Solutions Manual

For

Water-Resources Engineering
Third Edition

By

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Conventional solutions to all problems
Also includes Mathcad® solutions to selected problems contributed by Dixie M. Griffin, Jr.

Preface

This solutions manual contains the solutions to all end-of-chapter problems in Water-Resources Engineering, Third Edition. This manual should be treated as **confidential** by course instructors and/or their trustees, such as teaching assistants and graders. Unauthorized use of this solutions manual by students would normally be considered as cheating.

This solutions manual contains two sets of solutions: conventional solutions and Mathcad® solutions. The conventional solutions to all end-of-chapter problems were prepared by Dr. David A. Chin, using a calculator and/or electronic spreadsheet. Mathcad® solutions to selected problems were prepared by Dr. Dixie M. Griffin Jr. exclusively using Mathcad® software. Depending on the preference of the course instructor, students could be asked to solve problems in either format. The conventional solutions to all problems are presented first, and Mathcad® solutions to selected problems are presented thereafter.

Chapter 1

Introduction

1.1. The mean annual rainfall in Boston is approximately $\boxed{\text{1050 mm}}$, and the mean annual evapotranspiration is in the range of $\boxed{\text{380–630 mm}}$ (USGS). On the basis of rainfall, this indicates a $\boxed{\text{subhumid}}$ climate. The mean annual rainfall in Santa Fe is approximately $\boxed{\text{360 mm}}$ and the mean annual evapotranspiration is $\boxed{< \text{380 mm}}$. On the basis of rainfall, this indicates an $\boxed{\text{arid}}$ climate.

Chapter 2

Fundamentals of Flow in Closed Conduits

2.1. $D_1 = 0.1$ m, $D_2 = 0.15$ m, $V_1 = 2$ m/s, and

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$$
$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.15)^2 = 0.01767 \text{ m}^2$$

Volumetric flow rate, Q, is given by

$$Q = A_1V_1 = (0.007854)(2) = \boxed{0.0157 \text{ m}^3/\text{s}}$$

According to continuity,

$$A_1V_1 = A_2V_2 = Q$$

Therefore

$$V_2 = \frac{Q}{A_2} = \frac{0.0157}{0.01767} = \boxed{0.889 \text{ m/s}}$$

At 20°C, the density of water, ρ, is 998 kg/m^3, and the mass flow rate, $\dot{m}$, is given by

$$\dot{m} = \rho Q = (998)(0.0157) = \boxed{15.7 \text{ kg/s}}$$

2.2. From the given data: $D_1 = 200$ mm, $D_2 = 100$ mm, $V_1 = 1$ m/s, and

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$$
$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$$

The flow rate, Q_1, in the 200-mm pipe is given by

$$Q_1 = A_1V_1 = (0.0314)(1) = 0.0314 \text{ m}^3/\text{s}$$

and hence the flow rate, Q_2, in the 100-mm pipe is

$$Q_2 = \frac{Q_1}{2} = \frac{0.0314}{2} = \boxed{0.0157 \text{ m}^3/\text{s}}$$

The average velocity, V_2, in the 100-mm pipe is

$$V_2 = \frac{Q_2}{A_2} = \frac{0.0157}{0.00785} = \boxed{2 \text{ m/s}}$$

2.3. The velocity distribution in the pipe is

$$v(r) = V_0 \left[1 - \left(\frac{r}{R}\right)^2\right] \tag{1}$$

and the average velocity, $\bar{V}$, is defined as

$$\bar{V} = \frac{1}{A}\int_A V\, dA \tag{2}$$

where

$$A = \pi R^2 \qquad \text{and} \qquad dA = 2\pi r dr \tag{3}$$

Combining Equations 1 to 3 yields

$$\bar{V} = \frac{1}{\pi R^2}\int_0^R V_0\left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r dr = \frac{2V_0}{R^2}\left[\int_0^R r dr - \int_0^R \frac{r^3}{R^2} dr\right] = \frac{2V_0}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$
$$= \frac{2V_0}{R^2}\frac{R^2}{4} = \boxed{\frac{V_0}{2}}$$

The flow rate, Q, is therefore given by

$$Q = A\bar{V} = \boxed{\frac{\pi R^2 V_0}{2}}$$

2.4.

$$\beta = \frac{1}{A\bar{V}^2}\int_A v^2\, dA = \frac{4}{\pi R^2 V_0^2}\int_0^R V_0^2\left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right] 2\pi r dr$$
$$= \frac{8}{R^2}\left[\int_0^R r dr - \int_0^R \frac{2r^3}{R^2} dr + \int_0^R \frac{r^5}{R^4} dr\right] = \frac{8}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4}\right]$$
$$= \boxed{\frac{4}{3}}$$

2.5. $D = 0.2$ m, $Q = 0.06$ m^3/s, $L = 100$ m, $p_1 = 500$ kPa, $p_2 = 400$ kPa, $\gamma = 9.79$ kN/m^3.

$$R = \frac{D}{4} = \frac{0.2}{4} = 0.05 \text{ m}$$
$$\Delta h = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{500 - 400}{9.79} = 10.2 \text{ m}$$
$$\tau_0 = \frac{\gamma R \Delta h}{L} = \frac{(9.79 \times 10^3)(0.05)(10.2)}{100} = \boxed{49.9 \text{ N/m}^2}$$
$$A = \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 0.0314 \text{ m}^2$$
$$V = \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91 \text{ m/s}$$
$$f = \frac{8\tau_0}{\rho V^2} = \frac{8(49.9)}{(998)(1.91)^2} = \boxed{0.11}$$

2.6. $T = 20°\text{C}$, $V = 2$ m/s, $D = 0.25$ m, horizontal pipe, ductile iron. For ductile iron pipe, $k_s = 0.26$ mm, and

$$\frac{k_s}{D} = \frac{0.26}{250} = 0.00104$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998.2)(2)(0.25)}{(1.002 \times 10^{-3})} = 4.981 \times 10^5$$

From the Moody diagram:

$$\boxed{f = 0.0202 \text{ (pipe is smooth)}}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

Substituting for k_s/D and Re gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00104}{3.7} + \frac{2.51}{4.981 \times 10^5 \sqrt{f}} \right)$$

By trial and error leads to

$$\boxed{f = 0.0204}$$

Using the Swamee-Jain equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right]$$
$$= -2 \log \left[\frac{0.00104}{3.7} + \frac{5.74}{(4.981 \times 10^5)^{0.9}} \right]$$

which leads to

$$\boxed{f = 0.0205}$$

The head loss, h_f, over 100 m of pipeline is given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.0204 \frac{100}{0.25} \frac{(2)^2}{2(9.81)} = 1.66 \text{ m}$$

Therefore the pressure drop, Δp, is given by

$$\Delta p = \gamma h_f = (9.79)(1.66) = \boxed{16.3 \text{ kPa}}$$

If the pipe is 1 m lower at the downstream end, f would not change, but the pressure drop, Δp, would then be given by

$$\Delta p = \gamma (h_f - 1.0) = 9.79(1.66 - 1) = \boxed{6.46 \text{ kPa}}$$

2.7. From the given data: $D = 25$ mm, $k_s = 0.1$ mm, $\theta = 10°$, $p_1 = 550$ kPa, and $L = 100$ m. At 20°C, $\nu = 1.00 \times 10^{-6}$ m²/s, $\gamma = 9.79$ kN/m³, and

$$\frac{k_s}{D} = \frac{0.1}{25} = 0.004$$
$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.025)^2 = 4.909 \times 10^{-4} \text{ m}^2$$
$$h_f = f\frac{L}{D}\frac{Q^2}{2gA^2} = f\frac{100}{0.025}\frac{Q^2}{2(9.81)(4.909 \times 10^{-4})^2} = 8.46 \times 10^8 fQ^2$$

The energy equation applied over 100 m of pipe is

$$\frac{p_1}{\gamma} + \frac{V^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2 + h_f$$

which simplifies to

$$p_2 = p_1 - \gamma(z_2 - z_1) - \gamma h_f$$
$$p_2 = 550 - 9.79(100 \sin 10°) - 9.79(8.46 \times 10^8 fQ^2)$$
$$p_2 = 380.0 - 8.28 \times 10^9 fQ^2$$

(a) For $Q = 2$ L/min $= 3.333 \times 10^{-5}$ m³/s,

$$V = \frac{Q}{A} = \frac{3.333 \times 10^{-5}}{4.909 \times 10^{-4}} = 0.06790 \text{ m/s}$$
$$\text{Re} = \frac{VD}{\nu} = \frac{(0.06790)(0.025)}{1 \times 10^{-6}} = 1698$$

Since Re < 2000, the flow is laminar when $Q = 2$ L/min. Hence,

$$f = \frac{64}{\text{Re}} = \frac{64}{1698} = 0.03770$$
$$p_2 = 380.0 - 8.28 \times 10^9(0.03770)(3.333 \times 10^{-5})^2 = 380 \text{ kPa}$$

Therefore, when the flow is 2 L/min, the pressure at the downstream section is $\boxed{380 \text{ kPa}}$.
For $Q = 20$ L/min $= 3.333 \times 10^{-4}$ m³/s,

$$V = \frac{Q}{A} = \frac{3.333 \times 10^{-4}}{4.909 \times 10^{-4}} = 0.6790 \text{ m/s}$$
$$\text{Re} = \frac{VD}{\nu} = \frac{(0.6790)(0.025)}{1 \times 10^{-6}} = 16980$$

Since Re > 5000, the flow is turbulent when $Q = 20$ L/min. Hence,

$$f = \frac{0.25}{\left[\log\left(\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log\left(\frac{0.004}{3.7} + \frac{5.74}{16980^{0.9}}\right)\right]^2} = 0.0342$$
$$p_2 = 380.0 - 8.28 \times 10^9(0.0342)(3.333 \times 10^{-4})^2 = 349 \text{ kPa}$$

Therefore, when the flow is 2 L/min, the pressure at the downstream section is $\boxed{349 \text{ kPa}}$.

(b) Using the Colebrook equation with $Q = 20$ L/min,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right] = -2\log\left[\frac{0.004}{3.7} + \frac{2.51}{16980\sqrt{f}}\right]$$

which yields $\boxed{f = 0.0337}$. Comparing this with the Swamee-Jain result of $\boxed{f = 0.0342}$ indicates a difference of $\boxed{1.5\%}$, which is more than the 1% claimed by Swamee-Jain.

2.8. The Colebrook equation is given by

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Inverting and squaring this equation gives

$$\boxed{f = \frac{0.25}{\{\log[(k_s/D)/3.7 + 2.51/(\text{Re}\sqrt{f})]\}^2}}$$

This equation is "slightly more convenient" than the Colebrook formula since it is quasi-explicit in f, whereas the Colebrook formula gives $1/\sqrt{f}$.

2.9. The Colebrook equation is preferable since it provides greater accuracy than interpolating from the Moody diagram.

2.10. $D = 0.5$ m, $p_1 = 600$ kPa, $Q = 0.50$ m^3/s, $z_1 = 120$ m, $z_2 = 100$ m, $\gamma = 9.79$ kN/m^3, $L = 1000$ m, k_s (ductile iron) $= 0.26$ mm,

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$$
$$V = \frac{Q}{A} = \frac{0.50}{0.1963} = 2.55 \text{ m/s}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

where $k_s/D = 0.26/500 = 0.00052$, and at 20°C

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998)(2.55)(0.5)}{1.00 \times 10^{-3}} = 1.27 \times 10^6$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00052}{3.7} + \frac{2.51}{1.27 \times 10^6\sqrt{f}}\right)$$

which leads to

$$f = 0.0172$$

Applying the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since $V_1 = V_2$, and h_f is given by the Darcy-Weisbach equation, then the energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + f\frac{L}{D}\frac{V^2}{2g}$$

Substituting known values leads to

$$\frac{600}{9.79} + 120 = \frac{p_2}{9.79} + 100 + 0.0172\frac{1000}{0.5}\frac{(2.55)^2}{2(9.81)}$$

which gives

$$\boxed{p_2 = 684 \text{ kPa}}$$

If p is the (static) pressure at the top of a 30 m high building, then

$$p = p_2 - 30\gamma = 684 - 30(9.79) = 390 \text{ kPa}$$

This (static) water pressure is adequate for service.

2.11. The head loss, h_f, in the pipe is estimated by

$$h_f = \left(\frac{p_{\text{main}}}{\gamma} + z_{\text{main}}\right) - \left(\frac{p_{\text{outlet}}}{\gamma} + z_{\text{outlet}}\right)$$

where $p_{\text{main}} = 400$ kPa, $z_{\text{main}} = 0$ m, $p_{\text{outlet}} = 0$ kPa, and $z_{\text{outlet}} = 2.0$ m. Therefore,

$$h_f = \left(\frac{400}{9.79} + 0\right) - (0 + 2.0) = 38.9 \text{ m}$$

Also, since $D = 25$ mm, $L = 20$ m, $k_s = 0.15$ mm (from Table 2.1), $\nu = 1.00 \times 10^{-6}$ m^2/s (at 20°C), the combined Darcy-Weisbach and Colebrook equation (Equation 2.43) yields,

$$\begin{aligned} Q &= -0.965D^2\sqrt{\frac{gDh_f}{L}}\ln\left(\frac{k_s/D}{3.7} + \frac{1.774\nu}{D\sqrt{gDh_f/L}}\right) \\ &= -0.965(0.025)^2\sqrt{\frac{(9.81)(0.025)(38.9)}{20}}\ln\left[\frac{0.15/25}{3.7} + \frac{1.774(1.00\times10^{-6})}{(0.025)\sqrt{(9.81)(0.025)(38.9)/20}}\right] \\ &= 0.00265 \text{ m}^3/\text{s} = 2.65 \text{ L/s} \end{aligned}$$

The faucet can therefore be expected to deliver $\boxed{2.65 \text{ L/s}}$ when fully open.

2.12. From the given data: $Q = 300$ L/s $= 0.300$ m^3/s, $L = 40$ m, and $h_f = 45$ m. Assume that $\nu = 10^{-6}$ m^2/s (at 20°C) and take $k_s = 0.15$ mm (from Table 2.1). Substituting these data into Equation 2.43 gives

$$Q = -0.965D^2\sqrt{\frac{gDh_f}{L}}\ln\left(\frac{k_s/D}{3.7} + \frac{1.784\nu}{D\sqrt{gDh_f/L}}\right)$$

$$0.2 = -0.965D^2\sqrt{\frac{(9.81)D(45)}{(40)}}\ln\left(\frac{0.00015}{3.7D} + \frac{1.784(10^{-6})}{D\sqrt{(9.81)D(45)/(40)}}\right)$$

This is an implicit equation in D that can be solved numerically to yield $\boxed{D = 166 \text{ mm}}$.

2.13. Since $k_s = 0.15$ mm, $L = 40$ m, $Q = 0.3$ m^3/s, $h_f = 45$ m, $\nu = 1.00 \times 10^{-6}$ m^2/s, the Swamee-Jain approximation (Equation 2.44 gives

$$\begin{aligned} D &= 0.66\left[k_s^{1.25}\left(\frac{LQ^2}{gh_f}\right)^{4.75} + \nu Q^{9.4}\left(\frac{L}{gh_f}\right)^{5.2}\right]^{0.04} \\ &= 0.66\left\{(0.00015)^{1.25}\left[\frac{(40)(0.3)^2}{(9.81)(45)}\right]^{4.75} + (1.00\times10^{-6})(0.3)^{9.4}\left[\frac{40}{(9.81)(45)}\right]^{5.2}\right\}^{0.04} \\ &= 0.171 \text{ m} = \boxed{171 \text{ mm}} \end{aligned}$$

The calculated pipe diameter (171 mm) is about 3% higher than calculated by the Colebrook equation (166 mm).

2.14. The kinetic energy correction factor, α, is defined by

$$\int_A \rho\frac{v^3}{2}dA = \alpha\rho\frac{V^3}{2}A$$

or

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \tag{1}$$

Using the velocity distribution in Problem 2.3 gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_0^3\left[1 - \left(\frac{r}{R}\right)^2\right]^2 2\pi r\,dr \\ &= 2\pi V_0^3\int_0^R \left[1 - 3\left(\frac{r}{R}\right)^2 + 3\left(\frac{r}{R}\right)^4 - \left(\frac{r}{R}\right)^6\right] r\,dr \\ &= 2\pi V_0^3\int_0^R \left[r - \frac{3r^3}{R^2} + \frac{3r^5}{R^4} - \frac{r^7}{R^6}\right] dr \\ &= 2\pi V_0^3\left[\frac{r^2}{2} - \frac{3r^4}{4R^2} + \frac{r^6}{2R^4} - \frac{r^8}{8R^6}\right]_0^R \\ &= 2\pi R^2 V_0^3\left[\frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8}\right] \\ &= \frac{\pi R^2 V_0^3}{4} \end{aligned} \tag{2}$$

The average velocity, V, was calculated in Problem 2.3 as

$$V = \frac{V_0}{2}$$

hence

$$V^3 A = \left(\frac{V_0}{2}\right)^3 \pi R^2 = \frac{\pi R^2 V_0^3}{8} \tag{3}$$

Combining Equations 1 to 3 gives

$$\alpha = \frac{\pi R^2 V_0^3/4}{\pi R^2 V_0^3/8} = \boxed{2}$$

2.15. The kinetic energy correction factor, α, is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \tag{1}$$

Using the given velocity distribution gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_0^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{7}} 2\pi r\ dr \\ &= 2\pi V_0^3 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{3}{7}} r\ dr \end{aligned} \tag{2}$$

To facilitate integration, let

$$x = 1 - \frac{r}{R} \tag{3}$$

which gives

$$r = R(1 - x) \tag{4}$$

$$dr = -R\ dx \tag{5}$$

Combining Equations 2 to 5 gives

$$\begin{aligned} \int_A v^3 dA &= 2\pi V_0^3 \int_0^1 x^{\frac{3}{7}} R(1 - x)(-R) dx \\ &= 2\pi R^2 V_0^3 \int_0^1 x^{\frac{3}{7}}(1 - x) dx = 2\pi R^2 V_0^3 \int_0^1 (x^{\frac{3}{7}} - x^{\frac{10}{7}}) dx \\ &= 2\pi R^2 V_0^3 \left[\frac{7}{10} x^{\frac{10}{7}} - \frac{7}{17} x^{\frac{17}{7}}\right]_0^1 \\ &= 0.576\pi R^2 V_0^3 \end{aligned} \tag{6}$$

The average velocity, V, is given by (using the same substitution as above)

$$\begin{aligned} V &= \frac{1}{A}\int_A v\ dA \\ &= \frac{1}{\pi R^2}\int_0^R V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{7}} 2\pi r\ dr = \frac{2V_0}{R^2}\int_1^0 x^{\frac{1}{7}} R(1 - x)(-R) dx \\ &= 2V_0 \int_0^1 (x^{\frac{1}{7}} - x^{\frac{8}{7}}) dx = 2V_0 \left[\frac{7}{8} x^{\frac{8}{7}} - \frac{7}{15} x^{\frac{15}{7}}\right]_0^1 \\ &= 0.817 V_0 \end{aligned} \tag{7}$$

Using this result,

$$V^3A = (0.817V_0)^3\pi R^2 = 0.545\pi R^2 V_0^3 \tag{8}$$

Combining Equations 1, 6, and 8 gives

$$\alpha = \frac{0.576\pi R^2 V_0^3}{0.545\pi R^2 V_0^3} = \boxed{1.06}$$

The momentum correction factor, β, is defined by

$$\beta = \frac{\int_A v^2 dA}{AV^2} \tag{9}$$

In this case,

$$AV^2 = \pi R^2 (0.817V_0)^2 = 0.667\pi R^2 V_0^2 \tag{10}$$

and

$$\begin{aligned}\int_A v^2 dA &= \int_0^R V_0^2 \left(1 - \frac{r}{R}\right)^{\frac{2}{7}} 2\pi r\ dr \\ &= 2\pi V_0^2 \int_1^0 x^{\frac{2}{7}} R(1-x)(-R)dx = 2\pi R^2 V_0^2 \int_0^1 (x^{\frac{2}{7}} - x^{\frac{9}{7}})dx \\ &= 2\pi R^2 V_0^2 \left[\frac{7}{9}x^{\frac{9}{7}} - \frac{7}{16}x^{\frac{16}{7}}\right]_0^1 = 0.681\pi R^2 V_0^2 \end{aligned} \tag{11}$$

Combining Equations 9 to 11 gives

$$\beta = \frac{0.681\pi R^2 V_0^2}{0.667\pi R^2 V_0^2} = \boxed{1.02}$$

2.16. The kinetic energy correction factor, α, is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \tag{1}$$

Using the velocity distribution given by Equation 2.73 gives

$$\begin{aligned}\int_A v^3 dA &= \int_0^R V_0^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} 2\pi r\ dr \\ &= 2\pi V_0^3 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} r\ dr \end{aligned} \tag{2}$$

Let

$$x = 1 - \frac{r}{R} \tag{3}$$

which gives

$$r = R(1-x) \tag{4}$$

$$dr = -R\ dx \tag{5}$$

Combining Equations 2 to 5 gives

$$\begin{aligned}\int_A v^3 dA &= 2\pi V_0^3 \int_0^1 x^{\frac{3}{n}} R(1-x)(-R)dx \\ &= 2\pi R^2 V_0^3 \int_0^1 x^{\frac{3}{n}}(1-x)dx = 2\pi R^2 V_0^3 \int_0^1 (x^{\frac{3}{n}} - x^{\frac{3+n}{n}})dx \\ &= 2\pi R^2 V_0^3 \left[\frac{n}{3+n} x^{\frac{3+n}{n}} - \frac{n}{3+2n} x^{\frac{3+2n}{n}}\right]_0^1 \\ &= \frac{2n^2}{(3+n)(3+2n)} \pi R^2 V_0^3 \qquad (6)\end{aligned}$$

The average velocity, V, is given by

$$\begin{aligned}V &= \frac{1}{A}\int_A v\ dA \\ &= \frac{1}{\pi R^2}\int_0^R V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} 2\pi r\ dr = \frac{2V_0}{R^2}\int_1^0 x^{\frac{1}{n}} R(1-x)(-R)dx \\ &= 2V_0 \int_0^1 (x^{\frac{1}{n}} - x^{\frac{1+n}{n}})dx = 2V_0 \left[\frac{n}{1+n} x^{\frac{1+n}{n}} - \frac{n}{1+2n} x^{\frac{1+2n}{n}}\right]_0^1 \\ &= \left[\frac{2n^2}{(1+n)(1+2n)}\right] V_0 \qquad (7)\end{aligned}$$

Using this result,

$$V^3 A = \left[\frac{2n^2}{(1+n)(1+2n)}\right]^3 V_0^3 \pi R^2 = \frac{8n^6}{(1+n)^3(1+2n)^3} \pi R^2 V_0^3 \qquad (8)$$

Combining Equations 1, 6, and 8 gives

$$\begin{aligned}\alpha &= \frac{\frac{2n^2}{(3+n)(3+2n)} \pi R^2 V_0^3}{\frac{8n^6}{(1+n)^3(1+2n)^3} \pi R^2 V_0^3} \\ &= \boxed{\frac{(1+n)^3(1+2n)^3}{4n^4(3+n)(3+2n)}}\end{aligned}$$

Putting $n = 7$ gives $\boxed{\alpha = 1.06}$, the same result obtained in Problem 2.15.

2.17. $p_1 = 30$ kPa, $p_2 = 500$ kPa, therefore head, h_p, added by pump is given by

$$h_p = \frac{p_2 - p_1}{\gamma} = \frac{500 - 30}{9.79} = \boxed{48.0 \text{ m}}$$

Power, P, added by pump is given by

$$P = \gamma Q h_p = (9.79)(Q)(48.0) = \boxed{470 \text{ kW per m}^3\text{/s}}$$

2.18. $Q = 0.06$ m^3/s, $D = 0.2$ m, $k_s = 0.9$ mm (riveted steel), $k_s/D = 0.9/200 = 0.00450$, for 90° bend $K = 0.3$, for the entrance $K = 1.0$, at 20°C $\rho = 998$ kg/m^3, and $\mu = 1.00 \times 10^{-3}$ Pa·s, therefore

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.2)^2 = 0.0314 \text{ m}^2$$
$$V = \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91 \text{ m/s}$$
$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998)(1.91)(0.2)}{1.00 \times 10^{-3}} = 3.81 \times 10^5$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00450}{3.7} + \frac{2.51}{3.81 \times 10^5\sqrt{f}}\right)$$

which leads to

$$f = 0.0297$$

Minor head loss, h_m, is given by

$$h_m = \sum K\frac{V^2}{2g} = (1.0 + 0.3)\frac{(1.91)^2}{2(9.81)} = 0.242 \text{ m}$$

If friction losses, h_f, account for 90% of the total losses, then

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 9h_m$$

which means that

$$0.0297\frac{L}{0.2}\frac{(1.91)^2}{2(9.81)} = 9(0.242)$$

Solving for L gives

$$\boxed{L = 78.9 \text{ m}}$$

For pipe lengths shorter than the length calculated in this problem, the word "minor" should not be used.

2.19. From the given data: $p_0 = 480$ kPa, $v_0 = 5$ m/s, $z_0 = 2.44$ m, $D = 19$ mm $= 0.019$ m, $L = 40$ m, $z_1 = 7.62$ m, and $\sum K_m = 3.5$. For copper tubing it can be assumed that $k_s = 0.0023$ mm. Applying the energy and Darcy-Weisbach equations between the water main and the faucet gives

$$\frac{p_0}{\gamma} + z_0 - h_f - h_m = \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1$$
$$\frac{480}{9.79} + 2.44 - \frac{f(40)}{0.019}\frac{v^2}{2(9.81)} - 3.5\frac{v^2}{2(9.81)} = \frac{0}{\gamma} + \frac{v^2}{2(9.81)} + 7.62$$

which simplifies to

$$v = \frac{6.622}{\sqrt{107.3f - 0.2141}} \qquad (1)$$

The Colebrook equation, with $\nu = 1 \times 10^{-6}$ m^2/s gives

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}}\right]$$

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{0.0025}{3.7(19)} + \frac{2.51}{\frac{v(0.019)}{1\times10^{-6}}\sqrt{f}}\right]$$

$$\frac{1}{\sqrt{f}} = -2\log\left[3.556 \times 10^{-5} + \frac{1.321 \times 10^{-4}}{v\sqrt{f}}\right] \qquad (2)$$

Combining Equations 1 and 2 gives

$$\frac{1}{\sqrt{f}} = -2\log\left[3.556 \times 10^{-5} + \frac{1.995 \times 10^{-5}\sqrt{107.3f - 0.2141}}{\sqrt{f}}\right]$$

which yields

$$f = 0.0189$$

Substituting into Equation 1 yields

$$v = \frac{6.622}{\sqrt{107.3(0.0189) - 0.2141}} = 4.92 \text{ m/s}$$

$$Q = Av = \left(\frac{\pi}{4}0.019^2\right)(4.92) = 0.00139 \text{ m}^3\text{/s} = \boxed{1.39 \text{ L/s } (= 22 \text{ gpm})}$$

This flow is very high for a faucet. The flow would be reduced if other faucets are open, this is due to increased pipe flow and frictional resistance between the water main and the faucet.

2.20. From the given data: $z_1 = -1.5$ m, $z_2 = 40$ m, $p_1 = 450$ kPa, $\sum k = 10.0$, $Q = 20$ L/s $= 0.02$ m^3/s, $D = 150$ mm (PVC), $L = 60$ m, $T = 20$°C, and $p_2 = 150$ kPa. The combined energy and Darcy-Weisbach equations give

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left[\frac{fL}{D} + \sum k\right]\frac{V^2}{2g} \qquad (1)$$

where

$$V_1 = V_2 = V = \frac{Q}{A} = \frac{0.02}{\frac{\pi(0.15)^2}{4}} = 1.13 \text{ m/s} \qquad (2)$$

At 20°C, $\nu = 1.00 \times 10^{-6}$ m^2/s, and

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13)(0.15)}{1.00 \times 10^{-6}} = 169500$$

Since PVC pipe is smooth ($k_s = 0$), the friction factor, f, is given by

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right) = -2\log\left(\frac{2.51}{169500\sqrt{f}}\right)$$