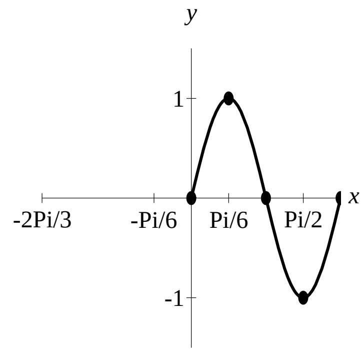
79. \$9, \$99, \$999, \$9,999, \$99,999, \$999,999, \$9,999,999 \$99,999,999, \$999,999,999

80. Let x be the number of minutes it would take the hare to pass the tortoise for the first time.

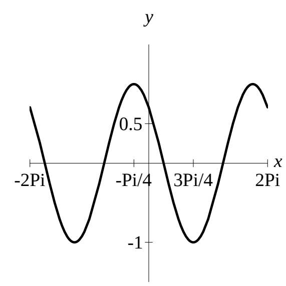
$$\begin{aligned} \frac{x}{6} &= \frac{x}{10} + 1 \\ 10x &= 6x + 60 \\ x &= 15 \text{ min.} \end{aligned}$$

Chapter 2 Test

1. Period $2\pi/3$, range $[-1, 1]$, amplitude 1, some points are $(0, 0)$, $\left(\frac{\pi}{6}, 1\right)$, $\left(\frac{\pi}{3}, 0\right)$, $\left(\frac{\pi}{2}, -1\right)$, $\left(\frac{2\pi}{3}, 0\right)$

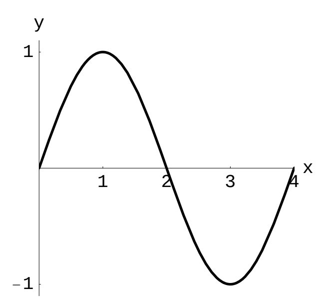


2. Period 2π, range $[-1, 1]$, amplitude 1, some points are $\left(-\frac{\pi}{4}, 1\right)$, $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{3\pi}{4}, -1\right)$, $\left(\frac{5\pi}{4}, 0\right)$, $\left(\frac{7\pi}{4}, 1\right)$

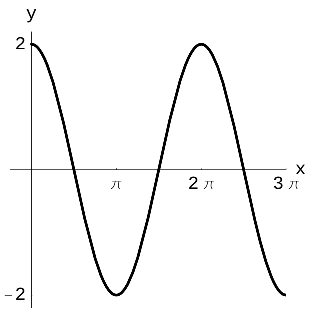


3. Since $B = \dfrac{\pi}{2}$, the period is 4.

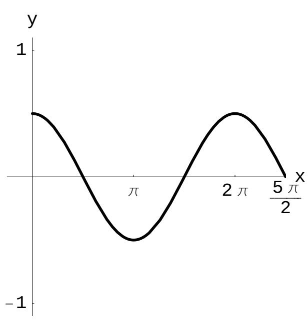
The range is $[-1, 1]$ and amplitude is 1. Some points are $(0, 0)$, $(1, 1)$, $(2, 0)$, $(3, -1)$, $(4, 0)$



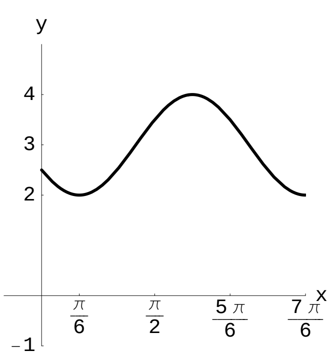
4. Period 2π, range $[-2, 2]$, amplitude 2, some points are $(\pi, -2)$, $\left(\frac{3\pi}{2}, 0\right)$, $(2\pi, 2)$, $\left(\frac{5\pi}{2}, 0\right)$, $(3\pi, -2)$



5. Period 2π, range $\left[-\frac{1}{2}, \frac{1}{2}\right]$, amplitude $\frac{1}{2}$, some points are $\left(\frac{\pi}{2}, 0\right)$, $\left(\pi, -\frac{1}{2}\right)$, $\left(\frac{3\pi}{2}, 0\right)$, $\left(2\pi, \frac{1}{2}\right)$, $\left(\frac{5\pi}{2}, 0\right)$



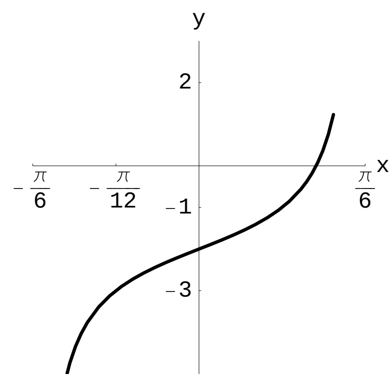
6. Period π since $B = 2$, range is $[-1+3, 1+3]$ or $[2, 4]$, amplitude is 1, some points are $\left(\frac{\pi}{6}, 2\right)$, $\left(\frac{5\pi}{12}, 3\right)$, $\left(\frac{2\pi}{3}, 4\right)$, $\left(\frac{11\pi}{12}, 3\right)$, $\left(\frac{7\pi}{6}, 2\right)$



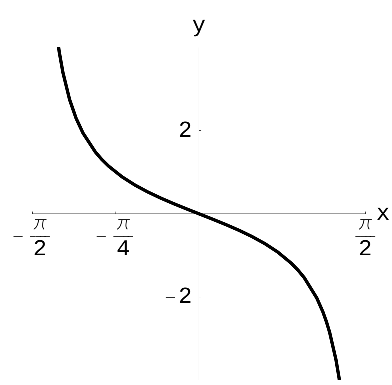
7. Note, amplitude is $A = 4$, period is 12 and so $B = \frac{\pi}{6}$, phase shift is $C = 3$, and the vertical shift downward is 2 units. Then

$$y = 4\sin\left(\frac{\pi}{6}(x-3)\right) - 2.$$

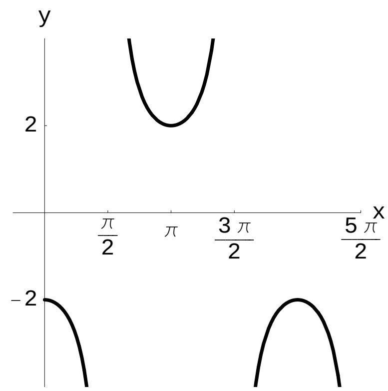
8. The period is $\frac{\pi}{3}$ since $B = 3$, by setting $3x = \frac{\pi}{2} + k\pi$ we get that the asymptotes are $x = \frac{\pi}{6} + \frac{k\pi}{3}$, and the range is $(-\infty, \infty)$.



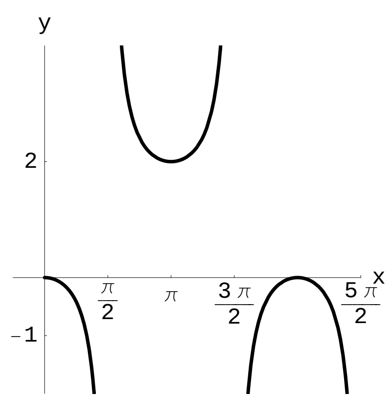
9. The period is π since $B = 1$, by setting $x + \frac{\pi}{2} = k\pi$ we get that the asymptotes are $x = -\frac{\pi}{2} + k\pi$, $x = -\frac{\pi}{2} + \pi + k\pi$, or $x = \frac{\pi}{2} + k\pi$, and the range is $(-\infty, \infty)$.



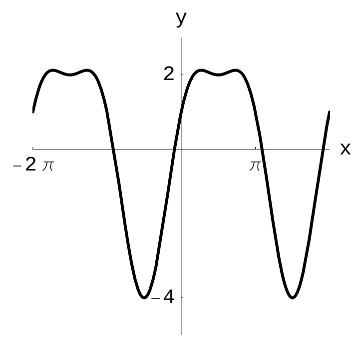
10. The period is $\frac{2\pi}{1}$ or 2π, by setting $x - \pi = \frac{\pi}{2} + k\pi$ we get $x = \frac{3\pi}{2} + k\pi$ and equivalently the asymptotes are $x = \frac{\pi}{2} + k\pi$, and the range is $(-\infty, -2] \cup [2, \infty)$.



11. The period is $\frac{2\pi}{1}$ or 2π, by setting $x - \frac{\pi}{2} = k\pi$ we get that the asymptotes are $x = \frac{\pi}{2} + k\pi$, and the range is $(-\infty, -1+1] \cup [1+1, \infty)$ or $(-\infty, 0] \cup [2, \infty)$.



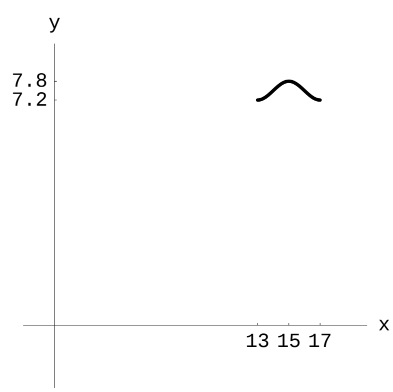
12. For each x-coordinate, the y-coordinate of $y = 3\sin(x) + \cos(2x)$ is obtained by adding the y-coordinates of $y_1 = 3\sin(x)$ and $y_2 = \cos(2x)$. For instance, $(\pi/2, 3\sin(\pi/2) + \cos(2 \cdot \pi/2)) = (\pi/2, 3 + (-1))$ or $(\pi/2, 2)$ is a point on the graph of $y = 3\sin(x) + \cos(2x)$.



13. Since the pH oscillates between 7.2 and 7.8, there is a vertical upward shift of 7.5 and $a = 0.3$. Note, the period is 4 days. Thus, $\frac{2\pi}{b} = 4$ or $b = \frac{\pi}{2}$. Since the pH is 7.2 on day 13, the pH is 7.5 on day 14. We can assume a right shift of 14 days. Hence, an equation is

$$y = 0.3\sin\left(\frac{\pi}{2}(x - 14)\right) + 7.5.$$

A graph of one cycle is given below.



Tying It All Together Chapters P-2

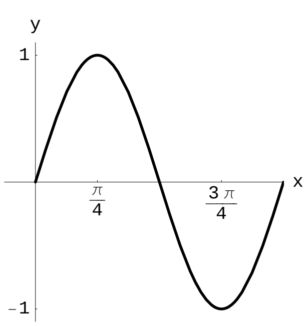
1.

θ deg	0	30	45	60	90	120	135	150	180
θ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\csc\theta$	und	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	und
$\sec\theta$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	und	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1
$\cot\theta$	und	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	und

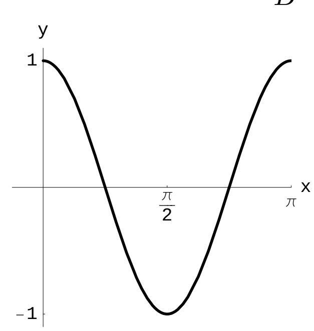
2.

θ rad	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
θ deg	180	210	225	240	270	300	315	330	360
$\sin\theta$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\csc\theta$	und	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	und
$\sec\theta$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	und	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
$\cot\theta$	und	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	und

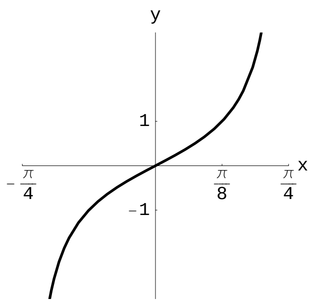
3. Domain $(-\infty, \infty)$, range $[-1, 1]$, and since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or π.



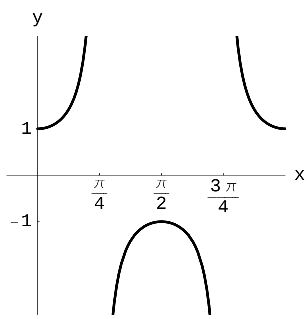
4. Domain $(-\infty, \infty)$, range $[-1, 1]$, and since $B = 2$ the period is $\frac{2\pi}{B} = \frac{2\pi}{2}$ or π.



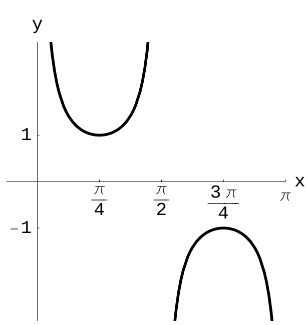
5. By setting $2x \neq \dfrac{\pi}{2} + k\pi$, the domain is $\left\{x \,:\, x \neq \dfrac{\pi}{4} + \dfrac{k\pi}{2}\right\}$, range is $(-\infty, \infty)$, and the period is $\dfrac{\pi}{B} = \dfrac{\pi}{2}$.



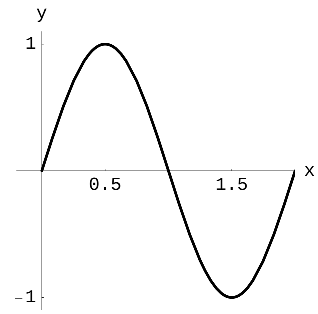
6. By setting $2x \neq \dfrac{\pi}{2} + k\pi$, we find that the domain is $\left\{x \,:\, x \neq \dfrac{\pi}{4} + \dfrac{k\pi}{2}\right\}$, the range is $(-\infty, -1] \cup [1, \infty)$, and the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2}$ or π.



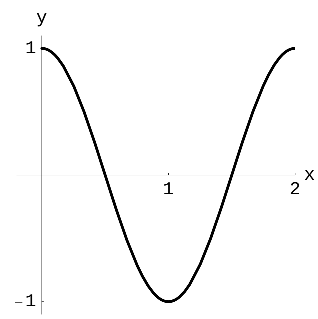
7. By setting $2x \neq k\pi$, we find that the domain is $\left\{x \,:\, x \neq \frac{k\pi}{2}\right\}$, the range is $(-\infty, -1] \cup [1, \infty)$, and the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{2}$ or π.



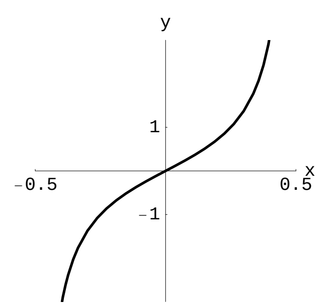
8. Domain $(-\infty, \infty)$, range $[-1, 1]$, and since $B = \pi$ the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi}$ or 2.



9. Domain $(-\infty, \infty)$, range $[-1, 1]$, and since $B = \pi$ the period is $\dfrac{2\pi}{B} = \dfrac{2\pi}{\pi}$ or 2.



10. By setting $\pi x \neq \dfrac{\pi}{2} + k\pi$, the domain is $\left\{x \,:\, x \neq \dfrac{1}{2} + k\right\}$, range is $(-\infty, \infty)$, and the period is $\dfrac{\pi}{B} = \dfrac{\pi}{\pi}$ or 1.



11. Odd, since $\sin(-x) = -\sin(x)$

12. Even, since $\cos(-x) = \cos(x)$

13. Odd, since $\tan(-x) = \dfrac{\sin(-x)}{\cos(-x)} = \dfrac{-\sin(x)}{\cos(x)} = -\tan(x)$, i.e., $\tan(-x) = -\tan(x)$

14. Odd, since $\cot(-x) = \dfrac{\cos(-x)}{\sin(-x)} = \dfrac{\cos(x)}{-\sin(x)} = -\cot(x)$, i.e., $\cot(-x) = -\cot(x)$

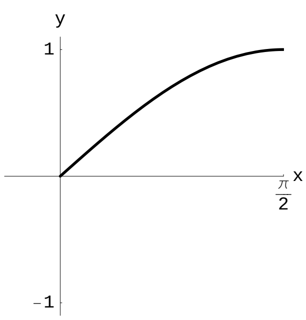
15. Even, since $\sec(-x) = \dfrac{1}{\cos(-x)} = \dfrac{1}{\cos(x)} = \sec(x)$, i.e., $\sec(-x) = \sec(x)$

16. Odd, since $\csc(-x) = \dfrac{1}{\sin(-x)} = \dfrac{1}{-\sin(x)} = -\csc(x)$, i.e., $\csc(-x) = -\csc(x)$

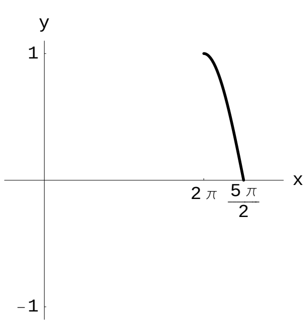
17. Even, since by Exercise 11 we get $\sin^2(-x) = (\sin(-x))^2 = (-\sin(x))^2 = (\sin(x))^2$, i.e., $\sin^2(-x) = \sin^2(x)$

18. Even, since by Exercise 12 we get $\cos^2(-x) = (\cos(-x))^2 = (\cos(x))^2$, i.e., $\cos^2(-x) = \cos^2(x)$

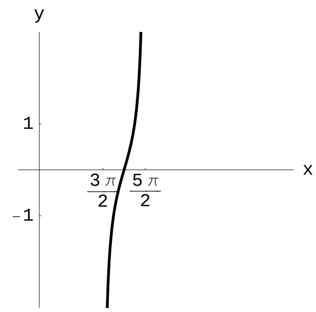
19. Increasing, as shown by the graph below.



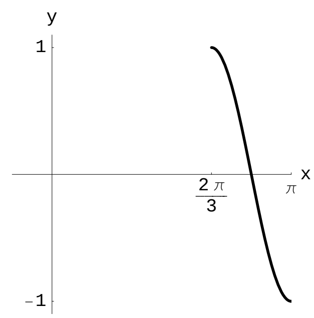
20. Decreasing, as shown by the graph below.



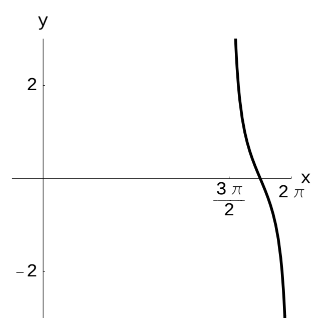
21. Increasing, as shown below.



22. Decreasing, as shown below.



23. Decreasing, as shown by the graph below.



24. Increasing, as shown by the graph below.